LHC and electroweak phase transitions

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Talk at Tuesday's "Werkstatt Seminar"

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References

[1] M. Carena, M. Quirós , C.E.M. Wagner *Open the Window for Electroweak Baryogenesis*, hep-ph/9603420

[2] M. Carena, G. Nardini, M. Quirós , C.E.M. Wagner *The Baryogenesis Window in the MSSM*, arXiv:0809.3760

[3] J. R. Espinosa, T. Konstandin, F. Riva *Strong Electroweak Phase Transitions in the Standard Model with a Singlet*, arXiv:1107.5441

In this talk we consider the electroweak phase transition in two models beyond the standard model. We are looking for the possibility of a first order phase transition. Why are we interested in this?

A first order phase transition is an out-of-equilibrium process and (as stated by the third Sakharov condition) this is needed for baryogenesis. But for baryongenesis we need an even stronger condition coming from the 'wash-out' criterium, that is, suppressing sphaleron processes in the broken phase that wash out the generated baryon asymmetry. The crucial parameter for this suppression is the order parameter $v(T_{\rm c})/T_{\rm c}$ which has to be larger than one for a successful preservation of the baryon asymmetry.

The effective Higgs potential in the SM

Recall the SM case (which we considered two weeks ago). The effective Higgs-potential reads

$$V_{\text{eff}}^{\text{SM}} = -m^2(T)\phi^2 - E_{\text{SM}}\, T\, \phi^3 + \frac{\lambda(T)}{2}\phi^4 + \ldots\,, \tag{1}$$

where $m^2(T)$ and $\lambda(T)$ each contain zero temperature and finite temperature corrections (the latter drive $m^2(T)$ negative at high T), whereas the cubic term comes only from finite temperature corrections due to the transversal gauge boson contribution. The cubic coefficient reads

$$E_{\text{SM}} \simeq \frac{2}{3}\, \left(\frac{2M_W^3 + M_Z^3}{\sqrt{2}\pi v^3}\right) \propto g^3\,. \tag{2}$$

(Numerically, $E_{\text{SM}} \simeq 0.018$.) This cubic term is crucial for the first order phase transition because it allows for a barrier in the effective potential.

The order parameter is

$$\frac{v(T_c)}{T_c} \simeq \frac{\sqrt{2}\, E_{\text{SM}}}{\lambda} \gtrsim 1\,, \tag{3}$$

where the one on the right-hand side reveals the demand of a strong first order phase transition. Another restriction of $v(T_c)/T_c$ originates from the validity of the perturbative expansion. The loop expansion parameter is $\theta = g^2T/M_W \sim gT/v$ and thus

$$\frac{v(T_c)}{T_c} \simeq \frac{\sqrt{2}\, E_{\text{SM}}}{\lambda} \sim \frac{g}{\theta} \gtrsim g\,. \tag{4}$$

(3) is the more restrictive condition here. For the measured value of m_t (that affects the loop corrections of the Higgs mass) and the lower bound on the Higgs mass from LEP (recall $m_h^2 = \lambda v^2$) a strong first order phase transition in the SM is clearly ruled out.

In the following we consider two different extensions of the SM and show their capability of providing a strong first order phase transition: The light stop scenario and the SM plus gauge singlet.

The light stop scenario

The stop mass matrix reads

$$m_{\tilde{t}}^2 = \begin{pmatrix} m_Q^2 + D_\mathrm{L}^2 + m_t^2 & 2m_t\widetilde{A}_t \\ 2m_t\widetilde{A}_t & m_U^2 + D_\mathrm{R}^2 + m_t^2 \end{pmatrix} , \tag{5}$$

where m_Q^2, m_U^2 are bilinear soft breaking parameters, $D_\mathrm{L,R}$ and m_t are the D-term and F-term contributions, respectively, and $\widetilde{A}_t = A_t - \mu/\tan\beta$ is the effective stop mixing parameter, wherein A_t in turn comes from the trilinear soft breaking term and $\mu/\tan\beta$ is another F-term contribution. The matrix possesses the eigenvalues

$$\begin{aligned} m_{1,2}^2 = &\frac{1}{2}\left(m_Q^2 + D_\mathrm{L}^2 + m_U^2 + D_\mathrm{R}^2 + 2m_t^2\right) \\ &\mp \sqrt{\frac{1}{4}\left(m_Q^2 + D_\mathrm{L}^2 - m_U^2 - D_\mathrm{R}^2\right)^2 + 4m_t^2\widetilde{A}_t^2} \, . \end{aligned} \tag{6}$$

In the MSSM the Higgs mass is restricted to not exceed M_Z at tree-level. Thus we need high loop corrections (from the stop) to push m_h upwards. This task is fulfilled by the left handed stop which is therefore considered to be very heavy in this scenario. On the other hand we want the right handed stop to be light (we'll see why in a minute). To obtain this setup we have to adjust the parameters. We consider

$$m_Q^2 \gg \text{all other parameters in (5)}$$

and expand the square root to first order in '(small parameters)/m_Q^2' obtaining

$$\begin{aligned} m_1^2 &\simeq m_U^2 + D_\mathrm{R}^2 + m_t^2\left(1 - \frac{\widetilde{A}_t^2}{m_Q^2}\right) , \\ m_2^2 &\simeq m_Q^2 + D_\mathrm{L}^2 + m_t^2\left(1 + \frac{\widetilde{A}_t^2}{m_Q^2}\right) . \end{aligned} \tag{7}$$

Now, let us come back to V_eff. What changes in comparison to the SM case?

$T=0$ 1-loop : { → heavier stop contributes to m, λ and pushes the Higgs mass up; → lighter stop: small contributions (unimportant) } $T=0$, 2-loop does not change picture

$$\begin{matrix} T\neq 0 \\ \text{1-loop} \end{matrix} : \begin{cases} \rightarrow \text{heavier stop: Boltzmann suppressed} \\ \quad \text{(unimportant)} \\ \rightarrow \text{lighter stop: plays a key role!} \end{cases}$$

So, let us write down the improved one-loop finite temperature effective potential:

$$V_{\rm eff}^{\rm MSSM} = -m^2(T)\phi^2 - T \left[E_{\rm SM}\,\phi^3 + \frac{\left(m_1^2 + \Pi_R(T)\right)^{3/2}}{2\pi} \right] + \frac{\lambda(T)}{2}\phi^4 + \dots\,, \quad (8)$$

where

$$\Pi_R(T) = \left\{ \frac{4}{9} g_3^2 + \frac{1}{6} y_t^2 \left[1 + \sin^2\beta \left(1 - \widetilde{A}_t^2/m_Q^2 \right) \right] + \left[\frac{1}{3} - \frac{1}{18} |\cos 2\beta| \right] g'^2 \right\} T^2 \quad (9)$$

is the finite temperature self-energy contribution.

Now comes the trick! In general the light stop does not induce a cubic term (as happens with the longitudinal components of the gauge fields). This is because the effective finite temperature mass does not vanish in the symmetric phase

$$\left(m_1^{\rm eff}\right)^2 (\phi = 0) = m_U^2 + \Pi_R(T). \quad (10)$$

But if one now adjusts m_U^2 to be negative and $m_U^2 \simeq -\Pi_R(T_{\rm c})$ one obtains approximately

$$m_1^2 + \Pi_R(T) \propto \phi^2\,, \quad (11)$$

which provides a cubic term when inserted in (8). The resulting $E_{\rm MSSM}$ then reads

$$E_{\rm MSSM} \simeq E_{\rm SM} + \frac{y_t^3 \sin^3\beta \left(1 - \widetilde{A}_t^2/m_Q^2 \right)^{3/2}}{2\pi}\,. \quad (12)$$

This means that in the limiting case of vanishing mixing, $E_{\rm MSSM}$ can be enhanced with respect to $E_{\rm MS}$ by a factor of up to

$$\frac{E_{\rm MSSM}}{E_{\rm SM}} \simeq 1 + \frac{m_t^3}{m_W^3} \simeq 10\,. \quad (13)$$

Recalling (3) and $m_h^2 = \lambda v^2$, this loosens the bound on the Higgs mass for a strong first order phase transition by a factor of up to roughly three.

Considering the loop expansion parameter $\theta = (y_t \sin\beta)^2\, T/m_t \sim y_t \sin\beta\, T/v$ results in the condition

$$\frac{v(T_c)}{T_c} \simeq \frac{\sqrt{2}\, E_{\rm SM}}{\lambda} \gtrsim y_t \sin\beta\,. \tag{14}$$

Hence, concerning the validity of perturbation theory the Higgs bound is loosened by a factor of up to m_t/m_W.

But this is not the whole story. Since m_U is now negative it might acquire a VEV. Such a VEV would obviously be color-breaking and this is something we clearly do not observe. To examine the situation is rather simple for $\widetilde{A}_t = 0$ but quite involving for the general case $\widetilde{A}_t \neq 0$. So let us just sketch the situations that can occur. Considering the VEVs $\langle\phi\rangle$ and $\langle U\rangle$, one has to ask:

- Which transition happens first?
- Which minimum is deeper?

You can distinguish four cases:

1. Instability: $\langle U\rangle$ first, $\langle U\rangle$ deeper
2. Two-step phase transition: $\langle U\rangle$ first, $\langle\phi\rangle$ deeper
3. Stability: $\langle\phi\rangle$ first, $\langle\phi\rangle$ deeper
4. Meta-stability: $\langle\phi\rangle$ first, $\langle U\rangle$ deeper

The first case is unacceptable while the second is proven to be unachievable. We are left with the latter ones. In fact it turns out that those scenarios that successfully provide a viable mass region for the Higgs and the stop are meta-stable, but sufficiently large life-times can be achieved. Figure 1 shows the allowed region in the m_h-$m_{\widetilde{t}_1}$-plane.

Can we find such a light stop at the LHC? Well, the so called co-annihilation region where the mass gap between the neutralino LSP and the light stop is less than about 30 GeV (and other SUSY particles are heavy) is extremely hard to explore. Since all other decay modes of the stop are either kinematically forbidden or disfavored (like the four-body decay $\widetilde{t} \to bjj\chi_1^0$

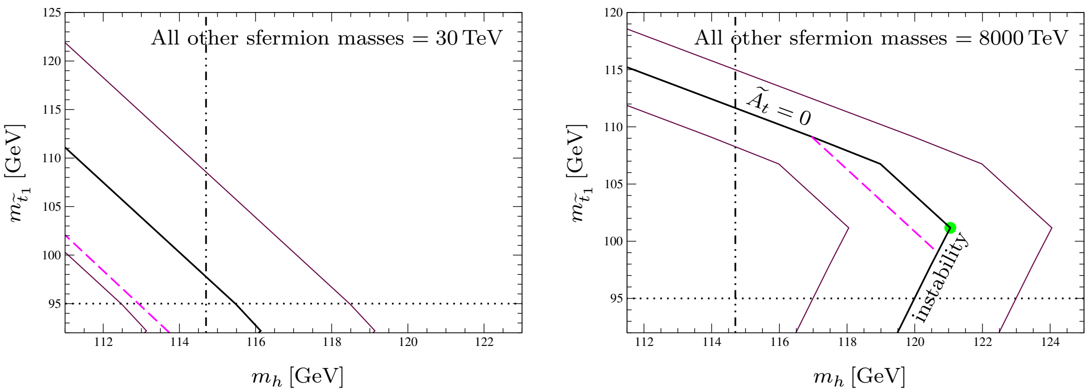


Figure 1: Allowed window for first order phase transition and a metastable vacuum solution. The allowed regions are to the left of the solid lines (for $\tan\beta \leq 15$) and dashed line (for $\tan\beta \leq 5$). In the case of $\tan\beta \leq 15$ the thick solid line displays the central value concerning the Higgs mass computation, while the maroon solid thin lines indicate its theoretical uncertainty by applying a $\pm 3\,\text{GeV}$ uncertainty. All sfermion masses except the right handed stop mass are set to 30 TeV (left panel) and 8000 TeV (right panel). $M_1 = M_2 = 100\,\text{GeV}$. Taken from [2].

or $\widetilde{t} \to bl\nu\chi_1^0$) only the loop-induced FCNC decay $\widetilde{t} \to c\chi_1^0$ is present. The resulting signature of this decay is two relatively soft c-jets plus MET which is not a very promising channel! So, if there is no strong production channel other than stop pair production in the reach of the LHC, the scenario can perfectly hide itself from observation—none of the current exclusion limits apply for this situation (see arXiv:1111.2250 or very recent arXiv: 1111.4467). With more integrated luminosity initial state radiation (that is the search for mono jet events) might, however, offer a certain handle on this scenario. In contrast, if the gluinos are within the reach of the LHC the scenario will show up in anomalous same-sign top pair events coming from the decay of the gluinos $\widetilde{g} \to t\,\widetilde{t}$.

SM plus gauge singlet

Consider the most general (renormalizable) tree-level potential for the SM Higgs field h and the singlet s with an additional $\mathbf{Z}_2$ symmetry $s \to -s$, containing the five parameters $\mu_h, \mu_s, \lambda_h, \lambda_s$ and λ_m,

$$V = -\frac{1}{2}\mu_h^2 h^2 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\lambda_m s^2 h^2 \,. \tag{15}$$

Within this potential it is possible to obtain two minima separated by a barrier at tree level! (In contrast to the SM and MSSM where the barrier is always due to the loop induced cubic term.)

But can we build a viable model with this?

Since the barrier is present at tree-level it is sufficient to only consider the leading term in the high temperature expansion of the effective potential

$$V_{1-\mathrm{loop}}^{T\neq 0} = \left(\frac{1}{2}c_h h^2 + \frac{1}{2}c_s s^2\right) T^2 + \dots \,, \tag{16}$$

where

$$\begin{aligned} c_h &= \frac{1}{48}\left[9g^2 + 3g'^2 + 2(6y_t^2 + 12\lambda_h + \lambda_m)\right] \,, \\ c_s &= \frac{1}{12}(2\lambda_m + 3\lambda_s + \dots) \equiv \frac{1}{4}\lambda_s + \delta c_s \,. \end{aligned} \tag{17}$$

The ellipses denote possible contributions from other particles coupled to the singlet. We can absorb the temperature evolution in a redefinition of $\mu_{h,s}$:

$$\begin{aligned} -\mu_h^2(T) &\equiv -\mu_h^2 + c_h(T^2 - T_c^2) \,, \\ \mu_s^2(T) &\equiv \mu_s^2 + c_s(T^2 - T_c^2) \,. \end{aligned} \tag{18}$$

Note that at very high temperatures the symmetry is restored. Let us consider the structure of the minima and how they evolve with temperature.

The stationary points of the potential will be determined by $\partial V/\partial h = 0$ and $\partial V/\partial s = 0$,

$$\begin{aligned} &\frac{\partial V}{\partial h} = 2h\frac{\partial V}{\partial h^2} = 2h\left(-\mu_h^2 + \lambda_h h^2 + \frac{1}{2}\lambda_m s^2\right) = 0 \\ &\Rightarrow \quad h = 0\,, \quad h^2 = D_h^2(s) = \frac{\mu_h^2}{\lambda_h} - \frac{\lambda_m}{2\lambda_h}s^2 \,, \end{aligned} \tag{19}$$

$$
\begin{aligned}
&\frac{\partial V}{\partial s} = 2s\frac{\partial V}{\partial s^2} = 2s\left(\mu_s^2 + \lambda_s s^2 + \frac{1}{2}\lambda_m h^2\right) = 0 \\
&\Rightarrow \quad s = 0\,, \quad h^2 = D_s^2(s) = -\frac{2\mu_s^2}{\lambda_m} - \frac{2\lambda_s}{\lambda_m}s^2\,.
\end{aligned}
\tag{20}
$$

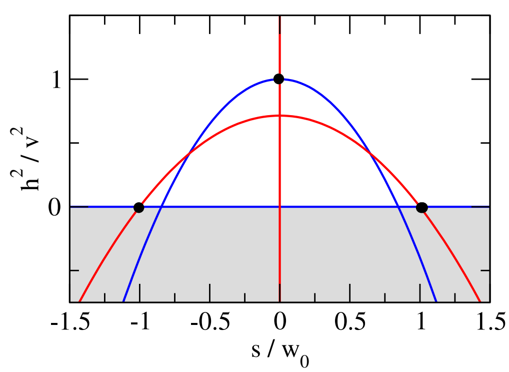
$\{D_h^2, h = 0\}$ and $\{D_s^2, s = 0\}$ are the curves along which the derivatives in the h and s direction, respectively, vanish. For an extremum both derivatives have to vanish. So we are looking for the intersection points of these curves.

To decide whether a stationary point is a minimum, maximum or saddle point we consider

$$
\left.\frac{\partial^2 V}{(\partial s)^2}\right|_{s=0} = 2\left.\frac{\partial V}{\partial s^2}\right|_{s=0} = 2\mu_s^2 + \lambda_m h^2 = \lambda_m\left(D_h^2(0) - D_s^2(0)\right)\,, \tag{21}
$$

$$
\left.\frac{\partial^2 V}{(\partial h)^2}\right|_{h=0} = 2\left.\frac{\partial V}{\partial h^2}\right|_{h=0} = -2\mu_h^2 + \lambda_m s^2 = -2\lambda_h D_h^2(s)\,. \tag{22}
$$

It turns out that each solution with two minima (separated by a barrier) contains one minimum at $h = 0, s \neq 0$ and the other at $h \neq 0, s = 0$. One of the symmetries is always broken. Let us make a picture. At T_c the curves look like

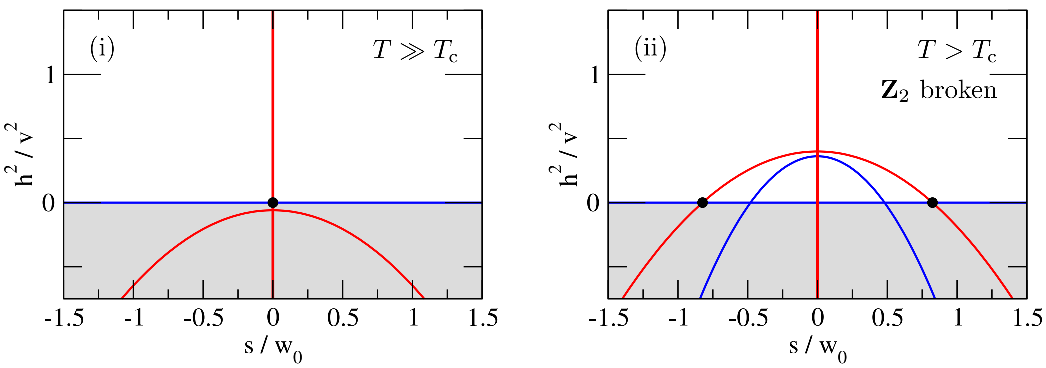


where the red and blue curves correspond to $\{D_h^2, h = 0\}$ and $\{D_s^2, s = 0\}$, respectivly. The dots denote the minima. Obviously $\mu_s^2 < 0$ at T_c.

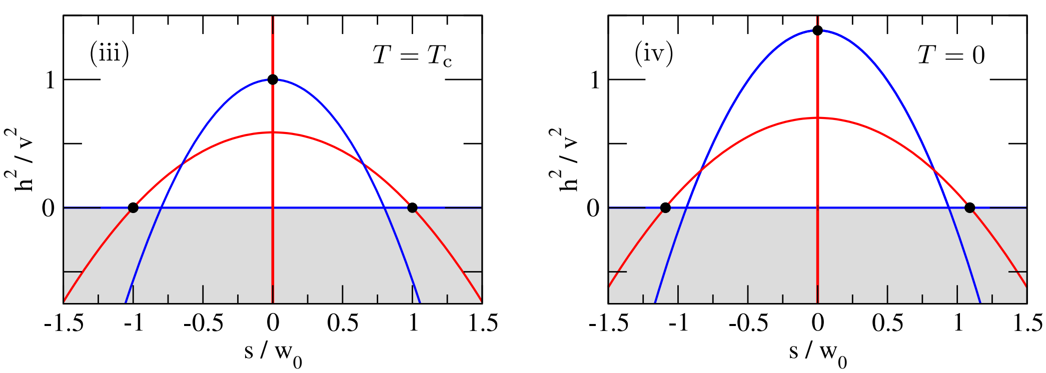
When temperature evolves the parabolas are only shifted, but the symmetry axis and the width remain the same:

$$
\frac{dD_h^2(s)}{dT^2} = -\frac{c_h}{\lambda_h}\,, \quad \frac{dD_s^2(s)}{dT^2} = -\frac{2c_s}{\lambda_m}\,. \tag{23}
$$

We illustrate this by the following pictures showing the evolution with decreasing temperature. (i) $T \gg T_\mathrm{c}$ in the symmetric phase. (ii) Still $T > T_\mathrm{c}$, but the $\mathbf{Z}_2$ symmetry is now broken, $w^2(T) \equiv \langle s^2(T) \rangle = -\mu_s^2(T)/\lambda_s$.



At some temperature the blue curve exceeds the red one at $s = 0$, $D_h^2(0) > D_s^2(0)$, thus, according to (21) a EW broken minimum occurs, $v^2(T) \equiv \langle h^2(T) \rangle = \mu_h^2(T)/\lambda_h$. (iii) At T_c both minima have equal depth and eventually (iv) at $T = 0$ the EW broken minimum is the deepest, $V(v, 0) < V(0, w)$.



Enabling such a behavior restricts the introduced parameters. Basically

1. D_s^2 parabola must be the wider one: $\lambda_m > 2\sqrt{\lambda_h \lambda_s}$
2. D_h^2 must evolve faster with temperature: $c_h/\lambda_h > 2c_s/\lambda_m$

These conditions can be combined to obtain

$$\frac{c_h}{c_s} > \sqrt{\frac{\lambda_h}{\lambda_s}}\,. \tag{24}$$

This condition coincides with the restriction that at $T_{\rm c}$ the EW broken minimum should decrease faster than the EW symmetric one

$$\left.\frac{d\Delta V_{\rm bs}(T)}{dT^2}\right|_{T_{\rm c}} > 0\,, \tag{25}$$

where $\Delta V_{\rm bs}(T) \equiv V[v^2(T),0] - V[0,w^2(T)]$.

Combining (24) with (17) now leads to a condition on the singlet quartic coupling,

$$\lambda_s > \lambda_{s,\rm min} \equiv \frac{4}{\lambda_h}\left[2c_h^2 - \lambda_h\delta c_s - 2c_h\sqrt{c_h^2 - \lambda_h\delta c_s}\right]\,, \tag{26}$$

while an upper bound comes from the width constraint (first restriction above):

$$\lambda_s < \lambda_{s,\rm max} \equiv \frac{\lambda_m^2}{4\lambda_h}\,. \tag{27}$$

Let us now write down the order parameter for this model. Therefore we consider

$$v^2(T) = \frac{\mu_h^2(T)}{\lambda_h} = \frac{\mu_h^2 + c_h(T^2 - T_c^2)}{\lambda_h} = v_{\rm EW}^2 - \frac{c_h}{\lambda_h}T^2\,, \tag{28}$$

where $v_{\rm EW} \equiv v(0)$. From this we can extract the important ratio $v(T_c)/T_c$ as

$$\frac{v(T_c)}{T_c} = \sqrt{\frac{v_{\rm EW}^2}{T_c^2} - \frac{c_h}{\lambda_h}}\,. \tag{29}$$

Note that the phase transition cannot be made arbitrarily strong by choosing low T_c, in which case $v(T_c) \simeq v_{\rm EW}$, since then the tunneling probability becomes small and furthermore the high-T approximation breaks down. However, one can easily achieve values well above one for a wide range of $T_{\rm c}$ and λ_h (see figure 2).

Finally, let us write down an expression for the singlet mass at $T=0$.

$$m_s^2 \equiv \left.\frac{\partial^2 V}{(\partial s)^2}\right|_{s=0}(T\!=\!0) = \mu_s^2(0) + \frac{1}{2}\lambda_m v_{\rm EW}^2 = \mu_s^2(T_{\rm c}) - c_s T_{\rm c}^2 + \frac{1}{2}\lambda_m v_{\rm EW}^2\,. \tag{30}$$

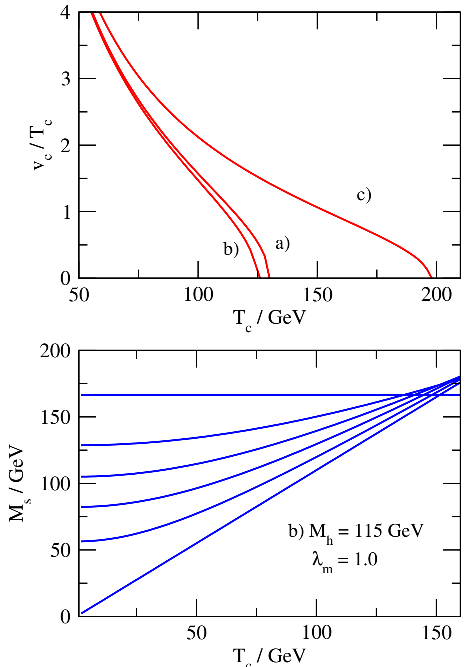


Figure 2: Ratio $v(T_c)/T_c$ (upper-left plot) and values of the singlet scalar mass (rest of plots) as a function of the critical temperature for the cases *a)* $m_h = 115$ GeV, $\lambda_m = 0.2$; *b)* $m_h = 115$ GeV, $\lambda_m = 1$; *c)* $m_h = 200$ GeV and $\lambda_m = 0.5$. Different masses correspond to different values of $\lambda_s \in (\lambda_{s,\text{min}}, \lambda_{s,\text{max}})$, with m_s increasing for lower λ_s. Taken from [3].

From $\Delta V_{\text{bs}}(T_{\text{c}}) = 0$ we obtain $w^2(T_{\text{c}})/v^2(T_{\text{c}}) = \sqrt{\lambda_h/\lambda_s}$ and thus

$$\mu_s^2(T_{\text{c}}) = -v^2(T_{\text{c}})\sqrt{\lambda_h\lambda_s} = -\left(v_{\text{EW}}^2 - \frac{c_h}{\lambda_h}T_{\text{c}}^2\right)\sqrt{\lambda_h\lambda_s}\,, \tag{31}$$

resulting in

$$m_s^2 = \left(\frac{1}{2}\lambda_m - \sqrt{\lambda_h\lambda_s}\right)v_{\text{EW}}^2 + \left(c_h\sqrt{\frac{\lambda_s}{\lambda_h}} - c_s\right)T_c^2\ . \tag{32}$$

The singlet mass squared is a simple linear combination of the two mass scales v_{EW}^2 and T_c^2 with positive coefficients. This is ensured by the allowed values of λ_s. $\lambda_{s,\text{min}}$ and $\lambda_{s,\text{max}}$ are exactly the limiting cases where the coefficient of T_c^2 and v_{EW}^2, respectively, would vanish.