06.12.11

Tunneling and Gravity

Workshop seminar, winter term 2011/2012.
Kai Schmitz

Outline:

(1) Goals of this talk
(2) Recap: the fate of the false vacuum.
(3) Motivation: why include gravity?
(4) Materialization of the bubble
(5) Minkowski bubble in de Sitter space
(6) Anti-de Sitter bubble in Minkowski space
(7) Safety of Minkowski vacua (Dvali)

Literature:

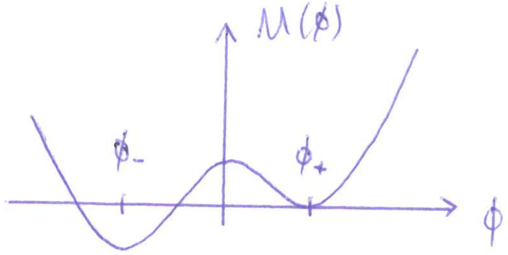
[1] Talk by Patrick Vaudrevange last week.
[2] Coleman; Phys. Rev. D15 (1977) 2929 – 2936.
[3] Coleman, Callan; Phys. Rev. D16 (1977) 1762 – 1768
[4] Coleman, Glaser, Martin; Commun. Math. Phys. 58 (1978) 211 – 221.
[5] Coleman, de Luccia; Phys. Rev. D21 (1980) 3305 – 3315.
[6] Parke; Phys. Lett. 121B (1983) 313 – 315.
[7] Coleman, Abbott; Nucl. Phys. B259 (1985) 170 – 174.
[8] Dvali; arXiv: 1107.0956 [hep-th].
[9] Vilenkin, Garriga, Ishlaer; arXiv: 1109.3422 [hep-th].

(1) Goals of this talk:

Last week: Field theory of a single scalar field ϕ w/ action

$$S = \int d^4x \left[\tfrac{1}{2} (\partial_\mu \phi)^2 - U(\phi) \right]$$

and U of the form



Classical theory:

Two homogeneous, stable ground states.

Quantum theory:

Decay of the false vacuum through barrier penetration.

$\Rightarrow$ Instantaneous materialization of a bubble of true vacuum within the false vacuum. Subsequently, expansion w/ a speed asymptotically approaching c, converting false into true vacuum.

Today: Include coupling to gravity.

$$\Rightarrow \quad S = \int d^4x \sqrt{-g} \left[\tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - U(\phi) - \tfrac{1}{2\kappa} R \right],$$

$$\kappa = 8\pi G = \frac{8\pi}{M_P^2}.$$

Goals: Effects of gravity on

- the production rate per unit spacetime volume (Γ/V),
- shape (critical bubble size),
- and further growth

of the bubble? In particular, influence on the stability of the false vacuum (CDL bound, argument by Dvali).

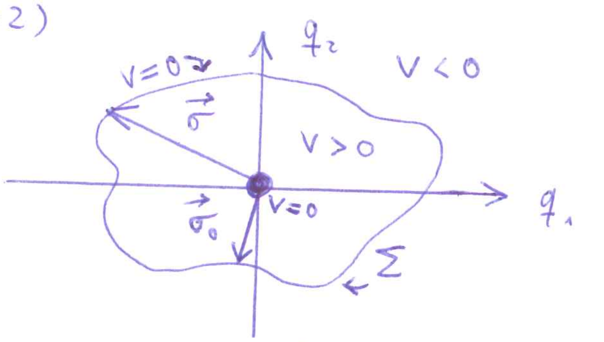
(2) Recap: the fate of the false vacuum:

Strategy to calculate Γ/V:

generalize results of N-dim. QM for particle of unit mass to QFT. Coleman: "Straight-forward transcription". ($\Rightarrow$ conclusion by analogy).

Tunneling in N dimensions:

($N=2$)



Contour plot of potential energy V in configuration space w/ coordinates q_i; $i = 1, \dots, N$.

$\vec{\sigma}_0$: minimal barrier penetration path.

In the semiclassical limit (small $\hbar$), the WKB approximation applies. One finds:

$$\Gamma/V = A\, e^{-B/\hbar} \left[1 + \mathcal{O}(\hbar) \right] \qquad (1)$$

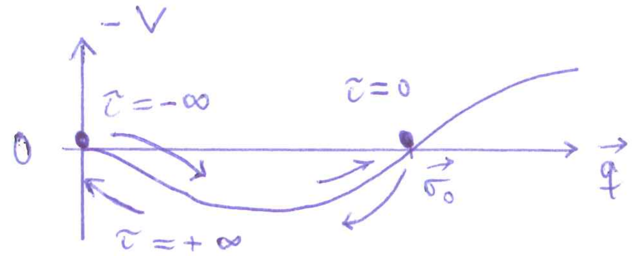
where B is given as

$$B = 2 \int_{\vec{0}}^{\vec{\sigma}_0} ds \sqrt{2V}$$

w/ $\vec{\sigma}_0$ minimizing the path functional $\int_{\vec{0}}^{\vec{\sigma}} ds \sqrt{2V}$. That is, $\delta_{\vec{\sigma}} \int_{\vec{0}}^{\vec{\sigma}_0} ds \sqrt{2V} = 0$.

$\Rightarrow$ Variational problem leading to EOM that looks like the Euler-Lagrange equation for the imaginary-time version of Hamilton's principle.

Solution of this EOM : "the bounce"



$\tau = it$, imaginary time.
$\vec{q}(-\tau) = \vec{q}(\tau)$.

$$\Rightarrow \quad B = 2\int_{-\infty}^{0} d\tau \, L_E \quad ; \quad L_E = \frac{1}{2}\left(\frac{d\vec{q}}{d\tau}\right)^2 + V$$

$$= \int_{-\infty}^{+\infty} d\tau \, L_E = S_E$$

$\Rightarrow$ B corresponds to the Euclidean action for the bounce.

generalization to QFT:

- Calculate Γ/V according to Eq. (1).
- $B = S_E(\phi) - S_E(\phi_+)$
- ϕ : "bounce solution" of the Euclidean EOM.

$\Rightarrow$ spherically symmetric, that is, $O(4)$-invariant. $\phi = \phi(\rho)$; $\rho = \sqrt{\tau^2 + \vec{x}^2}$.

$\Rightarrow$ situation looks like QM particle in 1D.

$\Rightarrow$ carry over interpretation from 1-dim. QM.

Euclidean action: $S_E = 2\pi^2 \int_0^\infty d\rho \, \rho^3 \left[\frac{1}{2}(\phi')^2 + U\right]$

EOM: $\phi'' + \frac{3}{\rho}\phi' = \frac{dU}{d\phi}$; $\phi' = \frac{d\phi}{d\rho}$.

Thin-wall approximation:

Radius of the bounce large compared to the characteristic range of variation of ϕ ; $U(\phi) = U_0(\phi) + \mathcal{O}(\varepsilon)$.

$$\Rightarrow \quad B = -\frac{1}{2}\pi^2 \bar{\rho}_0^{\,4}\, \varepsilon + 2\pi^2 \bar{\rho}_0^{\,3}\, S_1 .$$

$\bar{\rho}_0$: Integration constant introduced when solving the EOM for ϕ.

S_1 : Energy per unit surface area.

$$S_1 = \int_{\bar{\rho}_0 - \delta\rho}^{\bar{\rho}_0 + \delta\rho} d\rho \left[\tfrac{1}{2}(\phi')^2 + U_0(\phi) - U_0(\phi_+) \right]$$

$$= \frac{1}{4\pi\bar{\rho}_0^2} \int_{\bar{\rho}_0 - \delta\rho}^{\bar{\rho}_0 + \delta\rho} d\rho \, 4\pi\rho^2 \left[\tfrac{1}{2}(\phi')^2 + U_0(\phi) - U_0(\phi_+) \right].$$

The bounce corresponds to a stationary point of S_E. $\Rightarrow$ $\delta_{\bar{\rho}_0} B \stackrel{!}{=} 0$.

$$\Rightarrow \quad \bar{\rho}_0 = \frac{3S_1}{\varepsilon} \, , \qquad B_0 = \frac{27\pi^2 S_1^4}{2\varepsilon^3}.$$

Calculation of the coefficient A:

Starting point: Feynman's sum over histories. Note: wave-mechanical approach led to variational problem in imaginary time. Thus, now also Euclidean formulation.

$$\langle x_f | e^{-H\Delta\tau/\hbar} | x_i \rangle = N \int [dx] \, e^{-S_E/\hbar} \qquad (2)$$

Using $\mathbb{1} = \sum_n |n\rangle\langle n|$ where $H|n\rangle = E_n |n\rangle$,

$$\langle x_f | e^{-H\Delta\tau/\hbar} | x_i \rangle = \sum_n e^{-E_n\Delta\tau/\hbar} \langle x_f | n \rangle \langle n | x_i \rangle .$$

With $x_f = x_i$ and taking the limit $\Delta\tau \to \infty$, $\hbar \to 0$, we obtain

$$\langle x_i | e^{-H\Delta\tau/\hbar} | x_i \rangle = e^{-E_0\Delta\tau/\hbar} \, |\langle x_i | 0 \rangle|^2 + \ldots$$

$\Rightarrow$ That is, we can determine E_0, the ground-state energy of the initial state, by evaluating the RHS of Eq. (2) in the limit $\Delta\tau \to \infty$, $\hbar \to 0$ for all paths w/ boundary condition $x_f = x_i$. The functional integral is dominated by stationary points of the action $\hat{=}$ solutions of the Euclidean EOM w/ boundary condition $x_f = x_i$

$\Rightarrow$ Bounces!

$$N \int [dx]\, e^{-S_E/\hbar} \xrightarrow[\hbar \to 0]{\Delta\tau \to \infty} e^{-E_0 \Delta\tau/\hbar} \, |\langle x_i | 0 \rangle|^2$$

w/ $E_0 = \frac{1}{2}\hbar\omega - \hbar K e^{-B/\hbar}$; $\omega^2 = V''(x_i)$

$$|\langle x_i | 0 \rangle|^2 = \left(\frac{\omega}{\pi\hbar}\right)^{1/2}$$

K is defined such that:

$$N \int_{\substack{\text{one}\\\text{bounce}}} [dx]\, e^{-S_E/\hbar} = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega\Delta\tau/2}\, e^{-B/\hbar}\, \Delta\tau\, K$$

In order to determine K, distort integration over function space such that functional integral does not diverge.

$\Rightarrow$ $\operatorname{Im}\{K\} \neq 0$.

Calculate rate of vacuum decay as

$$\Gamma = -2 \operatorname{Im}\{E_0\}/\hbar$$
$$= 2 \operatorname{Im}\{K\}\, e^{-B/\hbar}$$

$\Rightarrow$ $\underline{\underline{A = 2 \operatorname{Im}\{K\}}}$

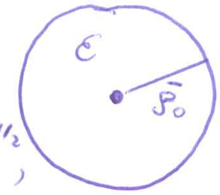
(for details cf. Ref. [1])

(3) Motivation: why include gravity?

Gravitational effects on vacuum decay:

Consider bubble of radius $\bar{\rho}_0$ and energy density ε. $\Rightarrow$ Schwarzschild radius

$$r_s = 2G \cdot \frac{4\pi}{3} \bar{\rho}_0^{\,3} \, \varepsilon$$



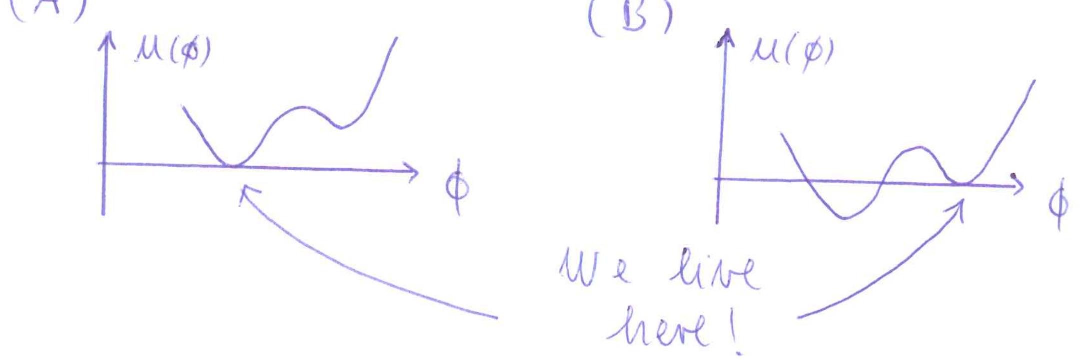
For $\bar{\rho}_0 = \Lambda = \left(\frac{8\pi G}{3}\varepsilon\right)^{-1/2} = \left(\frac{\kappa\varepsilon}{3}\right)^{-1/2}$,

we have $\bar{\rho}_0 = r_s = \Lambda$ and gravity has to be included. In general, the ratio $\bar{\rho}_0/\Lambda$ measures the importance of gravitational effects (cf. later).

Gravitational effects of vacuum decay:

Gravity assigns a meaning to the absolute zero of vacuum energy density: vanishing cosmological constant (CC). $\Rightarrow$ Minkowski vacuum.

Tunneling in u $\hat{=}$ changes in the CC. $\Rightarrow$ CC in the bubble different from the one outside the bubble. In our universe CC $\simeq 0$. Hence, focus on the following two cases:



(4) Materialization of the bubble:

Compute effects of gravity on B and $\bar{\rho}$. Changes in A irrelevant as theory non-renormalizable in any case.

Assumption: Gravity does not break any symmetry of the purely scalar problem; the solution corresponding to the bounce is still $O(4)$-invariant and has lower Euclidean action than any other nontrivial solution. However, if there are non-invariant bubble solutions of lower action or quantum gravitational decay channels that dominate over the semiclassical ones, the conclusions presented in the following are wrong.

Due to rotation invariance:

$$ds^2 = d\xi^2 + \rho^2(\xi)\, d\Omega_3^2 \qquad (3)$$

w/ ξ : radius coordinate

$d\Omega_3$: line element on a unit 3-sphere.

$\rho(\xi)$: curvature radius of a 3-sphere of radius ξ.

Note: metric invariant under translations in ξ direction (adding a constant to ξ, we do not need to change the form of the metric).

w/ $\phi = \phi(\xi)$ and ds as in Eq. (3), the action simplifies to:

$$S_E = 2\pi^2 \int d\xi \left[\rho^3 \left(\tfrac{1}{2}(\phi')^2 + U \right) + \frac{3}{\kappa}\left(\rho^2 \rho'' + \rho \rho'^2 - \rho \right) \right] \quad ; \quad ()' = \frac{d}{d\xi}$$

EOM for the scalar field ϕ:

(4) $$\phi'' + 3\frac{\rho'}{\rho}\phi' = \frac{dU}{d\phi} \qquad (\text{no gravity: } \rho(\xi) \to \xi.)$$

$\Rightarrow$ "Klein-Gordon equation in an expanding universe for an inverted potential".

EOM for the "scale factor" ρ:
(from Einstein's equations)

(5) $$\frac{\rho'^2}{\rho^2} = \frac{1}{\rho^2} + \frac{\kappa}{3}\left(\tfrac{1}{2}(\phi')^2 - U \right)$$

$\Rightarrow$ "Friedmann equation for an open ($k=-1$) FLRW universe".

Thin-wall approximation:

Neglect $3\frac{\rho'}{\rho}\phi'$ in Eq. (4); $U(\phi) = U_0(\phi) + O(\varepsilon)$.

Check validity of the thin-wall approximation:

- Away from the wall: $\phi' \simeq 0$.
- At the wall: ϕ varies over a range $\Delta\xi$.

$$\tfrac{1}{2}(\phi')^2 - U \leq \varepsilon$$

$$\Rightarrow \frac{\rho'^2}{\rho^2} \lesssim \frac{1}{\rho^2} + \frac{\kappa}{3}\varepsilon = \frac{1}{\rho^2} + \frac{1}{\Lambda^2} \qquad (\text{cf. Eq. (5)})$$

$\Rightarrow$ Demand: $\rho, \Lambda \gg \Delta\xi$.

However, no restriction on the ratio of $\bar{\rho}$ and Λ and hence on the importance of gravitational effects.

In the thin-wall approximation:
Same result as w/o gravity.

$$\int_{\phi(\bar{\xi})}^{\phi} d\phi \left\{ 2 \left(U_0 - U_0(\phi_\pm) \right) \right\}^{-1/2} = \xi - \bar{\xi} . \qquad (6)$$

$\Rightarrow$ ϕ goes monotonically from ϕ_- to ϕ_+; calculation of B can again be split into three parts.

Solve Eq. (5) w/ explicit result from Eq. (6).
Fix integration constant: $\bar{\rho} = \rho(\bar{\xi})$.
Note: $\bar{\xi}$ has no convention-independent meaning due to translation invariance. $\bar{\rho}$, however, does: it is the curvature radius of the bubble wall.

After integration by parts and eliminating ρ' w/ the help of Eq. (5):

$$S_E = 4\pi^2 \int d\xi \left(\rho^3 U - \frac{3\rho}{\kappa} \right)$$

$$B := S_E(\phi) - S_E(\phi_+) = B_{out} + B_{wall} + B_{in} .$$

$$B_{out} = 0$$

$$B_{wall} = 2\pi^2 \bar{\rho}^3 S_1$$

$$B_{in} = \frac{12\pi^2}{\kappa^2} \Big[U^{-1}(\phi_-) \left\{ \left(1 - \frac{\kappa}{3} \bar{\rho}^2 U(\phi_-)\right)^{3/2} - 1 \right\} - U^{-1}(\phi_+) \left\{ \left(1 - \frac{\kappa}{3} \bar{\rho}^2 U(\phi_+)\right)^{3/2} - 1 \right\} \Big]$$

For a more comprehensive discussion of this result, cf. Ref. [6].

Again determine $\bar{\rho}$ by demanding that

$$\delta_{\bar{\rho}} B = 0 .$$

Special cases:

(A) $U(\phi_+) = \varepsilon$; $U(\phi_-) = 0$

(B) $U(\phi_+) = 0$; $U(\phi_-) = -\varepsilon$

$$\Rightarrow \quad \bar{\rho} = \frac{\bar{\rho}_0}{1 + s(\bar{\rho}_0/2\Lambda)^2}$$

$$B = \frac{B_0}{[1 + s(\bar{\rho}_0/2\Lambda)^2]^2} \qquad s = \begin{cases} +1 ; & \text{case (A)} \\ -1 ; & \text{case (B)} \end{cases}$$

Discussion:

(A) More and smaller bubbles.

(B) Less and larger bubbles.

Vacuum decay totally quenched at

$$\bar{\rho}_0 = 2\Lambda \qquad \left(\bar{\rho}_0 = \frac{3 S_1}{\varepsilon} \; ; \; \Lambda = \left(\frac{k\varepsilon}{3}\right)^{-1/2} \right)$$

$$\Leftrightarrow \quad \varepsilon_{min} = \frac{3}{4} k S_1^2 = 6 \pi G S_1^2$$

$\Rightarrow$ CDL bound on ε. For $\varepsilon \leq \varepsilon_{min}$ gravity stabilizes the false vacuum.

Physical intuition for the CDL bound:

The process of barrier penetration does not violate the conservation of energy! Consider bubble of radius $\bar{\rho}$, neglect gravity:

$$E_{bubble} = E_{vol} + E_{surf}$$

$E_{vol} = -\frac{4\pi}{3}\bar{\rho}^3 \varepsilon$; negative volume energy.

$E_{surf} = +4\pi\bar{\rho}^2 S_1$; positive surface energy.

w/ $S_1 = \frac{\bar{\rho}_0}{3}\varepsilon$:

$$E_{Bubble} = \frac{4\pi}{3}\bar{\rho}^2(\bar{\rho}_0 - \bar{\rho})\varepsilon$$

$$= 0 \text{ for } \bar{\rho} = \bar{\rho}_0 \text{ as it should be.}$$

Now add gravitational corrections:

$$E_{Bubble} \longrightarrow E_{Bubble} + E_{grav} \overset{!}{=} 0$$

$$E_{Grav} = E_{Newton} + E_{geom}$$

$E_{Newton} = -\frac{\varepsilon\pi\bar{\rho}_0^5}{15\Lambda^2}$ → gravitational self-binding energy.

$E_{geom} = +\frac{2\pi\varepsilon\bar{\rho}_0^5}{5\Lambda^2}$ → geometrical correction taking care of the fact that due to $U(\phi_-) = -\varepsilon$ inside the bubble the volume of the bubble is smaller than in the flat case.

$\Rightarrow E_{grav} = +\frac{\pi\varepsilon\bar{\rho}_0^5}{3\Lambda^2}$ $\Rightarrow E_{Grav} > 0$

$\Rightarrow \bar{\rho} > \bar{\rho}_0$

$\Rightarrow$ Gravity diminishes the volume / surface ratio of the bubble; for $\varepsilon \leq \varepsilon_{min}$, $|E_{vol}|$ never catches up w/ E_{surf} to yield a zero-energy bubble, no matter how large $\bar{\rho}$ is.

(5) Minkowski bubble in de Sitter space:

Growth of the bubble w/o gravity:

Materialization in Euclidean space at time $t = \tau = 0$ w/ the center of the bubble located at $\vec{x} = \vec{0}$. Solution of the Minkowskian field equation is analytic continuation of Euclidean solution:

$$\phi(t, \vec{x}) = \phi\left((-t^2 + \vec{x}^2)^{-1/2}\right)$$

Bubble wall: $\vec{x}^2 - t^2 = \bar{\rho}_0^2$

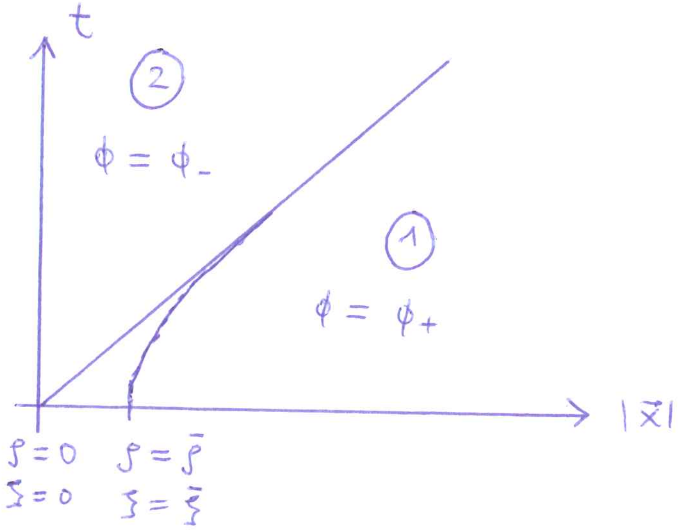
$\Rightarrow$ Velocity $v_{bubble} = \frac{d|\vec{x}|}{dt} = \frac{(|\vec{x}|^2 - \bar{\rho}_0^2)^{1/2}}{|\vec{x}|}$

$$\xrightarrow{t \gg \bar{\rho}_0} 1$$

Growth of the bubble w/ gravity:

Analytically continue both the scalar field and the metric to real times.

$\Rightarrow$ O(4) - invariant Euclidean manifold $\longrightarrow$ O(3,1) - invariant Minkowskian manifold.



In the thin-wall approximation, ϕ is simply ϕ_- inside and ϕ_+ outside the bubble.

① $ds^2 = -d\xi^2 - \rho^2(\xi)\, d\Omega_S^2$,

minus sign by convention.

$d\Omega_S$: line element on a unit hyperboloid w/ spacelike normal vector.

$$\rho'^2 = 1 - \frac{\kappa}{3}\rho^2\, U(\phi_+)$$

② $ds^2 = d\;^2 - [-i\rho(i\tau)]^2\, d\Omega_T^2$

$d\Omega_T$: line element on a unit hyperboloid w/ timelike normal vector.

$$\rho'^2 = 1 - \frac{\kappa}{3}\rho^2\, U(\phi_-)$$

<u>Decay of a false de Sitter vacuum:</u>

Special case (A) :

$U(\phi_-) = 0 \Rightarrow \rho(\xi) = \xi$.

$\Rightarrow$ Bubble interior is Minkowski space.

$U(\phi_+) = \varepsilon \Rightarrow \rho(\xi) = \Lambda \sin(\xi/\Lambda)$

$\Rightarrow$ Bubble exterior is de Sitter space.

Note : ρ bounded from above by Λ.

$\Rightarrow$ Λ radius of de Sitter space.

$\Rightarrow$ Explanation for the fact that $\bar{\rho} = \bar{\rho}(\bar{\rho}_0, \Lambda) \leq \Lambda$.

<u>Bubbles larger than Λ do not fit into the false de Sitter vacuum.</u>

(6) Anti-de Sitter bubble in Minkowski space:

Special case (B):

$U(\phi_+) = 0 \Rightarrow \rho(\xi) = \xi$

$\Rightarrow$ Bubble exterior is Minkowski space.

$U(\phi_-) = -\varepsilon \Rightarrow \rho(\xi) = \Lambda \sinh(\xi/\Lambda)$

$\Rightarrow -i\rho(i\tau) = \Lambda \sin(\tau/\Lambda)$

To avoid closed timelike curves, treat τ as an ordinary real variable rather than an angular variable. $\Rightarrow$ Metric describes covering space of a hyperboloid embedded into a 5-dim. Minkowski space w/ $O(3,2)$-invariant metric. $\Rightarrow$ Bubble interior is anti-de Sitter space. (open, expanding-and-contracting universe)

AdS bubble is dynamically unstable; tiny fluctuations in the initial field value convert the coordinate singularity $\tau = \pi\Lambda$ in the metric into a genuine (curvature) singularity. $\Rightarrow$ gravitational collapse.

Minkowskian EOMs:

$$\ddot{\phi} + 3\frac{\dot{\rho}}{\rho}\dot{\phi} + \frac{dU}{d\phi} = 0 \quad ; \quad \dot{\phi} = \frac{d\phi}{d\tau}$$

$$\dot{\rho}^2 = 1 + \frac{\kappa}{3}\rho^2\left(\tfrac{1}{2}(\dot{\phi})^2 + U\right)$$

Choose $\phi_- = 0$ and define $\mu^2 = \left.\frac{d^2U}{d\phi^2}\right|_{\phi_-}$.

Initial values at $\tau = 0$:

$\dot{\phi} \equiv 0$ (s.t. solution nonsingular, cf. $\rho = 0$ at $\tau = 0$ as well.)

Also required by Euclidean $O(4)$ symmetry.

$\Rightarrow \quad \ddot{\phi} = -\frac{dU}{d\phi} \quad \Rightarrow \quad \phi(\rho \gtrsim 0) \propto e^{-\mu\rho}$

As long as ϕ is exponentially small, we can continue to use

$$\dot{\rho} = 1 - \frac{\kappa}{3}\rho^2 \varepsilon \quad \Rightarrow \Lambda \sin(\tau/\Lambda).$$

Follow the evolution of ϕ according to its EOM throughout the expansion and subsequent contraction of the universe.

$\Rightarrow$ Remains exponentially small all the way down until $\rho/\Lambda \ll 1$ once again.

In this region, when $\tau \simeq \pi\Lambda - \rho$,

$$\phi \propto \rho^{-3/2} \cos((\pi\Lambda - \rho)\mu + \text{const.}).$$

This approximation breaks down as we approach again $\rho = 0$. Then, the $\frac{1}{2}\dot{\phi}^2$ term in the EOM for ρ cannot be neglected anymore. But, we still know that $\dot{\rho}^2 \geq 1 - \frac{\kappa}{3}\rho^2 \varepsilon$.

$\Rightarrow$ A second zero of ρ is inevitable!

For general ε, we do not hit $\dot{\phi} = 0$ at the second zero of ρ. ϕ becomes singular. Its oscillations become very large and its energy density grows w/o bound.

$\Rightarrow$ curvature singularity: ultimate fate of a metastable Minkowski vacuum is not transition to a new phase, but gravitational collapse on a time-scale of order 1.

(7) Safety of Minkowski vacua:

Dvali, July 2011:

For smooth V and $\bar{\rho} < \infty$, one can always deform a zero-energy AdS bubble in Minkowski space into a negative-energy AdS bubble.

$\Rightarrow$ Renders QFT inconsistent!

Thought experiment:

Consider the decay of the Minkowski vacuum into a a negative-energy bubble B_- and a positive-energy lump B_+ (e.g. wave-packet of the scalar field ϕ).

$$\text{Minkowski vacuum} \rightarrow B_-(p_\mu) + B_+(q_\mu)$$

Poincaré invariance implies:

$$d\Gamma(B_- B_+) = d\Gamma(p^2, q^2, pq)$$

Due to 4-momentum conservation:

$$q_\mu = -p_\mu \Rightarrow d\Gamma(B_- B_+) = d\Gamma(p^2)$$
$$= d\Gamma(m^2) = \text{const.}$$

$\Rightarrow \int d^3p \; d\Gamma(p^2)$ diverges!

Note that this usually does not happen due to conservation and positivity of the energy: $+\sqrt{\vec{p}^{\,2}+m^2} + \sqrt{\vec{q}^{\,2}+m^2} \stackrel{!}{=} \text{const.}$

$\Rightarrow$ Trouble due to localized object w/ negative (ADM) mass.

Stability of the Minkowski vacuum is a matter of consistency. Vacua below the Minkowski vacuum either do not exist or are above the CDL bound. The same reasoning should apply if the deviation from Minkowski is as tiny as in our vacuum.

$\Rightarrow$ Our vacuum should be stable or extremly long-lived!

Vilenkin et al., September 2011:

Yes: the asymptotic states of the expanding bubble at future infinity do include an infinite number of emitted particles whose positive energy is compensated by a negative energy in the bubble. But: each state corresponds to a certain average number of particles emitted per unit area per unit proper time. $\Rightarrow$ Such states are invariant under boosts. Integration over boost momenta is unnecessary!

Further argument: Production of particle-antiparticle pairs in a uniform electric field is a prototypical example of false-vacuum decay. None of the explicit calculations in the literature leads to vacuum instability.