The Affleck-Dine-Seiberg superpotential

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1 Motivation: Dynamical SUSY breaking

In realistic models, SUSY is broken at low energies. In order to keep attractive features of SUSY, as for instance loop cancellation at high energies, we would like SUSY to be broken spontaneously. In dynamical supersymmetry breaking this spontaneous breaking is realized via the vev of a composite field as a result of strong coupling [1, 2].

For SUSY to be natural solution to the hierarchy problem $m_H \ll M_{Pl}$, the scale of SUSY breaking has to be much smaller than the fundamental scale:

$$M_{SUSY} \ll M_{Pl}\,. \tag{1}$$

One hopes to explain this hierarchy analogue to the breaking of the chiral symmetry in QCD, where non-perturbative effects break the chiral symmetry dynamically and large hierarchies can be naturally explained via exponentials $\exp(-8\pi^2/g^2)$. Similarly, one is aiming for

$$M_{SUSY} = M_{Pl} e^{-\frac{8\pi^2}{g^2}} \ll M_{Pl}\,. \tag{2}$$

where the theory is supersymmetric at tree level and dynamically broken by quantum effects. Non-perturbative effects are the *only* way to break SUSY dynamically. This is due to powerful non-renormalization theorems in SUSY that render a superpotential supersymmetric to all orders in perturbation theory if it is supersymmetric at tree level.

In the following, we want to discuss non-perturbative corrections to the superpotential of SUSY QCD, mainly following [3, 4], that will turn out to be very important for dynamical SUSY breaking which will be discussed in the two remaining talks.

2 Preliminaries

2.1 Recap: The holomorphic gauge coupling

Last week, we have seen in Tobias's talk that we can represent an $SU(N)$ gauge supermultiplet a s chiral superfield

$$W_\alpha^a = -i\lambda_\alpha^a + \theta_\alpha D^a - (\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu}^a - (\theta\theta)(\sigma^\mu D_\mu \lambda^{a\dagger})_\alpha \,. \tag{3}$$

Defining the holomorphic gauge coupling

$$\tau = \frac{\Theta_{YM}}{2\pi} + \frac{4\pi i}{g^2}\,, \tag{4}$$

the SUSY Yang-Mills Lagrangian can be written as

$$\mathcal{L} = \frac{1}{16\pi i}\int d^2\theta\, \tau\, W_\alpha^a W_\alpha^a + h.c.\,. \tag{5}$$

The solution of the RG equation for the running coupling is

$$\tau_{\text{1-loop}} = \frac{b}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right)\,, \qquad b = 3N - F\,, \tag{6}$$

for the holomorphic intrinsic scale

$$\Lambda = |\Lambda| e^{i\Theta_{YM}/b}\,. \tag{7}$$

The symmetry

$$\Theta_{YM} \to \Theta_{YM} + 2\pi\, k\,, \qquad k \in \mathbb{Z}\,, \tag{8}$$

is a symmetry of the theory since it leaves the path integral invariant. It was shown, that τ only receives one-loop corrections (6) and non-perturbative n-instanton corrections

$$e^{-S_{\text{instanton}}} = \left(\frac{\Lambda}{\mu}\right)^b\,, \tag{9}$$

such that

$$\tau(\Lambda,\mu) = \frac{b}{2\pi i}\ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} a_n \left(\frac{\Lambda}{\mu}\right)^{bn}\,. \tag{10}$$

2.2 Non-renormalization theorems

In this section, we will show that renormalization is absent for an exemplary superpotential. The key ingredients are

- The holomorphy of the superpotential, i.e. W is only a function of chiral superfields, not of the complex conjugates.
- Interpreting all coupling constants as spurions, i.e. background fields.

- Invariance of the Lagrangian under R-symmetry and spurious symmetries.

If we calculate the Wilsonian superpotential, i.e. integrate out physics above a scale μ, the superpotential must then be a holomorphic function of these spurions/ couplings. Consider a theory of a chiral superfield ϕ renormalized at a scale Λ with superpotential

$$W_{\text{tree}} = \frac{m}{2}\phi^2 + \frac{\lambda}{3}\phi^3 \,. \tag{11}$$

Let's write down the symmetries of this theory. For the R-symmetry $U(1)_R$, the total charge of W has to be $R[W] = 2$ since the Lagrangian

$$\mathcal{L} = \int d^2\theta\, W \tag{12}$$

has to be invariant under $U(1)_R$ and $R[\theta] = 1 \Rightarrow R[d\theta] = -1$. We can also define an additional $U(1)_S$ which leaves the superpotential invariant if the fields have the following charges:

	$U(1)_S$	$U(1)_R$
ϕ	1	1
m	-2	0
λ	-3	-1

(13)

If we integrate out modes from Λ down to μ we get additional terms in the superpotential than those of eq. (11). However, they are restricted by holomorphy and the symmetries of (13) to be of the form

$$W_{\text{eff}} = m\phi^2\, f\left(\frac{\lambda\phi}{m}\right) = \sum_n a_n \lambda^n m^{1-n}\phi^{n+2} \,, \tag{14}$$

for some function f. Note, that the argument of f is $U(1)_R$ and $U(1)_S$ invariant while $m\phi^2$ is $U(1)_S$ invariant and has $U(1)_R$ charge two. To have a sensible weak coupling limit $\lambda \to 0$ and massless limit $m \to 0$ there should only appear positive powers of λ and m. So the only possibilities are $n = 0$ and $n = 1$ and the corresponding superpotential is

$$W_{\text{eff.}} = W_{\text{tree}} = \frac{m}{2}\phi^2 + \frac{\lambda}{3}\phi^3 \,, \tag{15}$$

so there is no renormalization. The argument generalizes to all tree level superpotentials. It is important to stress that even though W does not get renormalized the fields, masses and couplings do. It is just their combination in the superpotential that does not get renormalized.

2.3 Anomalies and instantons

An anomaly is a symmetry of the classical theory that is broken in the quantum theory. As we will see in this section instantons can cause this breaking. Consider a set of Weyl fermions ψ coupled to an $SU(N)$ gauge field B^a_μ:

$$S_{\text{fermion}} = \int d^4x\, i\bar{\psi}\bar{\sigma}^\mu(\partial_\mu + iB^a_\mu T^a)\psi \,. \tag{16}$$

Under a position dependent chiral rotation $\psi \to e^{i\alpha(x)}\psi$ we find

$$S_{\text{fermion}} \to S_{\text{fermion}} - \int d^4x\, \alpha(x)\partial_\mu(\underbrace{\bar{\psi}\bar{\sigma}^\mu\psi}_{j^\mu_A}) \,, \tag{17}$$

after integrating by parts. Varying the action with respect to α one obtains on the classical level

$$\partial_\mu j^\mu_A = 0\,. \tag{18}$$

On the quantum level one can show that the current j^μ_A of the global $U(1)_A$ is no longer conserved by evaluating fermion triangle diagrams with the global current and two gauge currents at the three vertices as in figure 1

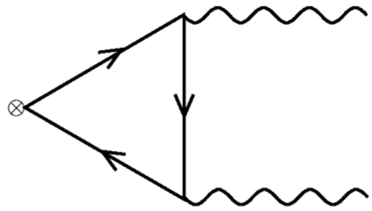


Figure 1:

One finds the one-loop result

$$\partial_\mu j^\mu_A \equiv A = \frac{1}{16\pi^2}\sum_r n_r\, T(r) F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu}\,, \tag{19}$$

with

$$(T^a_r)^m_l (T^b_r)^l_m = T(r)\delta^{ab}\,. \tag{20}$$

The index r labels the representation of the n_r fermions in the loop of figure 1. The result of eq. (19) is not corrected by higher-loops. Integrating over space-time gives

$$\int d^4x A = n_\psi - n_{\bar{\psi}} = \sum_r n_r\, 2T(r)\,, \tag{21}$$

where n_ψ and $n_{\bar{\psi}}$ are the number of fermion/ anti-fermion zero modes.

The appearance of $F\tilde{F}$ in eq. (19) is a hint towards the connection to instantons. Indeed, including

$$-\frac{\Theta_{YM}}{32\pi^2} F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu} \tag{22}$$

in eq. (16), the axial rotation $\psi \to e^{i\alpha(x)}\psi$ is equivalent to a shift

$$\Theta_{YM} \to \Theta_{YM} - \alpha\sum_r n_r\, 2T(r)\,, \tag{23}$$

which only is a symmetry of the path integral if the shift is 2π times an integer, see eq. (8). Hence, instantons can be seen as the source of the $U(1)_A$ breaking.

2.4 Gaugino condensation

Consider a pure $SU(N)$ super Yang-Mills, i.e. there is a gauge field strength $F^{a\,\mu\nu}$, a gaugino λ^a and an auxiliary field D^a. There is no matter, i.e. chiral supermultiplets.

Instantons break the $U(1)_R$ symmetry. This can be seen by calculating the mixed triangle anomaly between the $U(1)_R$ current and two gluons:

$$\int d^4x A = 1\cdot 2\cdot T(Ad) = 2N\,, \tag{24}$$

using $T(Ad) = N$. The R-charge of the gaugino is 1, so it transforms under $U(1)_R$ as

$$\lambda^a \to e^{i\alpha}\lambda^a\,, \tag{25}$$

which is equivalent to the shift

$$\Theta_{YM} \to \Theta_{YM} - 2N\,\alpha\,. \tag{26}$$

This is only a symmetry if

$$\alpha = \frac{k\pi}{N}\,, \qquad k \in \mathbb{Z}\,. \tag{27}$$

which means that the $U(1)_R$ is explicitly broken down to an $\mathbb{Z}_{2N}$ subgroup. Interpreting the holomorphic gauge coupling as a spurion chiral superfield we can define the following symmetry that leaves the path integral invariant:

$$\lambda^a \to e^{i\alpha}\lambda^a\,,\ \ \tau \to \tau + \frac{N\alpha}{\pi}\,. \tag{28}$$

We know that the effective superpotential transforms with R-charge 2, i.e.

$$W_{\text{eff}} \to e^{2i\alpha}W_{\text{eff}}\,. \tag{29}$$

Furthermore, we assume that there are no massless particles (this will be justified in section 3). The theory is asymptotically free in the UV and strongly coupled in the infrared. Hence, there will be color singlet composite states (mesons) in the infrared.
Now, the only term that we can write down in the superpotential that is consistent with eq. (29), the linear transformation of τ in eq. (28) and holomorphy is

$$\boxed{W_{\text{eff}} = a\mu^3\, e^{2\pi i\tau/N}} \tag{30}$$

where μ is an energy scale that we had to insert for dimensional reasons and a is a dimensionless constant that will be determined in section 3.

Since there are no massless degrees of freedom the effective potential eq. (30) holds all the way down to low energies. The simplest color singlet composite that can have a vacuum expectation value that does not break Lorentz invariance (which for example a fermion vev would do) is a gaugino condensate $\langle\lambda^a\lambda^a\rangle$. Remembering

$$W_{\text{eff}} = \frac{1}{16\pi i}\tau\, W^a_\alpha W^a_\alpha = \frac{1}{16\pi i}\tau\,(\lambda^a\lambda^a + \dots)\,, \tag{31}$$

the gaugino condensate is given by

$$\langle\lambda^a\lambda^a\rangle = 16\pi i\frac{\partial}{\partial\tau}W_{\text{eff}} = 16\pi i\frac{2\pi i}{N}a\mu^3 e^{2\pi i\tau/N}\,. \tag{32}$$

When inserting eq. (10) we can drop the non-perturbative corrections since they contribute only a phase to $\langle\lambda^a\lambda^a\rangle$. Setting $b = 3N$ we find

$$\boxed{\langle\lambda^a\lambda^a\rangle = -\frac{32\pi^2}{N}a\Lambda^3\,.} \tag{33}$$

This is what one also expects for dimensional reasons.

The presence of this gaugino condensate implies a spontaneous breaking of the spurious $\mathbb{Z}_{2N}$ symmetry since under $U(1)_R$:

$$\langle\lambda^a\lambda^a\rangle \to e^{2i\alpha}\langle\lambda^a\lambda^a\rangle = e^{2\pi i\frac{k}{N}}\langle\lambda^a\lambda^a\rangle \tag{34}$$

which is only invariant for $k = 0$ or $k = N$. Hence, the vacuum only respects a $\mathbb{Z}_2$ symmetry.

Supersymmetry is not broken in this setup. If it would be broken spontaneously, the spectrum would include a massless fermion in analogy to a goldstone boson, the Goldstino. So every theory which possesses a mass gap, which is the case in our setup, will not break supersymmetry.

There are N degenerate but distinct supersymmetric vacua. We can see this by considering

$$\langle(\lambda^a\lambda^a)^N\rangle \sim \Lambda^{3N} = \mu^{3N}\left(\frac{\Lambda}{\mu}\right)^b = \mu^{3N}e^{-S_{\text{instanton}}} = \mu^{3N}\left(\frac{|\Lambda|}{\mu}\right)^{3N} e^{i\Theta_{YM}} \tag{35}$$

where in the last step we have used eq. (7). Taking the Nth root of this expression, we see that the symmetry transformation $\Theta_{YM} \to \Theta_{YM} + 2\pi$ sweeps out N different values for $\langle\lambda^a\lambda^a\rangle$ which all differ by a phase, i.e. an instanton.

3 The ADS superpotential

We will now add matter to the pure SUSY $SU(N)$, so that we have $SU(N)$ SUSY QCD with $F < N$ flavors. The quarks Q and squarks Φ transform in the $SU(N)$ fundamental representation $\square$ and the anti-particles $\tilde{Q}$ and $\tilde{\Phi}$ in the anti-fundamental representation $\overline{\square}$. In total we have one vector and $2NF$ chiral supermultiplets. The global symmetry of this theory is an

$$SU(F)_L \times SU(F)_R \times U(1)_B \times U(1)_R\,. \tag{36}$$

The charges under these global symmetries are

	$SU(N)$	$SU(F)_L$	$SU(F)_R$	$U(1)_B$	$U(1)_R$
Φ	$\square$	$\square$	1	1	$(F-N)/F$
Q	$\square$	$\square$	1	1	$-N/F$
$\tilde{\Phi}$	$\overline{\square}$	1	$\overline{\square}$	-1	$(F-N)/F$
$\tilde{Q}$	$\overline{\square}$	1	$\overline{\square}$	-1	$-N/F$

(37)

Except for the R-symmetry which is of supersymmetric origin, this is similar to QCD where we have a chiral $SU(3)_L \times SU(3)_R \times U(1)_B$ global symmetry. On the perturbative level, there is again no superpotential

$$W_{\text{pert.}} = 0 \tag{38}$$

3.1 The moduli space of SUSY QCD

Solving the equations of motion, the D^a auxiliary field is given as

$$D^a = g(\Phi^{*jn}(T^a)^m_n\Phi_{mj} - \tilde{\Phi}^{jn}(T^a)^m_n\tilde{\Phi}^*_{mj})\,, \tag{39}$$

where $1 \le j \le F$ is a flavor index, $1 \le m, n \le N$ is a color index and $1 \le a \le N^2 - 1$ labels the adjoint representation.

In general, the D-term scalar potential for the squarks

$$V = \frac{1}{2}D^aD^a\,, \tag{40}$$

vanishes for all values of $\Phi \in (0,\infty)$, i.e. classically there are D-flat directions in moduli space. Solving $D^a = 0$ we can use $SU(N)$ gauge transformations to bring the matrix $\Phi^{*jn}\Phi_{mj}$ into a diagonal form. Then the $SU(F)$ invariance of $\Phi^{*jn}\Phi_{mj}$ can be used to obtain a simple form of Φ:

$$\langle \tilde{\Phi}^* \rangle = \langle \Phi \rangle = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix}, \tag{41}$$

with real *undetermined* vev's $v_1, \dots v_F$. Changing the values of the v_i corresponds to moving between different physical vacua as different vev's correspond to different particle spectra, for example gauge boson masses. Due to their gauge non-invariance the vev's break the gauge symmetry, generically from $SU(N)$ to $SU(N-F)$. In the super Higgs mechanism every broken generator corresponds to a supermultiplet that is eaten by the vector bosons to form a massive vector supermultiplet. Since there are

$$N^2 - 1 - ((N-F)^2 - 1) = 2NF - F^2 \tag{42}$$

broken generators, the number of uneaten chiral supermultiplets after super Higgsing is

$$2NF - (2NF - F^2) = F^2\,, \tag{43}$$

which can be parametrized in an $SU(N)$ gauge invariant $F \times F$ matrix field

$$M_i^j = \tilde{\Phi}^{jn}\Phi_{ni} \tag{44}$$

3.2 The spurious symmetry and the superpotential

For the theory we considered in section 2.4, there was an anomaly of the $U(1)_R$ symmetry. For SUSY QCD, there is no anomaly of $U(1)_R$ but an anomaly of the chiral symmetry $U(1)_A$. We see this by calculating the diagram of figure 1 with the global current and two gauge currents. For the $U(1)_R$ anomaly one finds

$$\int d^4x A_R = 2(R_\lambda T(Ad) + (R_\Phi - 1)T(\Box) \cdot 2F)\,. \tag{45}$$

The R-charge of the gaugino is 1 while we have some freedom in defining R_Φ: As long as there are no matter terms in the superpotential, there is no restriction on the R charge of the quarks and squarks. Using $T(Ad) = N$ and $T(\Box) = 1/2$ one finds that the anomaly eq. (45) vanishes for

$$R = \frac{F-N}{F} \tag{46}$$

In section 2.4 the anomaly is non-vanishing since there are no quarks which could cancel the contribution of the gauginos.

On the other hand, the gauginos do not contribute to the chiral anomaly while the quarks do:

$$\int d^4x A_A = 2 \cdot 2F \cdot T(\Box) = 2F\,. \tag{47}$$

So we have exchanged the $U(1)_R$ for an $U(1)_A$ anomaly. As in section 2.4 we can define a spurious symmetry that is a subgroup of the original $U(1)_A$ symmetry:

$$
\begin{aligned}
&Q \to e^{i\alpha} Q\,,\\
&\tilde{Q} \to e^{i\alpha}\tilde{Q}\,,\\
&\Theta_{YM} \to \Theta_{YM} + 2F\alpha\,, \qquad \Rightarrow \Lambda^b \to e^{i\,2F\,\alpha}\Lambda^b\,.
\end{aligned} \tag{48}
$$

We now want to write down the most general superpotential that is generated non-perturbatively and respects the R-symmetry and the spurious symmetry defined in eq. (48). The only $SU(F)$ invariants that we can use to build W and their respective charges are

	$U(1)_A$	$U(1)_R$
$W^a W^a$	0	2
Λ^b	$2F$	0
$\det M$	$2F$	$2(F-N)$

(49)

One obtains the charge of $\det M$ from the charge of Φ by multiplication with $2F$ since there are two Φ's in M and it is an $F \times F$ matrix. A general non-perturbative term in the Wilsonian superpotential must have the form

$$
(\Lambda^b)^n (W^a W^a)^m (\det M)^p \tag{50}
$$

The $U(1)_A$ charge of the superpotential must vanish while the $U(1)_R$ charge has to be two. So we get the equations:

$$
\begin{aligned}
0 &= n\,2F + p\,2F\,, &(51)\\
2 &= 2m + p\,2(F-N)\,, &(52)\\
\Rightarrow n &= -p = \frac{1-m}{N-F}\,. &(53)
\end{aligned}
$$

Note that m has to be integer and fulfill $m \geq 0$ so the $W^a W^a$ term does not violate locality. On the other hand, $n \geq 0$ has to be true to have a sensible weak coupling limit $\Lambda \to 0$. Hence, only $m = 0$ and $m = 1$ are allowed and the corresponding superpotential is

$$
\boxed{W = \underbrace{b\ln(\Lambda) W^a W^a}_{W_{\text{pert.}}} + \underbrace{C_{N,F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)}}_{W_{\text{ADS}}}} \tag{54}
$$

The first term is just the perturbative tree-level field strength term while the second term is the non-perturbative Affleck-Dine-Seiberg (ADS) superpotential [5]. $C_{N,F}$ is a renormalization scheme dependent dimensionless constant. Comparing the tree-level superpotential with eq. (10) we see that there are no non-perturbative corrections, i.e. instantons, that renormalize the gauge coupling τ.

What generates this superpotential? For $F = N - 1$ the ADS superpotential has the form

$$
W_{\text{ADS}} \sim \Lambda^b \sim e^{-S_{\text{instanton}}}\,, \tag{55}
$$

so instantons can generate it. For $F < N - 1$, W_{ADS} cannot be expressed as integer powers of Λ^b so instantons cannot generate the superpotential. We will see in the following two sections, that in this case gaugino condensation generates W_{ADS}.

3.3 Integrating out

We now want to construct effective theories from SUSY QCD by integrating out the flavor degrees of freedom. One possibility is to give a large vev v to the F squarks. The other possibility, to give a large mass term, will be explored in the next section. Well below the scale v we are then in pure $SU(N-F)$ SUSY Yang-Mills. Comparing the running holomorphic gauge coupling of the original theory and the low-energy effective theory we have:

$$\frac{8\pi^2}{g^2(\mu)} = b\ln\left(\frac{\mu}{\Lambda}\right), \qquad b = 3N - F\,, \tag{56}$$

$$\frac{8\pi^2}{g_L^2(\mu)} = b_L\ln\left(\frac{\mu}{\Lambda_L}\right), \qquad b_L = 3(N - F)\,. \tag{57}$$

At the scale $\mu = v$, which is the mass of the heavy gauge bosons that we integrated out, these couplings should match:

$$\frac{8\pi^2}{g^2(v)} = \frac{8\pi^2}{g_L^2(v)} \quad\Leftrightarrow\quad \left(\frac{\Lambda}{v}\right)^{3N-F} = \left(\frac{\Lambda_L}{v}\right)^{3(N-F)} \quad\Leftrightarrow\quad \frac{\Lambda^{3N-F}}{v^{2F}} = \Lambda_L^{3(N-F)}\,. \tag{58}$$

Inserting this into the formula for the ADS superpotential eq. (54) and using $\det M = v^{2F}$ we find

$$W_{\text{eff}} = C_{N,F}\Lambda_L^3\,, \tag{59}$$

which agrees with the result we derived in section 2.4 for the gaugino condensate. Note that the effective superpotential (59) does not depend on $\det M$ anymore, the gauge degrees of freedom which form the gaugino condensation and the F^2 gauge singlets described by M decouple in the infrared.

When deriving the gaugino condensation from pure SUSY Yang-Mills in section 2.4 we saw that by themselves the $SU(N-F)$ gauginos have an anomalous R-symmetry. This anomaly is absent in SUSY QCD. So how can a theory (pure SUSY Yang-Mills) as a low energy limit of another theory (SUSY QCD) have an anomaly that the original high energy theory did not have? The answer is that the two sectors are coupled by irrelevant operators which restore the original R-symmetry. To see this, note that the generic version of eq. (58) is

$$\frac{\Lambda^{3N-F}}{\det M} = \Lambda_L^{3(N-F)}\,. \tag{60}$$

Thus, the effective holomorphic coupling in the low-energy effective theory

$$\tau_L = \frac{3(N-F)}{2\pi i}\ln\left(\frac{\Lambda_L}{\mu}\right) \tag{61}$$

includes a term proportional to $\ln\det M$ which induces a Wess-Zumino term in the low-energy Lagrangian

$$\mathcal{L} = \frac{1}{32\pi^2}\int d^2\theta\,\ln(\det M)W^aW^a + h.c. \tag{62}$$

$$= \frac{1}{32\pi^2}\Big(\mathrm{Tr}(\mathcal{F}_{\mathcal{M}}M^{-1})\lambda^a\lambda^a + \mathrm{Arg}(\det M)F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu} + \mathcal{F}_M^2 + \dots\Big) + h.c.\,, \tag{63}$$

where $\mathcal{F}_M$ is the auxiliary field of the chiral superfield M. We have used $\ln(\det M) = \mathrm{Tr}(\ln M)$ and expanded $\ln M$ around its vev. Note, that under a rotation $\Phi \to e^{i\alpha}\Phi$, $\mathrm{Arg}(\det M)$ transforms as

$$\text{Arg}(\det M) \to \text{Arg}(\det M) + 2F\alpha\,. \tag{64}$$

and so the R-symmetry is indeed restored. Furthermore, $\text{Arg}(\det M)$ has the right transformation behavior to be the Nambu-Goldstone boson of the spontaneously broken R-symmetry.

Taking the equations of motion of eq. (62) we find

$$\mathcal{F}_{\mathcal{M}} = \frac{\partial W}{\partial M} = \frac{1}{32\pi^2} M^{-1} \langle \lambda^a \lambda^a \rangle\,, \tag{65}$$

so a non-trivial superpotential for M was created. The only possible superpotential that is consistent with holomorphy and symmetry is W_{ADS}. Hence W_{ADS} must be generated from gaugino condensation for $F < N-1$ flavors and

$$\mathcal{F}_M = \mathcal{F}_M^{ADS} = \frac{\partial W_{ADS}}{\partial M} = \frac{C_{N,F}}{N-F} \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}-1} \frac{-1}{(\det M)^2}\, \Lambda^{3N-F} M^{-1} \det M\,, \tag{66}$$

where we have used $\partial_M \det M = M^{-1} \det M$. Using $C_{N,F} = N - F$, which can be obtained from an $F = N-1$ instanton calculation, this simplifies to

$$\mathcal{F}_M = -M^{-1} \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}}\,, \tag{67}$$

such that

$$\boxed{\langle \lambda^a \lambda^a \rangle = -32\pi^2 \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{\frac{1}{N-F}} = -32\pi^2 \Lambda_L^3\,,} \tag{68}$$

generates the ADS superpotential. It is remarkable that gaugino condensation which is a purely strongly coupled phenomenon could be derived from the UV theory using the powerful constraints from holomorphy and symmetry.

3.4 Mass perturbations

We now give masses m^i_j to the flavors. Starting from the W_{ADS}, the superpotential must have the form

$$W = \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} f(t)\,, \tag{69}$$

with

$$t = m^i_j M^j_i \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{-1/(N-F)} \tag{70}$$

being $U(1)_R \times U(1)_A$ invariant. For a sensible small mass limit $m \to 0$ only positive powers of t can appear and for a sensible weak coupling limit $\Lambda \to 0$ no powers higher than one. Comparison with W_{ADS} then determines $f(t)$ to be

$$f(t) = C_{N,F} + t\,, \tag{71}$$

such that

$$W = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} + m^i_j M^j_i\,. \tag{72}$$

Integrating out the M^i_j degrees of freedom can be done effectively by solving the equations of motion

$$\frac{\partial W}{\partial M_i^j} = 0 \tag{73}$$

and plugging back in into the superpotential eq. (72). Using again $C_{N,F} = N - F$ the equations of motion give

$$M_i^j = \left(m^{-1}\right)_i^j \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} . \tag{74}$$

We take the determinant on both sides and plug the result back into eq. (74):

$$M_i^j = \left(m^{-1}\right)_i^j \left(\det m\, \Lambda^{3N-F}\right)^{1/N} . \tag{75}$$

If we would integrate out only one flavor with mass m, matching the holomorphic gauge couplings at the scale $\mu = m$ would give the relation

$$\left(\frac{\Lambda}{m}\right)^{3N-F} = \left(\frac{\Lambda_L}{m}\right)^{3N-F+1} \quad \Leftrightarrow \quad \Lambda^{3N-F} m = \Lambda_L^{3N-F+1} . \tag{76}$$

Integrating out all masses this generalizes to

$$\Lambda^{3N-F} \det m = \Lambda_L^{3N} \tag{77}$$

Now we can insert equations (77) and (75) into eq. (72) to obtain the effective superpotential

$$W_{\text{eff}} = (N - F + F)\left(\Lambda_L^{3N}\right)^{1/N} = N e^{2\pi i k/N} \Lambda_L^3 , \qquad k = 1, \ldots, N . \tag{78}$$

This agrees with the result (30) we found in section 2.4 and hence our assumption of a mass-gap was justified. We can once more derive the gaugino condensate to be

$$\boxed{\langle \lambda^a \lambda^a \rangle = -32\pi^2 e^{2\pi i k/N} \Lambda_L^3 , \qquad k = 1, \ldots, N .} \tag{79}$$

The appearance of the N-th root tells us that there are N distinct supersymmetric vacua which correspond to different phases of the vev of M, see eq. (75). This is consistent with what we found at the end of section 2.4. Furthermore, note that we could determine the coefficient a in eq. (33) from the UV theory.

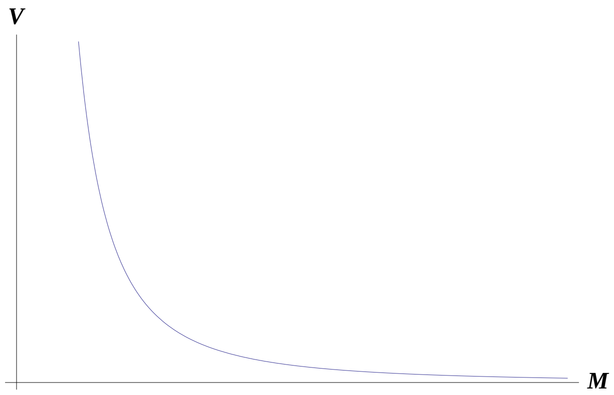
Finally, let us mention that we could have integrated out only one or a few flavors either by large vevs or large masses. In this case, we would have obtained an effective theory with less flavor and in the case of large vevs also less gauge group rank. The effective superpotential should again be of the ADS form with $N_L \leq N$ and $F_L < F$. The ADS superpotential passes this consistency check.

4 Vacuum structure

Let us now discuss the scalar potential that is generated by W_{ADS}. If we take for simplicity $\det M \sim M^F$ we get

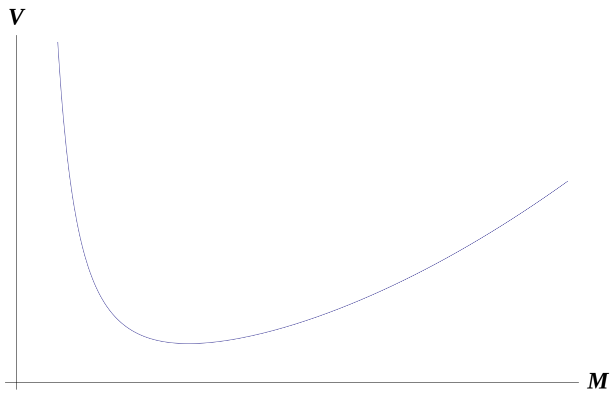
$$V_{ADS} = \left|\frac{\partial W_{ADS}}{\partial M}\right|^2 \sim |M|^{\frac{-2N}{N-F}} \tag{80}$$

and since $F < N$ this is of the following form:



This is called a runaway potential since it has its minimum at $\langle M\rangle = \infty$. This minimum is $V(\langle M\rangle = \infty) = 0$ so the vacuum does not break supersymmetry which is consistent with our earlier statement that the theory possesses N supersymmetric vacua.

Clearly it is not very satisfying that $\langle M\rangle = \infty$ and also ultimately we are looking for a way to generate a non-supersymmetric vacuum. One possibility is that there are corrections to the Kähler potential. For a non-trivial Kähler potential the scalar potential is given by $V = W^\dagger_{\bar{a}}(K^{-1})^{\bar{a}b}W_b$ which can lead to meta-stable, i.e. local minima. Globally the runaway direction persists. Another possibility is of course adding tree level mass terms, as we have already done in section 3.4. Those will generate a quadratic potential for large M such that we can obtain a stable minimum for M at a finite value with spontaneously broken supersymmetry:



Constructing realistic models of supersymmetry breaking will be discussed in the next seminars.

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