Dihadron Production in Semi-inclusive Deep Inelastic Scattering: A Theoretical Overview

S. Gliske

High Energy Physics Division
Argonne National Lab
HERMES and STAR Collaborations

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Outline

I. Background
II. Lund/Artru and Gluon Radiation Models
III. Partial Wave Expansion of the Fragmentation Functions
IV. Cross Section
V. Spectator Model for Fragmentation

Background

Main Topics

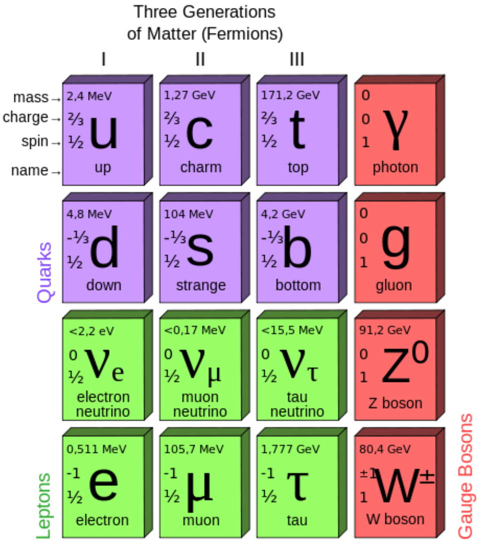
Spin Fundamental quantum number (more fundamental than mass). The group theory is identical to angular momentum.

Proton Bound state of quarks and gluons, has spin 1/2 and mass 0.9 GeV

Quark Fundamental particle, fermion (spin 1/2), interacts via "all" fundamental forces

Gluon Fundamental particle, boson (spin 1), carries the strong nuclear force.

Standard Model of Particle Physics



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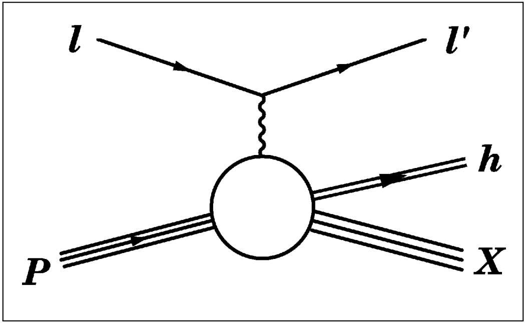
Proton Models

- Early data suggested the proton was made of 2 u-quarks and 1 d-quark
- Pauli exclusion principle implies the spins of the u-quarks must be oppositely aligned
- The spin of the proton is then $\frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$
- Problem: data later showed that the quark masses equal only 10% of the proton mass.
  - Other 90% is binding energy, i.e. more quarks and gluons (called the sea)
  - The “original” 2 u and 1 d are called “valence quarks”
- How much do the quarks then contribute to the spin of the proton?
  - First measurements suggested 20-30%—The Spin Crisis!

Semi-Inclusive Deep Inelastic Scattering (SIDIS)



- Scattering: lepton interacting with proton
- Inelastic: produce additional particles
- Deep: highly off-shell virtual photon, probes internal structure of the proton
- Semi-Inclusive: the lepton and a few of the target fragments are measured

Experimental Access to Quark Spin

Deep Inelastic Scattering (DIS) $e^{\pm} + p \rightarrow e^{\pm} + X$

Inclusive DIS $e^{\pm} + p \rightarrow h + X$ ($h = \pi^{\pm}, \pi^{0}, K^{\pm}$, etc.)

Semi-Inclusive DIS (SIDIS) $e^{\pm} + p \rightarrow e^{\pm} + h + X$

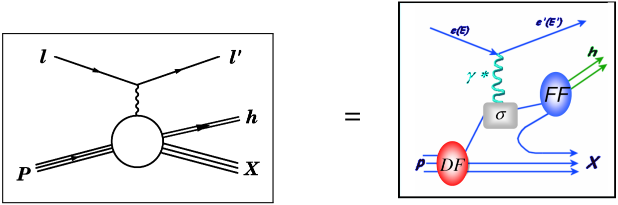
More SIDIS $e^{\pm} + p \rightarrow e^{\pm} + H + X$, with H a system of hadrons, e.g. $\pi^{+}\pi^{0}$ or $K^{+}K^{-}$.

Inclusive pp $p + p \rightarrow h + X$

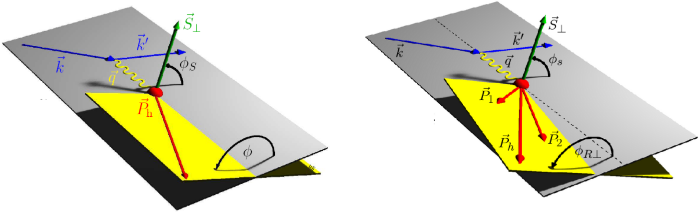
- Note: when colliding an electron or positron into a proton, it is not the electron that "hits" the proton, but rather a high energy photon
  - At HERMES, the cleanest data usually has the photon momentum between 30-90% of the lepton beam momentum.
  - The effective wavelength at HERMES was then between 50 to 150 am, while the other HERA experiment reached wavelengths near 1 am.
- When colliding two protons, it is possible for quarks, anti-quarks, and gluons from each of the protons to interact.
  - The results are more difficult to interpret, as several contributions of the above can contribute.

Cross Section Factorization

- SIDIS cross section can be written $\sigma^{ep\to ehX} = \sum_q DF \otimes \sigma^{eq\to eq} \otimes FF$

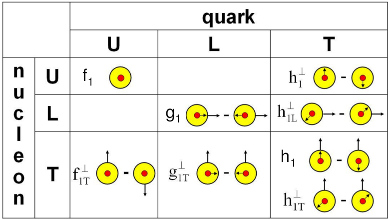


- Access integrals of DFs and FFs through Fourier moments of ϕ_h, ϕ_S, $\phi_{R\perp}$ and Legendre polynomials in $\cos\vartheta$.



Distribution and Fragmentation Functions

Distribution Functions (DF)



Fragmentation Functions (FF)

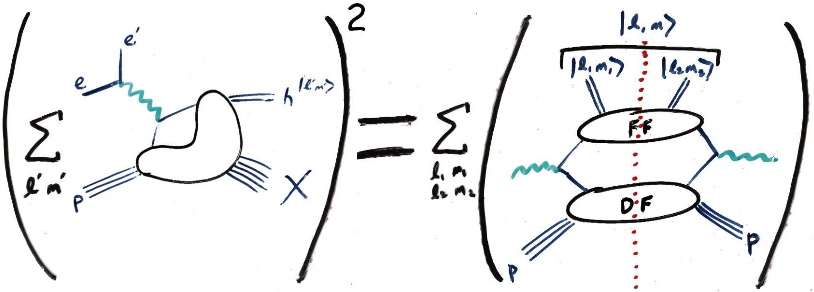
quark	
Unpol.	**Pol.**
D_1	$H_1^\perp$

- Many more distributions at higher twist (an expansion in terms of the Q^2, the rest mass of the virtual photon)
- f_1 is the unpolarized distribution, g_1 the helicity distribution, and h_1 the transversity distribution.
- The $f_{1T}^\perp$ (the Sivers function) is related to orbital angular momentum of quarks.
- The pretzelocity function $h_{1T}^\perp$ is related to the shape of the proton.
- The Boer-Mulders function has polarized quarks in an unpolarized proton
- The “polarized” fragmentation function is known as the Collins function

Twist

- Twist is rigorously defined as the difference between the order and the spin of an operator in an operator-product expansion.
- In practice, twist describes the scaling with a relevant mass quantity divided by Q
- Leading twist is twist-2, i.e. an overall factor proportional to $1/Q^2$
- Higher twist also implies diagrams with more vertices and effects, even at leading order in α_S

Optical Theorem



- Amplitudes of different $|l', m'\rangle$ are summed before amplitude is squared.
- Analog two-dihadron amplitude includes sum the states of both dihadrons.
- Note: cross sections and physical quantities usually prefer direct-sum over direct-product bases.
  - E.g., physical meson states are basis elements $|0, 0\rangle$ and $|1, 0\rangle$, not basis elements $\left|\frac{1}{2}, \frac{1}{2}\right\rangle \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$, $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle \left|\frac{1}{2}, \frac{1}{2}\right\rangle$.
  - New expansion: in terms of the $|l, m\rangle$ state of the two Dihadron system.

Transverse Momentum

- In the γ-proton center-of-mass frame, the proton is moving with a large velocity.
- Initially, all effects from the motion of the quarks in directions transverse to the direction of the proton were considered completely negligible. (See Kane, *et. al* 1979)
- Thus the quarks are all assumed to be moving collinear with the proton (a 'cold' system)
- An inclusive asymmetry A_N was found to be non-zero at several experiments at varying energies, with the only explanation being transverse momentum dependent (TMD) effects.
  - Two theories were suggested: one by D. Sivers with the TMD effect in the proton, and one by J. Collins with the TMD effect in the factorization process.
- Data taken 2002-2005 at HERMES fully demonstrate both of these transverse momentum effects (and others) at HERMES
  - Concurring results from other experiments are also now available.

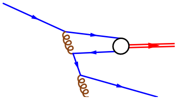
The Lund/Artru and Gluon Radiation Models

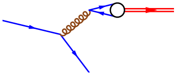
Preliminaries

- Collinear SIDIS Dihadron cross section
  - Collinear access to transversity through two transverse target moments.
  - Transversity is coupled with "Collins-like" fragmentation functions $H_{1,OT}^{\sphericalangle\, sp}$ and $H_{1,LT}^{\sphericalangle\, pp}$, associated with *sp* and *pp* interference.
- TMD SIDIS Dihadron cross section
  - The Lund/Artru string fragmentation model predicts Collins function for pseudo-scalar and vector meson final states have opposite signs.
- Two types of fragmentation are usually defined

  Favored: struck quark present in the observed particles.

  Disfavored: struck quark not present in the observed particles.

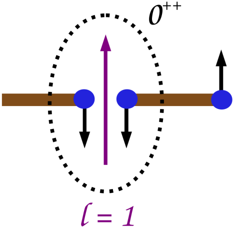




Quark Spin and Meson Polarizations

- Mesons have one valence quark and one valence anti-quark
- The spins of the valence quark and anti-quark can be either aligned or anti-aligned
- One can either write the spins in the
  - Direct product basis: $\left|\frac{1}{2}, \pm\frac{1}{2}\right\rangle \left|\frac{1}{2}, \pm\frac{1}{2}\right\rangle$
  - Direct sum basis: $|1, m\rangle$ or $|0, 0\rangle$.
- One often writes $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$.
- In either case, there exists four basis elements
- The mass eigenstates are those of the direct sum basis
  - $|1, m\rangle$ represent three polarization of vector mesons
  - $|0, 0\rangle$ represent the one polarization of pseudo-scalar mesons
- For each pseudo-scalar meson, there exists a vector meson with identical quark content, only differing in the polarization of the quarks (up to mixing of mass flavor eigenstates)

Lund/Artru String Fragmentation Model



- Favored fragmentation modeled as the breaking of a gluon flux tube between the struck quark and the remnant.
- Assume that the flux tube breaks into a $q\bar{q}$ pair with quantum numbers equal to the vacuum.

- Expect mesons overlapping with $\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle$ and $\left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle$ states to prefer "quark left".
  - $|0,0\rangle =$ pseudo-scalar mesons.
  - $|1,0\rangle =$ longitudinally polarized vector mesons.
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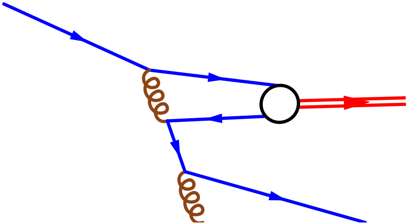
  - $|1,\pm1\rangle =$ transversely polarized vector mesons.
- For the two ρ_T's, "the Collins function" should have opposite sign to that for π
- For ρ_L, "the Collins function" is zero.

Lund/Artru String Fragmentation Model



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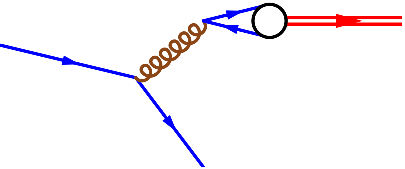
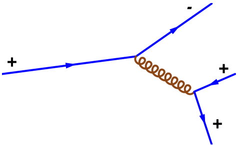
Gluon Radiation Fragmentation Model

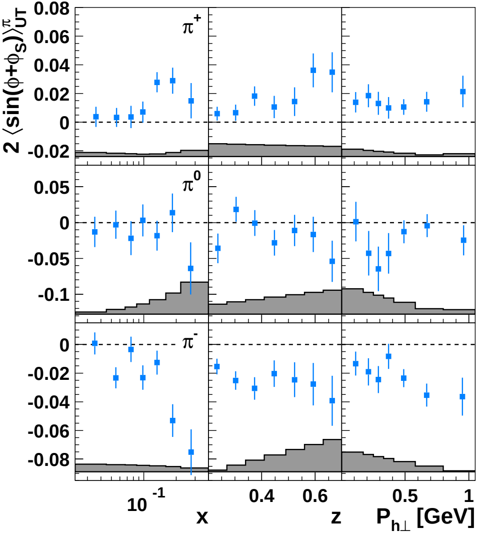


- Disfavored frag. model: assume produced diquark forms the observed meson
- Assume additional final state interaction to set pseudo-scalar quantum numbers
- Assume no additional interactions in dihadron production.
- Exists common sub-diagram between this model and the Lund/Artru model.
- Keeping track of quark polarization states, sub-diagram for disfavored $|1,1\rangle$ diquark production identical to sub-diagram for favored $\left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle$ diquark production.



- Implies that the disfavored Collins function for transverse vector mesons also has opposite sign as the favored pseudo-scalar Collins function
  - Thus fav. = disfav. for Vector Mesons
  - Data suggests fav. $\approx$ -disfav. for pseudo-scalar mesons.

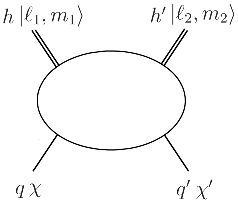
HERMES Collins Moments for Pions



- Final result published in January A. Airapetian et al, Phys. Lett. B 693 (2010) 11-16. arXiv:1006.4221 (hep-ex)
- Significant π^- asymmetry implies $H_1^{\perp,disf} \approx -H_1^{\perp,fav}$
- Pions have small, but non-zero asymmetry
- Expect Collins moments negative for $\rho^{\pm}$.
- Would like uncertainties on dihadron moments on the order of 0.02.

Partial Wave Analysis

Fragmentation Functions and Spin/Polarization



- Leading twist Fragmentation functions are related to number densities
  - Amplitudes squared rather than amplitudes
- Difficult to relate Artru/Lund prediction with published notation and cross section.
- Propose new convention for fragmentation functions
  - Functions entirely identified by the polarization states of the quarks, χ and χ'
  - Any final-state polarization, i.e. $|\ell_1, m_1\rangle\,|\ell_2, m_2\rangle$, contained within partial wave expansion of fragmentation functions
- Exists exactly two fragmentation functions
  - D_1, the unpolarized fragmentation function ($\chi = \chi'$)
  - $H_1^{\perp}$, the polarized (Collins) fragmentation function ($\chi \neq \chi'$)
- New partial waves analysis proposed, using direct sum basis $|\ell, m\rangle$ rather than the direct product basis $|\ell_1, m_1\rangle\,|\ell_2, m_2\rangle$.

Partial Wave Expansion

- Fragmentation functions expanded into partial waves in the direct sum basis according to

$$D_1 = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos\vartheta) e^{im(\phi_R - \phi_k)} D_1^{|\ell,m\rangle}(z, M_h, |\boldsymbol{k}_T|),$$

$$H_1^{\perp} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos\vartheta) e^{im(\phi_R - \phi_k)} H_1^{\perp|\ell,m\rangle}(z, M_h, |\boldsymbol{k}_T|),$$

- Each term in pseudo-scalar and dihadron cross section uniquely related to a specific partial wave $|\ell, m\rangle$.
- Cross section looks the same for all final states, excepting certain partial waves may or may not be present
  - Pseudo-scalar production is $\ell = 0$ sector
  - Dihadron production is $\ell = 0, 1, 2$ sector
  - Given the pseudo-scalar cross section (at any twist) can extrapolate cross section for other final states

Rigorous Definitions

- Fragmentation Correlation Matrix

$$\Delta_{mn}(P_h, S_h; k) = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{ik\cdot x} \langle 0 | \Psi_m(x) | P_h, S_h; X \rangle \langle P_h, S_h; X | \overline{\Psi}_n(0) | 0 \rangle$$

- Trace Notation

$$\Delta^{[\Gamma]}(z, M_h, |\boldsymbol{k}_T|, \cos\vartheta, \phi_R - \phi_k) = 4\pi \frac{z|\boldsymbol{R}|}{16M_h} \int dk^+ \, \mathrm{Tr}\left[\Gamma \Delta(k, P_h, R)\right]\Big|_{k^- = P_h^- / z}.$$

- Define fragmentation functions via trace relations

	Previous Definitions		New Definition
FF	**Pseudo-Scalar**	**Dihadron**	**All Final States**
D_1	$\Delta^{[\gamma^-]}$	$\Delta^{[\gamma^-]}$	$\Delta^{[\gamma^-(1+i\gamma^5)]}$
$G_1^\perp$	$--$	$\propto \Delta^{[\gamma^-\gamma^5]}$	$--$
$H_1^\perp$	$\Delta^{[(\sigma^{1-})\gamma^5]}$	$\Delta^{[(\sigma^{1-})\gamma^5]}$	$\Delta^{[(\sigma^{1-}+i\sigma^{2-})\gamma^5]}$
$\bar{H}_1^{\sphericalangle}$	$--$	$\propto \Delta^{[(\sigma^{2-})\gamma^5]}$	$--$

Relation with Previous Notation

- Real part of fragmentation function similar
- New definition of D_1 & $H_1^{\perp}$
  - Adds "imaginary" part to D_1 & $H_1^{\perp}$, instead of introducing new functions.
  - Functions are complex valued and depend on Q^2, z, $|k_T|$, M_h, $\cos\vartheta$, $(\phi_R - \phi_k)$.
- Comparing with similar trace definitions, e.g. PRD 67:094002, yields the relations

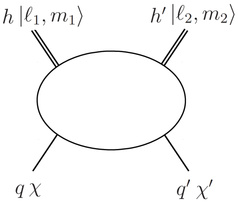
$$D_1\Big|_{Gliske} = \left[D_1 + i\frac{|\boldsymbol{R}||\boldsymbol{k}_T|}{M_h^2}\sin\vartheta\sin(\phi_R - \phi_k)\, G_1^{\perp}\right]_{other},$$

$$H_1^{\perp}\Big|_{Gliske} = \left[H_1^{\perp} + \frac{|\boldsymbol{R}|}{|\boldsymbol{k}_T|}\sin\vartheta e^{i(\phi_R-\phi_k)}\, \bar{H}_1^{\sphericalangle}\right]_{other} = \frac{|\boldsymbol{R}|^2}{|\boldsymbol{k}_T|^2}H_1^{\sphericalangle}\Big|_{other},$$

- Note: there are inconsistencies in the literature between definitions of $H_1^{\sphericalangle}$, $\bar{H}_1^{\sphericalangle}$, and $H_1'^{\sphericalangle}$.

Where is "the Collins function?"

- Consider direct sum vs. direct product basis



$$\begin{aligned}\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2} &= \left(\frac{1}{2}\otimes\frac{1}{2}\right)\otimes\left(\frac{1}{2}\otimes\frac{1}{2}\right),\\ &= (1\oplus 0)\otimes(1\oplus 0)\,,\\ &= 2\oplus 1\oplus 1\oplus 1\oplus 0\oplus 0.\end{aligned}$$

- Three $\ell = 1$ and two $\ell = 0$ cannot be separated experimentally
  - Theoretically distinguishable via Generalized Casimir Operators
- Longitudinal vector meson state $|1,0\rangle\,|1,0\rangle$ is a mixture of $|2,0\rangle$ and $|0,0\rangle$
  - Cannot access, due to $\ell = 0$ multiplicity
  - Model predictions for longitudinal vector mesons not testable
- Transverse vector meson states $|1,\pm1\rangle\,|1,\pm1\rangle$ are exactly $|2,\pm2\rangle$
  - Models predict dihadron $H_1^{\perp|2,\pm2\rangle}$ has opposite sign as pseudo-scalar $H_1^{\perp}$.
  - Cross section has direct access to $H_1^{\perp|2,\pm2\rangle}$
- Note: the usual IFF, related to $H_1^{\perp|1,1\rangle}$ is not pure *sp*, but also includes *pp* interference.

Cross Section

Dihadron Twist-3 Cross Section

$$
\begin{aligned}
d\sigma_{UU} \;=\; & \frac{\alpha^2 M_h P_{h\perp}}{2\pi x y Q^2}\left(1+\frac{\gamma^2}{2x}\right) \\
& \times \sum_{\ell=0}^{2}\Bigg\{ A(x,y)\sum_{m=0}^{\ell}\Big[P_{\ell,m}\cos(m(\phi_h-\phi_R))\Big(F_{UU,T}^{P_{\ell,m}\cos(m(\phi_h-\phi_R))}+\epsilon F_{UU,L}^{P_{\ell,m}\cos(m(\phi_h-\phi_R))}\Big)\Big] \\
& + B(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\cos((2-m)\phi_h+m\phi_R)F_{UU}^{P_{\ell,m}\cos((2-m)\phi_h+m\phi_R)} \\
& + V(x,y)\sum_{m=-\ell}^{\ell}P_{\ell,m}\cos((1-m)\phi_h+m\phi_R)F_{UU}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_R)}\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
d\sigma_{UT} \;=\; & \frac{\alpha^2 M_h P_{h\perp}}{2\pi x y Q^2}\left(1+\frac{\gamma^2}{2x}\right)|S_\perp|\sum_{\ell=0}^{2}\sum_{m=-\ell}^{\ell}\Bigg\{A(x,y)\Big[P_{\ell,m}\sin((m+1)\phi_h-m\phi_R-\phi_S)) \\
& \times\Big(F_{UT,T}^{P_{\ell,m}\sin((m+1)\phi_h-m\phi_R-\phi_S)}+\epsilon F_{UT,L}^{P_{\ell,m}\sin((m+1)\phi_h-m\phi_R-\phi_S)}\Big)\Big] \\
& + B(x,y)\Big[P_{\ell,m}\sin((1-m)\phi_h+m\phi_R+\phi_S)F_{UT}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_R+\phi_S)} \\
& \quad + P_{\ell,m}\sin((3-m)\phi_h+m\phi_R-\phi_S)F_{UT}^{P_{\ell,m}\sin((3-m)\phi_h+m\phi_R-\phi_S)}\Big] \\
& + V(x,y)\Big[P_{\ell,m}\sin(-m\phi_h+m\phi_R+\phi_S)F_{UT}^{P_{\ell,m}\sin(-m\phi_h+m\phi_R+\phi_S)} \\
& \quad + P_{\ell,m}\sin((2-m)\phi_h+m\phi_R-\phi_S)F_{UT}^{P_{\ell,m}\sin((2-m)\phi_h+m\phi_R-\phi_S)}\Big]\Bigg\}.
\end{aligned}
$$

Structure Functions, Unpolarized

$$F_{UU,L}^{P_{\ell,m}\cos(m\phi_h - m\phi_R)} = 0,$$

$$F_{UU,T}^{P_{\ell,m}\cos(m\phi_h - m\phi_R)} = \begin{cases} \mathcal{I}\left[f_1 D_1^{|\ell,0\rangle}\right] & m = 0, \\ \mathcal{I}\left[2\cos(m\phi_h - m\phi_k)\, f_1 \left(D_1^{|\ell,m\rangle} + D_1^{|\ell,-m\rangle}\right)\right] & m > 0, \end{cases}$$

$$F_{UU}^{P_{\ell,m}\cos((2-m)\phi_h + m\phi_R)} = -\mathcal{I}\left[\frac{|\boldsymbol{p}_T||\boldsymbol{k}_T|}{MM_h}\cos\left((m-2)\phi_h + \phi_p + (1-m)\phi_k\right) h_1^{\perp} H_1^{\perp|\ell,m\rangle}\right],$$

$$\begin{aligned} F_{UU}^{P_{\ell,m}\cos((1-m)\phi_h + m\phi_R)} = -\frac{2M}{Q}\mathcal{I}\Bigg[& \frac{|\boldsymbol{k}_T|}{M_h}\cos((m-1)\phi_h + (1-m)\phi_k) \\ & \times \left(xhH_1^{\perp|\ell,m\rangle} + \frac{M_h}{M} f_1 \frac{\tilde{D}^{\perp|\ell,m\rangle}}{z}\right) \\ & + \frac{|\boldsymbol{p}_T|}{M}\cos((m-1)\phi_h + \phi_p - m\phi_k) \\ & \times \left(xf^{\perp} D_1^{|\ell,m\rangle} + \frac{M}{M_h} h_1^{\perp}\frac{\tilde{H}^{|\ell,m\rangle}}{z}\right)\Bigg]. \end{aligned}$$

- Can test Lund/Artru model with $F_{UU}^{\sin^2\vartheta\cos(2\phi_R)}$, $F_{UU}^{\sin^2\vartheta\cos(4\phi_h - 2\phi_R)}$ via Boer-Mulder's function

Twist-2 Structure Functions, Transverse Target

$$F_{UT,L}^{P_{\ell,m}\sin((m+1)\phi_h-m\phi_R-\phi_S)} = 0$$

$$F_{UT,T}^{P_{\ell,m}\sin((m+1)\phi_h-m\phi_R-\phi_S)} = -\mathcal{I}\left[\frac{|\boldsymbol{p}_T|}{M}\cos\left((m+1)\phi_h-\phi_p-m\phi_k\right)\right.$$
$$\left.\times\left(f_{1T}^{\perp}\left(D_1^{|\ell,m\rangle}+D_1^{|\ell,-m\rangle}\right)+\chi(m)g_{1T}\left(D_1^{|\ell,m\rangle}-D_1^{|\ell,-m\rangle}\right)\right)\right],$$

$$F_{UT}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_R+\phi_S)} = -\mathcal{I}\left[\frac{|\boldsymbol{k}_T|}{M_h}\cos\left((m-1)\phi_h-\phi_p-m\phi_k\right)h_1 H_1^{\perp|\ell,m\rangle}\right],$$

$$F_{UT}^{P_{\ell,m}\sin((3-m)\phi_h+m\phi_R-\phi_S)} = \mathcal{I}\left[\frac{|\boldsymbol{p}_T|^2|\boldsymbol{k}_T|}{M^2M_h}\cos\left((m-3)\phi_h+2\phi_p-(m-1)\phi_k\right)h_{1T}^{\perp}H_1^{\perp|\ell,m\rangle}\right].$$

- Can test Lund/Artru model with $F_{UT}^{\sin^2\vartheta\,\sin(-\phi_h+2\phi_R+\phi_S)}$ and $F_{UT}^{\sin^2\vartheta\,\sin(3\phi_h-2\phi_R+\phi_S)}$ via transversity
- In theory, could also test Lund/Artru and gluon radiation models with $F_{UT}^{\sin^2\vartheta\,\sin(\phi_h+2\phi_R-\phi_S)}$ and $F_{UT}^{\sin^2\vartheta\,\sin(5\phi_h-2\phi_R-\phi_S)}$ via pretzelocity
- Data from SIDIS pseudo-scalar production indicate pretzelocity very small or possibly zero

Collinear Assumption and Structure Functions

- TMD Structure function for the $|1,1\rangle$ A_{UT} moment

$$F_{UT}^{\sin\vartheta\,\sin(\phi_R+\phi_S)}(x,y,z,P_{h\perp},\boldsymbol{p}_T,\boldsymbol{k}_T) \quad = \quad -\Im\left[\frac{|\boldsymbol{k}_T|}{M_h}\cos\left(\phi_p-\phi_k\right)\,h_1(x,p_T)\,H_1^{\perp|1,1\rangle}(z,zk_T)\right]$$

- Collinear assumption implies

$$\int d\phi_h\,dP_{h\perp}\,F_{UT}^{\sin\vartheta\,\sin(\phi_R+\phi_S)}(x,y,z,P_{h\perp},\boldsymbol{p}_T,\boldsymbol{k}_T) \quad \approx \quad h_1(x)\,H_1^{\perp|1,1\rangle\ (1)}(z),$$

with

$$h_1(x)=\int dp_T\,h_1(x,p_T), \qquad H_1^{\perp|1,1\rangle\ (1)}(z)=\int dk_T\,\frac{|\boldsymbol{k}_T|}{M_h}H_1^{\perp|1,1\rangle}(z,zk_T).$$

Collinear versus TMD Moments

- It is not the particulars of the DF or FF that make a moment survive in the collinear case, but rather the $\sum m = 0$ (necessary condition).
  - Moments with $h_1^\perp H_1^{\perp|\ell,m\rangle}$ (Boer-Mulders moments)
    - $h_1^\perp$ has $\chi \neq \chi'$, and thus $\Delta m = -1$
    - $H_1^\perp$ similarly has $\Delta m = -1$.
    - Final state polarization must have $m = 2$ in order that $\sum m = 0$.
    - Only surviving moment in collinear dihadron production is $|2, 2\rangle$.
  - Moments with $h_1 H_1^{\perp|\ell,m\rangle}$ (Collins moments)
    - h_1 has $\Delta m = 0$.
    - $H_1^\perp$ again has $\Delta m = -1$.
    - Collinear moments are $|1, 1\rangle$, $|2, 1\rangle$.
- Can also look for the m which cancels the ϕ_h dependence

$$F_{UU}^{P_{\ell,m}\cos((2-m)\phi_h+m\phi_R)} = -\mathfrak{I}\left[\frac{|\boldsymbol{p}_T||\boldsymbol{k}_T|}{MM_h}\cos\left((m-2)\phi_h+\phi_p+(1-m)\phi_k\right) h_1^\perp H_1^{\perp|\ell,m\rangle}\right],$$

$$F_{UT}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_R+\phi_S)} = -\mathfrak{I}\left[\frac{|\boldsymbol{k}_T|}{M_h}\cos\left((m-1)\phi_h-\phi_p-m\phi_k\right) h_1 H_1^{\perp|\ell,m\rangle}\right],$$

Spectator Model of Dihadron Fragmentation

Collinear Dihadron Spectator Model

- Exists only one model for polarized dihadron fragmentation functions
  - 2006 publication of A. Bacchetta and M. Radici from INFN-Pavia *Phys. Rev.* D74 (2006)
  - Focuses on collinear fragmentation
- The model is a spectator model
  - Optical theorem used to compute the scattering amplitude of $p\gamma^*\bar{p}'\gamma'^* \rightarrow H\bar{H}'$.
  - A single particle "spectator" is assumed to mediate between $p\ \gamma\ H$ and $\bar{p}\ \gamma\ \bar{H}$ vertices.
  - Spectator forced to be on-shell, with mass $M_s \propto M_h$.
- Model assumes single spectator for both hadron pairs and vector mesons.
  - This causes the amplitudes to be summed, rather than the cross sections
- The leading twist fragmentation correlation matrix is computed from the tree level diagram.
- Integration over transverse momenta is performed before extracting fragmentation functions via trace relationships.

TMD Dihadron Spectator Model

- One can use the same correlator to extract TMD fragmentation functions
  - Just do not integrate over transverse momentum.
  - Convenient to apply new partial wave analysis after Dirac trace algebra.
  - Numeric studies show need for additional $\boldsymbol{k}_T$ cut-off.
- Original model intended for $\pi^+\pi^-$ pairs
  - Adding flavor dependence allows generalization to $\pi^+\pi^0$, $\pi^-\pi^0$ pairs.
  - Slight change to vertex function allows generalization to K^+K^- pairs.
  - Slight change to vertex function and allows generalization to K^+K^- pairs.
- Unfortunately, the model only includes partial waves of the Collins function for $\ell < 2$.
  - Instead, one can set $|2, \pm 2\rangle$ partial waves proportional to either $H_1^{\perp|\ell,m\rangle}$ for $\ell \leq 1$ or to $D_1^{|\ell,m\rangle}$ for $\ell \leq 2$.

Fragmentation Correlation Function

- The tree-level diagram implies the following fragmentation correlation function

$$
\begin{aligned}
\Delta^q(k, P_h, R) \quad = \quad & \Big\{ |F^s|^2 e^{-2\frac{k^2}{\Lambda_s^2}} \not{k} \left(\not{k} - \not{P}_h + M_s\right) \not{k} \\
& + |F^p|^2 e^{-2\frac{k^2}{\Lambda_p^2}} \not{k}\not{R} \left(\not{k} - \not{P}_h + M_s\right) \not{R}\not{k} \\
& + F^{s*} F^p e^{-2\frac{k^2}{\Lambda_{sp}^2}} \not{k} \left(\not{k} - \not{P}_h + M_s\right) \not{R}\not{k} \\
& + F^s F^{p*} e^{-2\frac{k^2}{\Lambda_{sp}^2}} \not{k}\not{R} \left(\not{k} - \not{P}_h + M_s\right) \not{k} \Big\} \\
& \times \frac{1}{(2\pi)^3} \frac{1}{k^4} \delta\left((k - P_h)^2 - M_s^2\right) e^{-2\frac{k_T^2}{\Lambda_b^2}} .
\end{aligned}
$$

- The cut-offs are imposed by assuming certain vertex functions.
- Fragmentation functions can be obtained by applying trace-definitions.

Results of the Model Calculation

$$\frac{16\pi^2 M_h k^4}{|\boldsymbol{R}|} D_1^{|0,0\rangle} = \left(\frac{z^2|\boldsymbol{k}_T|^2 + M_s^2}{1-z}\right)\left[|F^s|^2\, e^{-2\frac{k^2}{\Lambda_s^2}} - R^2\,|F^p|^2\, e^{-2\frac{k^2}{\Lambda_p^2}}\right]$$

$$\frac{16\pi^2 M_h k^4}{|\boldsymbol{R}|} D_1^{|1,1\rangle} = -2M_s|\boldsymbol{R}||\boldsymbol{k}_T|\left[\mathrm{Re}\left(F^{s*}F^p\right) e^{-2\frac{k^2}{\Lambda_{sp}^2}}\right]$$

$$\frac{16\pi^2 M_h k^4}{|\boldsymbol{R}|} D_1^{|1,0\rangle} = -2\frac{M_s|\boldsymbol{R}|}{zM_h}\left(M_h^2 + z^2|\boldsymbol{k}_T|^2\right)\left[\mathrm{Re}\left(F^{s*}F^p\right) e^{-2\frac{k^2}{\Lambda_{sp}^2}}\right]$$

$$\frac{16\pi^2 M_h k^4}{|\boldsymbol{R}|} D_1^{|2,2\rangle} = |\boldsymbol{k}_T|^2|\boldsymbol{R}|^2\left[|F^p|^2\, e^{-2\frac{k^2}{\Lambda_p^2}}\right],$$

$$\frac{16\pi^2 M_h k^4}{|\boldsymbol{R}|} D_1^{|2,1\rangle} = \frac{|\boldsymbol{k}_T||\boldsymbol{R}|^2}{zM_h}\left(M_h^2 + z^2|\boldsymbol{k}_T|^2 + \frac{1}{2}z^2k^2\right)\left[|F^p|^2\, e^{-2\frac{k^2}{\Lambda_p^2}}\right],$$

$$\frac{16\pi^2 M_h k^4}{|\boldsymbol{R}|} D_1^{|2,0\rangle} = \left(\frac{|\boldsymbol{R}|^2}{z^2M_h^2}\left(M_h^2 + z^2|\boldsymbol{k}_T|^2\right)\left(M_h^2 + z^2|\boldsymbol{k}_T|^2 + z^2k^2\right) - 2|\boldsymbol{k}_T|^2|\boldsymbol{R}|^2\right)\left[|F^p|^2\, e^{-2\frac{k^2}{\Lambda_p^2}}\right],$$

$$D_1^{|\ell,-m\rangle} = D_1^{|\ell,m\rangle}.$$

Model Calculation for Fragmentation Functions

$$\frac{8\pi^2 k^4}{|\boldsymbol{R}|} H_1^{\perp|1,1\rangle} = -\frac{|\boldsymbol{R}|}{|\boldsymbol{k}_T|}\left(k^2 + |\boldsymbol{k}_T|^2\right)\left(\left(1-z^2\right)k^2 - z^2|\boldsymbol{k}_T|^2\right) \times \left[\operatorname{Im}\left(F^{s*}F^{p}\right) e^{-2\frac{k^2}{\Lambda_{sp}^2}}\right],$$

$$\frac{8\pi^2 k^4}{|\boldsymbol{R}|} H_1^{\perp|1,0\rangle} = \frac{1}{z} M_h |\boldsymbol{R}| \left(zk^2 - 2\left(M_h^2 + z^2(k^2 + |\boldsymbol{k}_T|^2)\right)\right) \times \left[\operatorname{Im}\left(F^{s*}F^{p}\right) e^{-2\frac{k^2}{\Lambda_{sp}^2}}\right],$$

$$\frac{8\pi^2 k^4}{|\boldsymbol{R}|} H_1^{\perp|1,-1\rangle} = -M_h^2 |\boldsymbol{R}||\boldsymbol{k}_T| \left[\operatorname{Im}\left(F^{s*}F^{p}\right) e^{-2\frac{k^2}{\Lambda_{sp}^2}}\right].$$

- Note again the absence of the $H_1^{\perp|2,m\rangle}$ partial waves.

Conclusions and Summary

Conclusions and Summary

- The Lund/Artru and (new) gluon radiation model
  - Can verify the predictions regarding the signs of certain structure functions
- New partial wave analysis
  - Increases understanding and aids in interpretation
  - Simplifies notation
  - Allows computation of the sub-leading twist cross section
- TMD Spectator Model for Dihadron Fragmentation
  - Only available model for TMD polarized dihadron production
  - Unfortunately, predicts $|2, \pm 2\rangle$ states to be zero.