Extraction of Helicity Amplitude Ratios from Exclusive ρ^0-meson Electroproduction on Transversely Polarized Proton

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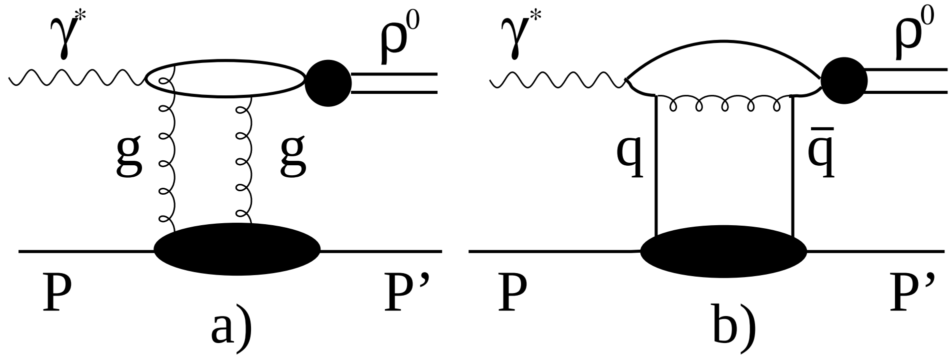
Physics Motivation



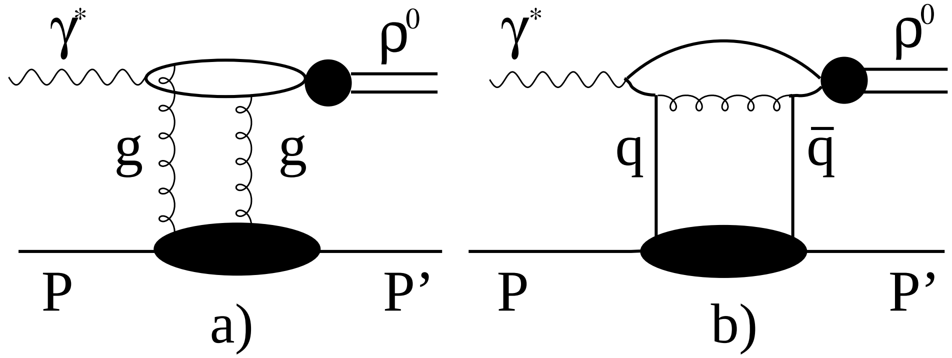
- $\gamma^* + N \to V + N$ is a perfect reaction to study both vector-meson ($V = \rho^0,\ \phi,\ \omega,\ \ldots$) production mechanism and hadron (nucleon) structure.

- Properties of Spin-Density Matrix Elements (SDMEs).
  SDMEs are dimensionless coefficients in the angular distribution of final particles and therefore can be extracted from data.

- SDMEs are expressible in terms of ratios of helicity amplitudes $F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$
  of the $\gamma^*\ (\lambda_\gamma) + N\ (\lambda_N) \to V\ (\lambda_V) + N\ (\lambda'_N)$ reaction,
  hence the ratios can be obtained from angular distribution of final particles.

- Generalized Parton Distributions (GPDs) of the nucleon can be obtained from the helicity amplitude $F_{00} \equiv F_{0\frac{1}{2}0\frac{1}{2}}$ $(\gamma_L \to V_L)$ for which factorization theorem is proved.

Physics Motivation

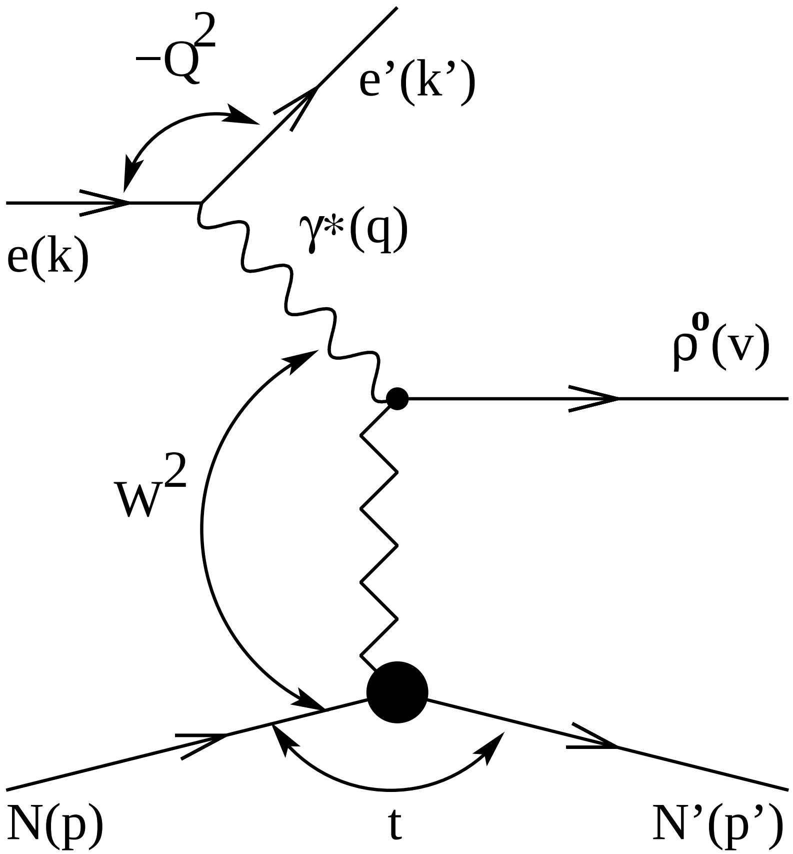


- Generalized Parton Distributions (GPDs)
  Quark GPDs: $H_q(x, \xi, t)$, $E_q(x, \xi, t)$, ...
  Gluon GPDs: $H_g(x, \xi, t)$, $E_g(x, \xi, t)$, ...
  H_q and H_g can be obtained from nucleon helicity non-flip amplitudes ($\lambda_N = \lambda'_N$).
  E_q and E_g can be extracted from nucleon helicity-flip amplitudes ($\lambda_N \neq \lambda'_N$).
  Therefore data on transversely polarized target are to be analyzed.
- Relations by Ji

$$\frac{1}{2}\int_{-1}^{1} dx x[H_q(x, \xi, t \to 0) + E_q(x, \xi, t \to 0)] =< J_q >,$$
$$\int_{-1}^{1} dx x[H_g(x, \xi, t \to 0) + E_g(x, \xi, t \to 0)] =< J_g > .$$

To use the Ji relations $E_q(x, \xi, t \to 0)$ and $E_g(x, \xi, t \to 0)$ are to be extracted from data on transversely polarized targets at $t \neq 0$.

Phenomenological description of reaction $e + N \to e' + V + N'$



$$\mathrm{QED}: \quad \mathrm{e}(\lambda) \to \mathrm{e}'(\lambda') + \gamma^*(\lambda_\gamma),$$
$$\mathrm{QCD}: \gamma^*(\lambda_\gamma) + \mathrm{N}(\lambda_\mathrm{N}) \to \mathrm{V}(\lambda_\mathrm{V}) + \mathrm{N}'(\lambda'_\mathrm{N}).$$

The helicity amplitude of the reaction $\gamma^* + N \to V + N$

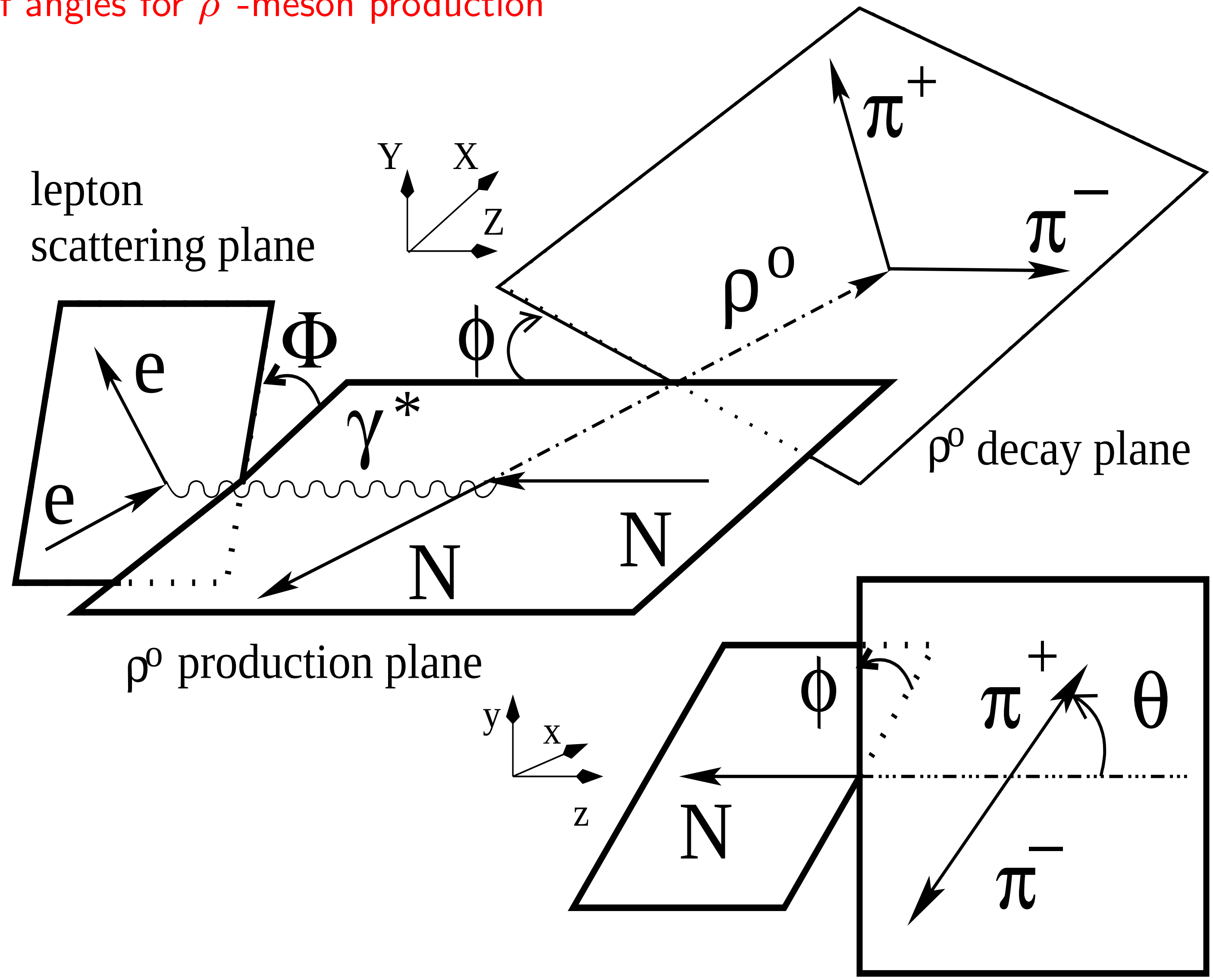
$$F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$$
$$= (-1)^{\lambda_\gamma} \langle v \lambda_V p' \lambda'_N | J^\sigma_{(h)} | p \lambda_N \rangle e^{(\lambda_\gamma)}_\sigma .$$

$J^\sigma_{(h)}$ is the electromagnetic current of hadrons;
$e^{(\lambda_\gamma)}_\sigma$ is the photon polarization four-vector;
$\lambda_\gamma = \pm 1$ transverse virtual photon,
$\lambda_\gamma = 0$ longitudinal virtual photon.
$E^{(\lambda_V)}_\sigma$ is the vecor meson polarization vector;
$\lambda_V = \pm 1$ transverse vector meson,
$\lambda_V = 0$ longitudinal vector meson.
Amplitude decomposition into Natural (NPE) and Unnatural Parity Exchange (UPE) Amplitudes (18=10+8)

$$F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} = T_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} + U_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$$

Phenomenological description of reaction $e + N \to e' + V + N'$

- Definition of angles for ρ^0-meson production



- For ρ^0-meson production $\vec{n} = \vec{p}_{\pi^+}/|\vec{p}_{\pi^+}|,\ Y_{1\lambda_V}(\vec{n}) = Y_{1\lambda_V}(\theta, \phi)$.

Phenomenological description of reaction $e + N \to e' + V + N'$

- Spin-Density Matrix Elements $\varrho_{\lambda_V \lambda'_V}$ of the vector meson can be extracted from the angular distribution of final particles

$$|\rho^0; J = 1, M > \to |\pi^+\pi^-; L = 1, M > \to Y_{1M}(\theta, \phi)$$

$$\mathcal{W}(\Phi, \Psi, \theta, \phi) = \sum_{\lambda_V, \lambda'_V} Y_{1\lambda_V}(\theta, \phi) \varrho_{\lambda_V \lambda'_V}(\Phi, \Psi) Y^*_{1\lambda'_V}(\theta, \phi)$$

- Relation between spin-density matrix of virtual photon $\rho_{\lambda_\gamma \lambda'_\gamma} = \rho_{\lambda_\gamma \lambda'_\gamma}(\Phi)$, the nucleon $\tau_{\lambda'_N \lambda_N} = \tau_{\lambda'_N \lambda_N}(\Psi)$ and that of vector meson (ρ^0 meson) $\varrho_{\lambda_V \lambda'_V}$:

$$\varrho_{\lambda_V \lambda'_V} = \sum \frac{F_{\lambda_V \mu_N \lambda_\gamma \lambda_N} \; \rho_{\lambda_\gamma \lambda'_\gamma} \; \tau_{\lambda_N \lambda'_N} F^*_{\lambda'_V \mu_N \lambda'_\gamma \lambda'_N}}{2\mathcal{N}},$$

  where Ψ is the angle between the transverse polarization vector, $\vec{P}_T$ and the lepton scattering plane.

- SDMEs in the Diehl representation ($u^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$, $n^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$, $s^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$, $l^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma}$) are the Fourier coefficients in decomposition of Φ and Ψ dependences of spin-density matrix of vector meson $\varrho_{\lambda_V \lambda'_V}(\Phi, \Psi)$

The HERMES Experiment

ρ^0-meson production by longitudinally polarized beam on transversely polarized proton

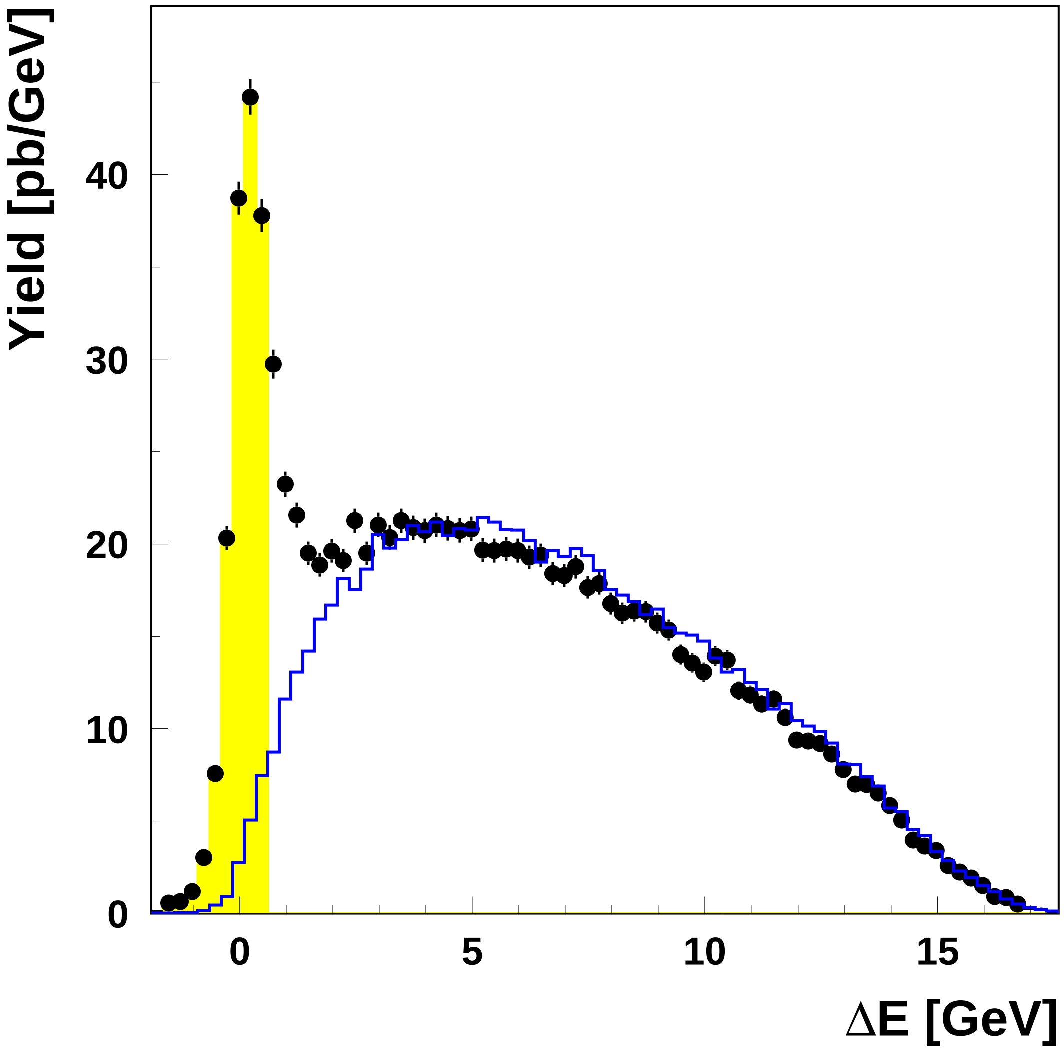
- Longitudinally polarized electron/positron beam with energy of 27.6 GeV.
  $0.15 < |P_B| < 0.80$.

- 6.3 GeV $> W >$ 3.0 GeV, $< W >= 4.73$ GeV;
  $Q^2 > 1$ GeV^2, $< Q^2 >= 1.93$ GeV^2;
  $-t' = -(t - t_{min}) < 0.4$ GeV^2, $< -t' >= 0.132$ GeV^2.

- Recoil nucleon was not detected. Missing mass criterion was used.
  $\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$; $-1.0 \text{ GeV} < \Delta \text{E} < 0.8$ GeV;
  M_X mass of recoil system; M_p proton mass.

- 8741 events with exclusive ρ^0-mesons produced with unpolarized and longitudinally polarized beam ($< |P_B| > \approx 0.3 \pm 0.02$) on transversely polarized proton ($|\vec{P}_T| \approx 0.72 \pm 0.06$) were obtained.

ΔE distribution for ρ^0 meson production



$7\% <$ fraction of background $< 23\%$ for increasing $-t'$ is subtracted, $< f_{bg} >= 11\%$.

Extraction of Helicity Amplitude Ratios

$$T_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} = [F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} + (-1)^{\lambda_\gamma - \lambda_V} F_{-\lambda_V \lambda'_N -\lambda_\gamma \lambda_N}]/2;$$ exchanges with pomeron, ρ, a_2, ...

$$U_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} = [F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} - (-1)^{\lambda_\gamma - \lambda_V} F_{-\lambda_V \lambda'_N -\lambda_\gamma \lambda_N}]/2;$$ exchanges with π, a_1, ...

Amplitudes without nucleon helicity flip:

$$T^{(1)}_{\lambda_V \lambda_\gamma} \equiv T_{\lambda_V \frac{1}{2} \lambda_\gamma \frac{1}{2}} = T_{\lambda_V -\frac{1}{2} \lambda_\gamma -\frac{1}{2}}, \quad U^{(1)}_{\lambda_V \lambda_\gamma} \equiv U_{\lambda_V \frac{1}{2} \lambda_\gamma \frac{1}{2}} = -U_{\lambda_V -\frac{1}{2} \lambda_\gamma -\frac{1}{2}},$$

Amplitudes with nucleon helicity flip:

$$T^{(2)}_{\lambda_V \lambda_\gamma} \equiv T_{\lambda_V \frac{1}{2} \lambda_\gamma -\frac{1}{2}} = -T_{\lambda_V -\frac{1}{2} \lambda_\gamma \frac{1}{2}}, \quad U^{(2)}_{\lambda_V \lambda_\gamma} \equiv U_{\lambda_V \frac{1}{2} \lambda_\gamma -\frac{1}{2}} = U_{\lambda_V -\frac{1}{2} \lambda_\gamma \frac{1}{2}},$$

Angular distribution is dimensionless quantity, hence it may depend on the helicity amplitude ratios only.

Amplitude ratios:

$$t^{(1)}_{\lambda_V \lambda_\gamma} = T^{(1)}_{\lambda_V \lambda_\gamma}/T^{(1)}_{00}, \; t^{(2)}_{\lambda_V \lambda_\gamma} = T^{(2)}_{\lambda_V \lambda_\gamma}/T^{(1)}_{00}, \; u^{(1)}_{\lambda_V \lambda_\gamma} = U^{(1)}_{\lambda_V \lambda_\gamma}/T^{(1)}_{00}, \; u^{(2)}_{\lambda_V \lambda_\gamma} = U^{(2)}_{\lambda_V \lambda_\gamma}/T^{(1)}_{00}.$$

Total number of independent complex amplitude ratios is 17 (34 real functions).

Small amplitudes can be reliably extracted if there is product of those by the amplitude $T^{(1)}_{00}$ or $T^{(1)}_{11}$ being dominant at large Q^2 and small $|t|$.

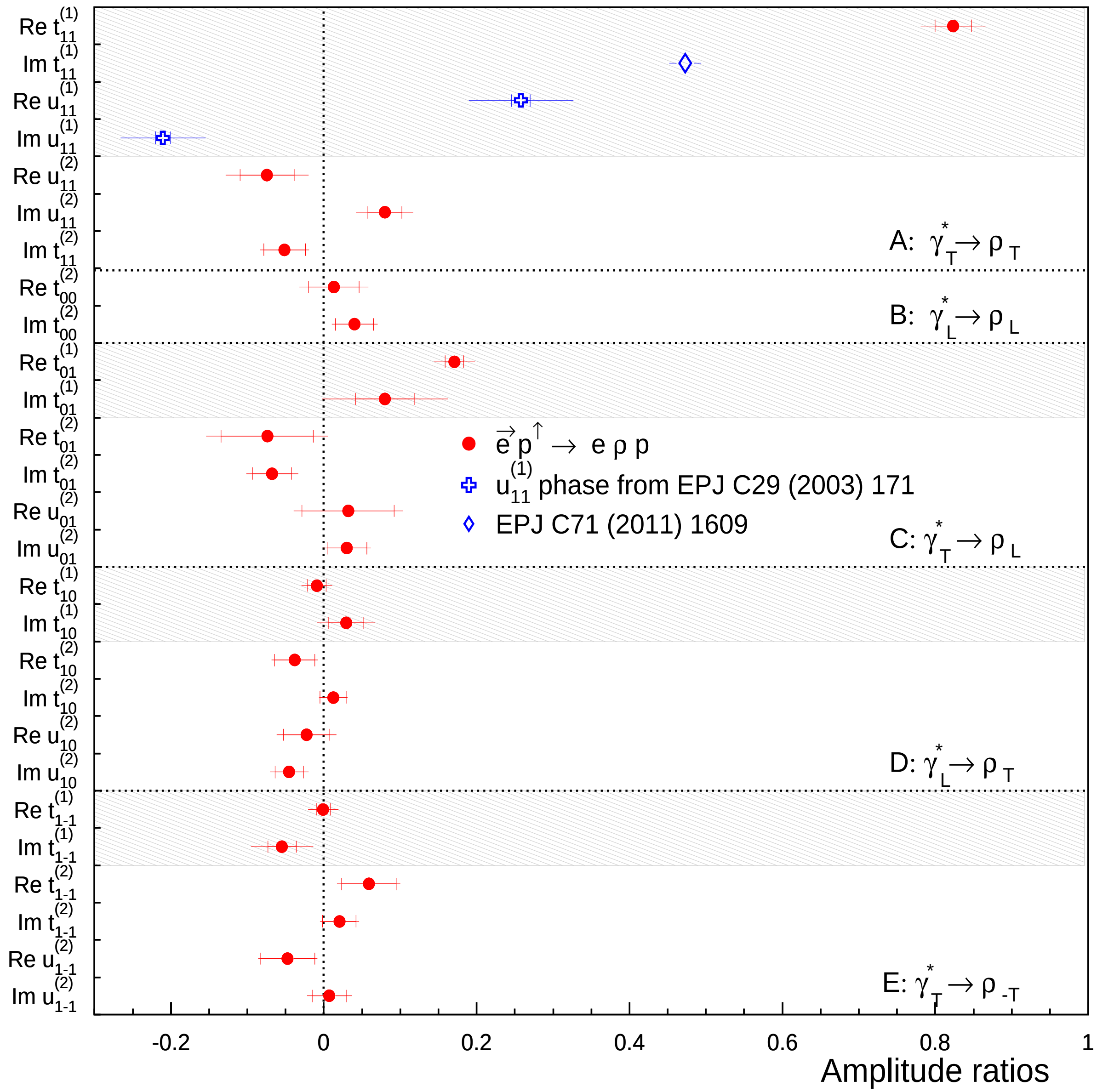
For longitudinally polarized beam and transversely polarized target only 25 parameters can be reliably extracted.

For the present data, the phase shifts of $T^{(1)}_{11}$ and $U^{(1)}_{11}$ are fixed from previous HERMES data.

Ratios $u^{(1)}_{10}$, $u^{(1)}_{10}$, $u^{(1)}_{1-1}$ are not obtained from present data since they are multiplied by small factor $\sqrt{1-\epsilon}S_L$ with the longitudinal (with respect of virtual photon) target polarization $S_L < 0.04$.

Results are published in Eur. Phys. J. C77 (2017) 378; (arXiv:hep-ex 1702.00345)

Extraction of Helicity Amplitude Ratios

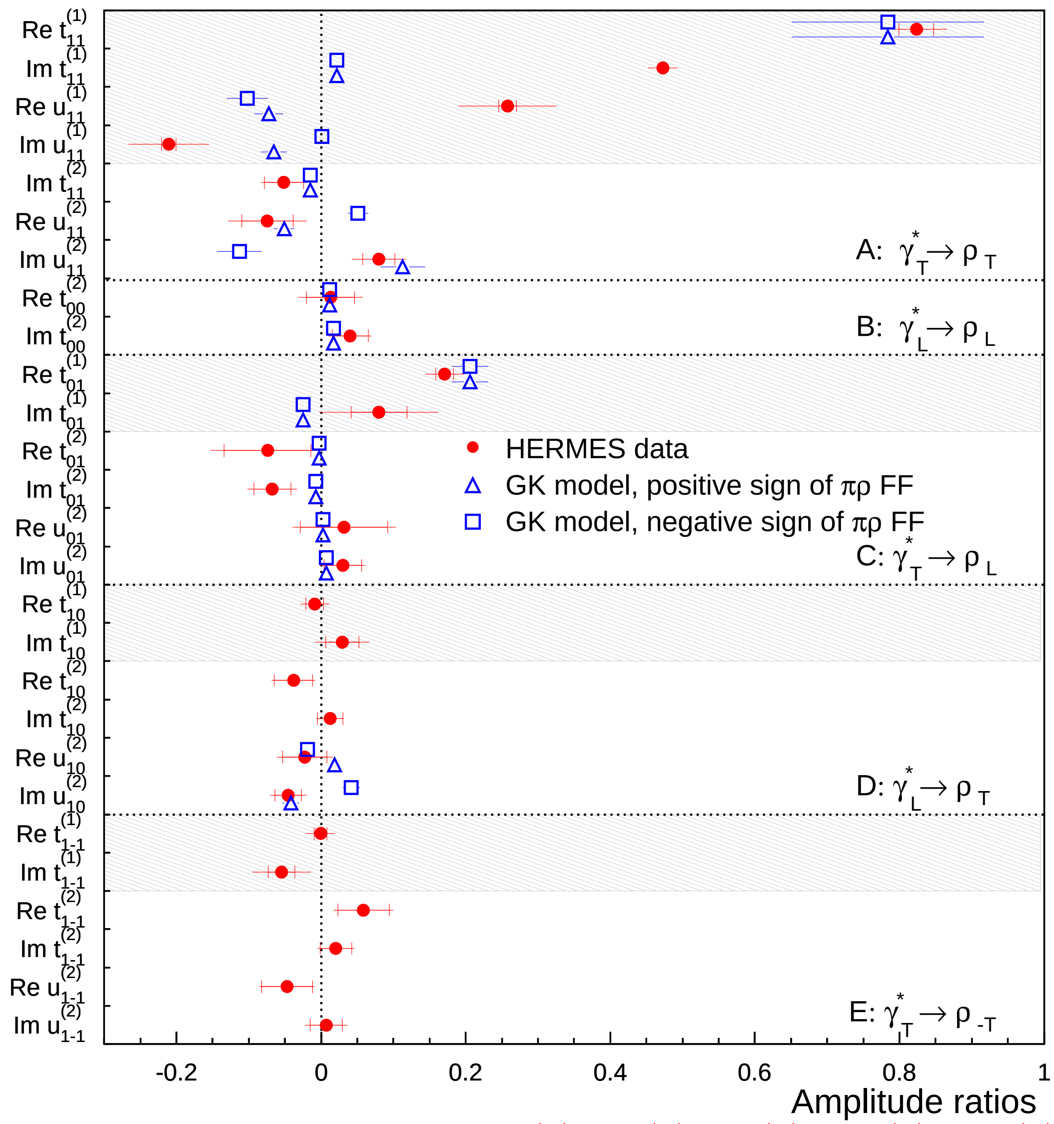


Ratios of amplitudes without nucleon-helicity flip are shown in shaded areas.

Phase of $u_{11}^{(1)}$ from EPJ C29 (2003) 171; $\mathrm{Im}[\mathrm{t}_{11}^{(1)}]$ from EPJ C71 (2011) 1609.

Comparison of Extracted Amplitude Ratios with Goloskokov-Kroll Model

- Comparison of amplitude ratios from the Goloskokov-Kroll model (blue squares and triangles) with amplitude ratios (red points) obtained in Eur. Phys. J. C77 (2017) 378; (arXiv:hep-ex 1702.00345)



In the GK Model, amplitudes $T_{10}^{(1)}$, $T_{10}^{(2)}$, $T_{1-1}^{(1)}$, $T_{1-1}^{(2)}$, $U_{1-1}^{(1)}$, $U_{1-1}^{(2)}$ are put equal to zero.

Comparison of Calculated SDMEs with Directly Extracted SDMEs

- SDMEs in the Diehl Representation

$$u^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* \Big],$$

$$l^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* \Big],$$

$$s^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* \Big],$$

$$n^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* \Big],$$

where ϵ is the ratio of longitudinal and transverse photon fluxes and the normalization factors are

$$\mathcal{N}_T = \frac{1}{2} \sum_{\lambda'_V, \lambda'_N, \lambda_N} \Big[|T_{\lambda'_V \lambda'_N 1 \lambda_N}|^2 + |U_{\lambda'_V \lambda'_N 1 \lambda_N}|^2 \Big], \quad \mathcal{N}_L = \frac{1}{2} \sum_{\lambda'_V, \lambda'_N, \lambda_N} \Big[|T_{\lambda'_V \lambda'_N 0 \lambda_N}|^2 + |U_{\lambda'_V \lambda'_N 0 \lambda_N}|^2 \Big]$$

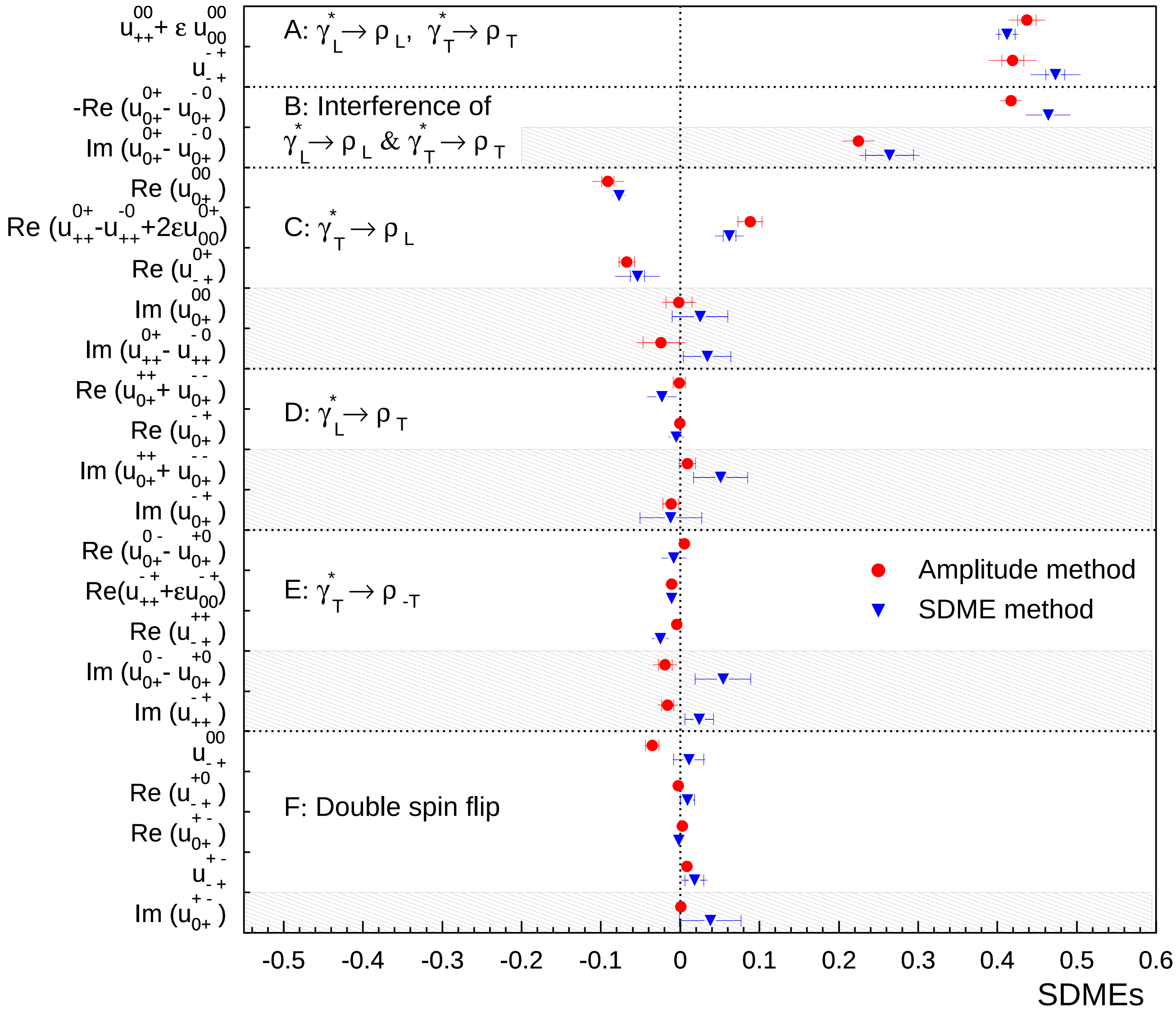
The NPE amplitudes $T_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N}$ and UPE amplitudes $U_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N}$ are defined by

$$T_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} = \frac{1}{2} \Big[F_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} + (-1)^{\lambda_N - \lambda'_N} F_{\lambda'_V -\lambda'_N \lambda_\gamma -\lambda_N} \Big],$$

$$U_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} = \frac{1}{2} \Big[F_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} - (-1)^{\lambda_N - \lambda'_N} F_{\lambda'_V -\lambda'_N \lambda_\gamma -\lambda_N} \Big].$$

Comparison of Calculated SDMEs with Directly Extracted SDMEs

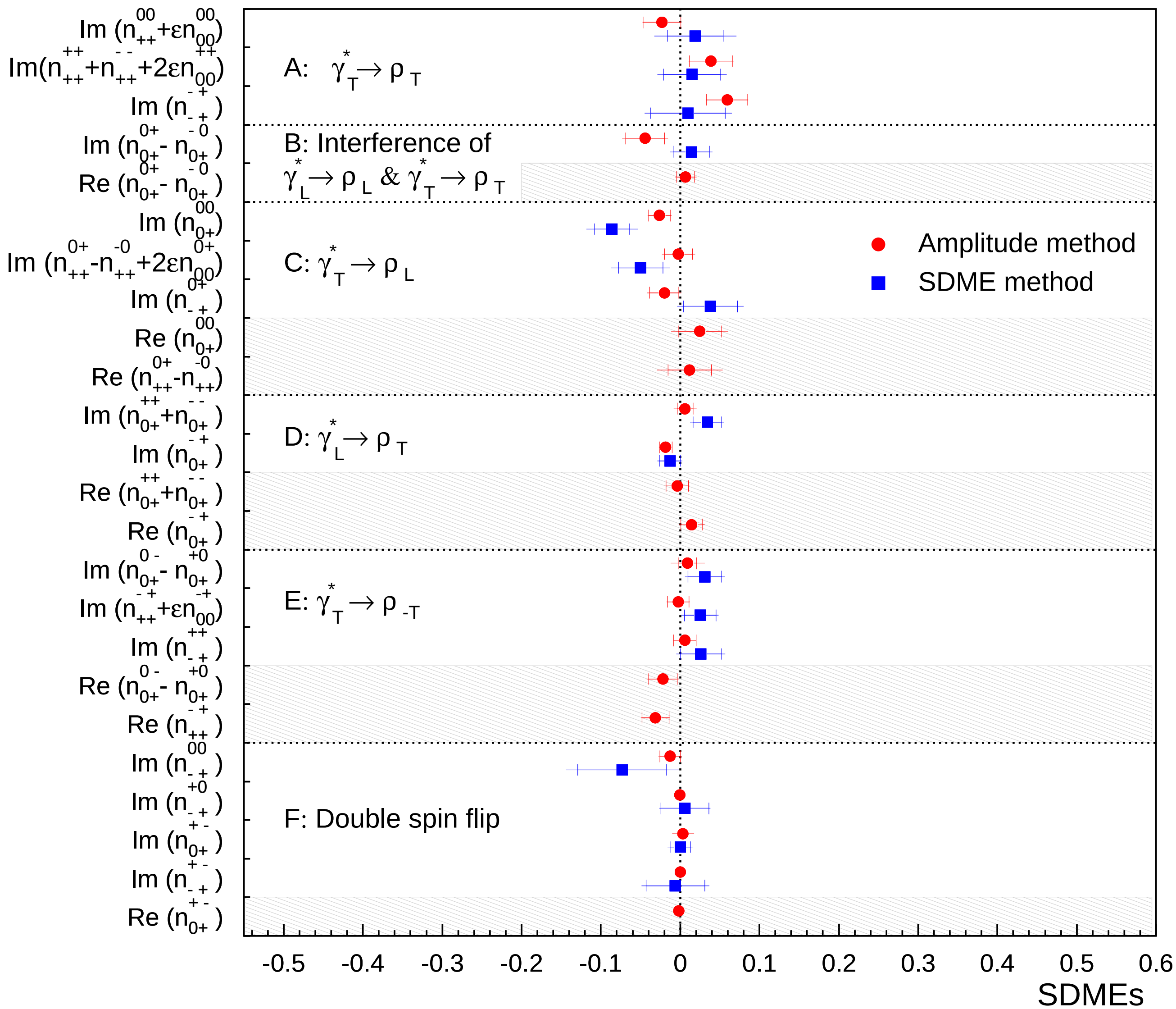
- Comparison of calculated with amplitude ratios (red points) extracted in Eur.Phys.J. C77(2017)378 and "direct" (blue triangles) SDMEs $u^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} \propto t^{(1)}_{\lambda_V \lambda_\gamma} t^{(1)*}_{\lambda'_V \lambda'_\gamma}$ in the Diehl representation



"Polarized" SDMEs (obtainable only with longitudinally polarized beam) are shown in shaded areas.

Comparison of Calculated SDMEs with Directly Extracted SDMEs

- Comparison of calculated with amplitude ratios (red points) extracted in Eur.Phys.J. C77(2017)378 and "direct" (blue squares) SDMEs $n^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} \propto t^{(1)}_{\lambda_V \lambda_\gamma} t^{(2)*}_{\lambda'_V \lambda'_\gamma}$ in the Diehl representation



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Comparison of Calculated SDMEs with Directly Extracted SDMEs

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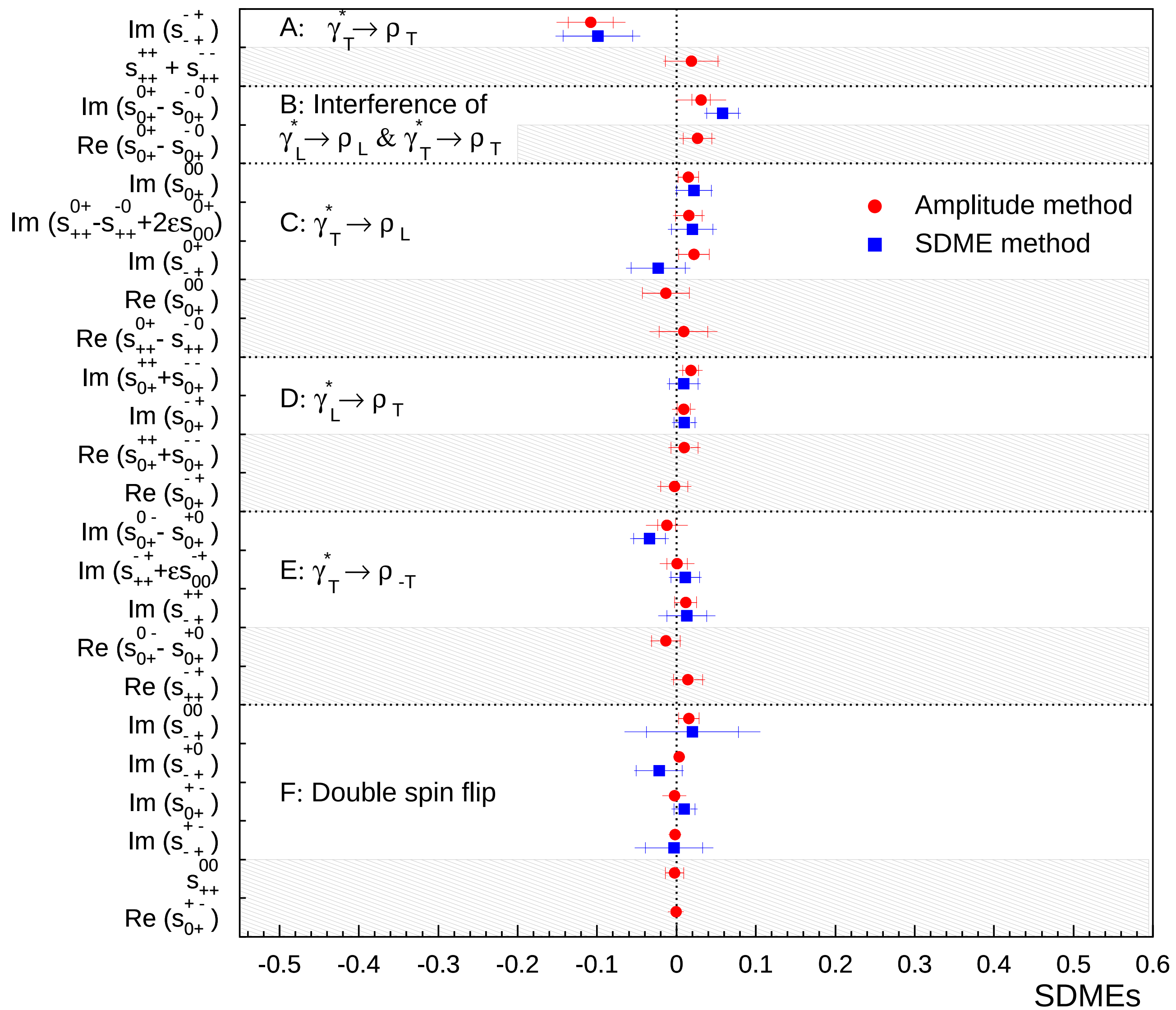

"Polarized" SDMEs (obtainable only with longitudinallypolarized beam) are shown in shaded areas.

Summary and Conclusions

- Exclusive vector-meson electroproduction in DIS is studied at HERMES using a longitudinally polarized electron/positron beam and unpolarized or transversely polarized hydrogen target with $|\vec{P}_T| = 0.72 \pm 0.06$ in the kinematic region $Q^2 > 1.0$ GeV^2, 3.0 GeV $< W <$ 6.3 GeV, and $-t' < 0.2$ GeV^2.
- For the first time, the amplitude analysis of the ρ^0-meson electroproduction on the transversely polarized proton is performed by HERMES.
- Using an unbinned maximum likelihood method, information on 25 real functions (real or imaginary parts of helicity amplitude ratios) is obtained.
- Results for amplitudes without the nucleon-helicity flip are in good agreement with those of previous HERMES amplitude analysis of the data on unpolarized targets, while ratios of amplitudes with nucleon-helicity flip to $T_{0\frac{1}{2}0\frac{1}{2}}$ are extracted for the first time.
- Comparison of obtained amplitude ratios with calculations in the Goloskokov-Kroll model demonstrates a good agreement for the real parts of the ratios of helicity amplitudes without the nucleon-helicity flip to $T_{0\frac{1}{2}0\frac{1}{2}}$, while the results for the imaginary parts (the phase differences of the amplitudes) disagree with the theoretical calculations.
- The comparison with the GK model shows the importance of pion-pole contribution to unnatural-parity-exchange amplitudes, besides the positive sign of the $\pi\rho$ transition form factor is much more preferable than the negative sign.
- SDMEs calculated with the extracted amplitude ratios are in good agreement with those obtained directly from the HERMES data.

Unbinned Maximum Likelihood Method

- No background corrections

$\ln \mathcal{L} = \sum_i^I \ln[\mathcal{W}(\mathcal{R}, (P_b)_i, (P_T)_i, \Phi_i, \Psi_i, \theta_i, \varphi_i))/N_i],$

$N_i = K_1 + K_2(P_b)_i + K_3(P_T)_i + K_4(P_b)_i(P_T)_i$

$(P_b)_i$ beam polarization, $(P_T)_i$ target polarization for i-th event,
$\mathcal{R}$ set of amplitude ratios, I denotes a total number of experimental events.

$N_{++} = \frac{1}{L}\sum_{m=1}^{L} \mathcal{W}(\mathcal{R}, (P_b = 1), (P_T = 1), \Phi_m, \Psi_m, \theta_m, \varphi_m)$

N_{+-} corresponds to $P_b = 1$, $P_T = -1$, N_{-+} to $P_b = -1$, $P_T = 1$ etc.
K_1, K_2, K_3, and K_4 are linear combinations of N_{++}, N_{+-}, N_{-+}, and N_{--}.
L is a total number of MC events with the uniform angular distribution of the generated events.

- Likelihood function with background corrections

$\ln \mathcal{L}_{tot} = \sum_i^I \ln\Big[(1 - f_{bg})\frac{\mathcal{W}(\mathcal{R},(P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{N_i} + f_{bg}\frac{\mathcal{W}_{bg}((P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{N_i^{bg}}\Big]$

Angular distribution, $\mathcal{W}_{bg}$ of background events is assumed to be independent of polarizations P_b and P_T, f_{bg} is the fraction of reconstructed background events.

$N_i^{bg} = \frac{1}{L}\sum_{m=1}^{L} \mathcal{W}_{bg}((P_b = 0), (P_T = 0), \Phi_m, \Psi_m, \theta_m, \varphi_m),$

$\ln \mathcal{L}_{tot} = \sum_i^I \ln\Big[\frac{(1-g_{bg})\mathcal{W}(\mathcal{R},(P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)+g_{bg}\mathcal{W}_{bg}((P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{(1-g_{bg})N_i+g_{bg}N_i^{bg}}\Big],$

where g_{bg} is the fraction of background events in 4π (before interaction of particles with detector).