Ratios of Helicity Amplitudes for Exclusive ρ^0 Electroproduction on Transversely Polarized Proton

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- Phenomenological description of reaction $e + N \to e' + V + N$
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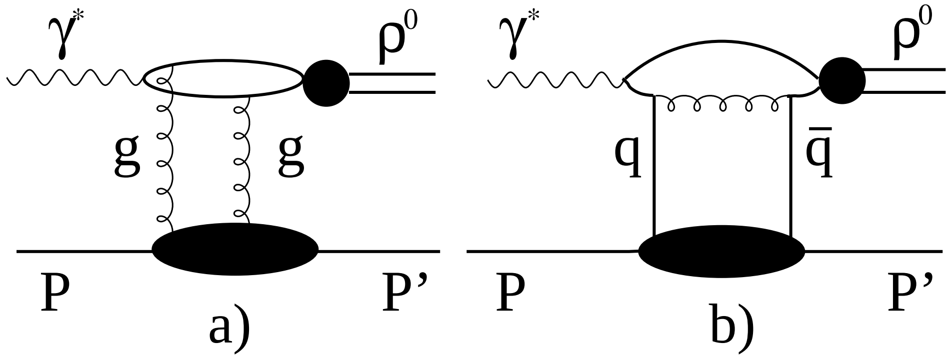
Physics Motivation



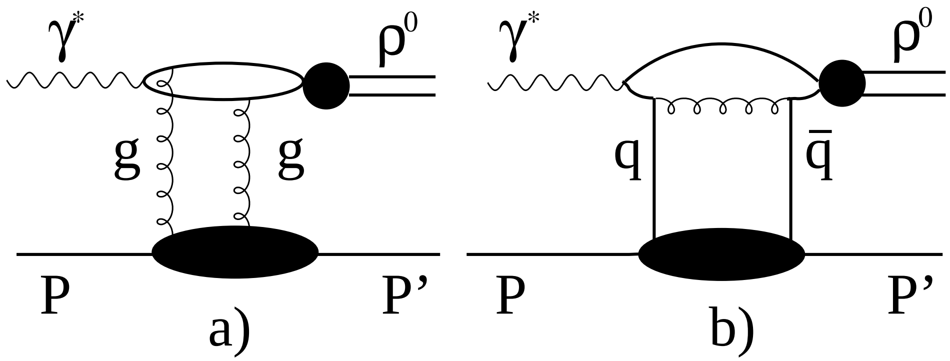
- $\gamma^* + N \rightarrow V + N$ is a perfect reaction to study both vector-meson ($V = \rho^0,\ \phi,\ \omega,\ \ldots$) production mechanism and hadron (nucleon) structure.

- Properties of Spin-Density Matrix Elements (SDMEs).
  SDMEs are dimensionless coefficients in the angular distribution of final particles and therefore can be extracted from data.

- SDMEs are expressible in terms of ratios of helicity amplitudes $F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$ of the $\gamma^* + N \rightarrow V + N$ reaction, hence the ratios can be obtained from angular distribution of final particles.

- Generalized Parton Distributions (GPDs) of the nucleon can be obtained from the helicity amplitude $F_{00} \equiv F_{0\frac{1}{2}0\frac{1}{2}}$ ($\gamma_L \rightarrow V_L$) for which factorization theorem is proved.

Physics Motivation

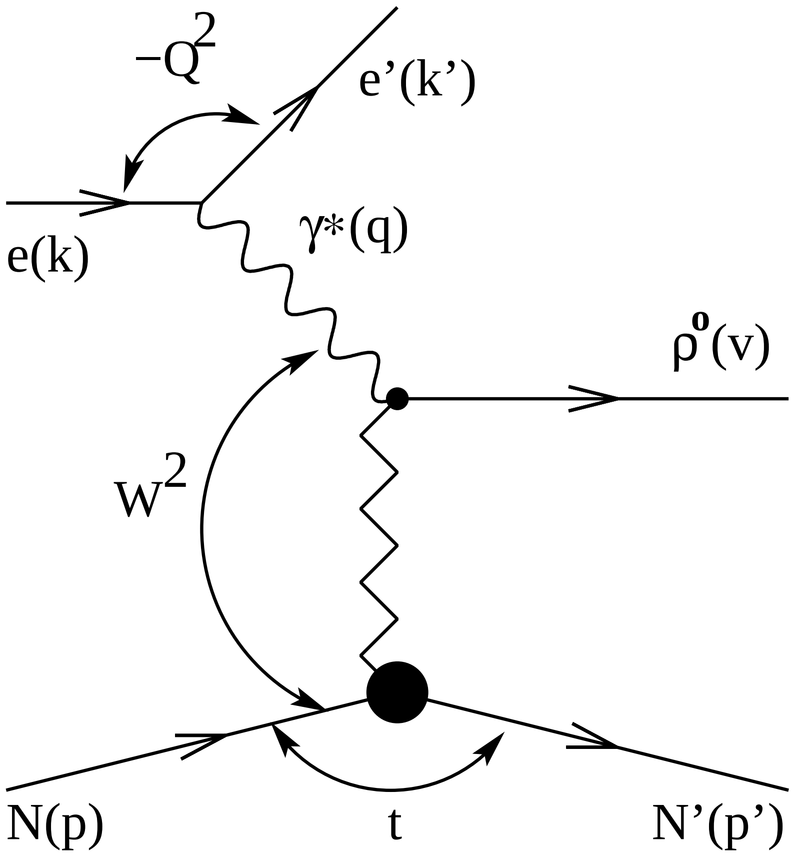


- Generalized Parton Distributions (GPDs)
  Quark GPDs: $H_q(x, \xi, t)$, $E_q(x, \xi, t)$, ...
  Gluon GPDs: $H_g(x, \xi, t)$, $E_g(x, \xi, t)$, ...
  H_q and H_g can be obtained from nucleon helicity non-flip amplitudes ($\lambda_N = \lambda'_N$).
  E_q and E_g can be extracted from nucleon helicity-flip amplitudes ($\lambda_N \neq \lambda'_N$).
- Relations by Ji

$$\frac{1}{2}\int_{-1}^{1} dx x[H_q(x, \xi, t \to 0) + E_q(x, \xi, t \to 0)] =< J_q >,$$
$$\int_{-1}^{1} dx x[H_g(x, \xi, t \to 0) + E_g(x, \xi, t \to 0)] =< J_g > .$$

To use the Ji relations $E_q(x, \xi, t \to 0)$ and $E_g(x, \xi, t \to 0)$ are to be extracted from data on transversely polarized targets at $t \neq 0$.

Phenomenological description of reaction $e + N \to e' + V + N'$



$$\text{QED}: \quad \mathrm{e}(\lambda) \to \mathrm{e}'(\lambda') + \gamma^*(\lambda_\gamma),$$

$$\text{QCD}: \gamma^*(\lambda_\gamma) + \mathrm{N}(\lambda_\mathrm{N}) \to \mathrm{V}(\lambda_\mathrm{V}) + \mathrm{N}'(\lambda'_\mathrm{N}).$$

The helicity amplitude of the reaction $\gamma^* + N \to V + N$

$$F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$$

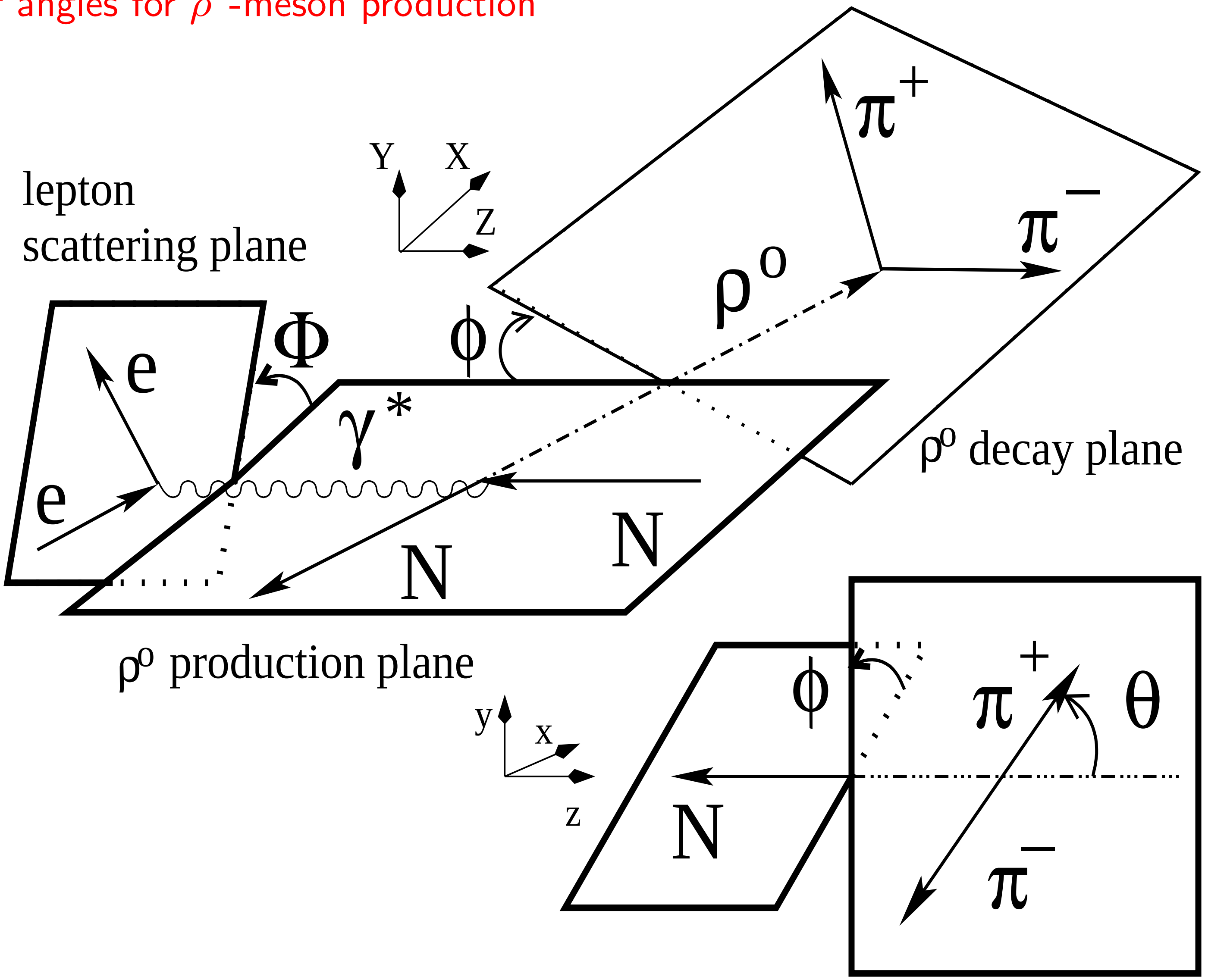
$$= (-1)^{\lambda_\gamma} \langle v \lambda_V p' \lambda'_N | J^\sigma_{(h)} | p \lambda_N \rangle e^{(\lambda_\gamma)}_\sigma.$$

$J^\sigma_{(h)}$ is the electromagnetic current of hadrons;
$e^{(\lambda_\gamma)}_\sigma$ is the photon polarization four-vector;
$\lambda_\gamma = \pm 1$ transverse virtual photon,
$\lambda_\gamma = 0$ longitudinal virtual photon.
$E^{(\lambda_V)}_\sigma$ is the vecor meson polarization vector;
$\lambda_V = \pm 1$ transverse vector meson,
$\lambda_V = 0$ longitudinal vector meson.

Amplitude decomposition into Natural (NPE) and Unnatural Parity Exchange (UPE) Amplitudes (18=10+8)

$$F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} = T_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} + U_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$$

- Definition of angles for ρ^0-meson production



- For ρ-meson production $\vec{n} = \vec{p}_{\pi^+}/|\vec{p}_{\pi^+|}$, $\mathrm{Y}_{1\lambda_V}(\vec{n}) = Y_{1\lambda_V}(\theta, \phi)$.

Phenomenological description of reaction $e + N \to e' + V + N'$

- Spin-Density Matrix Elements $\varrho_{\lambda_V \lambda'_V}$ of the vector meson can be extracted from the angular distribution of final particles

$$|\rho^0; J=1, M> \to |\pi^+\pi^-; L=1, M> \to Y_{1M}(\theta,\phi)$$

$$\mathcal{W}(\Phi,\Psi,\theta,\phi) = \sum_{\lambda_V,\lambda'_V} Y_{1\lambda_V}(\theta,\phi)\varrho_{\lambda_V\lambda'_V}(\Phi,\Psi)Y^*_{1\lambda'_V}(\theta,\phi)$$

- Relation between spin-density matrix of virtual photon $\rho_{\lambda_\gamma\lambda'_\gamma} = \rho_{\lambda_\gamma\lambda'_\gamma}(\Phi)$, the nucleon $\tau_{\lambda'_N\lambda_N} = \tau_{\lambda'_N\lambda_N}(\Psi)$ and that of vector meson (ρ^0 meson) $\varrho_{\lambda_V\lambda'_V}$:

$$\varrho_{\lambda_V\lambda'_V} = \sum \frac{F_{\lambda_V\mu_N\lambda_\gamma\lambda_N}\ \rho_{\lambda_\gamma\lambda'_\gamma}\ \tau_{\lambda_N\lambda'_N} F^*_{\lambda'_V\mu_N\lambda'_\gamma\lambda'_N}}{2\mathcal{N}},$$

  where Ψ is the angle between the transverse polarization vector, $\vec{P}_T$ and the lepton scattering plane.

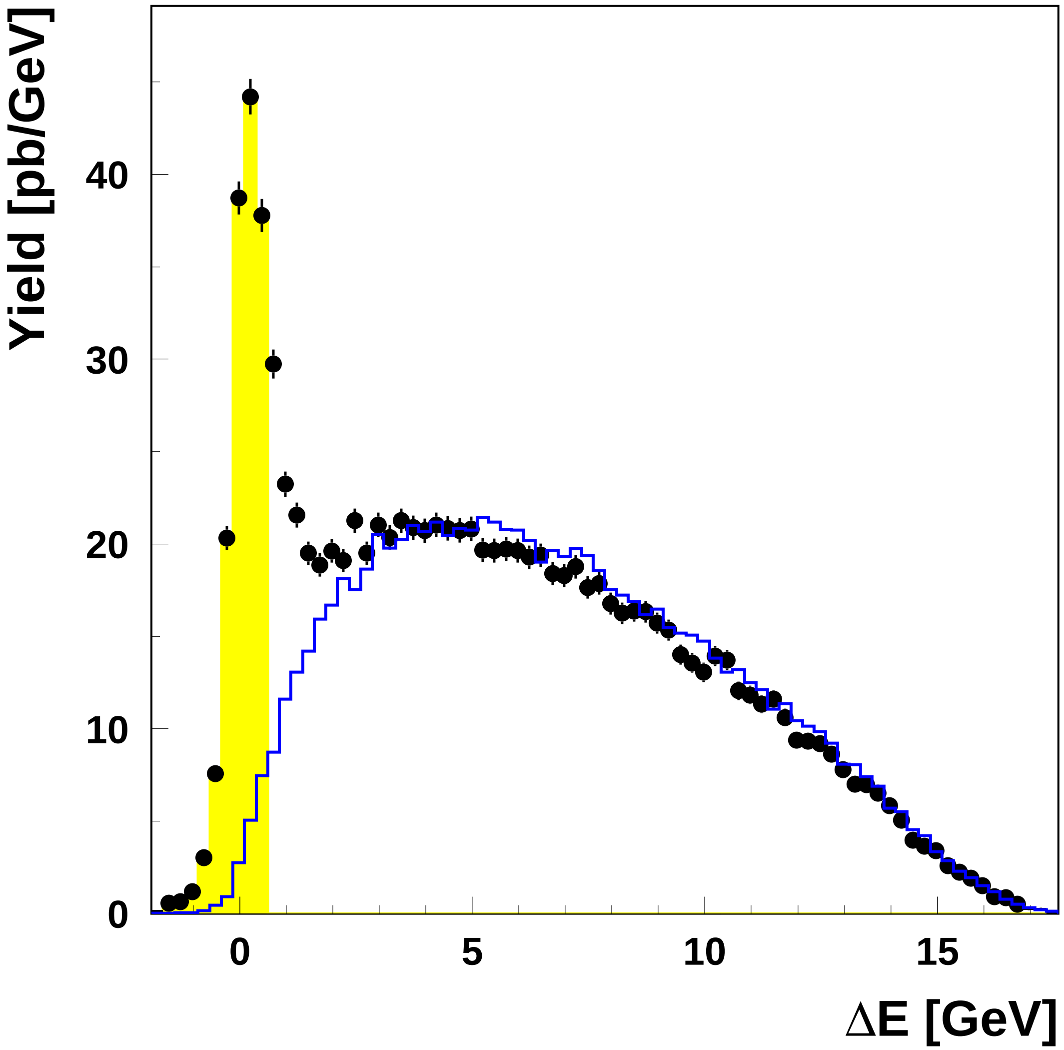
- SDMEs in the Diehl representation ($u^{\lambda_V\lambda'_V}_{\lambda_\gamma\lambda'_\gamma}$, $n^{\lambda_V\lambda'_V}_{\lambda_\gamma\lambda'_\gamma}$, $s^{\lambda_V\lambda'_V}_{\lambda_\gamma\lambda'_\gamma}$, $l^{\lambda_V\lambda'_V}_{\lambda_\gamma\lambda'_\gamma}$) are the Fourier coefficients in decomposition of Φ and Ψ dependences of spin-density matrix of vector meson $\varrho_{\lambda_V\lambda'_V}(\Phi,\Psi)$

The HERMES Experiment

ρ^0-meson production by longitudinally polarized beam on transversely polarized proton

- Longitudinally polarized electron/positron beam with energy of 27.6 GeV.
  $0.15 < |P_B| < 0.80$.
- 6.3 GeV $> W > 3.0$ GeV,
  $Q^2 > 1$ GeV^2,
  $-t' = -(t - t_{min}) < 0.4$ GeV^2.
- Recoil nucleon was not detected. Missing mass criterion was used.
  $\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$; $-1.0 \text{ GeV} < \Delta \text{E} < 0.8$ GeV;
  M_X mass of recoil system; M_p proton mass.
- 8741 events with exclusive ρ-mesons produced with unpolarized and longitudinally polarized beam ($< |P_B| > \approx 0.3$) on transversely polarized proton ($|\vec{P}_T| \approx 0.72 \pm 0.06$) were obtained.

ΔE distribution for ρ^0 meson production



$7\% <$ fraction of background $< 23\%$ for increasing $-t'$ is subtracted, $< f_{bg} >= 11\%$.

Extraction of Helicity Amplitude Ratios

$$T_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} = [F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} + (-1)^{\lambda_\gamma - \lambda_V} F_{-\lambda_V \lambda'_N -\lambda_\gamma \lambda_N}]/2;$$ exchanges with pomeron, ρ, a_2, ...

$$U_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} = [F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} - (-1)^{\lambda_\gamma - \lambda_V} F_{-\lambda_V \lambda'_N -\lambda_\gamma \lambda_N}]/2;$$ exchanges with π, a_1, ...

Amplitudes without nucleon helicity flip:

$$T^{(1)}_{\lambda_V \lambda_\gamma} \equiv T_{\lambda_V \frac{1}{2} \lambda_\gamma \frac{1}{2}} = T_{\lambda_V -\frac{1}{2} \lambda_\gamma -\frac{1}{2}}, \;\; U^{(1)}_{\lambda_V \lambda_\gamma} \equiv U_{\lambda_V \frac{1}{2} \lambda_\gamma \frac{1}{2}} = -U_{\lambda_V -\frac{1}{2} \lambda_\gamma -\frac{1}{2}},$$

Amplitudes with nucleon helicity flip:

$$T^{(2)}_{\lambda_V \lambda_\gamma} \equiv T_{\lambda_V \frac{1}{2} \lambda_\gamma -\frac{1}{2}} = -T_{\lambda_V -\frac{1}{2} \lambda_\gamma \frac{1}{2}}, \;\; U^{(2)}_{\lambda_V \lambda_\gamma} \equiv U_{\lambda_V \frac{1}{2} \lambda_\gamma -\frac{1}{2}} = U_{\lambda_V -\frac{1}{2} \lambda_\gamma \frac{1}{2}},$$

Angular distribution is dimensionless quantity, hence it may depend on the helicity amplitude ratios only.

Amplitude ratios:

$$t^{(1)}_{\lambda_V \lambda_\gamma} = T^{(1)}_{\lambda_V \lambda_\gamma}/T^{(1)}_{00}, \; t^{(2)}_{\lambda_V \lambda_\gamma} = T^{(2)}_{\lambda_V \lambda_\gamma}/T^{(1)}_{00}, \; u^{(1)}_{\lambda_V \lambda_\gamma} = U^{(1)}_{\lambda_V \lambda_\gamma}/T^{(1)}_{00}, \; u^{(2)}_{\lambda_V \lambda_\gamma} = U^{(2)}_{\lambda_V \lambda_\gamma}/T^{(1)}_{00}.$$

Total number of independent amplitude ratios is 17 (34 real functions).

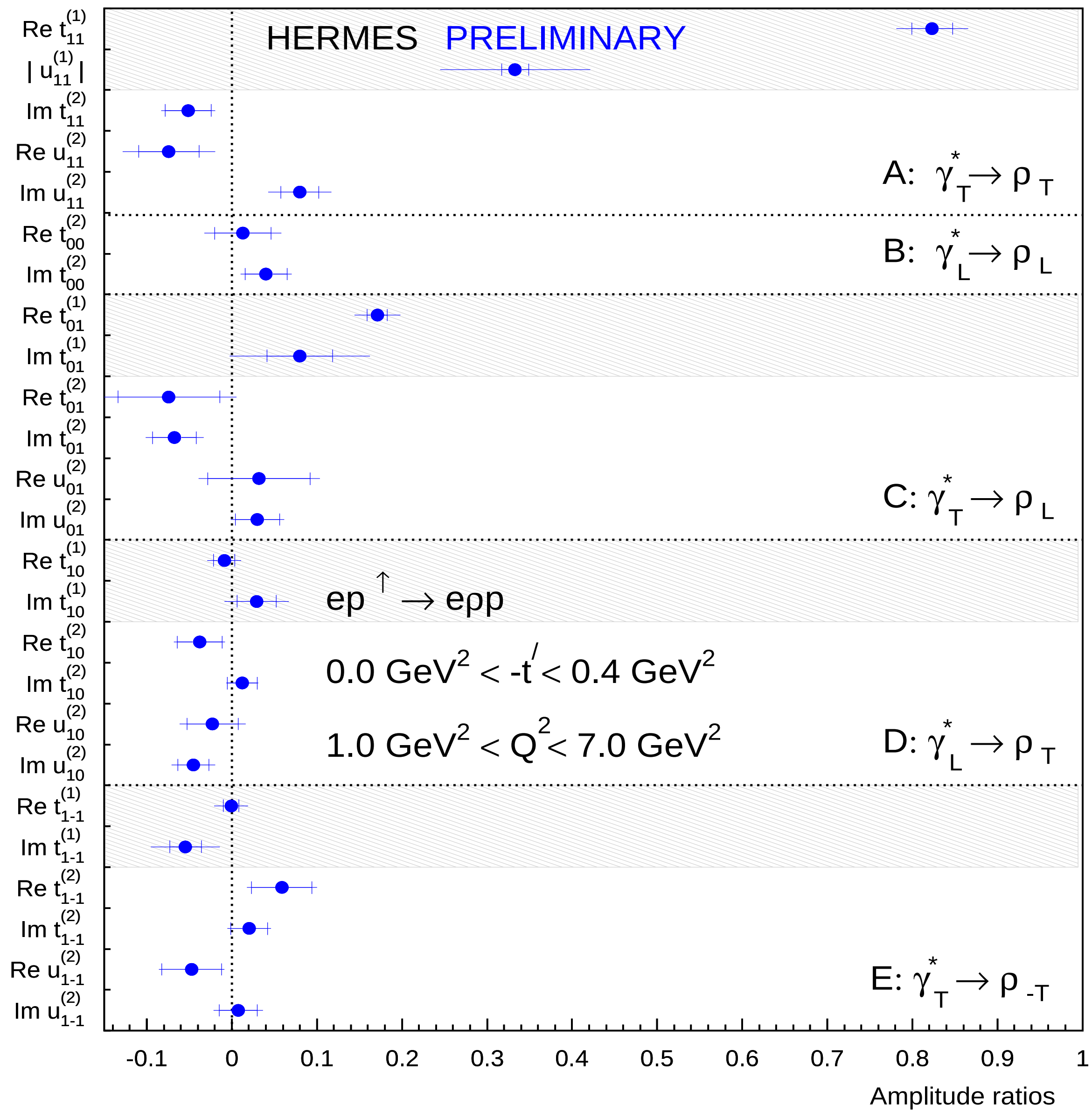
Small amplitudes can be reliably extracted if there is product of those by the amplitude $T^{(1)}_{00}$ or $T^{(1)}_{11}$ being dominant at large Q^2 and small $|t|$.

For longitudinally polarized beam and transversely polarized target only 25 parameters can be reliably extracted.

For the present data, the phase shifts of $T^{(1)}_{11}$ and $U^{(1)}_{11}$ are fixed from previous HERMES data.

Ratios $u^{(1)}_{10}$, $u^{(1)}_{10}$, $u^{(1)}_{1-1}$ are not obtained from present data since they are multiplied by small factor $\sqrt{1-\epsilon} S_L$ with the longitudinal (with respect of virtual photon) target polarization $S_L < 0.04$.

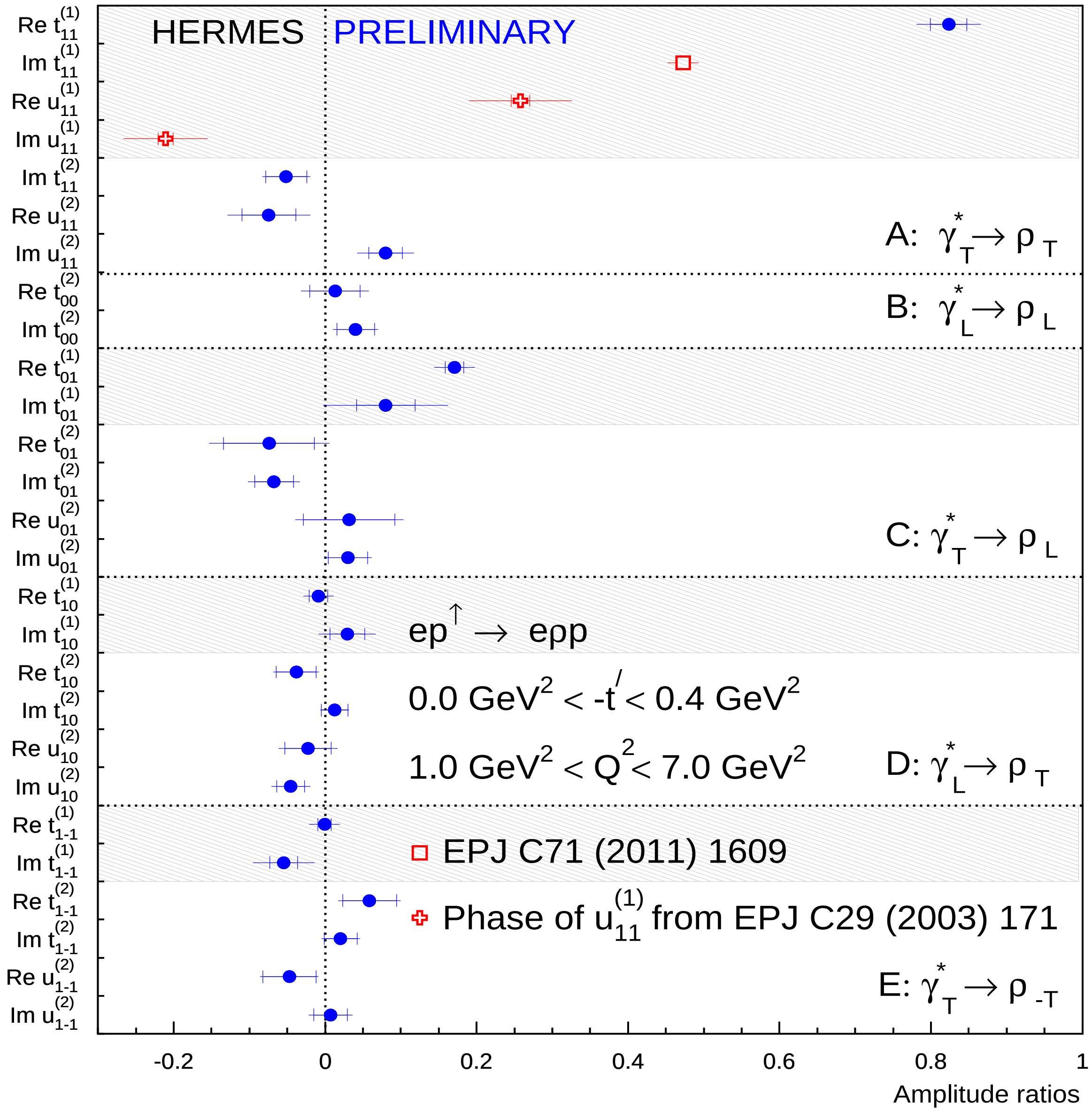
Extraction of Helicity Amplitude Ratios



Ratios of amplitudes without nucleon-helicity flip are shown in shaded areas.

They were also obtained in previous HERMES analysis (Eur. Phys. J. C71 (2011) 1609).

Extraction of Helicity Amplitude Ratios



Ratios of amplitudes without nucleon-helicity flip are shown in shaded areas.

Phase of $u_{11}^{(1)}$ from EPJ C29 (2003) 171; $\mathrm{Im}[\mathrm{t}_{11}^{(1)}]$ from EPJ C71 (2011) 1609.

Comparison of Calculated with Directly Extracted SDMEs

- SDMEs in the Diehl Representation

$$u^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* \Big],$$

$$l^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma \frac{1}{2}})^* \Big],$$

$$s^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* \Big],$$

$$n^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} = (\mathcal{N}_T + \epsilon \mathcal{N}_L)^{-1} \sum_{\sigma=\pm\frac{1}{2}} \Big[T_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (T_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* + U_{\lambda_V \sigma \lambda_\gamma \frac{1}{2}} (U_{\lambda'_V \sigma \lambda'_\gamma -\frac{1}{2}})^* \Big],$$

where ϵ is ratio of longitudinal and transverse photon fluxes and the normalization factors are

$$\mathcal{N}_T = \frac{1}{2} \sum_{\lambda'_V, \lambda'_N, \lambda_N} \Big[|T_{\lambda'_V \lambda'_N 1 \lambda_N}|^2 + |U_{\lambda'_V \lambda'_N 1 \lambda_N}|^2 \Big], \ \mathcal{N}_L = \frac{1}{2} \sum_{\lambda'_V, \lambda'_N, \lambda_N} \Big[|T_{\lambda'_V \lambda'_N 0 \lambda_N}|^2 + |U_{\lambda'_V \lambda'_N 0 \lambda_N}|^2$$

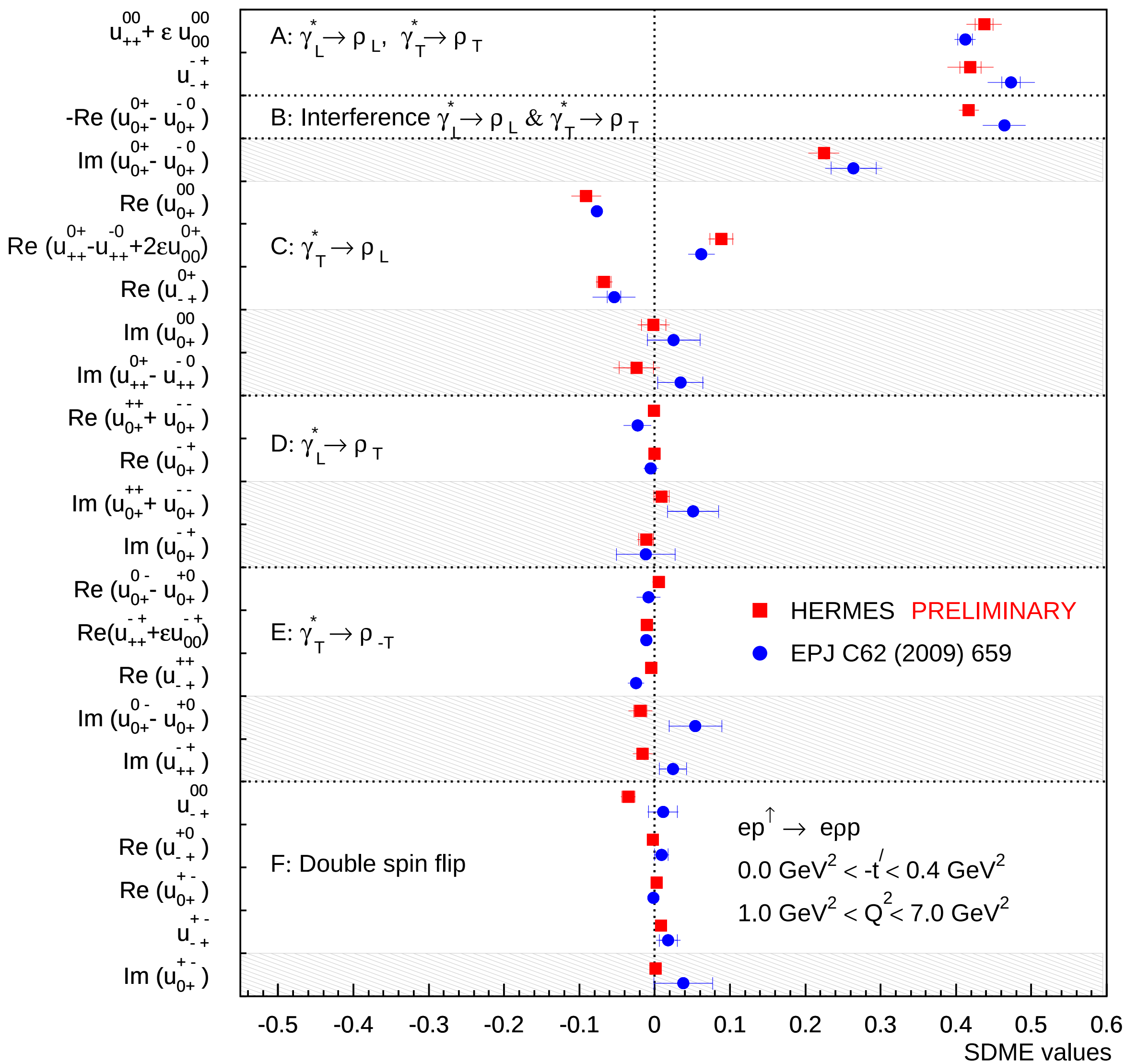
The NPE amplitudes $T_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N}$ and UPE amplitudes $U_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N}$ are defined by

$$T_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} = \frac{1}{2} \Big[F_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} + (-1)^{\lambda_N - \lambda'_N} F_{\lambda'_V - \lambda'_N \lambda_\gamma - \lambda_N} \Big],$$

$$U_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} = \frac{1}{2} \Big[F_{\lambda'_V \lambda'_N \lambda_\gamma \lambda_N} - (-1)^{\lambda_N - \lambda'_N} F_{\lambda'_V - \lambda'_N \lambda_\gamma - \lambda_N} \Big].$$

Comparison of Calculated with Directly Extracted SDMEs

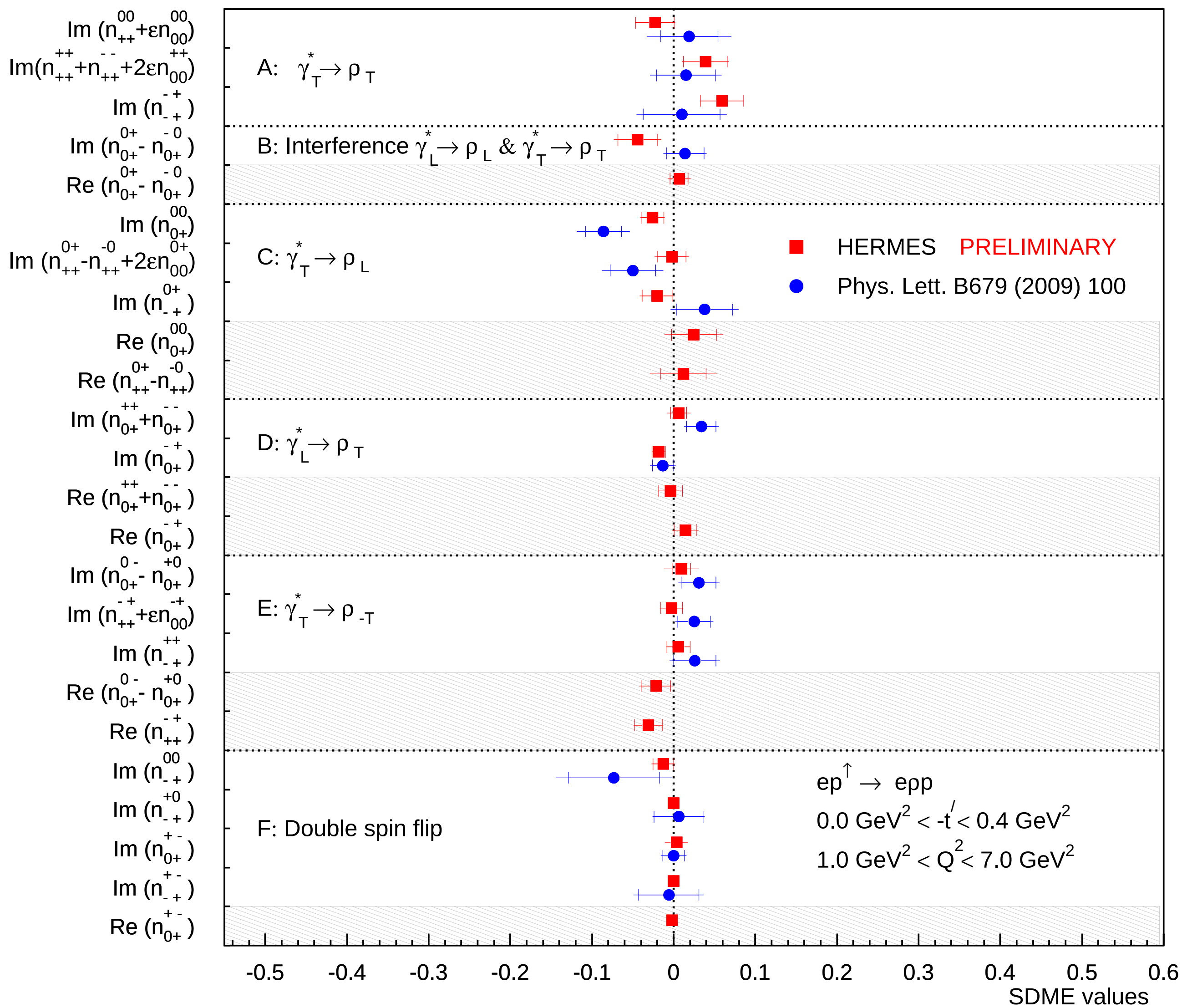
- Comparison of calculated with extracted amplitude ratios (red squares) and "direct" (blue points) SDMEs $u^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} \propto t^{(1)}_{\lambda_V \lambda_\gamma} t^{(1)*}_{\lambda'_V \lambda'_\gamma}$ in the Diehl representation



"Polarized" SDMEs (obtainable only with longitudinally polarized beam) are shown in shaded areas.

Comparison of Calculated with Directly Extracted SDMEs

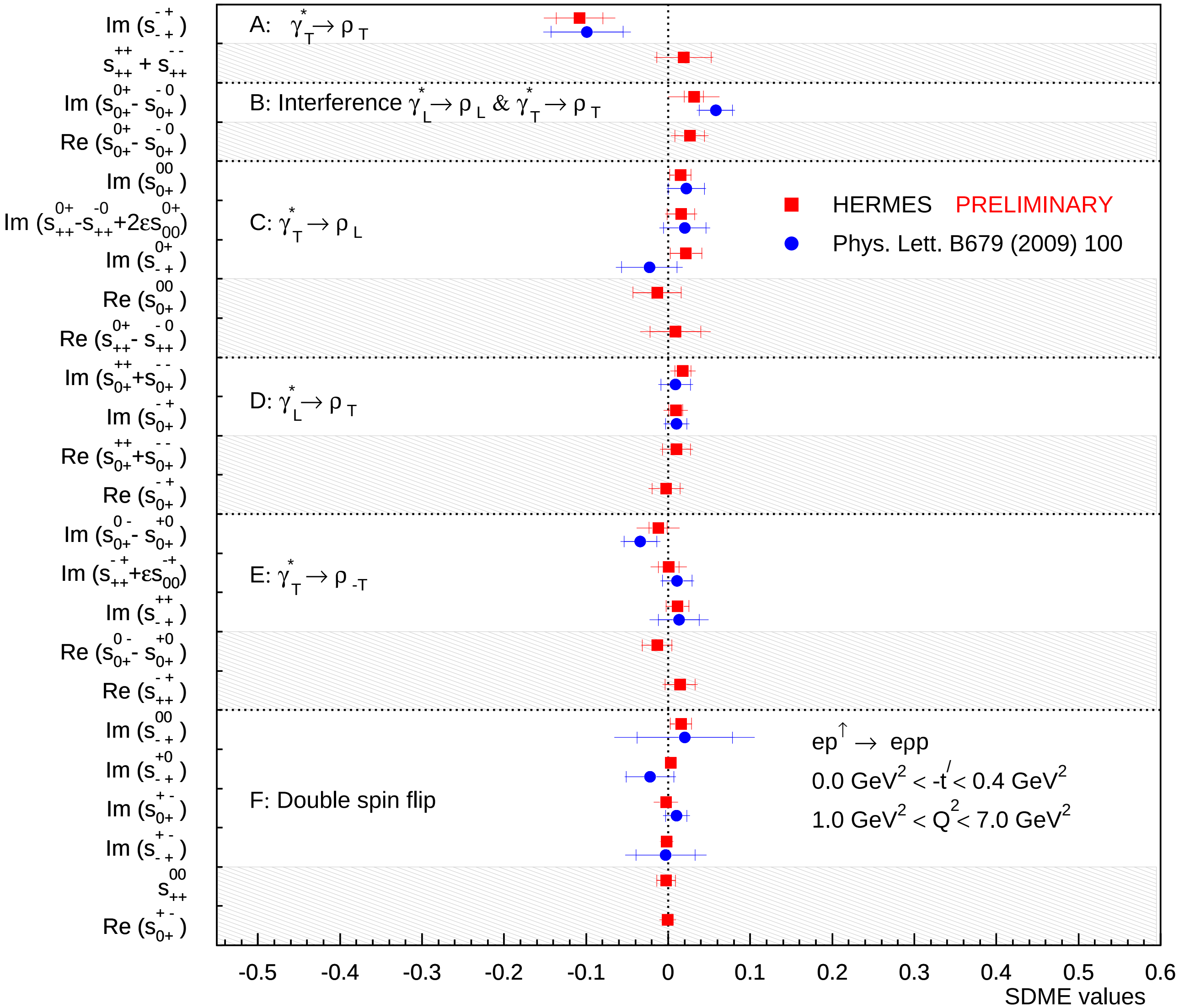
- Comparison of calculated with extracted amplitude ratios (red squares) and "direct" (blue points) SDMEs $n^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} \propto t^{(1)}_{\lambda_V \lambda_\gamma} t^{(2)*}_{\lambda'_V \lambda'_\gamma}$ in the Diehl representation



"Polarized" SDMEs are shown in shaded areas.

Comparison of Calculated with Directly Extracted SDMEs

- Comparison of calculated with extracted amplitude ratios (red squares) and "direct" (blue points) SDMEs $s^{\lambda_V \lambda'_V}_{\lambda_\gamma \lambda'_\gamma} \propto t^{(1)}_{\lambda_V \lambda_\gamma} u^{(2)*}_{\lambda'_V \lambda'_\gamma}$ in the Diehl representation



"Polarized" SDMEs are shown in shaded areas.

Summary

- Exclusive vector-meson electroproduction in DIS is studied at HERMES using a longitudinally polarized electron/positron beam and unpolarized or transversely polarized hydrogen target with $|\vec{P}_T| = 0.72 \pm 0.06$ in the kinematic region $Q^2 > 1.0$ GeV^2, 3.0 GeV $< W < 6.3$ GeV, and $-t' < 0.2$ GeV^2.

- For the first time, the amplitude analysis of the ρ^0-meson electroproduction on the transversely polarized proton is performed by HERMES.

- Using an unbinned maximum likelihood method, information on 25 real functions (real or imaginary parts of helicity amplitude ratios) is obtained.

- Results for amplitudes without the nucleon-helicity flip are in good agreement with those of previous HERMES amplitude analysis, while ratios of amplitudes with nucleon-helicity flip to $T_{0\frac{1}{2}0\frac{1}{2}}$ are extracted for the first time.

- SDMEs calculated with the extracted amplitude ratios are in good agreement with those obtained directly from the HERMES data.

Unbinned Maximum Likelihood Method

- Calculation of "background" SDMEs

  Two Monte Carlo (MC) sets.

  First (normalization) MC set: uniform angular distribution ($\cos\theta$, Φ, ϕ). Number of events is N_{MC}.

  Second (background pseudo-data) MC set for calculation of a set S_{bg} of 15 background SDMEs. Number of events N_{PD}.

  Log-likelihood function for background pseudo-data events for unpolarized (U) beam

  $$-\ln L(S_{bg}) = -\sum_{i=1}^{N_{PD}} \ln \frac{\mathcal{W}^U(S_{bg},\Phi_i,\phi_i,\cos\theta_i)}{\widetilde{\mathcal{N}}_{bg}(S_{bg})},$$

  $$\widetilde{\mathcal{N}}_{bg}(S_{bg}) = \sum_{j=1}^{N_{MC}} \mathcal{W}^U(S_{bg},\Phi_j,\phi_j,\cos\theta_j).$$

- Calculation of physical SDMEs

  N total number of experimental events in exclusive region.

  S set of 23 SDMEs for unpolarized target and longitudinally (L) polarized beam.

  $$-\ln L(S) = -\sum_{i=1}^{N} \ln\left[\frac{(1-f_{bg})*\mathcal{W}^{U+L}(S,\Phi_i,\phi_i,\cos\theta_i)}{\widetilde{\mathcal{N}}(S,S_{bg})} + \frac{f_{bg}*\mathcal{W}^U(S_{bg},\Phi_i,\phi_i,\cos\theta_i)}{\widetilde{\mathcal{N}}(S,S_{bg})}\right]$$

  f_{bg} fraction of background events in experimental events in exclusive region.

  The total normalization factor

  $$\widetilde{\mathcal{N}}(S,S_{bg}) = \sum_{j=1}^{N_{MC}}[(1-f_{bg})*\mathcal{W}^{U+L}(S,\Phi_j,\phi_j,\cos\theta_j) + f_{bg}*\mathcal{W}^U(S_{bg},\Phi_j,\phi_j,\cos\theta_j)]$$

Unbinned Maximum Likelihood Method

- No background corrections

$\ln \mathcal{L} = \sum_i^I \ln[\mathcal{W}(\mathcal{R}, (P_b)_i, (P_T)_i, \Phi_i, \Psi_i, \theta_i, \varphi_i))/N_i]$,

$N_i = K_1 + K_2(P_b)_i + K_3(P_T)_i + K_4(P_b)_i(P_T)_i$

$(P_b)_i$ beam polarization, $(P_T)_i$ target polarization for i-th event,
$\mathcal{R}$ set of amplitude ratios.

$N_{++} = \frac{1}{L}\sum_{m=1}^{L} \mathcal{W}(\mathcal{R}, (P_b = 1), (P_T = 1), \Phi_m, \Psi_m, \theta_m, \varphi_m)$

N_{+-} corresponds to $P_b = 1$, $P_T = -1$, N_{-+} to $P_b = -1$, $P_T = 1$ etc.
K_1, K_2, K_3, and K_4 are linear combinations of N_{++}, N_{+-}, N_{-+}, and N_{--}.

- Likelihood function with background corrections

$\ln \mathcal{L}_{tot} = \sum_i^I \ln\Big[(1 - f_{bg})\frac{\mathcal{W}(\mathcal{R},(P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{N_i} + f_{bg}\frac{\mathcal{W}_{bg}((P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{N_i^{bg}}\Big]$

Angular distribution, $\mathcal{W}_{bg}$ of background events is assumed to be independent of polarizations P_b and P_T, f_{bg} fraction of reconstructed background events.

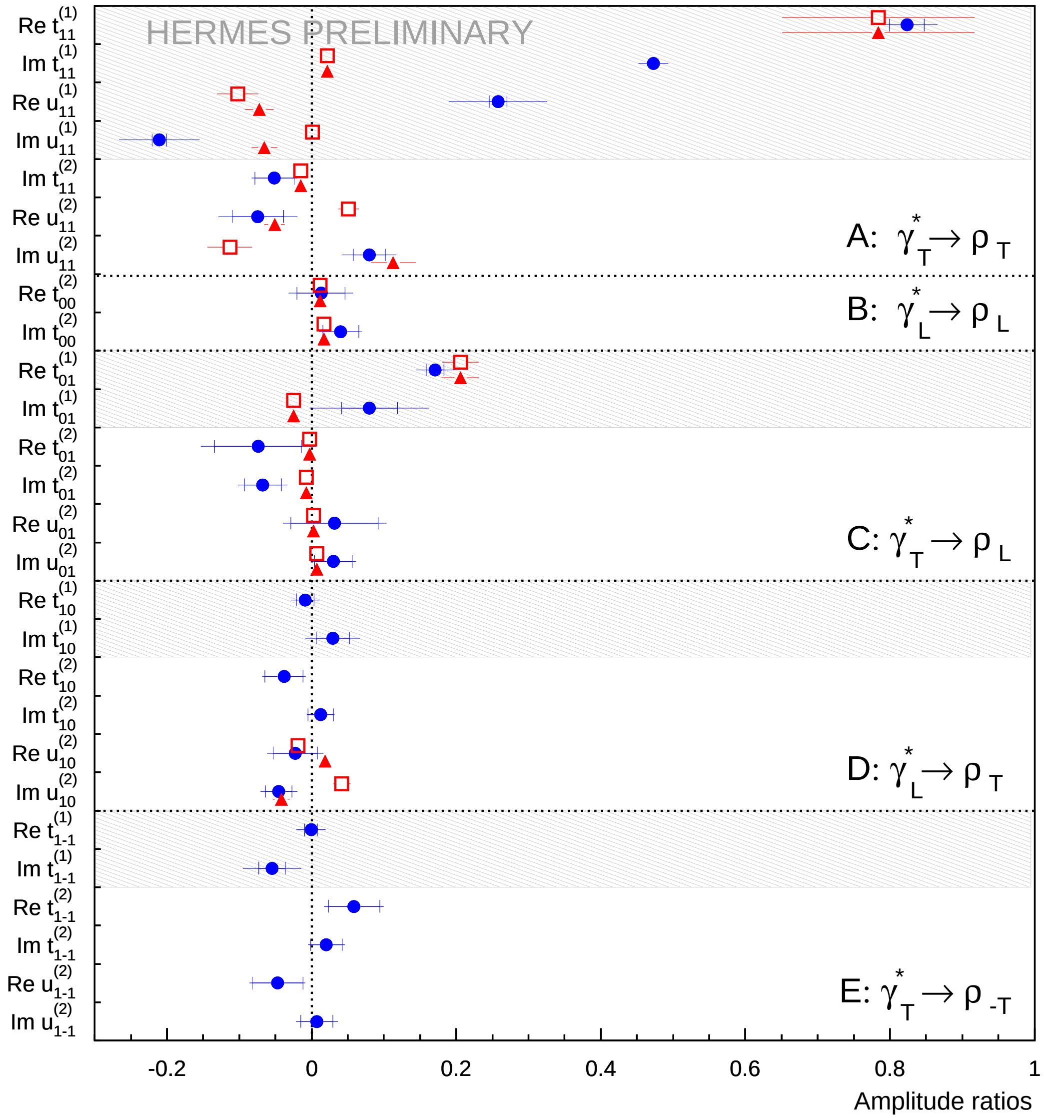
$N_i^{bg} = \frac{1}{L}\sum_{m=1}^{L} \mathcal{W}_{bg}((P_b = 0), (P_T = 0), \Phi_m, \Psi_m, \theta_m, \varphi_m)$

$\ln \mathcal{L}_{tot} = \sum_i^I \ln\Big[\frac{(1-g_{bg})\mathcal{W}(\mathcal{R},(P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)N_i+g_{bg}\mathcal{W}_{bg}((P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{(1-g_{bg})N_i+g_{bg}N_i^{bg}}\Big]$

g_{bg} is the fraction of background in 4π (before interaction of particles with detector)

Comparison of Extracted Amplitude Ratios with GK-Model Amplitudes

- Comparison of amplitude ratios from the Goloskokov-Kroll model (red squares) with extracted amplitude ratios (blue points)



In the GK Model, amplitudes $T^{(1)}_{10}$, $T^{(2)}_{10}$, $T^{(1)}_{1-1}$, $T^{(2)}_{1-1}$, $U^{(1)}_{1-1}$, $U^{(2)}_{1-1}$ are put equal to zero.