# Spin-density matrix elements in exclusive $\omega$ electroproduction on $^1$H and $^2$H targets at 27.5 GeV beam energy

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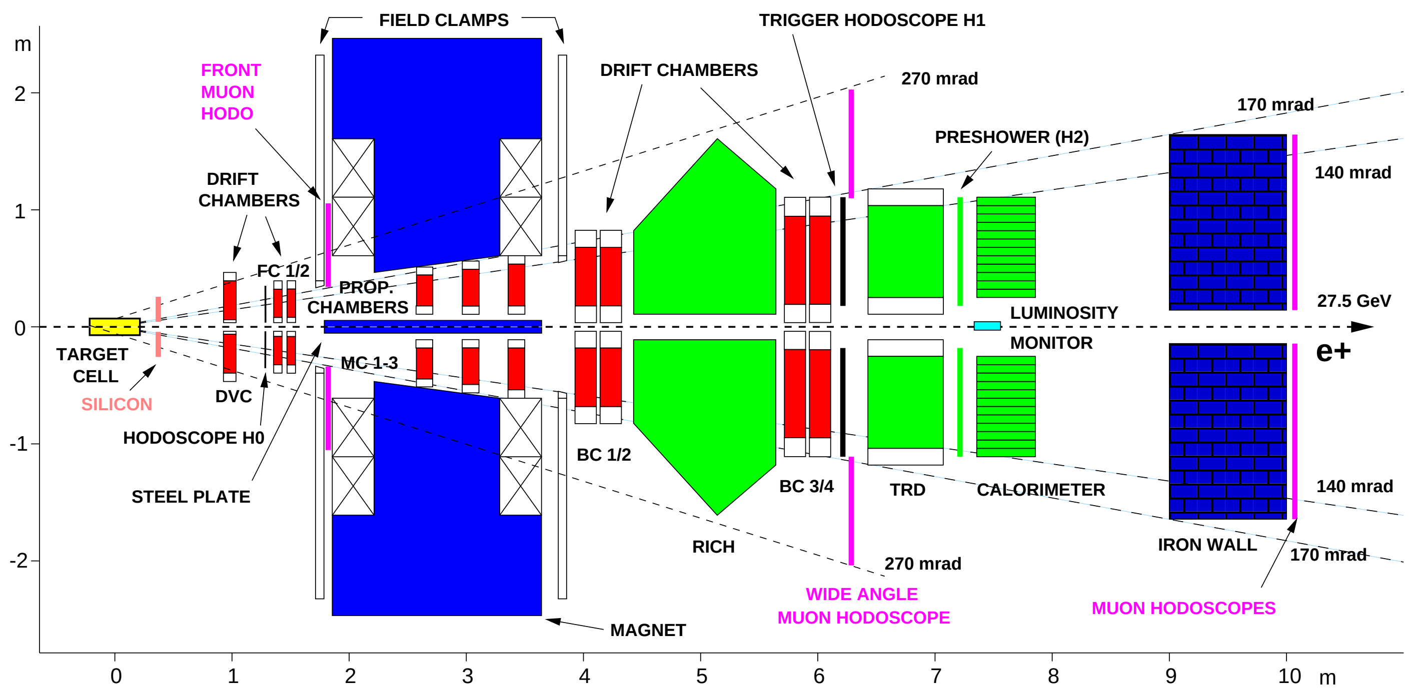
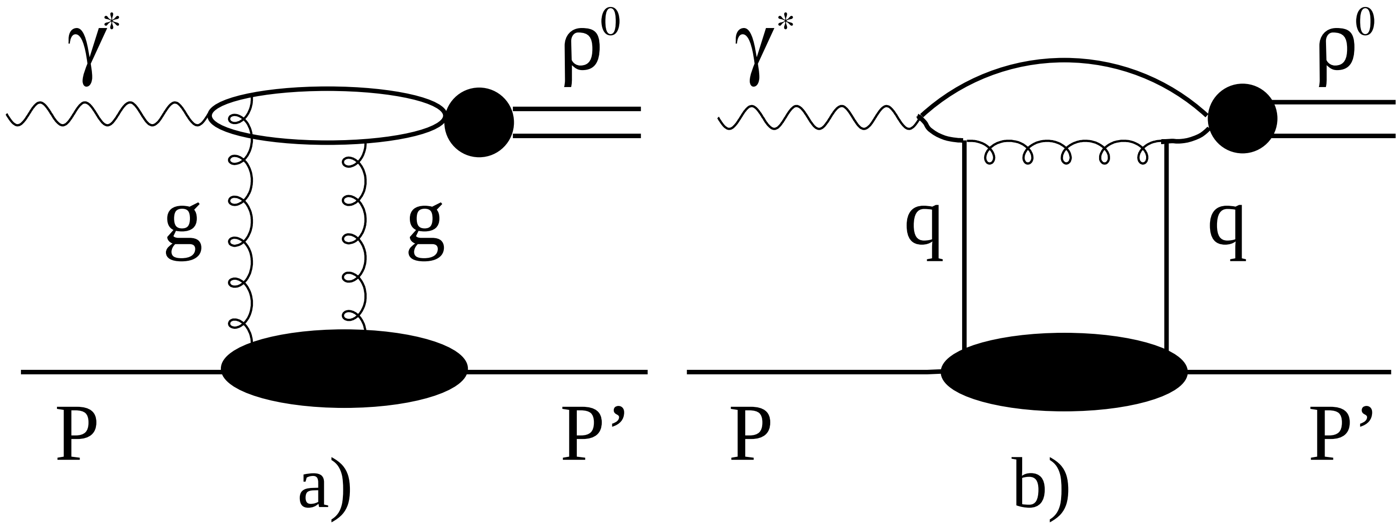
# HERMES FOREVER!



# Physics Motivation




- $\gamma^* + N \to V + N$ is a perfect reaction to study both vector-meson $(V = \rho^0,\ \phi,\ \omega)$ production mechanism and hadron structure.
- Properties of Spin-Density Matrix Elements (SDMEs).
  SDMEs are coefficients in the angular distribution of final hadrons and therefore can be extracted from data.
  SDMEs are expressible in terms of ratios of helicity amplitudes of $\gamma^* + N \to V + N$ reaction, hence ratios can be estimated from angular distribution of final hadrons.

- Generalized Parton Distributions (GPDs) of the nucleon can be obtained from the amplitude $F_{00} \equiv F_{0\frac{1}{2}0\frac{1}{2}}$ $(\gamma_L \to V_L\ )$ for which factorization theorem is proved.
  GPDs permit to calculate the contribution of the total angular momentum of a quark of some flavour or gluon to the nucleon spin (Ji's sum rule).

# Phenomenological description of reaction $e + N \to e' + \omega^0 + N$

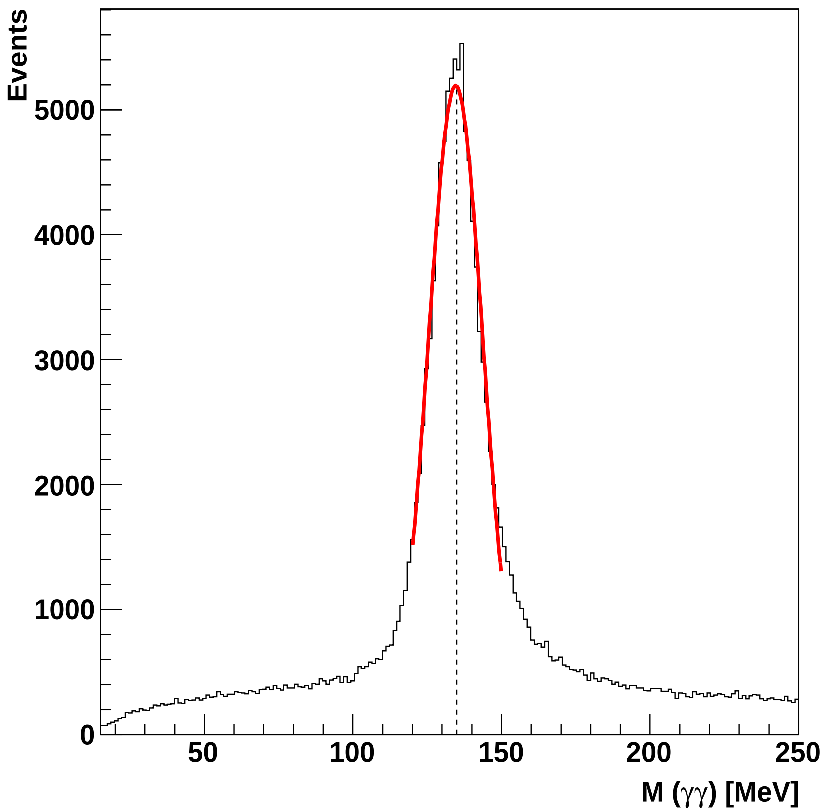
- First process $e \to e + \gamma^*$ (QED). Spin-density matrix of virtual photon $\rho_{\lambda_\gamma \lambda'_\gamma}$.
- Second process $\gamma^* + N \to \omega + N$ (QCD). Helicity amplitudes $F_{\lambda_\omega \lambda'_N \lambda_\gamma \lambda_N}$.
  Spin-Density Matrix of $\omega$: $r = \frac{F \rho F^+}{\mathcal{N}}$. $\rho = \sum \left( c_\alpha \Sigma^\alpha \right)$, $r^\alpha = F \Sigma^\alpha F^+$, $r = \sum c_\alpha r^\alpha$.
- Third process $\omega \to \pi^+ + \pi^- + \pi^0 (\to \gamma + \gamma)$ (Quantum mechanics).
  $\omega$: $J^P = 1^-$. P($3\pi$) $= -1$, orbital motion $1^+$: $Y_{1\lambda_\omega}(\vec{n})$,
  where $\vec{n}$ is a unit normal to $\omega$-decay plane.
- Spin-Density Matrix Elements (SDMEs) $r_{\lambda_\omega \lambda'_\omega}$ of the $\omega$ meson are extracted from the angular distribution of decay pions
  $W(\Phi, \Theta, \phi) = \sum_{\lambda_\omega, \lambda'_\omega} Y_{1\lambda_\omega}(\vec{n}) r_{\lambda_\omega \lambda'_\omega} Y^*_{1\lambda'_\omega}(\vec{n})$.
- Quantities (SDMEs) $r^\alpha_{\lambda_\omega \lambda'_\omega}$ can be calculated from the relation
  $r^\alpha_{\lambda_\omega \lambda'_\omega} = \frac{1}{2N_\alpha} \sum F_{\lambda_\omega \lambda'_N \lambda_\gamma \lambda_N} \; \Sigma^\alpha_{\lambda_\gamma \lambda'_\gamma} \; F^*_{\lambda'_\omega \lambda'_N \lambda'_\gamma \lambda_N}$,
  where $\Sigma^\alpha$ is a set of 9 matrixes:

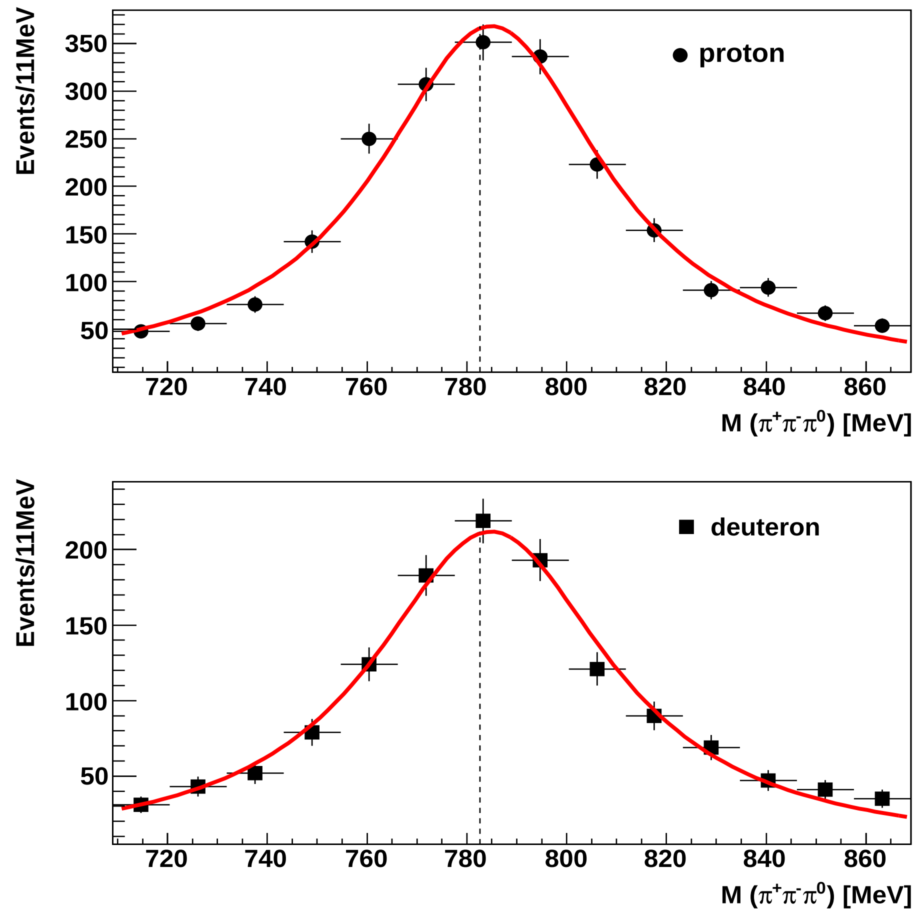
$$\Sigma^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \Sigma^1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \Sigma^5 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

# The HERMES Experiment

- Longitudinally polarized electron/positron beam with energy of 27.6 GeV.
- Momentum resolution $\Delta P/P < 1.5\%$. Efficiency of electron identification 98%.
- Only three tracks of charged particles in any event, two calorimeter clusters $(\gamma, \gamma)$.
- Energy of scattered electron (positron) $E'_e > 3.5$ GeV.
- 6.3 GeV $> W > 3.0$ GeV, $Q^2 > 1$ GeV$^2$, $-t' = -(t - t_{max}) < 0.2$ GeV$^2$.
- Recoil nucleon was not detected. Missing mass criterion was used.
  $\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$; $-1.0 \text{ GeV} < \Delta\text{E} < 0.8$ GeV;
  $M_X$ $(M_p)$ mass of recoil system (proton).
- 0.11 GeV $< M(\gamma\gamma) <$ 0.16 GeV; $0.71 \text{ GeV} < \text{M}(\pi^+\pi^-\pi^0) <$ 0.87 GeV.
- $16\% <$ fraction of background $< 26\%$ for increasing $-t'$.
- 2260 events with exclusive $\omega$ meson produced on proton and 1332 on deuteron were accumulated in 1996 - 2007 years.
- A. Airapetian et al. (the HERMES collaboration), Eur. Phys. J. C74, 3110 (2014).

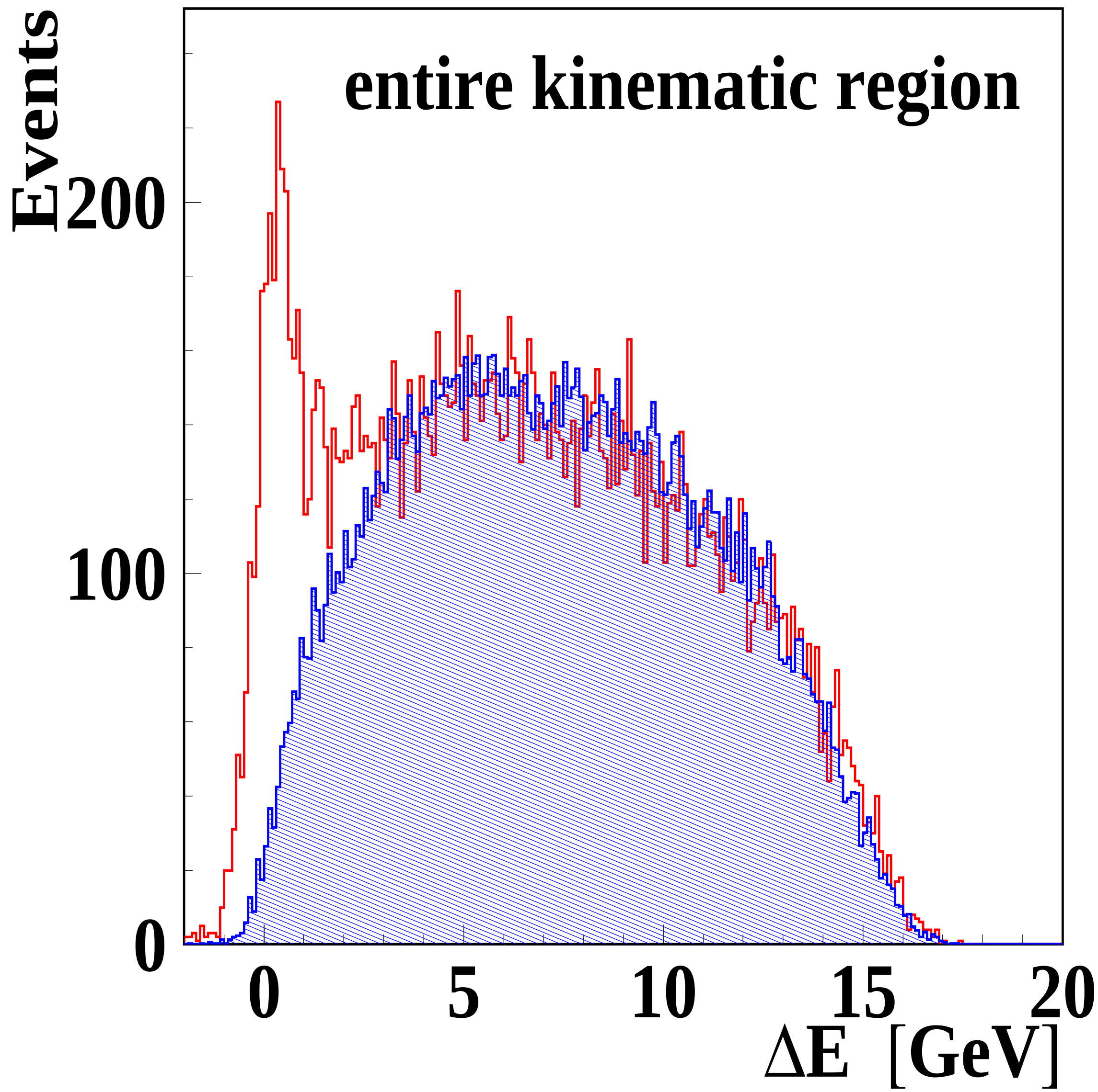
- Mass distributions for $\pi^0$ and $\omega$ decays




Left panel: $\gamma\gamma$ mass distribution for $\pi^0$ decay ($m_{\pi^0} = 134.7 \pm 19.9$ MeV).
Right panel: $\pi^+, \pi^-, \pi^0$ mass distribution for $\omega$ decay.
$M_\omega = 784.8 \pm 55.8$ MeV (proton target),
$M_\omega = 784.6 \pm 58.2$ MeV (deuteron target).

## $\Delta E$ distribution for $\omega$ meson production



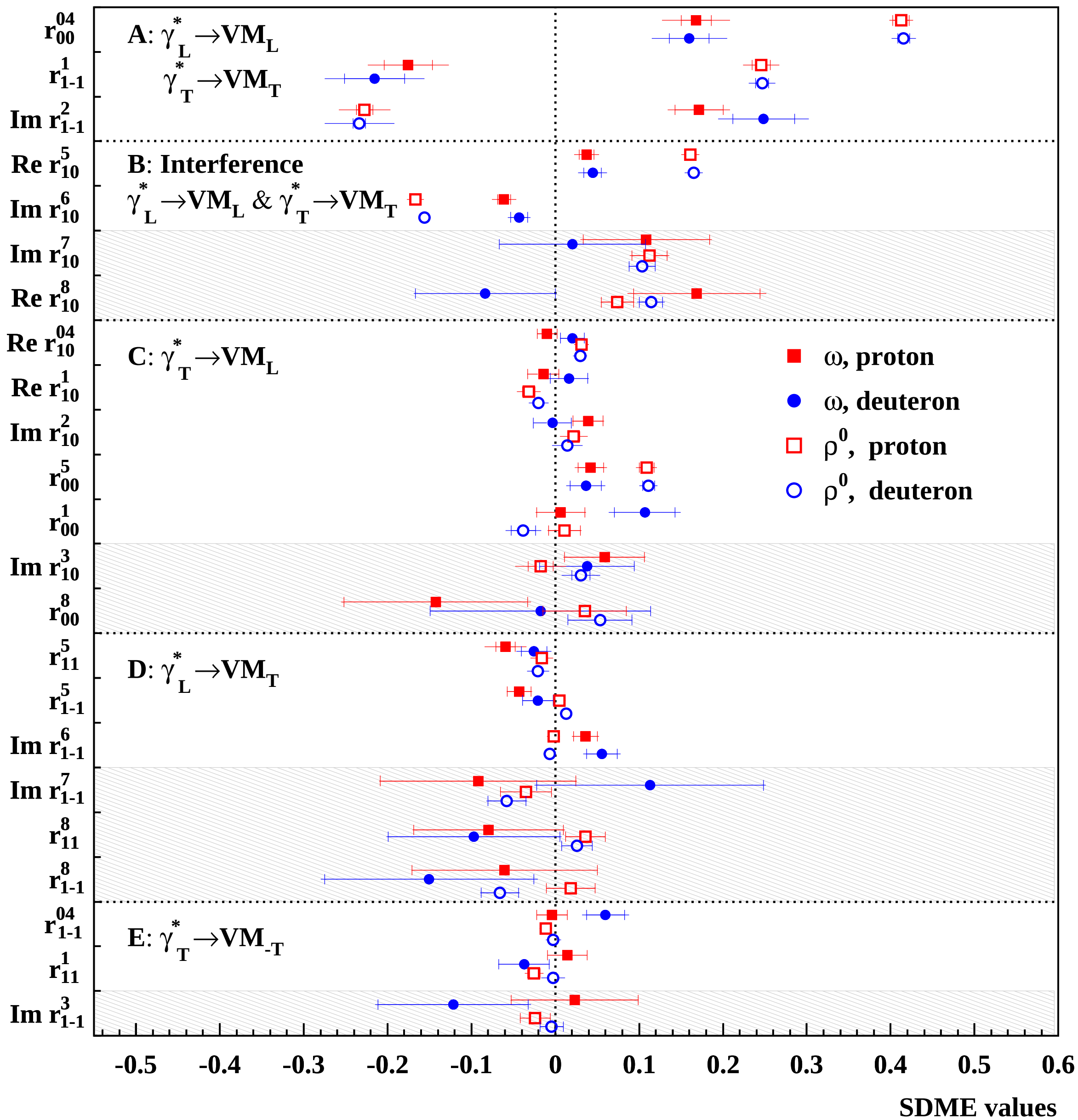

$\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$. $M_X$ and $M_p$ are masses of recoil system and proton, respectively.

## Results on Spin-Density Matrix Elements

- S-Channel Helicity Conservation (SCHC):
  helicity amplitudes with $\lambda_V = \lambda_\gamma$ are dominant ($T_{00}$, $T_{11}$, $U_{11}$).
  $T_{\lambda_V \mu_N \lambda_\gamma \lambda_N}$ is Natural Parity Exchange (NPE) amplitude.
  $U_{\lambda_V \mu_N \lambda_\gamma \lambda_N}$ is Unnatural Parity Exchange (UPE) amplitude.
  $$T_{\lambda_V \mu_N \lambda_\gamma \lambda_N} = \Big[F_{\lambda_V \mu_N \lambda_\gamma \lambda_N} + (-1)^{\mu_N - \lambda_N} F_{\lambda_V -\mu_N \lambda_\gamma -\lambda_N}\Big]/2,$$
  $$U_{\lambda_V \mu_N \lambda_\gamma \lambda_N} = \Big[F_{\lambda_V \mu_N \lambda_\gamma \lambda_N} - (-1)^{\mu_N - \lambda_N} F_{\lambda_V -\mu_N \lambda_\gamma -\lambda_N}\Big]/2,$$
- Class A of SDMEs: Main terms proportional to $|T_{00}|^2$ or $|T_{11}|^2$.
  Class B: Main terms proportional to $\mathrm{Re}[T_{00}T^*_{11}]$ or $\mathrm{Im}[T_{00}T^*_{11}]$.
  Class C: Main terms proportional to $T_{01}$ ($\sim \sqrt{-t'}/M_p$ at small $-t'$).
  Class D: Main terms proportional to $T_{10}$ ($\sim \sqrt{-t'}/M_p$ at small $-t'$).
  Class E: Main terms proportional to $T_{1-1}$ ($-t'/M_p^2$ at small $-t'$).
  If SCHC is valid all elements of classes C, D, E are zero.
- Check of SCHC relations for Class-A and B SDMEs for the proton
  $r^1_{1-1} + \mathrm{Im}[r^2_{1-1}] = -0.004 \pm 0.038 \pm 0.015,$
  $\mathrm{Re}[r^5_{10}] + \mathrm{Im}[r^6_{10}] = -0.024 \pm 0.013 \pm 0.004,$
  $\mathrm{Im}[r^7_{10}] - \mathrm{Re}[r^8_{10}] = -0.060 \pm 0.100 \pm 0.018,$
  and deuteron
  $r^1_{1-1} + \mathrm{Im}[r^2_{1-1}] = 0.033 \pm 0.049 \pm 0.016,$
  $\mathrm{Re}[r^5_{10}] + \mathrm{Im}[r^6_{10}] = 0.001 \pm 0.016 \pm 0.005,$
  $\mathrm{Im}[r^7_{10}] - \mathrm{Re}[r^8_{10}] = 0.104 \pm 0.110 \pm 0.023.$

# Results on Spin-Density Matrix Elements

- Comparison of results for $\omega$ and $\rho^0$ mesons for entire kinematic region




"Polarized" SDMEs are measured by HERMES for the first time.

# Results on Spin-Density Matrix Elements

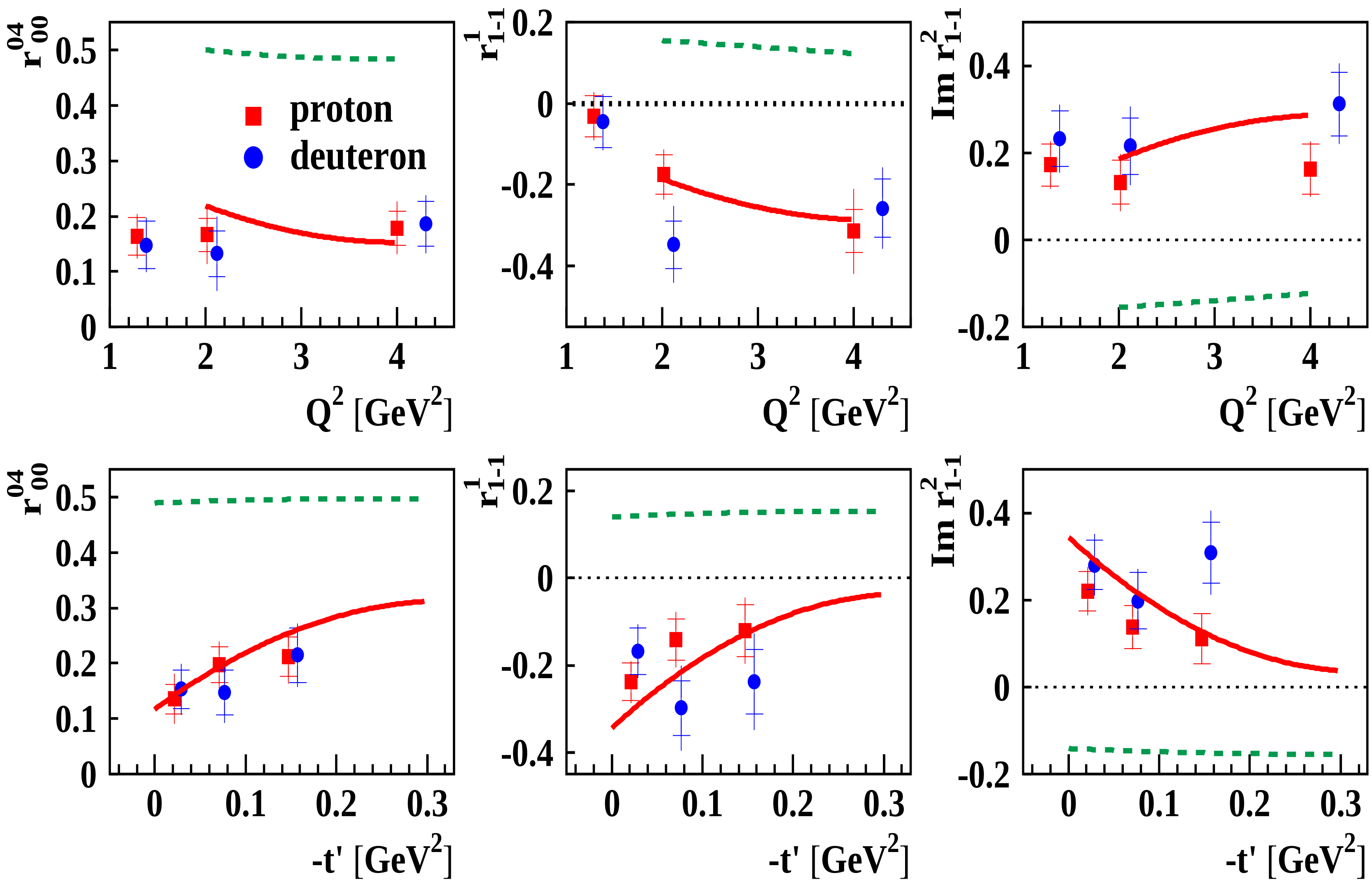
- Enigma of the $\rho-\omega$ difference for class A SDMEs
  $\mathrm{Im}\{r^2_{1-1}\} - r^1_{1-1} = \frac{1}{\mathcal{N}}(-|T_{1\frac{1}{2}1\frac{1}{2}}|^2 - |T_{1-\frac{1}{2}1\frac{1}{2}}|^2 + |U_{1\frac{1}{2}1\frac{1}{2}}|^2 + |U_{1-\frac{1}{2}1\frac{1}{2}}|^2) > 0.???$
  $T_{\lambda_V\mu_N\lambda_\gamma\lambda_N}$ is Natural Parity Exchange (NPE) amplitude.
  $U_{\lambda_V\mu_N\lambda_\gamma\lambda_N}$ is Unnatural Parity Exchange (UPE) amplitude.
  In Regge Phenomenology, NPE amplitudes are due to exchanges of Pomeron, $\rho$, $\omega$, $f_2$, $a_2$, ... reggeons ($J^P = 0^+,\ 1^-,\ 2^+, ...$).
  UPE amplitudes are due to exchanges of $\pi$, $a_1$, ... reggeons ($J^P = 0^-,\ 1^+,\ 2^-, ...$).
  Pion exchange at intermediate $W$ and $Q^2$.
  Vertices $\gamma\omega\pi$ and $\pi NN$ are big $\Rightarrow |U_{11}| > |T_{11}|$.

- How to make the $U_{11}$ amplitude large (dominant) at large $Q^2$?
  First step: hard $\rho^0$ production (big $Q^2$).
  Modulus of amplitude $|T_{11}(\gamma + N \to \rho^0 + N)| > |T_{11}(\gamma + N \to \omega + N)|$.
  Second step: pion exchange in final state $\rho^0 + N \to \omega + N$.
  $\rho^0 \to \omega + \pi^0$, $\pi^0 + N \to N$. Vertices $\rho^0\omega\pi$ and $\pi NN$ are big.
  Peripherical pion exchange (soft) is combined with hard $\rho^0$ production at high $Q^2$.

- Factorization theorem
  FT is proved for $T_{00}$ $(\gamma_L \to V_L)$ only. $U_{00} \equiv 0$.
  Ivanov-Kirshner: $|F_{00}/F_{11}| \propto Q$ at $Q \to \infty$. $F_{00} = T_{00}$, $F_{11} = T_{11} + U_{11}$.
  It is true for the "direct" amplitudes without final state interaction.
  Fractional contribution of pion exchange goes to zero at $W \to \infty$.

# Kinematic Dependences of SDMEs

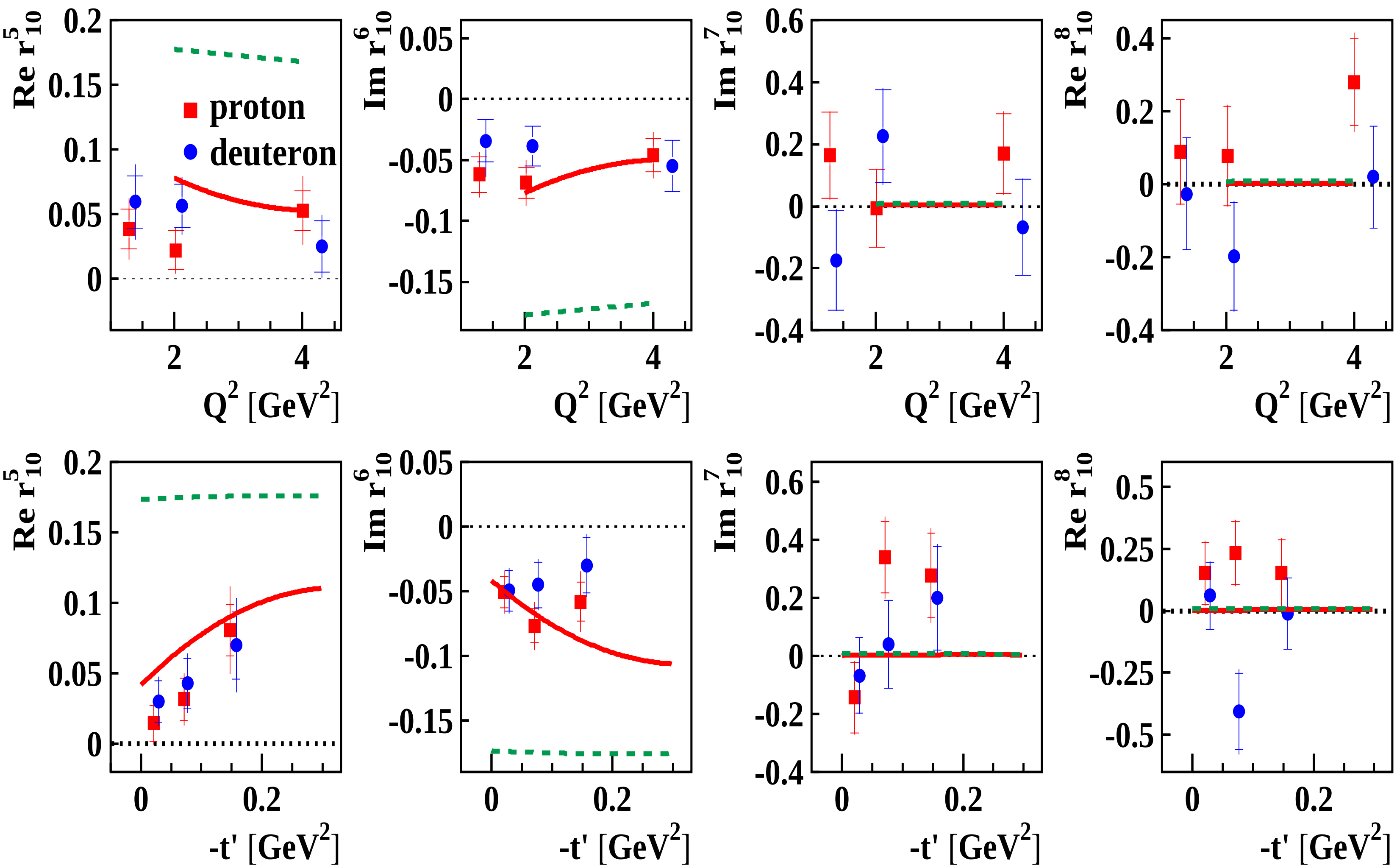
- Kinematic dependences of class-A SDMEs




Red solid (green dashed) curves are obtained with (without) pion pole contribution in the Goloskokov-Kroll model.

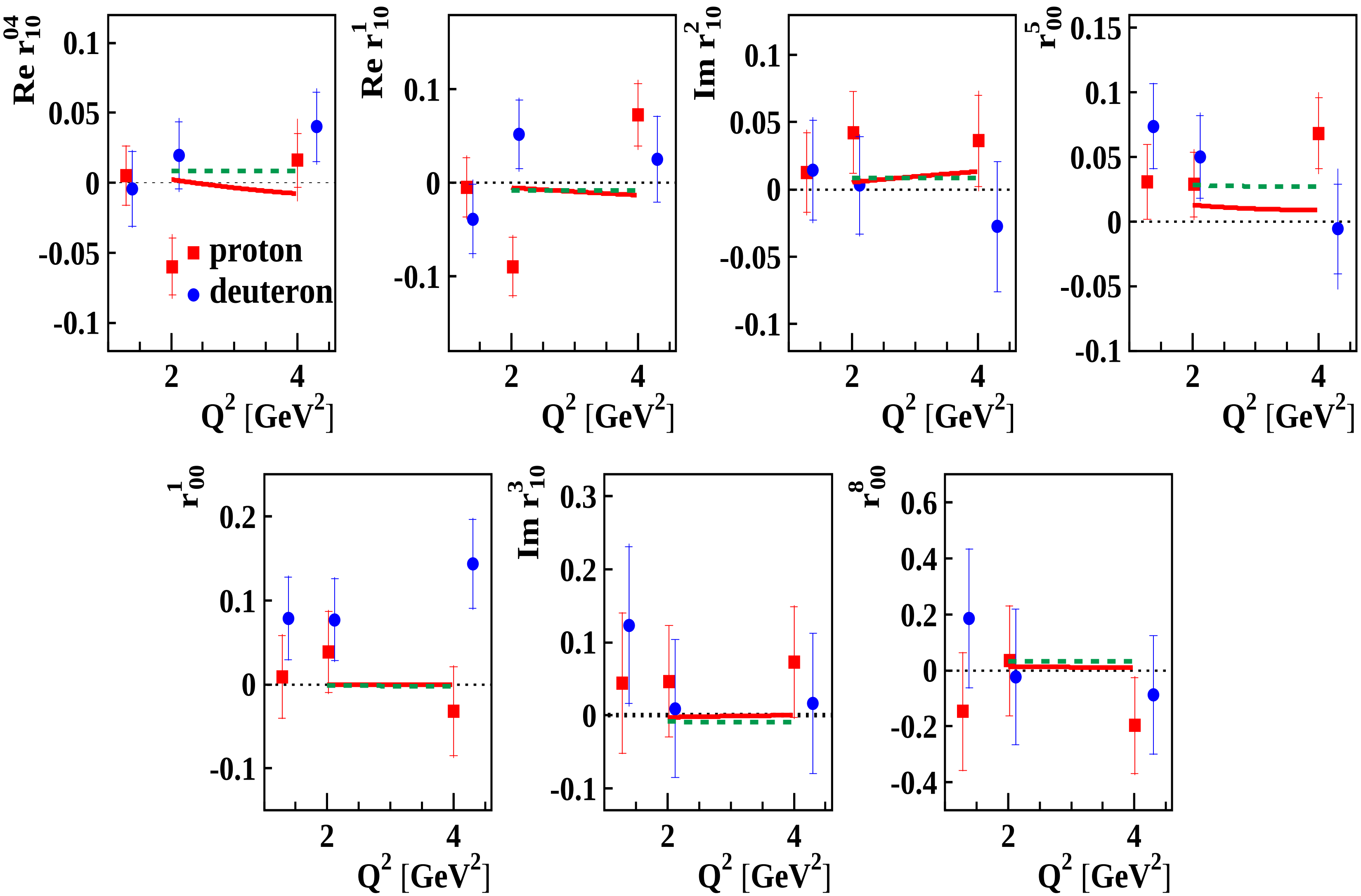
# Kinematic Dependences of SDMEs

- Kinematic dependences of class-B SDMEs



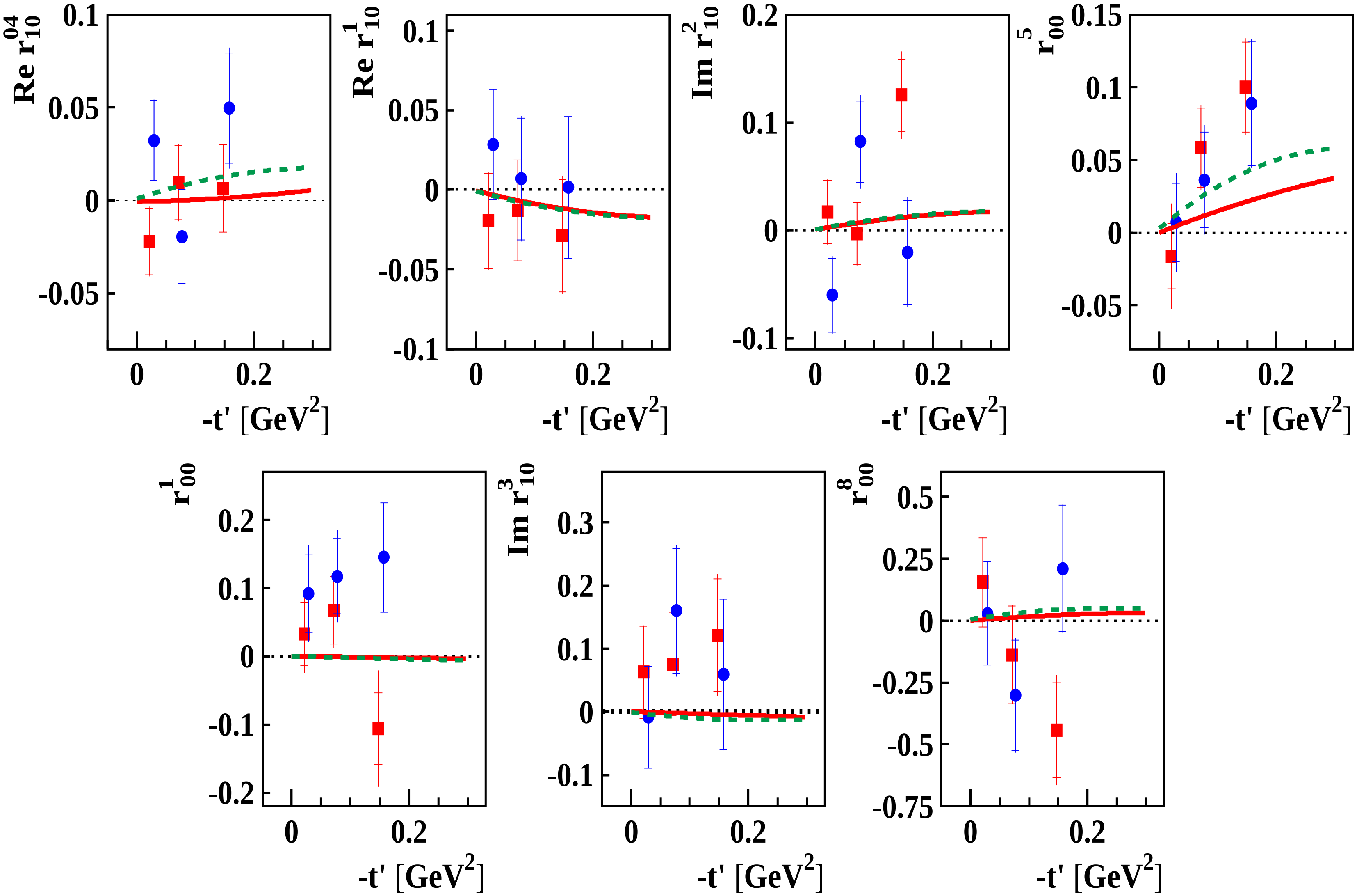

Red solid (green dashed) curves are obtained with (without) pion pole contribution in the Goloskokov-Kroll model.

- Kinematic dependences of class-C SDMEs



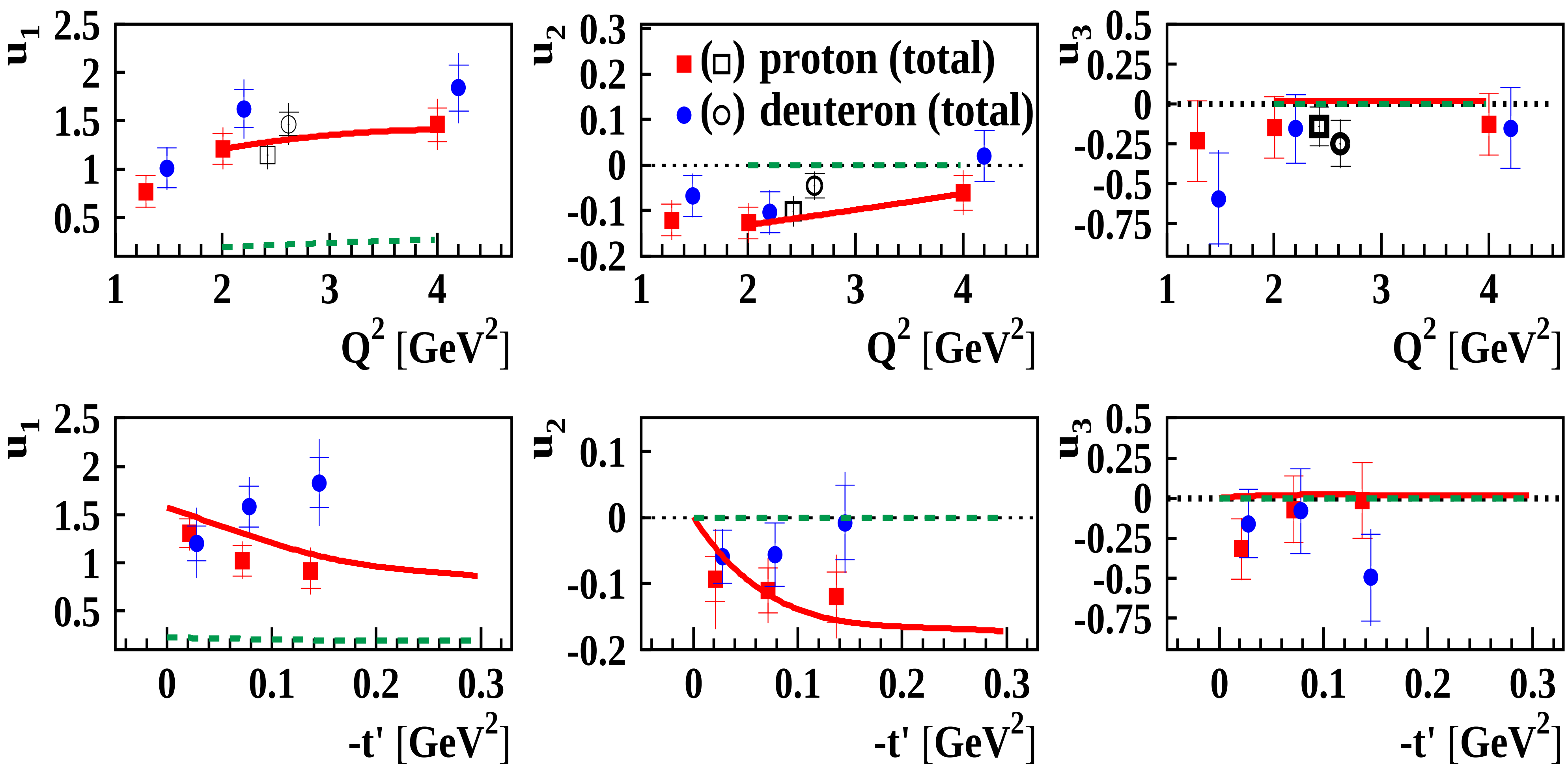

Red solid (green dashed) curves are obtained with (without) pion pole contribution in the Goloskokov-Kroll model.

- Kinematic dependences of class-C SDMEs




Red solid (green dashed) curves are obtained with (without) pion pole contribution in the Goloskokov-Kroll model.

# Test of Unnatural-Parity Exchange



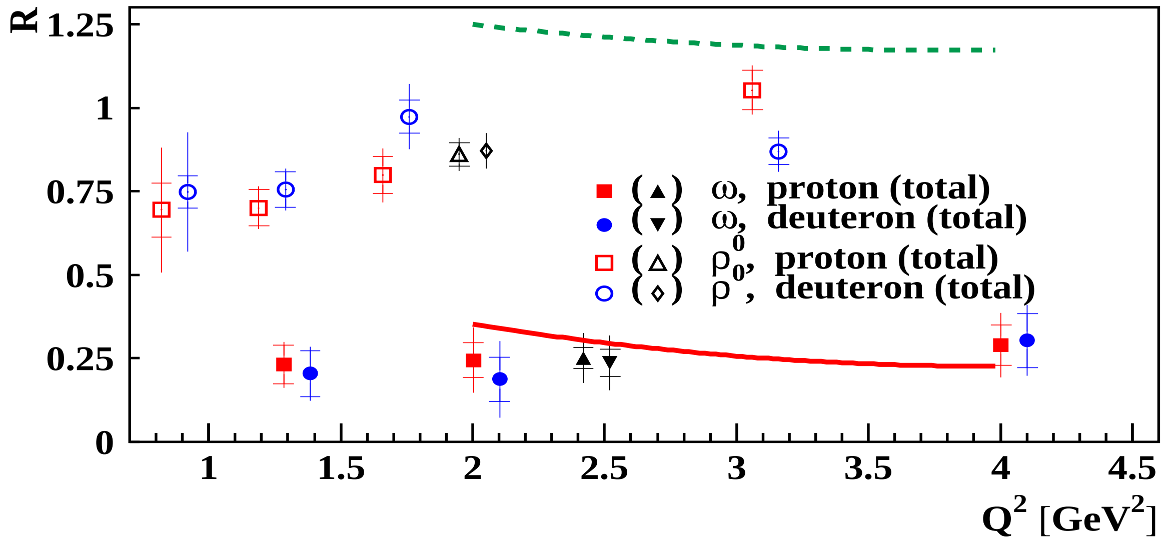

$u_1 = 1 - r^{04}_{00} + 2r^{04}_{1-1} - 2r^{1}_{11} - 2r^{1}_{1-1}$, $\quad u_1 = \widetilde{\sum} \frac{4\epsilon |U_{10}|^2 + 2|U_{11} + U_{-11}|^2}{\mathcal{N}}$, $\epsilon = \frac{N^{\gamma}_{L}}{N^{\gamma}_{T}} \approx 0.8$,

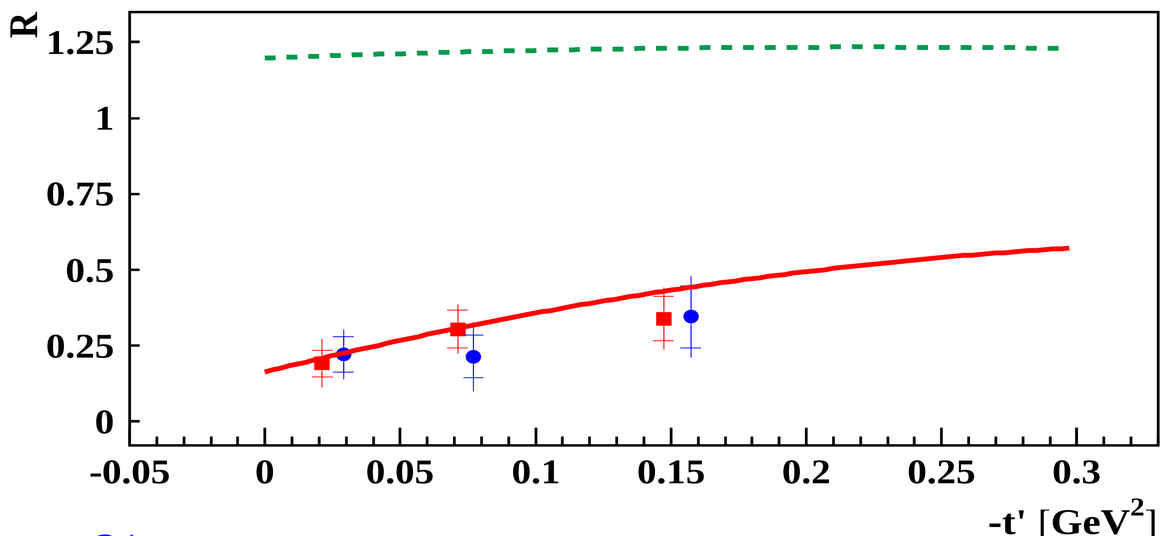
$u_2 = r^{5}_{11} + r^{5}_{1-1}$, $u_3 = r^{8}_{11} + r^{8}_{1-1}$, $\quad u_2 + iu_3 = \sqrt{2}\widetilde{\sum} \frac{(U_{11} + U_{-11})U^{*}_{10}}{\mathcal{N}}$.

Red solid (green dashed) curves are obtained with (without) pion pole contribution in the Goloskokov-Kroll model.

# Longitudinal-to-Transverse Cross-Section Ratio

Kinematic dependence of $R = d\sigma_L/d\sigma_T$

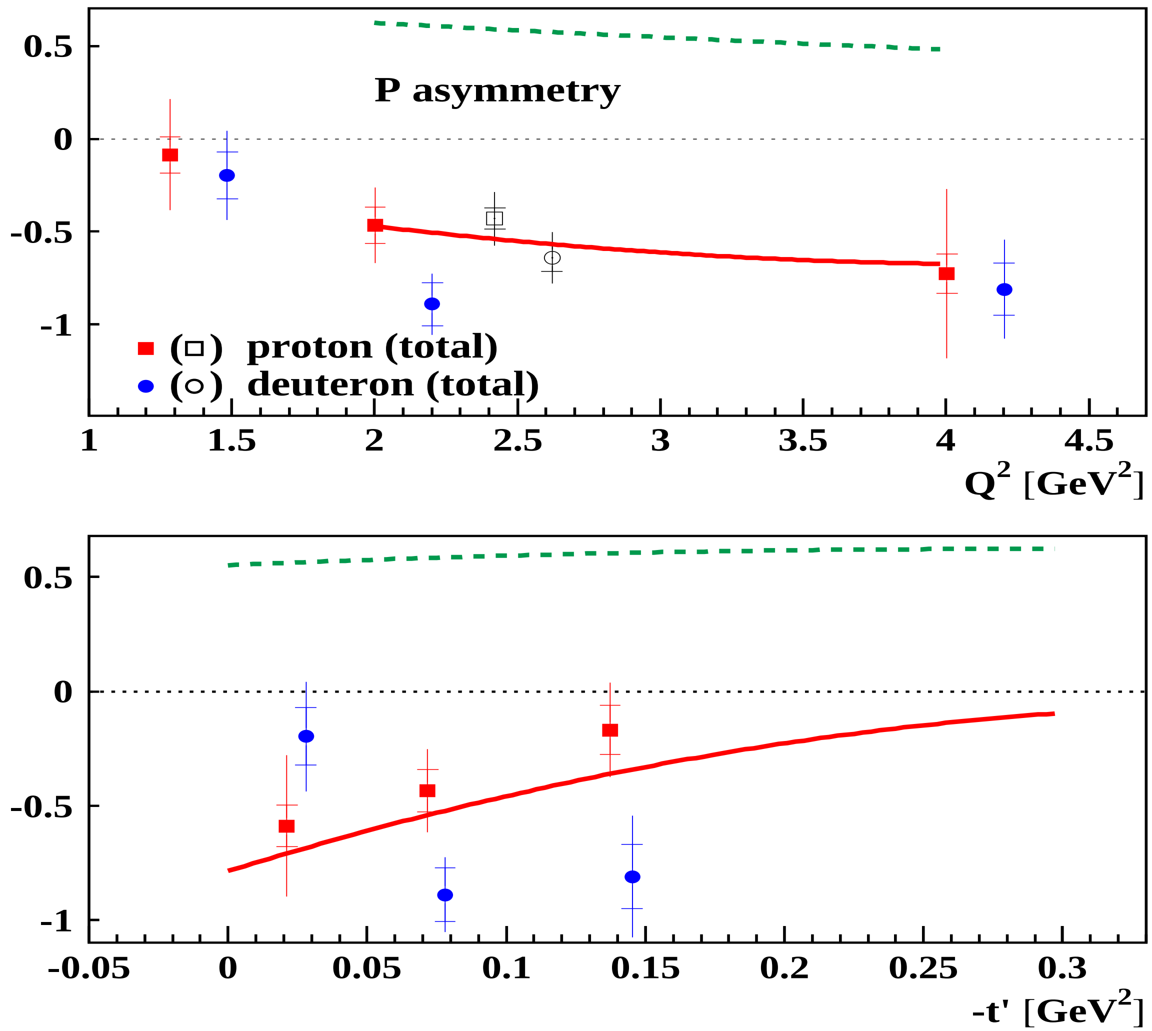



$$R = \frac{\widetilde{\sum}_{\lambda_\omega} |F_{\lambda_\omega 0}|^2}{\widetilde{\sum}_{\lambda_\omega} |F_{\lambda_\omega 1}|^2} \approx \frac{1}{\epsilon} \frac{r^{04}_{00}}{1 - r^{04}_{00}} \approx |T_{00}|^2 / [|T_{11}|^2 + |U_{11}|^2].$$

Curves show result of Goloskokov-Kroll calculations.

## UPE-to-NPE asymmetry



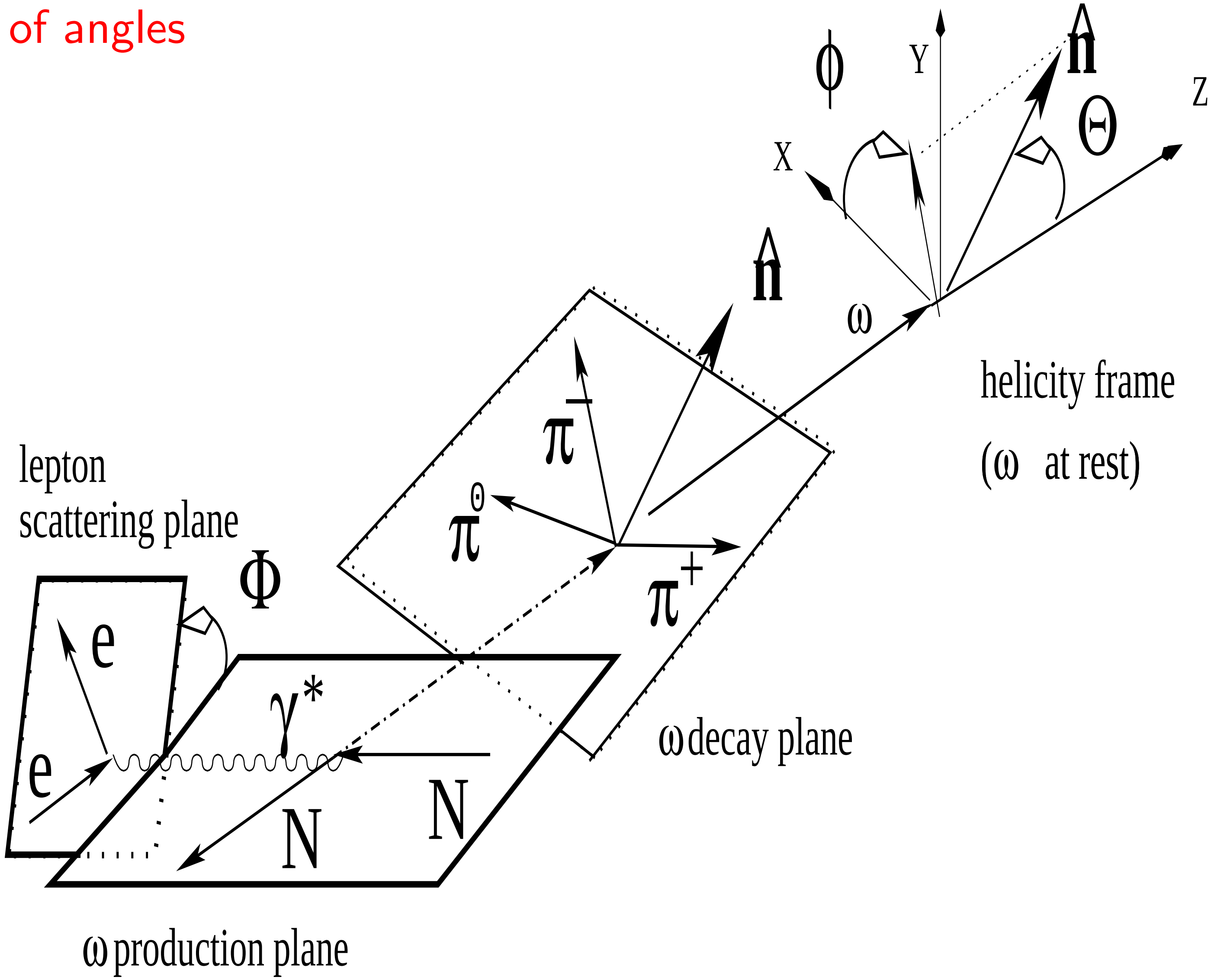

$$P = \frac{d\sigma_T^N - d\sigma_T^U}{d\sigma_T^N + d\sigma_T^U} \approx \frac{2r^1_{1-1} - r^1_{00}}{1 - r^{04}_{00}}$$

Curves show result of Goloskokov-Kroll calculations.

# Summary

- Exclusive $\omega$ electroproduction is studied at HERMES using a longitudinally polarized electron/positron beam and unpolarized hydrogen and deuterium targets in the kinematic region $Q^2 > 1.0$ GeV$^2$, $3.0$ GeV $< W < 6.3$ GeV, and $-t' < 0.2$ GeV$^2$.
- Using an unbinned maximum likelihood method, 15 unpolarized, and, for the first time, 8 polarized spin density matrix elements are extracted.
- No significant differences between proton and deuteron results are seen.
- While the values of class-A and B SDMEs agree with the hypothesis of $s$-channel helicity conservation, the class-C SDME $r^5_{00}$ indicates a violation of SCHC.
- Using the SDMEs $r^1_{1-1}$ and $\mathrm{Im}\{r^2_{1-1}\}$ it is shown that $|U_{11}|^2 > |T_{11}|^2$.
- The importance of UPE transitions is also shown by considering $u_1$, $u_2$, $u_3$ and $P$. This suggests that at HERMES energies $\pi^0$, $a_1$... exchanges play a significant role.
- The ratio $R = d\sigma_L/d\sigma_T$ between longitudinal and transverse virtual-photon cross-sections is determined to be $R = 0.25 \pm 0.03 \pm 0.07$ for the $\omega$ meson.
- Two possible hierachies of amplitudes are shown to correspond to the obtained SDME values. The most probable hierachy is $|U_{11}|^2 > |T_{00}|^2 \sim |T_{11}|^2$ $\gg |U_{10}|^2 \sim |T_{01}|^2 \sim |U_{01}|^2 \gg |T_{10}|^2, |T_{1-1}|^2, |U_{1-1}|^2$.

- Definition of angles



- Dependence of angular distribution on SDMEs

$$\mathcal{W}^{U+L}(\Phi,\phi,\cos\Theta) = \mathcal{W}^{U}(\Phi,\phi,\cos\Theta) + P_b\mathcal{W}^{L}(\Phi,\phi,\cos\Theta)$$

$$\mathcal{W}^{U}(\Phi,\phi,\cos\Theta) =$$
$$\frac{3}{8\pi^2}\Big[\frac{1}{2}(1-r^{04}_{00}) + \frac{1}{2}(3r^{04}_{00}-1)\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^{04}_{10}\}\sin 2\Theta\cos\phi - r^{04}_{1-1}\sin^2\Theta\cos 2\phi$$
$$-\ \epsilon\cos 2\Phi\Big(r^1_{11}\sin^2\Theta + r^1_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^1_{10}\}\sin 2\Theta\cos\phi - r^1_{1-1}\sin^2\Theta\cos 2\phi\Big)$$
$$-\ \epsilon\sin 2\Phi\Big(\sqrt{2}\mathrm{Im}\{r^2_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^2_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$
$$+$$
$$\sqrt{2\epsilon(1+\epsilon)}\cos\Phi\Big(r^5_{11}\sin^2\Theta + r^5_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^5_{10}\}\sin 2\Theta\cos\phi - r^5_{1-1}\sin^2\Theta\cos 2\phi\Big)$$
$$+\ \sqrt{2\epsilon(1+\epsilon)}\sin\Phi\Big(\sqrt{2}\mathrm{Im}\{r^6_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^6_{1-1}\}\sin^2\Theta\sin 2\phi\Big)\Big],$$

$$\mathcal{W}^{L}(\Phi,\phi,\cos\Theta) = \frac{3}{8\pi^2}\Big[\sqrt{1-\epsilon^2}\Big(\sqrt{2}\mathrm{Im}\{r^3_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^3_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$
$$+\ \sqrt{2\epsilon(1-\epsilon)}\cos\Phi\Big(\sqrt{2}\mathrm{Im}\{r^7_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^7_{1-1}\}\sin^2\Theta\sin 2\phi\Big)$$
$$+\ \sqrt{2\epsilon(1-\epsilon)}\sin\Phi\Big(r^8_{11}\sin^2\Theta + r^8_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^8_{10}\}\sin 2\Theta\cos\phi -$$
$$r^8_{1-1}\sin^2\Theta\cos 2\phi\Big)\Big].$$

# Unbinned Maximum Likelihood Method

- Calculation of "background" SDMEs
  Two Monte Carlo (MC) sets.
  First (normalization) MC set: uniform angular distribution ($\cos\Theta$, $\Phi$, $\phi$). Number of events is $N_{MC}$.
  Second (background pseudo-data) MC set for calculation of a set $S_{bg}$ of 15 background SDMEs. Number of events $N_{PD}$.
  Log-likelihood function for background pseudo-data events for unpolarized (U) beam
  $-\ln L(S_{bg}) = -\sum_{i=1}^{N_{PD}} \ln \frac{\mathcal{W}^U(S_{bg},\Phi_i,\phi_i,\cos\Theta_i)}{\widetilde{\mathcal{N}}_{bg}(S_{bg})}$,
  $\widetilde{\mathcal{N}}_{bg}(S_{bg}) = \sum_{j=1}^{N_{MC}} \mathcal{W}^U(S_{bg},\Phi_j,\phi_j,\cos\Theta_j)$.

- Calculation of physical SDMEs
  $N$ total number of experimental events in exclusive region.
  $S$ set of 23 SDMEs for unpolarized target and longitudinally (L) polarized beam.
  $-\ln L(S) = -\sum_{i=1}^{N} \ln\left[\frac{(1-f_{bg})*\mathcal{W}^{U+L}(S,\Phi_i,\phi_i,\cos\Theta_i)}{\widetilde{\mathcal{N}}(S,S_{bg})} + \frac{f_{bg}*\mathcal{W}^U(S_{bg},\Phi_i,\phi_i,\cos\Theta_i)}{\widetilde{\mathcal{N}}(S,S_{bg})}\right]$
  $f_{bg}$ fraction of background events in experimental events in exclusive region.
  The total normalization factor
  $\widetilde{\mathcal{N}}(S,S_{bg}) =$
  $\sum_{j=1}^{N_{MC}}[(1-f_{bg})*\mathcal{W}^{U+L}(S,\Phi_j,\phi_j,\cos\Theta_j) + f_{bg}*\mathcal{W}^U(S_{bg},\Phi_j,\phi_j,\cos\Theta_j)]$

# Unbinned Maximum Likelihood Method

- No background corrections

$\ln \mathcal{L} = \sum_i^I \ln[\mathcal{W}(\mathcal{R}, (P_b)_i, (P_T)_i, \Phi_i, \Psi_i, \theta_i, \varphi_i))/N_i]$,

$N_i = K_1 + K_2(P_b)_i + K_3(P_T)_i + K_4(P_b)_i(P_T)_i$

$(P_b)_i$ beam polarization, $(P_T)_i$ target polarization for $i$-th event,
$\mathcal{R}$ set of amplitude ratios.

$N_{++} = \frac{1}{L}\sum_{m=1}^L \mathcal{W}(\mathcal{R}, (P_b = 1), (P_T = 1), \Phi_m, \Psi_m, \theta_m, \varphi_m)$

$N_{+-}$ corresponds to $P_b = 1$, $P_T = -1$, $N_{-+}$ to $P_b = -1$, $P_T = 1$ etc.
$K_1$, $K_2$, $K_3$, and $K_4$ are linear combinations of $N_{++}$, $N_{+-}$, $N_{-+}$, and $N_{--}$.

- Likelihood function with background corrections

$\ln \mathcal{L}_{tot} = \sum_i^I \ln\left[(1 - f_{bg})\frac{\mathcal{W}(\mathcal{R},(P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{N_i} + f_{bg}\frac{\mathcal{W}_{bg}((P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{N_i^{bg}}\right]$

Angular distribution, $\mathcal{W}_{bg}$ of background events is assumed to be independent of polarizations $P_b$ and $P_T$, $f_{bg}$ fraction of reconstructed background events.

$N_i^{bg} = \frac{1}{L}\sum_{m=1}^L \mathcal{W}_{bg}((P_b = 0), (P_T = 0), \Phi_m, \Psi_m, \theta_m, \varphi_m)$

$\ln \mathcal{L}_{tot} = \sum_i^I \ln\left[\frac{(1-g_{bg})\mathcal{W}(\mathcal{R},(P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)N_i + g_{bg}\mathcal{W}_{bg}((P_b)_i,(P_T)_i,\Phi_i,\Psi_i,\theta_i,\varphi_i)}{(1-g_{bg})N_i + g_{bg}N_i^{bg}}\right]$

$g_{bg}$ is the fraction of background in $4\pi$ (before interaction of particles with detector)