Polarized Inclusive Structure Functions and Fits to g_1 Data

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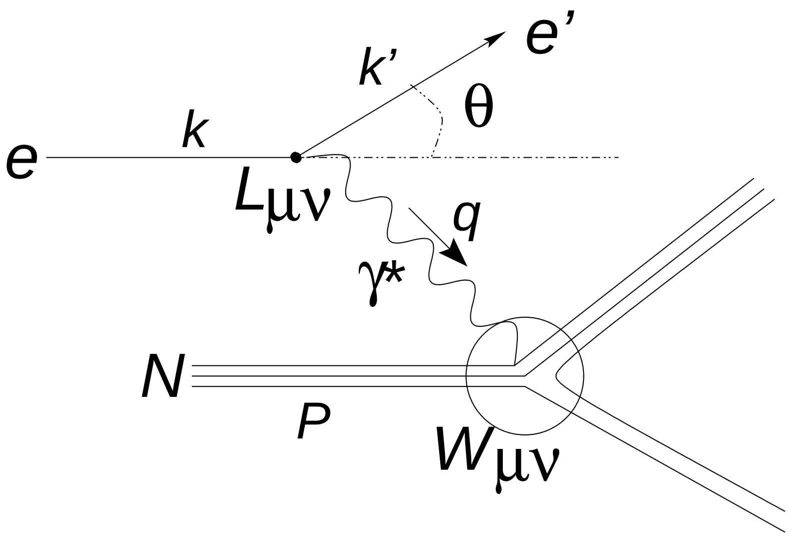
- ⋆ DIS and Inclusive Physics
- ⋆ Experiments
- ⋆ Structure Functions g_1, g_2, b_1
- ⋆ QCD Fits and Polarized Parton Distributions

Deep Inelastic Scattering



$Q^2 = -q^2 = 4EE' \sin^2\left(\frac{\theta}{2}\right)$ 4-momentum transfer

$x = Q^2/(2M\nu)$ momentum fraction carried by the struck parton

$y = (E - E')/E$ fraction of momentum transfer

$$\frac{d^2\sigma}{d\Omega dE} = \frac{\alpha^2}{2M_p Q^4}\frac{E'}{E}\mathbf{L}_{\mu\nu}\mathbf{W}^{\mu\nu}$$

$\mathbf{L}_{\mu\nu}$ is exact in QED

$$\mathbf{W}^{\mu\nu} = -g^{\mu\nu}\mathbf{F_1} + \frac{p^\mu p^\nu}{\nu}\mathbf{F_2} + \frac{i}{\nu}\epsilon^{\mu\nu\lambda\sigma}q^\lambda s^\sigma \mathbf{g_1} + \frac{i}{\nu^2}\varepsilon^{\mu\nu\lambda\sigma}q^\lambda(p\cdot q s^\sigma - s\cdot q p^\sigma)\mathbf{g_2} -$$
$$\underbrace{-r_{\mu\nu}\mathbf{b_1} + \frac{1}{6}(s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu})\mathbf{b_2} + \frac{1}{2}(s_{\mu\nu} - u_{\mu\nu})\mathbf{b_3} + \frac{1}{2}(s_{\mu\nu} - t_{\mu\nu})\mathbf{b_4}}_{\text{spin 1}}$$

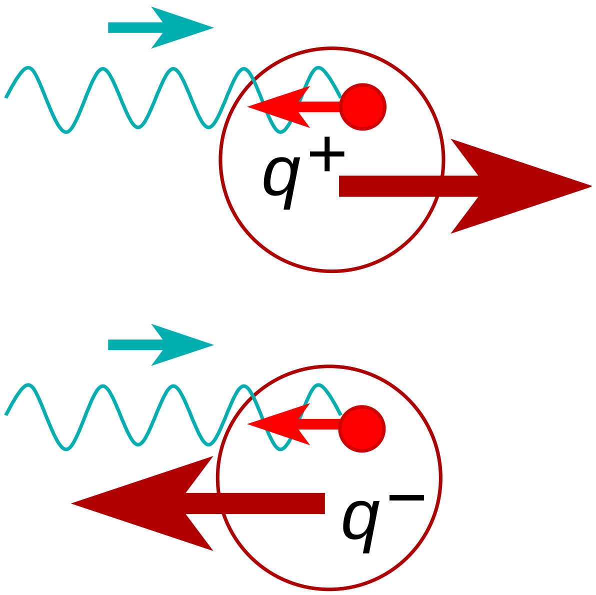
The hadronic tensor has to be parameterized introducing $\mathbf{F_1}$, $\mathbf{F_2}$ (unpolarized structure functions) and $\mathbf{g_1}$, $\mathbf{g_2}$, $\mathbf{b_1}$, ... (polarized structure functions).

$r_{\mu\nu}, s_{\mu\nu}, t_{\mu\nu}, u_{\mu\nu}$ dependent on target tensor polarization

High Energy Spin Physics Experiments

Lab	Experiment	Year	Beam			Target			Measure
			Energy	Type	P_B	Type	P_T	f	
SLAC	E80	75	10-16 GeV	e^-	0.85	H-butanol	0.50	0.13	A_1^p
	E130	80	16-23 GeV	e^-	0.81	H-butanol	0.58	0.15	A_1^p
	E142	92	19-26 GeV	e^-	0.39	^{3}He	0.35	0.35	g_1^n
	E143	93	10-29 GeV	e^-	0.85	NH_3	0.70	0.15	g_1^p
	"	"	"	"	"	ND_3	0.25	0.24	g_1^d
	E154	95	48 GeV	e^-	0.83	^{3}He	0.38	0.55	g_1^n
	E155	97	48 GeV	e^-	0.81	NH_3	0.80	0.15	g_1^p
	"	"	"	"	"	LiD	0.22	0.36	g_1^d
	E155X	99	29/32 GeV	e^-	0.81	NH_3	0.70	0.16	g_2^p
	"	"	"	"	"	LiD	0.22	0.36	g_2^d
CERN	EMC	85	200 GeV	μ^+	0.79	NH_3	0.78	0.16	g_1^p
	SMC	92	100 GeV	μ^+	0.81	D-butanol	0.40	0.19	$g_1^{d,n}$
	"	93	190 GeV	μ^+	0.80	H-butanol	0.86	0.12	g_1^p, g_2^p
	"	94/95	"	μ^+	0.80	D-butanol	0.50	0.20	g_2^d
	"	96	"	μ^+	0.80	NH_3	0.89	0.16	g_1^p
DESY	HERMES	95	28 GeV	e^+	0.55	^{3}He	0.46	1.00	g_1^n
	"	96/97	"	"	"	H	0.88	1.00	g_1^p
	"	98 (99/00)	"	e^- (e^+)	"	D	0.85	1.00	g_1^d, b_1^d
	"	≥ 01	"	$e^\pm$	"	H	0.85	1.00	
CERN	COMPASS	≥ 01	160 GeV	μ^+	0.80	NH_3	0.90	0.16	
		"	"	"	0.50	LiD	0.50	0.50	
BNL	RHIC	≥ 01	200 GeV	p	0.70	200 GeV p	0.70	1.0	

The structure function g_1



In the quark parton model:

$$\mathbf{g_1}(x) = \tfrac{1}{2} \Sigma_f \, e_f^2 \, \Delta q_f(x)$$

$$\mathbf{g_1^d} = \tfrac{1}{2}(\mathbf{g_1^p} + \mathbf{g_1^n}) \left(1 - \tfrac{3}{2}\omega_d\right)$$

with $\omega_d = 0.05 \pm 0.01$

$$\mathbf{g_1^{^3He}} = \rho_n \mathbf{g_1^n} + 2\rho_p \mathbf{g_1^p}$$

with $\rho_n = 0.86 \pm 0.02$

and $\rho_p = -0.028 \pm 0.004$

$$A_{||} = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} = \frac{1}{P_b P_t} \frac{N^{\uparrow\downarrow} L^{\uparrow\uparrow} - N^{\uparrow\uparrow} L^{\uparrow\downarrow}}{N^{\uparrow\downarrow} L^{\uparrow\uparrow} + N^{\uparrow\uparrow} L^{\uparrow\downarrow}}$$ measured asymmetry

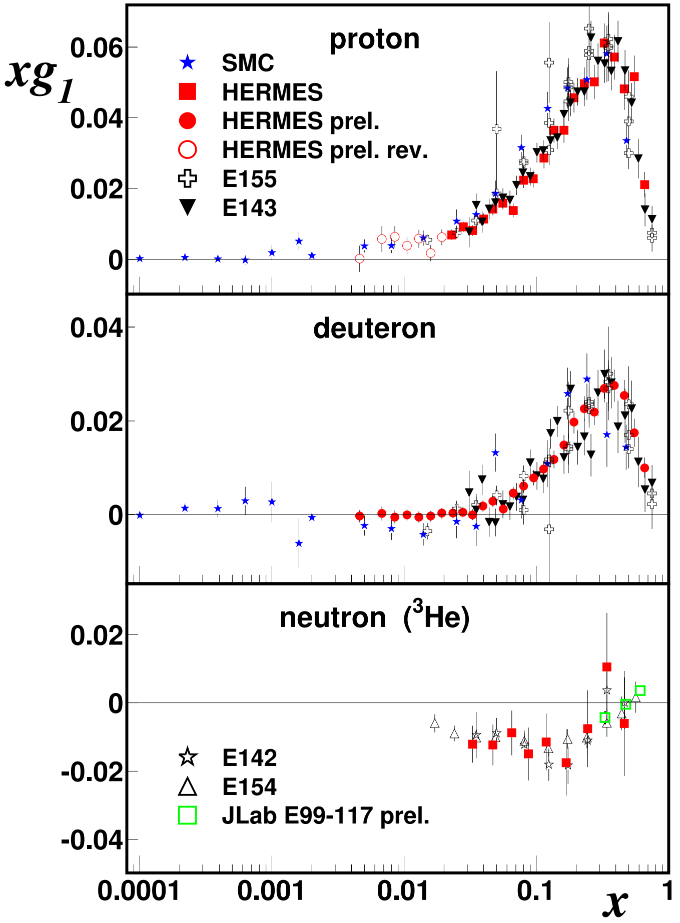
$$\frac{\mathbf{g_1}}{\mathbf{F_1}} = \frac{1}{1+\gamma^2} \left[\frac{A_{||}}{D} + (\gamma - \eta) A_2 \right]$$ structure function ratio

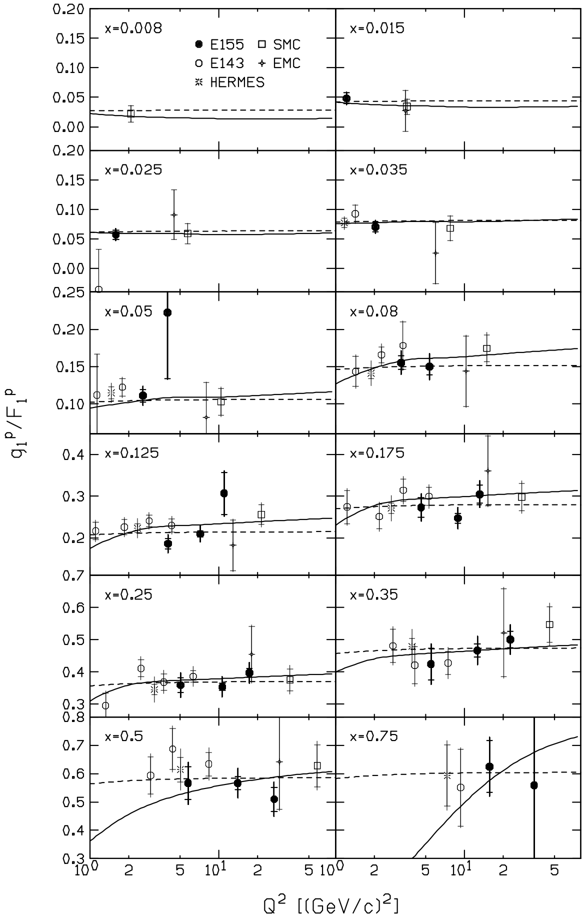
where D, γ, η are kinematic factors.

$$\mathbf{g_1} = \left(\frac{\mathbf{g_1}}{\mathbf{F_1}} \right)_{meas} \cdot \mathbf{F_1}^{param}$$ extraction of g_1

$F_1 = \frac{(1+\gamma^2) F_2}{2x(1+R)} \rightarrow F_2$: Nucl.Phys.**B364** (1995) 107, $R = \sigma_L / \sigma_T$: Phys.Lett. **B250** (1990) 193

World data on g_1





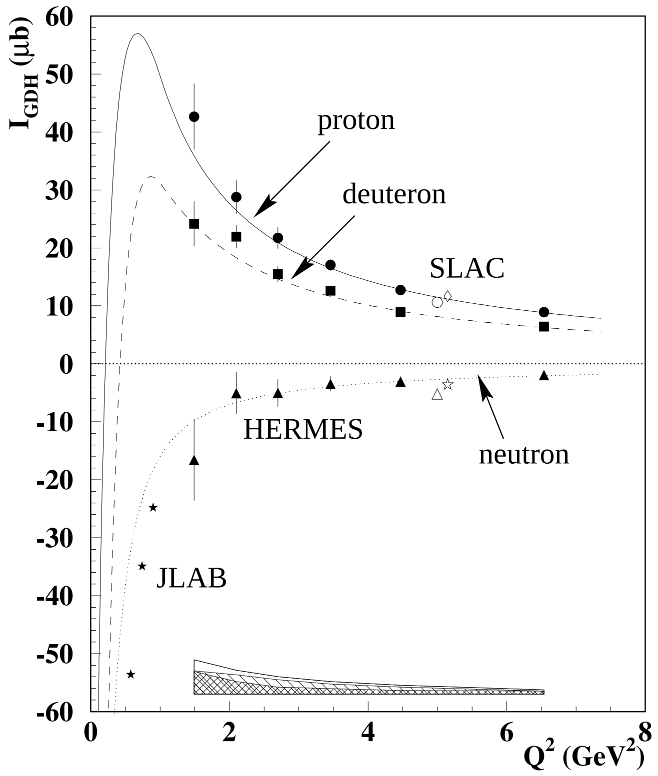
E155, Phys.Lett.**B493** (2000) 19

E155 fit to g_1^p/F_1^p data:

$$\frac{g_1^p}{F_1^p} = x^{0.7}(0.817 + 1.014x - 1.489x^2)\left(1 - \frac{0.04\pm0.06}{Q^2}\right)$$

Within experimental errors g_1 and F_1 have the same Q^2 evolution.

GDH Sum Rule



$$\boxed{\int_{\nu_0}^{\infty}[\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)]\frac{d\nu}{\nu} = -\frac{2\pi^2\alpha}{M^2}k^2}$$

Q^2 Generalization

$$\Longrightarrow$$

$$I_{GDH}(Q^2) = \int_{\nu_0}^{\infty}(\sigma_{\uparrow\downarrow}(\nu, Q^2) - \sigma_{\uparrow\uparrow}(\nu, Q^2))\frac{d\nu}{\nu}$$

$$= \frac{8\pi^2\alpha}{M}\int_0^{x_0}\frac{\mathbf{g_1}(x, Q^2) - \gamma^2\mathbf{g_2}(x, Q^2)}{K}\frac{dx}{x}$$

$$I_{GDH}^{\gamma\to 0}(Q^2) = \frac{16\pi^2\alpha}{Q^2}\Gamma_1 \qquad (\Gamma_1 = \int \mathbf{g_1}dx)$$

Eur.Phys.J.**C26** (2003) 527

It connects static ground state properties with helicity structure as measured in DIS and in the Resonance Region.

Experiments done with real (MAMI in Mainz) and virtual (HERMES, JLAB, SLAC) photons.

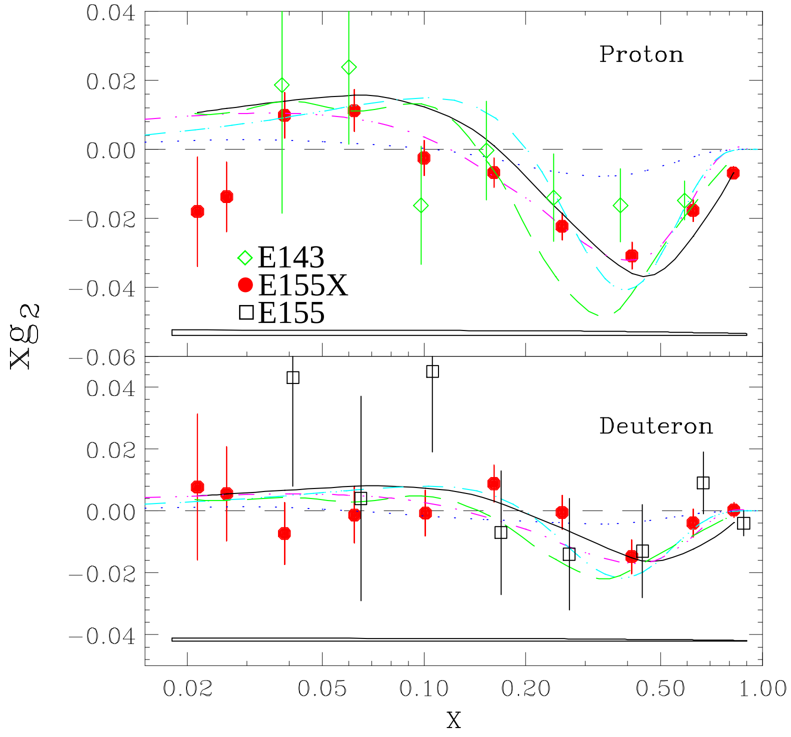
The structure function $\mathbf{g}_2$

It's measured with transversely polarised targets.

Measurements by E155X are consistent with a Wandzura and Wilczek contribution alone:

$$\mathbf{g_2}(x,Q^2) = \underbrace{\mathbf{g_2}(x,Q^2)^{WW}(x,Q^2)}_{\text{twist } 2} - \int_x^1 \frac{\partial}{\partial y}\left(\frac{m}{M}\underbrace{h_T(y,Q^2)}_{\text{twist } 2} + \underbrace{\xi(y,Q^2)}_{\text{twist } 3}\right)\frac{dy}{y} = \mathbf{g_2}^{WW} - \bar{g}_2$$

$$\mathbf{g_2}^{WW}(x,Q^2) = -\mathbf{g_1}(x,Q^2) + \int_x^1 \mathbf{g_1}(y,Q^2)\frac{dy}{y}$$



Phys.Lett **B553** (2003) 18.

$$d_2 = 3\int_0^1 dx x^2 \bar{g}_2(x,Q^2) = \begin{array}{l} 0.0032 \pm 0.0017\ \text{(p)} \\ 0.0079 \pm 0.0048\ \text{(d)} \end{array} \quad \text{(SLAC avgd)}$$

Burkhardt-Cottingham Sum Rule: $\int_0^1 \mathbf{g}_2 dx = 0 \quad \rightsquigarrow$

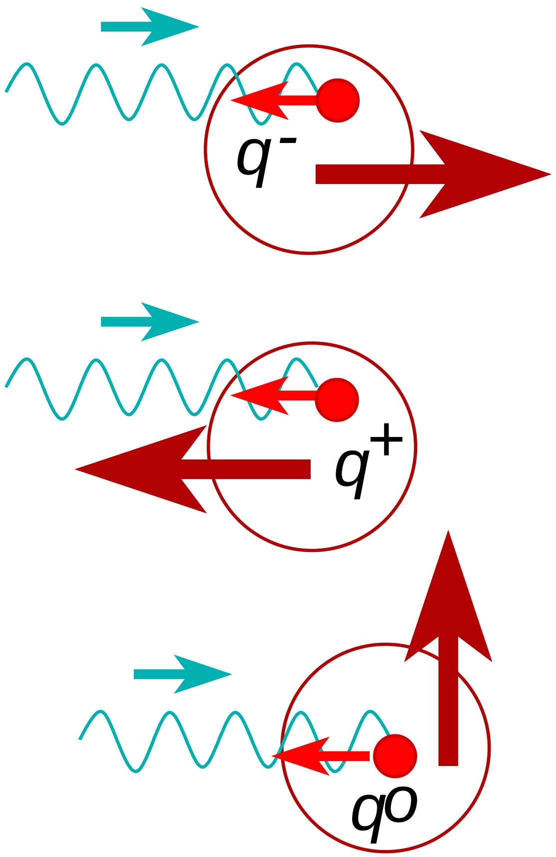
$$\int_{x\ \text{meas}} \mathbf{g}_2 dx = \begin{array}{l} -0.042 \pm 0.008\ \text{(p)} \\ -0.006 \pm 0.011\ \text{(d)} \end{array} \quad \text{(SLAC avgd)}$$

Efremov-Leader-Teryaev S.R $\int_0^1 x(g_1^v + 2g_2^v) = 0 \quad \rightsquigarrow$

$$\int_0^1 x(g_1^p + 2g_2^p - g_1^n + g_2^n) = 0$$

$$\int_{\text{meas}} = -0.011 \pm 0.008$$

The structure function b_1 (spin 1 targets)



In the quark-parton model:

$$\mathbf{b_1}(x) = \frac{1}{2}\sum_f e_f^2[2q_f^0 - (q_f^+ + q_f^-)]$$

b_1 enters in $W_{\mu\nu}^{SYM}$

$\rightsquigarrow$ not sensitive to P_b and D

If $b_1 \neq 0$ then the asymmetry $A_{||}$ has to be corrected:

$$A_{||} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} = \frac{D\frac{\mathbf{g_1}}{\mathbf{F_1}}}{1 - \frac{1}{3}\frac{\mathbf{b_1}}{\mathbf{F_1}}}$$

It can be studied in spin-1 targets as D, ^{6}Li, ^{14}N.

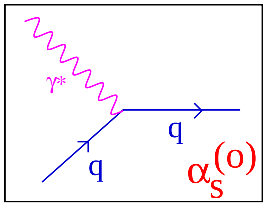
Sum rule:

$$\int \mathbf{b_1} dx = \frac{1}{6}\delta Q_{sea}$$

First direct measurement from HERMES which shows that $\mathbf{b_1}$ is positive, growing towards low x and has a $\simeq 1\%$ effect on the extraction of $\mathbf{g_1^d}$.

$\Rightarrow$ Talk by Michelle Stancari

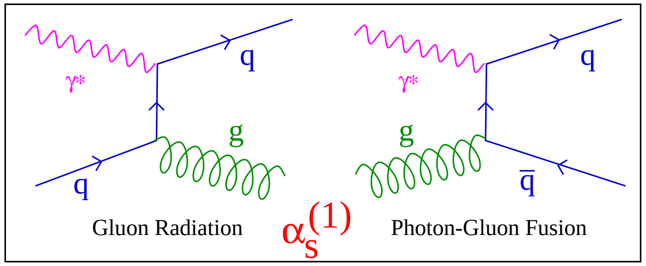
Beyond the Naive Parton Model



$\Longrightarrow$ No gluons

$$g_1^0(x) = \frac{1}{2}\sum e_q^2\,\Delta q(x)$$

$$\Downarrow$$



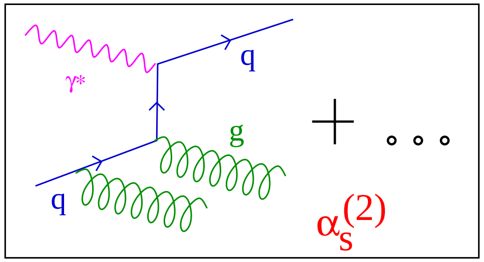
$\Longrightarrow$ Quarks are re-defined

with the inclusion of ΔG

(weak dependence)

$$g_1^{LO}(x,Q^2) = \frac{1}{2}\sum e_q^2\,\Delta q(x,Q^2)$$

$$\Downarrow$$



$\Longrightarrow$ g_1 becomes

explicitly ΔG dependent

$$g_1^{NLO}(x,Q^2) = g_1^{LO} + \frac{\alpha_s}{2\pi}\,\frac{1}{2}\sum e_q^2\left[\Delta q(x,Q^2)\otimes C_q + \Delta G(x,Q^2)\otimes C_G\right]$$

Splitting Functions and Evolution Equations

In NLO there are two independent NS distributions, plus $\Delta\Sigma$ and ΔG:

$$\Delta q^p_{NS} = \tfrac{1}{2}\,(2\Delta u - \Delta d - \Delta s)$$
$$\Delta q^n_{NS} = \tfrac{1}{2}\,(2\Delta d - \Delta u - \Delta s)$$
$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

Their Q^2 dependence is regulated by the evolution equations:

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix}P^S_{qq} & 2n_f P_{qG} \\ P^S_{Gq} & P_{GG}\end{pmatrix} \otimes \begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$
$$\frac{d}{d\ln Q^2}\Delta q_{NS} = \frac{\alpha_s}{2\pi} P^{NS}_{qq} \otimes \Delta q_{NS}$$

- Each distribution is parameterized at a starting Q^2_0:

$$\Delta f = N_f\,\eta\,x^{\alpha}\,(1-x)^{\beta}\,(\,1 + \gamma x + \rho\sqrt{x}\,)$$

- It is evolved at Q^2_{meas} using the evolution equations, where g_1^{calc} is calculated.
- The χ^2 is minimized:

$$\chi^2 = \sum_{\text{data}} \frac{(\mathbf{g_1}^{\text{meas}} - \mathbf{g_1}^{\text{calc}})^2}{\sigma^2_{\text{stat}}}$$

- The parameters α, β, γ... are evaluated.

QCD fits

	$\Delta f(x, Q_0^2)$	
BB	$\eta A x^{\alpha}(1-x)^{\beta}(1+\gamma x)$	Nucl.Phys. **B636** (2002) 225
LSS	$\eta A x^{\alpha+1} f_{MRST99}$	Eur.Phys.J.**C23** (2002) 479
GRSV	$\eta A x^{\alpha}(x-1)^{\beta} f_{GRV98}$	Phys. Rev.**D63** (2001) 094005
AAC	$A x^{\alpha}(1+\gamma x^{\lambda}) f_{GRV98}$	Phys. Rev.**D62** (2000) 034017

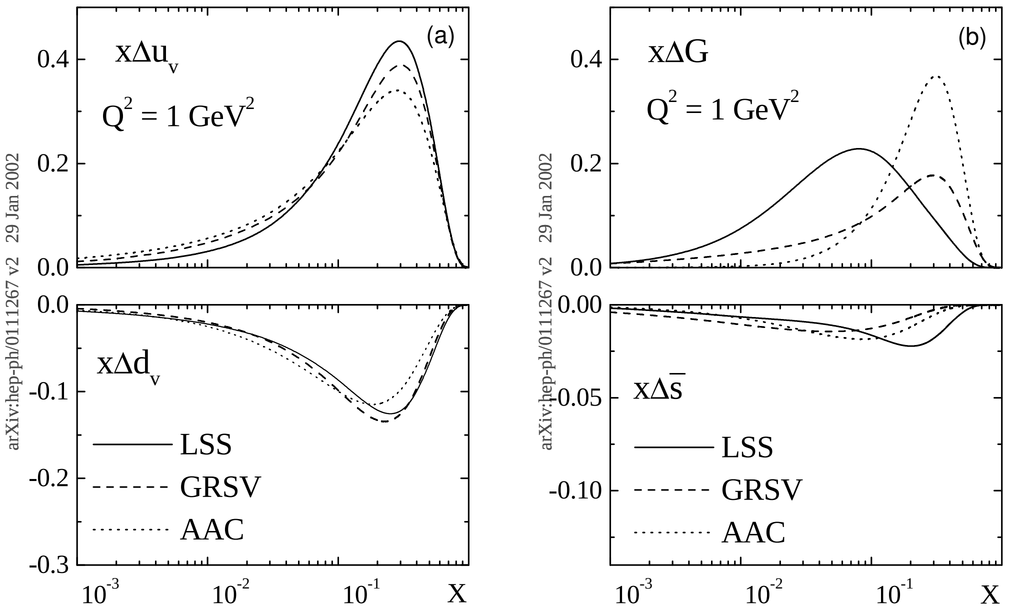
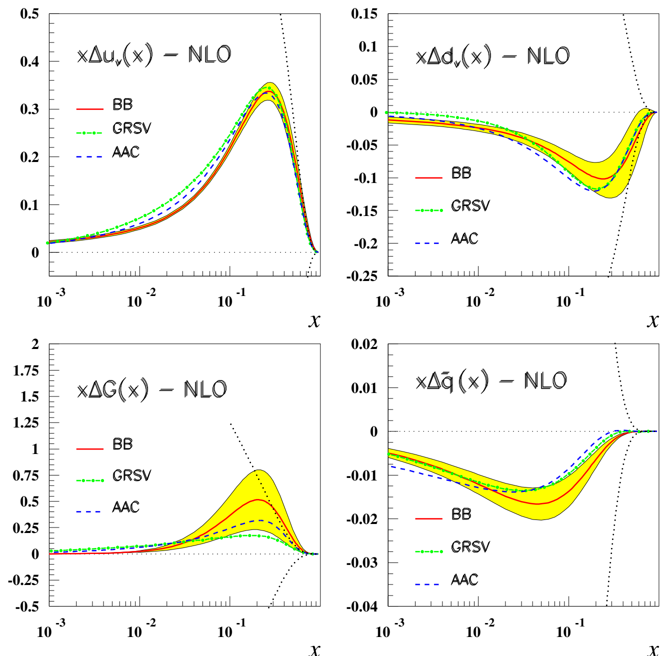


Fig. 8 (a) Fig. 8(b)



Eur.Phys.J.**C23** (2002) 479

Nucl.Phys. **B636** (2002) 225

Results show that ΔG and Δs are not strongly constrained by data, while Δu_v and Δd_v are well determined

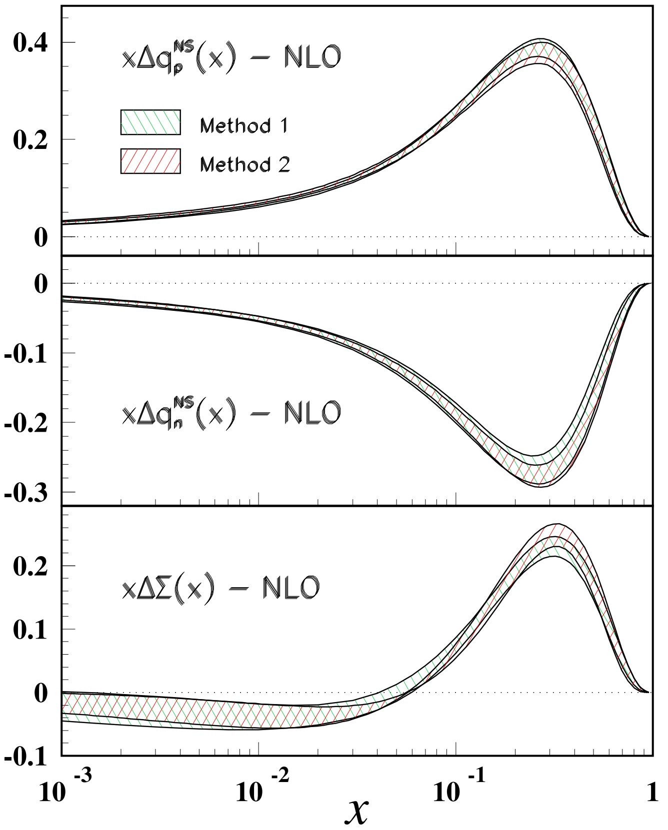
HERMES QCD fits (Parameterizations)

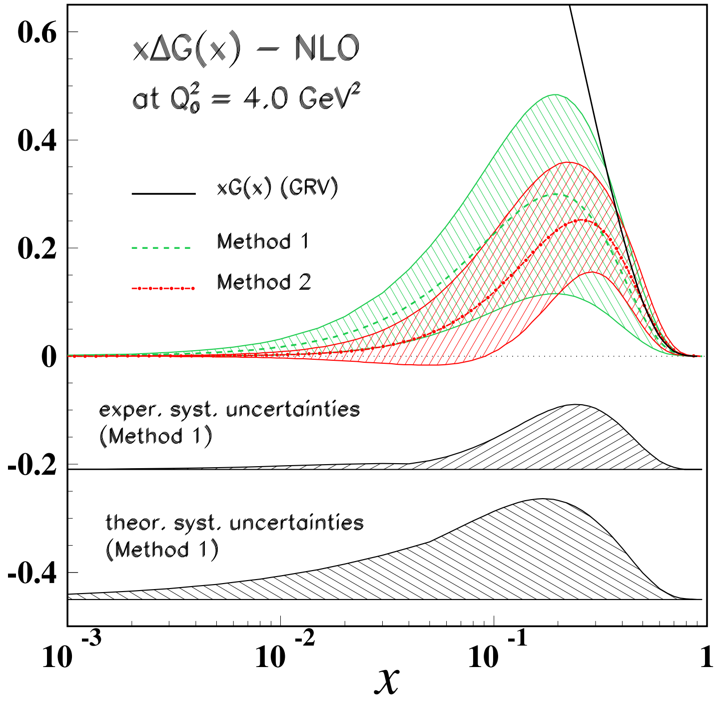
$$x\Delta q_i(x,Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x+\rho_i x^{\frac{1}{2}})$$

Method 1	Method 2
$\overline{MS}$	$\overline{MS}$
Mellin Transform	Finite differences
$\Delta u_v, \Delta d_v, \Delta\bar{q}_s, \Delta G$	$\Delta q^p_{NS}, \Delta q^n_{NS}, \Delta\Sigma, \Delta G$
symmetric sea: $\Delta\bar{q}_s =$ $\Delta\bar{u}_s = \Delta\bar{d}_s = \Delta s = \Delta\bar{s}$	no assumption (in the fit)
η_{u_v}, η_{d_v} fixed by F,D $\gamma_{u_v}, \gamma_{d_v} \neq 0$ fixed $\left.\begin{array}{l} a_G = a_{sea}+1 \\ \left.\frac{b_{\bar{q}_s}}{b_G}\right\vert_{pol} = \left.\frac{b_{\bar{q}_s}}{b_G}\right\vert_{unpol}\end{array}\right\\} *$ $b_{\bar{q}_s} = 8.08, b_G = 5.61$ $\gamma_{\bar{q}_s} = 0, \gamma_G = 0$ $\rho = 0$ for all densities $\rightarrow$ 7 fit parameters	$\eta_{q_p^{NS}}, \eta_{q_n^{NS}}$ fixed by F,D $\gamma_{q_p^{NS}} = \gamma_{q_n^{NS}} \neq 0$ fixed $\\{$ no such relations $b_G = 5.61$ $\gamma_\Sigma \neq 0$ fixed, $\gamma_G = 0$ $\rho = 0$ for all densities $\rightarrow$ 7 fit parameters
$\Lambda^{(4)}_{QCD} = 291 \pm 30\ MeV$ $Q_0^2 = 4\ GeV^2$ data: $Q^2 > 1\ GeV^2$	$\alpha_s(M_Z^2) = 0.117 \pm 0.002$ $Q_0^2 = 4\ GeV^2$ data: $Q^2 > 1\ GeV^2$

* lead to positivity for $\Delta\bar{q}_s$ and ΔG

HERMES QCD FITS (Results)





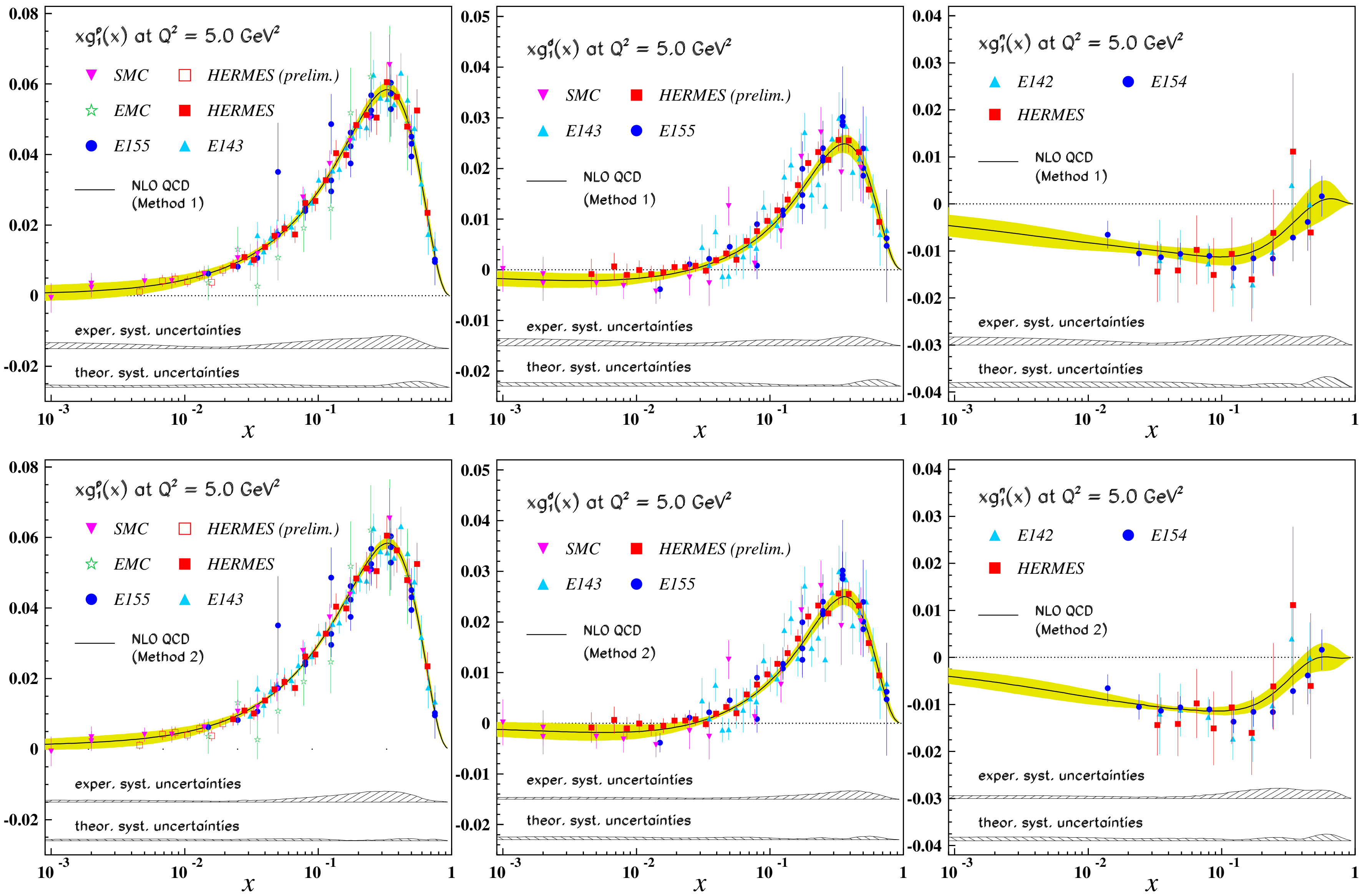
$\Delta G = 0.616 \pm 0.388$(stat) (method 1)

$\Delta G = 0.408 \pm 0.228$(stat) (method 2)

Singlet and non-singlet distributions agree very well.

The two gluon distributions agree within errors. Differences may be due to slightly different assumptions

HERMES QCD fits (Mellin vs Finite Differences)



Very good agreement between the two analyses with the measured $g_1^{p,d,n}$

Conclusions and Outlook

- The status of knowledge of inclusive structure functions $\mathbf{g}_1^{p,d,n}$ has been reviewed.

  - $\mathbf{g}_1 \rightarrow$ latest precise measurements by HERMES
  - $\mathbf{g}_2 \rightarrow$ latest precise measurements by SLAC

- Fits to g_1 world data provide good insight into polarized parton distributions but still limited sensitivity to ΔG and $\Delta s \rightsquigarrow$ RHIC and COMPASS