Extraction of GPD $H(\xi, \xi, t)$ from single spin asymmetry

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- DVCS Cross Section
- Parameterization of GPDs
- Projected Statistical Accuracy
- Contribution of GPDs to the DVCS cross section
- Extraction of $Im\mathcal{H}$
- Conclusions

DVCS Cross Section

We follow to papers

A.V. Belitsky et al., Phys.Lett. B510, 117 (2001)

A.V. Belitsky et al., hep-ph/0112108

$e(k)p(P_1) \to e(k')p(P_2)\gamma(q_2)$

$$\frac{d\sigma}{dx_{\mathrm{B}}dyd|t|d\phi} = \frac{\alpha^3 x_{\mathrm{B}} y}{8\,\pi\,Q^2}\left(1 + \frac{4M^2x_{\mathrm{B}}^2}{Q^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2$$

$$\mathcal{T}^2 = |\mathcal{T}_{BH}|^2 + |\mathcal{T}_{VCS}|^2 + \mathcal{I}$$

$q_1 = k - k'$, $Q^2 = -q_1^2$, $x_{\mathrm{B}} = -q_1^2/(2P_1 \cdot q_1)$,
$y = P_1 \cdot q_1/P_1 \cdot k$, $t = (P_2 - P_1)^2$ and ϕ is the azimuthal angle between lepton and hadron scattering planes.

$$\mathcal{I} = \frac{1}{x_{\mathrm{B}}y^3\mathcal{P}_1\mathcal{P}_2(-t)}\left\{\frac{t}{Q^2}c_0^{\mathcal{I}} + \sum_{m=1}^{2} K^m\left[c_m^{\mathcal{I}}\cos(m\phi) + \lambda s_m^{\mathcal{I}}\sin(m\phi)\right]\right\}$$

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{1}{y^2Q^2}\left\{c_0^{\mathrm{DVCS}} + K\left[c_1^{\mathrm{DVCS}}\cos(\phi) + \lambda\, s_1^{\mathrm{DVCS}}\sin(\phi)\right]\right\}$$

$c_1^{\mathcal{I}}$, $s_1^{\mathcal{I}}$, c_0^{DVCS} arise at the twist-two level.

$c_0^{\mathcal{I}}$, $c_2^{\mathcal{I}}$, $s_2^{\mathcal{I}}$, c_1^{DVCS}, s_1^{DVCS} are given in terms of twist-two and twist-three GPDs.

DVCS Cross Section

For an unpolarized nucleon target:

$$
\begin{aligned}
c^{\mathcal{I}}_{1,\mathrm{unp}} &= -8(2-2y+y^2)\mathrm{Re}\,\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\left(\mathcal{F}\right),\\
s^{\mathcal{I}}_{1,\mathrm{unp}} &= 8y(2-y)\mathrm{Im}\,\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\left(\mathcal{F}\right),\\
c^{\mathcal{I}}_{2,\mathrm{unp}} &= -16\frac{2-y}{2-x_{\mathrm{B}}}\mathrm{Re}\,\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\left(\mathcal{F}^{\mathrm{eff}}\right),\\
s^{\mathcal{I}}_{2,\mathrm{unp}} &= 16\frac{y}{2-x_{\mathrm{B}}}\mathrm{Im}\,\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\left(\mathcal{F}^{\mathrm{eff}}\right).
\end{aligned}
$$

$\mathcal{F} \equiv \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$ — twist-two Compton form factors.

$\mathcal{F}^{\mathrm{eff}} = -2\xi\left(\frac{1}{1+\xi}\mathcal{F} + \mathcal{F}^3_+ - \mathcal{F}^3_-\right)$ is an ‘effective’ twist-three CFF.

The twist-two function

$$
\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\left(\mathcal{F}\right) = F_1\mathcal{H} + \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}(F_1+F_2)\widetilde{\mathcal{H}} - \frac{t}{4M^2}F_2\mathcal{E}
$$

In the following we set $\mathcal{F}^3_+ = 0$, $\mathcal{F}^3_- = 0$.

DVCS Cross Section

The Compton form factors are related to GPDs $F_i \equiv \{H_i, \ldots, \widetilde{E}_i\}$:

$$\mathcal{F}(\xi, t) = \sum_{i=u,d,\ldots} C_i^{(\mp)}(\xi, x) \otimes F_i(x, \xi, t).$$

The leading order coefficient functions

$$\xi\, C_i^{(\mp)}(\xi, x) = Q_i^2 (1 - x/\xi - i\epsilon)^{-1} \mp (x \to -x).$$

Then

$$\begin{aligned}
\mathrm{Im}\, \mathcal{H}_1 &= -\pi \sum_q e_q^2 \big(H(\xi, \xi, t) - H(-\xi, \xi, t) \big)\,, \\
\mathrm{Im}\, \widetilde{\mathcal{H}}_1 &= -\pi \sum_q e_q^2 \big(\widetilde{H}(\xi, \xi, t) + \widetilde{H}(-\xi, \xi, t) \big)\,, \\
\mathrm{Re}\, \mathcal{H}_1 &= \sum_q e_q^2 \Big[P \int_{-1}^{+1} H(x, \xi, t) \left(\frac{1}{x - \xi} + \frac{1}{x + \xi} \right) dx \Big]\,, \\
\mathrm{Re}\, \widetilde{\mathcal{H}}_1 &= \sum_q e_q^2 \Big[P \int_{-1}^{+1} \widetilde{H}(x, \xi, t) \left(\frac{1}{x - \xi} - \frac{1}{x + \xi} \right) dx \Big]\,,
\end{aligned}$$

P denotes Cauchy's principal value.

$\mathcal{E}_1$ and $\widetilde{\mathcal{E}}_1$ can be expressed through E and $\widetilde{E}$ analogously.

Parameterization of GPDs

Basically we follow the ansatz:
M. Vanderhaeghen, P.A.M. Guichon, M. Guidal, Phys.Rev. **D60**, 094017 (1999).
K. Goeke, M.V. Polyakov, M. Vanderhaeghen, hep-ph/0106012, 2001

In a simplest approach GPDs — independent of the ξ.

$$\begin{aligned} H^u(x,\xi,t) &= u(x)\ F_1^u(t)\,/\,2\ , \\ H^d(x,\xi,t) &= d(x)\ F_1^d(t)\ , \\ H^s(x,\xi,t) &= 0\ . \end{aligned}$$

$$\begin{aligned} \widetilde{H}^u(x,\xi,t) &= \Delta u_V(x)\ g_A(t)/g_A(0)\ , \\ \widetilde{H}^d(x,\xi,t) &= \Delta d_V(x)\ g_A(t)/g_A(0)\ , \\ \widetilde{H}^s(x,\xi,t) &= 0 \end{aligned}$$

$F_1^u = 2F_1^p + F_1^n,\ F_1^d = F_1^p + 2F_1^n$

E and $\widetilde{E}$ have no exp. measurable limit at $\xi \to 0$.
E is taken in a form analogous to H:

$$\begin{aligned} E^u(x,\xi,t) &= u(x)\ F_2^u(t)\,/\,2\ , \\ E^d(x,\xi,t) &= d(x)\ F_2^d(t)\ , \\ E^s(x,\xi,t) &= 0 \end{aligned}$$

The function $\widetilde{E}$ is modelled to be due to the pion pole:

$$\widetilde{E}^u(x,\xi,t) = -\widetilde{E}^d(x,\xi,t) = \frac{1}{2}\,\widetilde{E}_{\pi-pole}(x,\xi,t)\ ,$$

$$\widetilde{E}_{\pi-pole}(x,\xi,t) = \theta\,(-\xi \le x \le \xi)\ \ h_A(t)\,\frac{1}{\xi}\,\Phi\left(\frac{x}{\xi}\right)\ ,$$

$\Phi(z) = 3/4(1-z^2)$ — pion distribution amplitude
$h_A(t) = \frac{4M^2 g_A}{m_\pi^2 - t}$

Parameterization of GPDs

To introduce a dependence of the GPDs on ξ, the DD formalism is used

D. Müller et al., Fortsch. Phys. **42**, 101 (1994)
A.V. Radyushkin, Phys.Rev. **D56**, 5524 (1997)

$$H^q_{DD}(x,\xi) = \int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dt\ \delta(x-y-t\xi)\ h(y,t)\ q(y) \ .$$

$q(y)$ - ordinary quark distribution

$$h(y,t) = \frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)} \frac{\left[(1-|y|)^2 - t^2\right]^b}{(1-|y|)^{2b+1}} \ ,$$

b — free parameter

Analogously for functions $\widetilde{H}$ and E.

Unfactorized model: Goeke, Polyakov, Vanderhaeghen, 2001; Burkardt 2001

$$H^q(x,\xi=0,t) = \frac{1}{x^{\alpha' t}} q(x),$$

where $\alpha' \approx 0.8$ GeV^{-2}.

$$H^q_{DD}(x,\xi) = \int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dt\ \delta(x-y-t\xi)\ h(y,t)\ q(y) \frac{1}{y^{\alpha' t}} \ .$$

Parameterization of GPDs

The contribution of the D-term

(M.V. Polyakov, C. Weiss, Phys.Rev. **D60**, 114017 (1999))

is taken according to the chiral quark soliton model calculation

(N. Kivel, M.V. Polyakov, M. Vanderhaeghen, Phys.Rev. **D63**, 114014 (2001)).

$$
\begin{aligned}
H^q(x,\xi) &= H^q_{DD}(x,\xi) + \theta(\xi - |x|)\,\frac{1}{N_f}\,D(\frac{x}{\xi})\,, \\
E^q(x,\xi) &= E^q_{DD}(x,\xi) - \theta(\xi - |x|)\,\frac{1}{N_f}\,D(\frac{x}{\xi})\,.
\end{aligned}
$$

$q(x)$ — MRST98 (at $Q^2 = 2$ GeV^2)
$\Delta q(x)$ — LSS98 (at $Q^2 = 1$ GeV^2)

A scale dependence of GPDs is neglected.

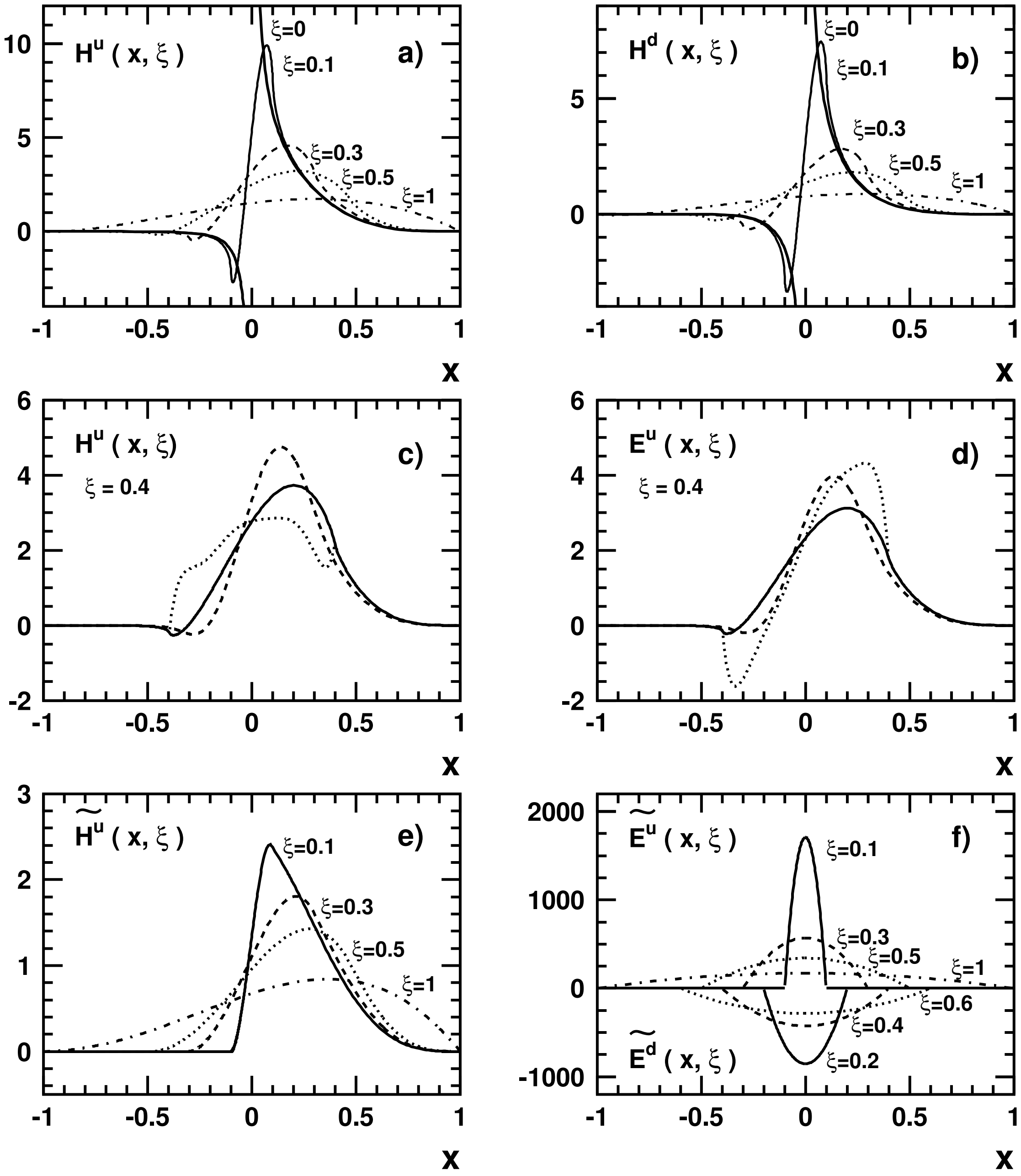
A few versions of GPD models were tried:

- A) $H(x,\xi)$, $\widetilde{H}(x,\xi)$, and $E(x,\xi)$ are ξ-independent.
- B) The ξ dependence of $H(x,\xi)$, $\widetilde{H}(x,\xi)$, and $E(x,\xi)$ is generated through the DD formalism with $b = 1$.
- C) same as B), but with the parameter $b = 3$.
- D) same as B), but the contribution of the D-term to $H(x,\xi)$ and $E(x,\xi)$ is included additionally.
- E) same as D), but with the parameter $b = 3$.
- F) unfactorized model for $H(x,\xi,t)$, $b = 1$.
- G) same as F), but with the parameter $b = 3$.

For $\widetilde{E}(x,\xi,t)$ the pion pole expression is used for all versions.

Parameterization of GPDs

Dependence of the GPDs on x and ξ (at $t = 0$).



a) $H^u(x,\xi)$ for different values of ξ in version B).
b) The same as a) but for d-quarks.
c) $H^u(x,\xi)$ at $\xi = 0.4$
solid line - version B), dashed line - version C), dotted line - version D).
d) The same as c) but for $E^u(x,\xi)$.
e) $\widetilde{H}^u(x,\xi)$ for different values of ξ in version B).
f) $\widetilde{E}^u(x,\xi)$ and $\widetilde{E}^d(x,\xi)$ for different values of ξ.

DVCS Projections
Unpolarized Proton Target

I) Lepton charge asymmetry

$$
\begin{aligned}
d\Delta\sigma_{ch} \quad &\equiv \quad d\sigma(e^+p) - d\sigma(e^-p) \sim \\
&\text{Re} \quad \left\{ F_1\mathcal{H}_1 + \frac{x_B}{2-x_B}(F_1+F_2)\widetilde{\mathcal{H}}_1 - \frac{t}{4M^2}F_2\mathcal{E}_1 \right\}
\end{aligned}
$$

II) Beam helicity asymmetry using a polarized positron beam:

$$
\begin{aligned}
d\Delta\sigma_{LU} \quad &\equiv \quad d\sigma(\overrightarrow{e^+}p) - d\sigma(\overleftarrow{e^+}p) \sim \\
&\text{Im} \quad \left\{ F_1\mathcal{H}_1 + \frac{x_B}{2-x_B}(F_1+F_2)\widetilde{\mathcal{H}}_1 - \frac{t}{4M^2}F_2\mathcal{E}_1 \right\}
\end{aligned}
$$

Future measurements of DVCS at HERMES:

$L = 2$ fb^{-1} , 1 year of running, $P_B = 50\%$.

Geometrical acceptance of the spectrometer.

Kinematical cuts:

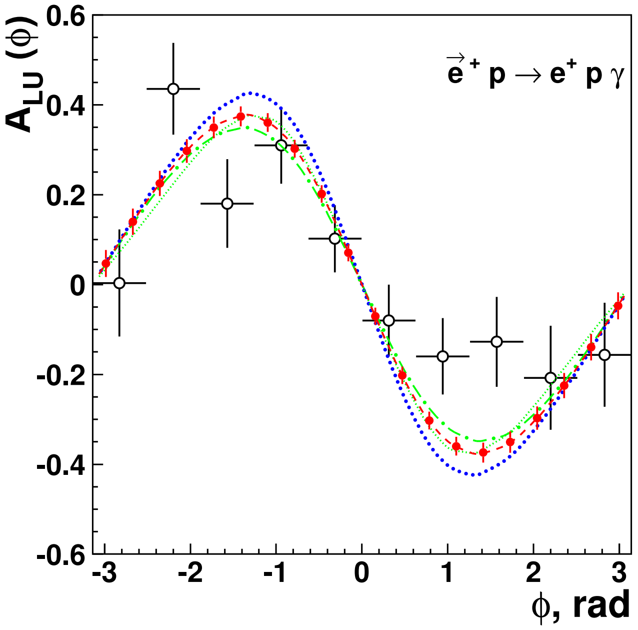
$E_e > 3.5$ GeV, $W^2 > 4$ GeV^2, $Q^2 > 1$ GeV^2,

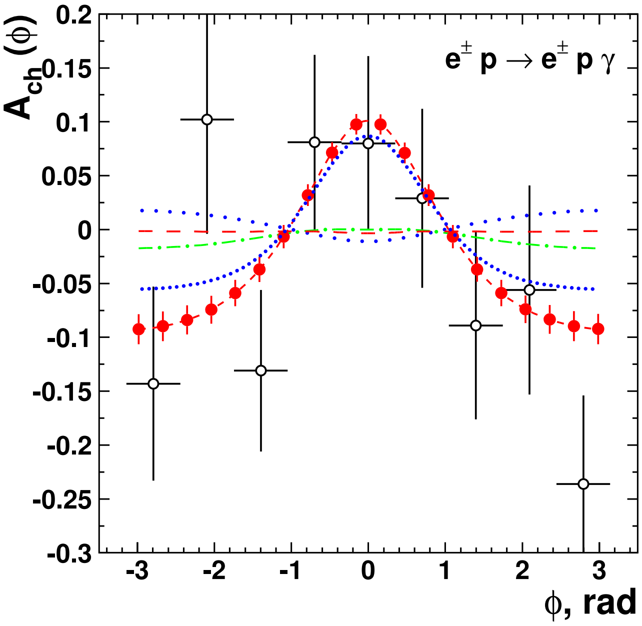
$E_\gamma > 0.8$ GeV, $P_p > 0.1$ GeV,

$0.35 < \theta_p^{lab} < 1.35$ rad, $15 < \Theta_{\gamma\gamma^*}^{lab} < 70$ mrad.

DVCS Projections
Asymmetry vs. Azimuthal Angle

A) - dash-dotted line
B) - long space dotted line
C) - long space dashed line
D) - dotted line
E) - dashed line
F) - dotted line



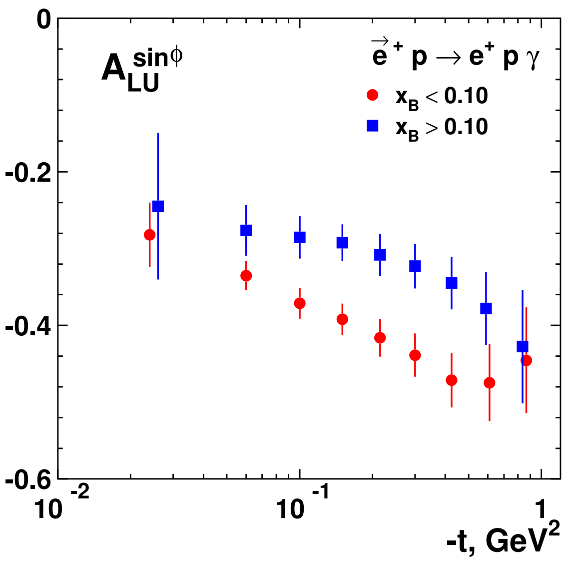


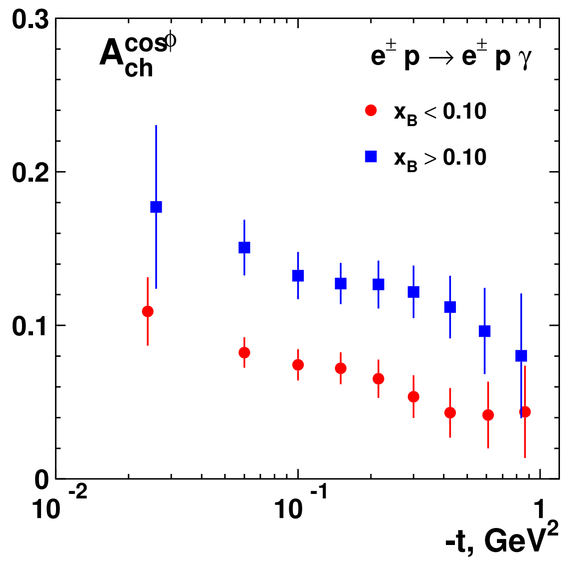
DVCS Projections

$\sin\phi$ / $\cos\phi$ Moments

$$A_{LU}^{\sin\phi} = 2 \cdot \frac{\int_0^{2\pi} d\phi \, \sin\phi \{d\sigma(e^{\uparrow}p)/d\phi \,-\, d\sigma(e^{\downarrow}p)/d\phi\}}{\int_0^{2\pi} d\phi \, \{d\sigma(e^{\uparrow}p)/d\phi \,+\, d\sigma(e^{\downarrow}p)/d\phi\}}$$

$$A_{ch}^{\cos\phi} = 2 \cdot \frac{\int_0^{2\pi} d\phi \, \cos\phi \{d\sigma(e^{+}p)/d\phi \,-\, d\sigma(e^{-}p)/d\phi\}}{\int_0^{2\pi} d\phi \, \{d\sigma(e^{+}p)/d\phi \,+\, d\sigma(e^{-}p)/d\phi\}}$$

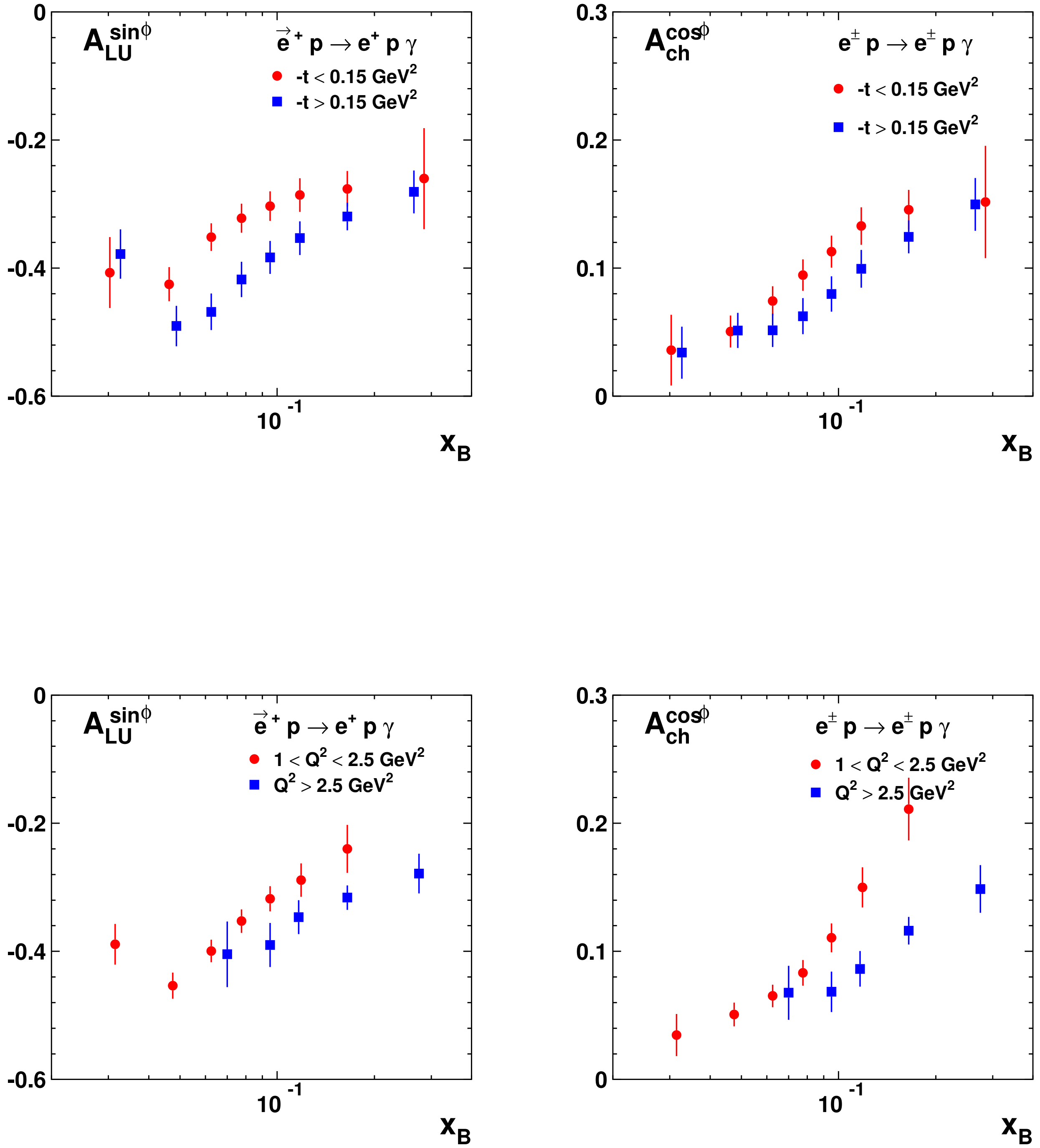




Asymmetry as a function of x_B and t based upon GPDs version E).

DVCS Projections

$\sin\phi$ / $\cos\phi$ Moments



Contribution of GPDs to the DVCS cross section

Unpolarized DVCS cross-section:

$$
\begin{aligned}
\mathcal{C}^{\mathrm{DVCS}}_{\mathrm{unp}}\left(\mathcal{F},\mathcal{F}^*\right) = \frac{1}{(2-x_{\mathrm{B}})^2}\Bigg\{ & 4(1-x_{\mathrm{B}})\left(\mathcal{H}\mathcal{H}^* + \widetilde{\mathcal{H}}\widetilde{\mathcal{H}}^*\right) \\
& - x_{\mathrm{B}}^2\left(\mathcal{H}\mathcal{E}^* + \mathcal{E}\mathcal{H}^* + \widetilde{\mathcal{H}}\widetilde{\mathcal{E}}^* + \widetilde{\mathcal{E}}\widetilde{\mathcal{H}}^*\right) \\
& - \left(x_{\mathrm{B}}^2 + (2-x_{\mathrm{B}})^2\frac{t}{4M^2}\right)\mathcal{E}\mathcal{E}^* \\
& - x_{\mathrm{B}}^2\frac{t}{4M^2}\widetilde{\mathcal{E}}\widetilde{\mathcal{E}}^*\Bigg\},
\end{aligned}
$$

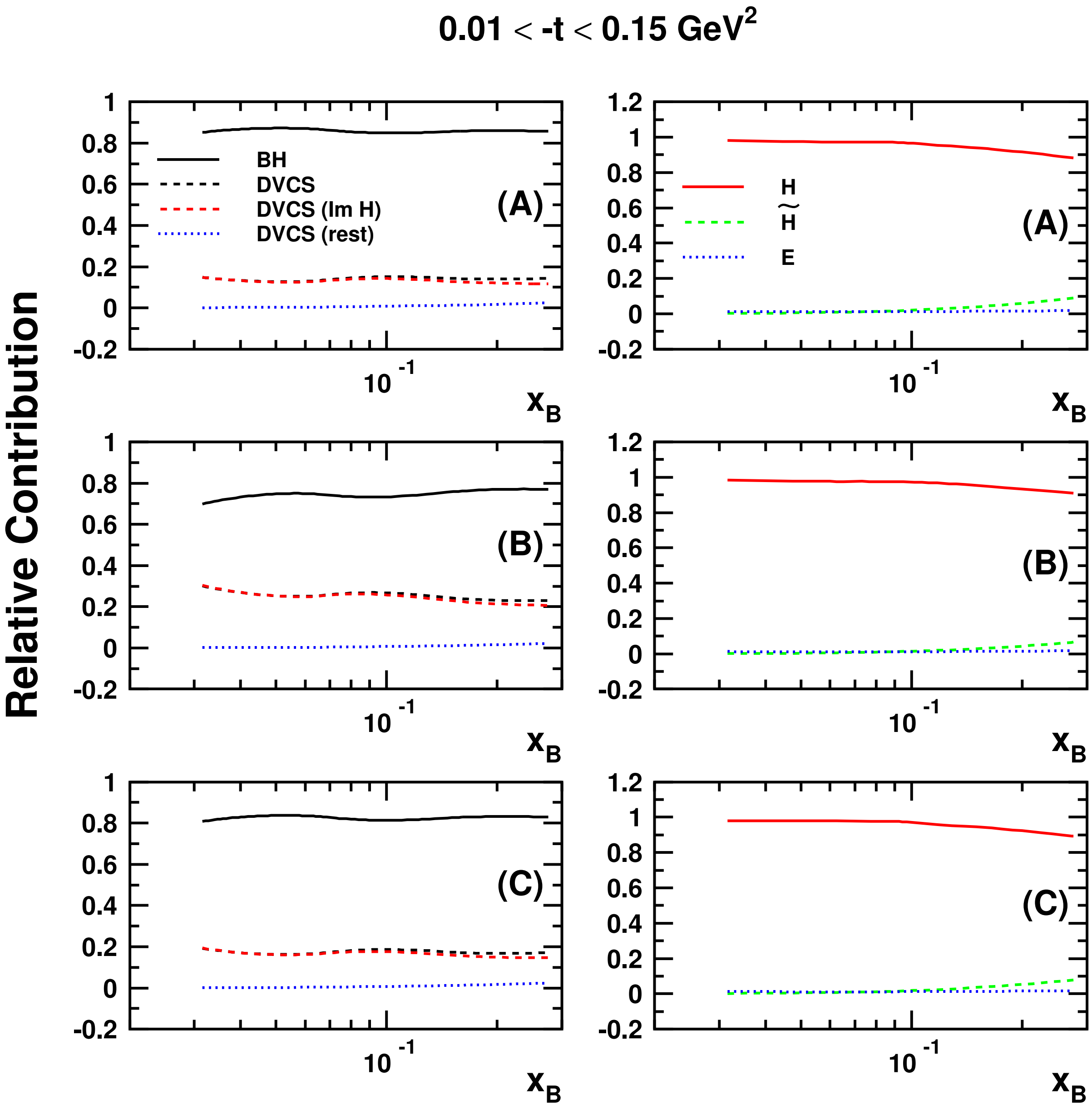
BH-DVCS interference term:

$$
\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}\left(\mathcal{F}\right) = F_1\mathcal{H} + \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}(F_1+F_2)\widetilde{\mathcal{H}} - \frac{t}{4M^2}F_2\mathcal{E}
$$

$A_{LU}^{\sin\phi}$ depends on all GPDs, but numerically the main contribution comes from $Im\mathcal{H}$ only (at small t).

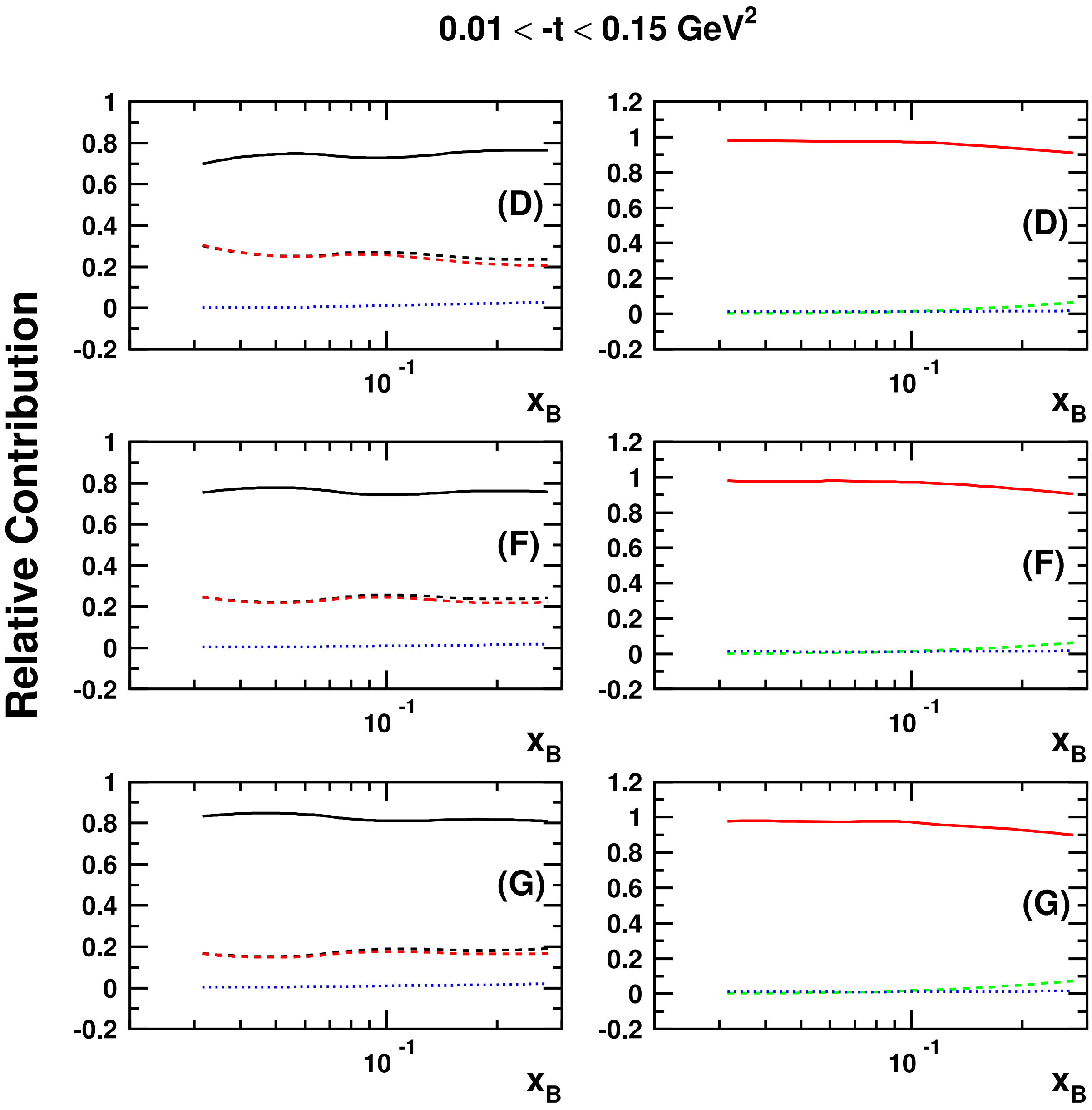
Consider region of $0.01 < -t < 0.15$ GeV^2.

Contribution of GPDs to the DVCS cross section



$\int d\Omega \frac{d\sigma^{DVCS}}{d\Omega}$ $\int d\Omega \sin\phi \frac{d\sigma^{I}}{d\Omega}$

Contribution of GPDs to the DVCS cross section



$\int d\Omega \, \frac{d\sigma^{DVCS}}{d\Omega}$ $\int d\Omega \, \sin\phi \frac{d\sigma^{I}}{d\Omega}$

Contribution of GPDs to the DVCS cross section

$A_{LU}^{\sin\phi}$ gets significant contribution from $Im\mathcal{H}$ only. Therefore

$$A_{LU}^{\sin\phi} = 2 \cdot \frac{\int d\Omega \, \sin\phi \frac{d\sigma^{I}}{d\Omega} \cdot Im\mathcal{H}}{\int d\Omega \, \frac{d\sigma^{BH}}{d\Omega} + \int d\Omega \, \frac{d\sigma^{DVCS}}{d\Omega} \cdot (Im\mathcal{H})^2} \, ,$$

where $d\sigma^{I}/d\Omega$ and $d\sigma^{DVCS}/d\Omega$ are calculated with
$Im\mathcal{H} = 1$, $Re\mathcal{H} = 0$, $\widetilde{\mathcal{H}} = 0$, $\mathcal{E} = 0$, $\widetilde{\mathcal{E}} = 0$.

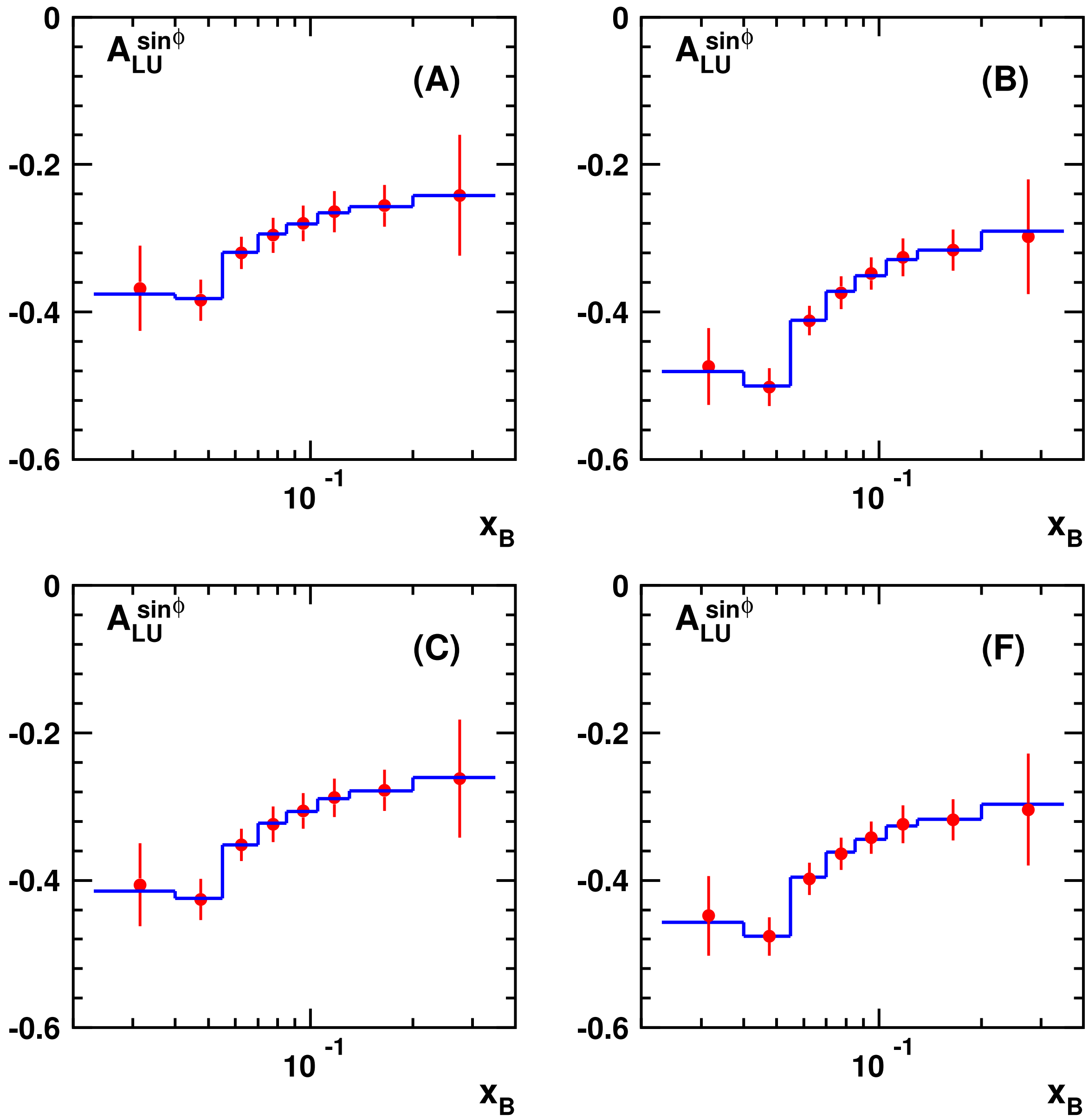
At small ξ, $Im\mathcal{H}(\xi, t=0)$ one may parameterize:
A. Freund, M. McDermott, 2001

$$Im\mathcal{H}(\xi, t=0) \; = \; c_1 \xi^{c2}$$

Let's fit the *experimentally measured* moments with this anzats.

Extraction of $Im\mathcal{H}$.
Fit to $A_{LU}^{\sin\phi}$

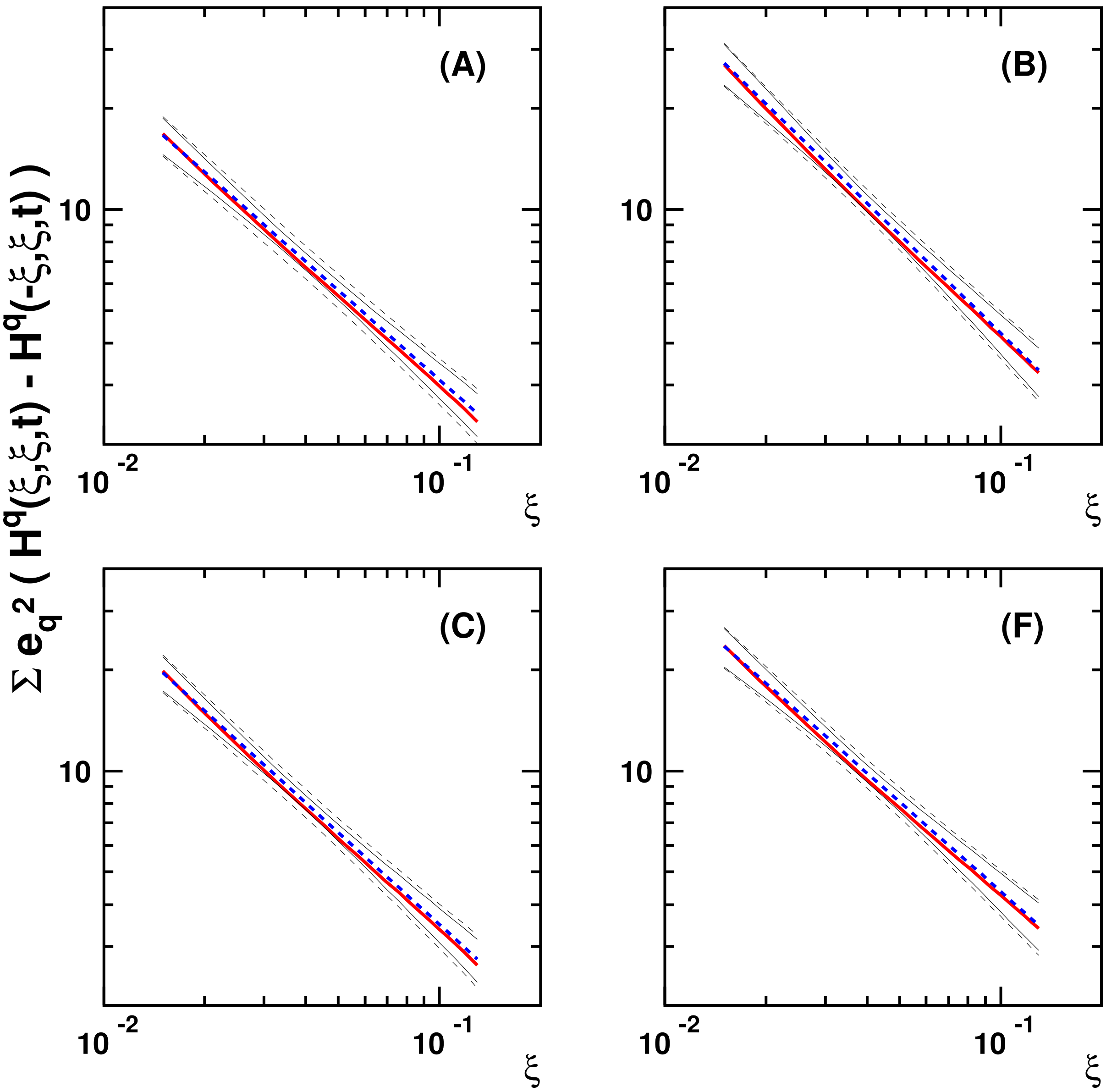
$0.01 < -t < 0.15$ GeV^2



χ^2 / NDF is $(4 \div 6)/6$ typically

Extraction of $Im\mathcal{H}$.
1σ error bands.

$$0.01 < -t < 0.15 \text{ GeV}^2$$



Red line comes from an input model.

Blue line is obtained with the fit.

Conclusions

- HERMES upgraded with a recoil detector will able to measure the DVCS-BH interference term with a good accuracy.
- A possibility of HERMES to measure GPD $H(\xi, \xi, t)$ has been studied.
- Single beam-spin asymmetry moment $A_{LU}^{\sin\phi}$ gets the most significant contribution from $Im\mathcal{H}$.
- Simple anzats, $Im\mathcal{H}(\xi) = c_1\xi^{c2}$, allows one to describe a dependence of $A_{LU}^{\sin\phi}$ on x_B at small $-t$ and to get $Im\mathcal{H}(\xi)$ in a parameterized form.
- Further studies are required on possibilities to reconstruct a t-dependence of $Im\mathcal{H}$.