NLO QCD Analysis of the Polarized Structure Function g_1

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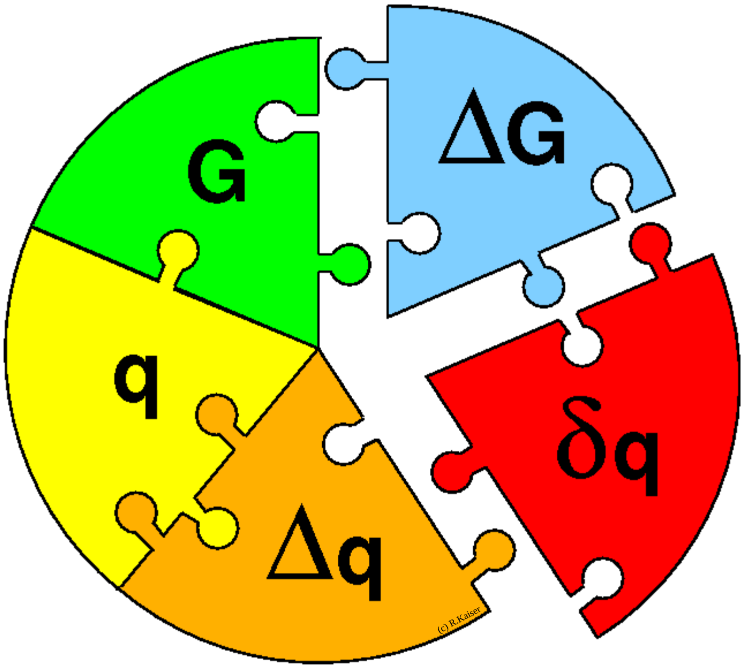
Motivation

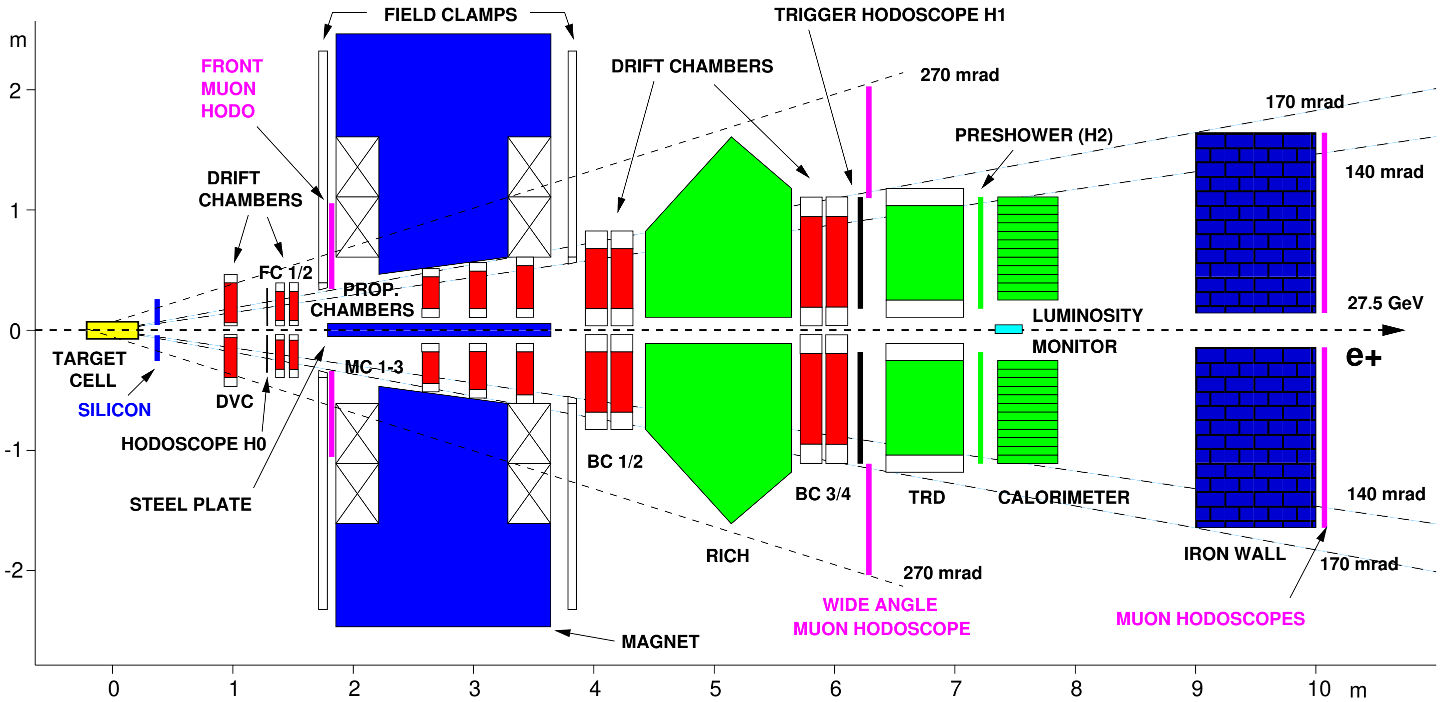
Why do we study Polarized Deep Inelastic Scattering ?

- Study of short distance structure of nucleon spin
- Understand and finally solve the spin puzzle



- Test of perturbative QCD in spin sector: Λ_{QCD}
- Test of fundamental and less fundamental sum rules

The HERMES Experiment at DESY



- **Beam:** transversally self-polarizing 27.5 GeV $e^{+/-}$-beam of HERA, rotated to longitudinal spin orientation at the HERMES IP; $P_{\text{beam}} \approx 0.53 \pm 0.02$

- **Target:** cooled open-ended storage cell inside the beam pipe, longitudinally polarized pure (H, D, ^{3}He) gas atoms of $(7 - 33) \times 10^{13}$nucleons/cm^2; $P_{\text{H}} = 0.824 \pm 0.035$, $P_{\text{D}} = 0.844 \pm 0.037$, $P_{\text{He}} = 0.46 \pm 0.02$

- **Tracking:** forward dipole magnet spectrometer with 57 chamber planes $(40\,\text{mrad} < \theta < 220\,\text{mrad})$; resolution: $\delta\theta < 0.6$ mrad, $\delta p/p < 1.5\%$

- **Particle ID:** Cerenkov detector, TRD, preshower (H2), lead-glass calorimeter; lepton identification efficiency $> 98\%$, hadron contamination $< 1\%$

- **Fast Trigger**: scintillator hodoscopes H0, H1, H2; calorimeter energy threshold 1.4(3.5) GeV

QCD Analysis Formalism

- $g_1(x,Q^2)$ in terms of a MELLIN convolution of the polarized densities Δf_j and the polarized Wilson coefficients ΔC_j^A:

$$g_1(x,Q^2) = \frac{1}{2}\sum_{j=1}^{N_f} e_j^2 \int_x^1 \frac{dz}{z}\left[\frac{1}{N_f}\,\Delta\Sigma\left(\frac{x}{z},\mu_f^2\right)\Delta C_q^S\left(z,\frac{Q^2}{\mu_f^2}\right)\right.$$
$$+\,\Delta G\left(\frac{x}{z},\mu_f^2\right)\Delta C^G\left(z,\frac{Q^2}{\mu_f^2}\right)$$
$$\left.+\,\Delta q_j^{NS}\left(\frac{x}{z},\mu_f^2\right)\Delta C_q^{NS}\left(z,\frac{Q^2}{\mu_f^2}\right)\right],$$

  with the singlet density $\Delta\Sigma$

$$\Delta\Sigma\left(z,\mu_f^2\right) = \sum_{j=1}^{N_f}\left[\Delta q_j\left(z,\mu_f^2\right)+\Delta\bar{q}_j\left(z,\mu_f^2\right)\right],$$

  the non–singlet densities Δq_j^{NS}

$$\Delta q_j^{NS}\left(z,\mu_f^2\right) = \Delta q_j\left(z,\mu_f^2\right)+\Delta\bar{q}_j\left(z,\mu_f^2\right)-\frac{1}{N_f}\Delta\Sigma\left(z,\mu_f^2\right),$$

  the gluon density ΔG, and the factorization scale μ_f.

- The polarized quark density $\Delta q_j = q_j^{\uparrow\downarrow} - q_j^{\uparrow\uparrow}$.
- The renormalization scale μ_r of $a_s(\mu_r^2)$.

$\Longrightarrow$ $g_1(x,Q^2)$ will be presented using the three light flavors, i.e. $N_f = 3\,(u,d,s)$. Charm absorbed into ΔG.

Evolution Equations

- The evolution equations are given by

$$\frac{\partial \Delta q_i^{\mathrm{NS}}(x,Q^2)}{\partial \log Q^2} = \Delta P_{NS}^{-}(x,a_s) \otimes \Delta q_i^{\mathrm{NS}}(x,Q^2)$$

$$\frac{\partial}{\partial \log Q^2}\begin{pmatrix} \Delta\Sigma(x,Q^2) \\ \Delta G(x,Q^2) \end{pmatrix} = \Delta \boldsymbol{P}(x,a_s) \otimes \begin{pmatrix} \Delta\Sigma(x,Q^2) \\ \Delta G(x,Q^2) \end{pmatrix}$$

with the polarized splitting functions

$$\Delta P_{\mathrm{NS}}^{-}(x,a_s) = a_s \Delta P_{\mathrm{NS}}^{(0)}(x) + a_s^2 \Delta P_{\mathrm{NS}}^{-(1)}(x) + \mathcal{O}(a_s^3)$$

$$\Delta \boldsymbol{P}(x,a_s) \equiv \begin{pmatrix} \Delta P_{qq}(x,Q^2) & \Delta P_{qg}(x,Q^2) \\ \Delta P_{gq}(x,Q^2) & \Delta P_{gg}(x,Q^2) \end{pmatrix}$$

$$= a_s \Delta \boldsymbol{P}^{(0)}(x) + a_s^2 \Delta \boldsymbol{P}^{(1)}(x) + \mathcal{O}(a_s^3)$$

and $\otimes$ the MELLIN convolution

$$[A \otimes B](x) = \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) A(x_1) B(x_2)$$

- The polarized Wilson coefficient functions $\Delta C_i(x,\alpha_s(Q^2))$ and the polarized splitting functions $\Delta P_{ij}(x,\alpha_s(Q^2))$ are known in the $\overline{MS}$ scheme up to NLO. [W.L. van Neerven and E.B. Zijlstra, Nucl. Phys. B417 (1994) 61, R. Mertig and W.L. van Neerven, Z. Phys. C70 (1996) 637, W. Vogelsang, Phys. Rev. D54 (1996) 2023]

$\Longrightarrow$ $\boxed{\text{A complete NLO QCD analysis possible.}}$

Evolution in MELLIN space

- The evolution equations are solved analytically in MELLIN–N space.

- The non–singlet solution:

$$\Delta q^{\rm NS}(N,a_s) = \left(\frac{a_s}{a_0}\right)^{-P_{\rm NS}^{(0)}/\beta_0}\left[1-\frac{1}{\beta_0}(a_s-a_0)\right.$$
$$\left.\times\left(P_{\rm NS}^{-(1)}-\frac{\beta_1}{\beta_0}P_{\rm NS}^{(0)}\right)\right]\Delta q^{\rm NS}(N,a_0)$$

and the singlet solution:

$$\begin{pmatrix}\Delta\Sigma(N,a_s)\\ \Delta G(N,a_s)\end{pmatrix} = \left[\mathbf{1}+a_s\boldsymbol{U}_1(N)\right]\boldsymbol{L}(N,a_s,a_0)\left[\mathbf{1}-a_0\boldsymbol{U}_1(N)\right]$$
$$\times\begin{pmatrix}\Delta\Sigma(N,a_0)\\ \Delta G(N,a_0)\end{pmatrix},$$

where $a_s = a_s(Q^2)$ and $a_0 = a_s(Q_0^2)$.

$\Longrightarrow$ Note: The input and the evolution parts factorize.

- The input densities are evolved to the scale Q^2, respectively to the coupling $\alpha_s(Q^2)$. An inverse MELLIN–transformation to x–space is then performed by a contour integral in the complex plane around all singularities.

[W.Furmanski and R.Petronzio, Z.Phys.**C11** (1982) 293, M.Glück, E.Reya, and A.Vogt, Z.Phys.**C48** (1990) 471, J.Blümlein and A.Vogt, Phys.Rev.**D58** (1998) 014020.]

Parameterization

- General shape for the parameterization of the polarized parton densities at the input scale Q_0^2:

$$\boxed{x\Delta q_i(x,Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x + \rho_i x^{\frac{1}{2}})}$$

- Normalization:

$$\begin{aligned} A_i^{-1} &= \left(1+\gamma_i \frac{a_i}{a_i+b_i+1}\right) B(a_i, b_i+1) \\ &\quad + \rho_i B(a_i+\frac{1}{2}, b_i+1) \end{aligned}$$

such that $$\int_0^1 dx \Delta q_i(x,Q_0^2) = \eta_i$$

is the first moment of $\Delta q_i(x,Q_0^2)$.

- The MELLIN–N moments for complex arguments N :

$$\begin{aligned} \mathbf{M}[\Delta q_i(x,Q_0^2)](N) &= \eta_i A_i \left(1+\gamma_i \frac{N-1+a_i}{N+a_i+b_i}\right) \\ &\quad \times B(N-1+a_i, b_i+1) \\ &\quad + \rho_i B\left(N+a_i-\frac{1}{2}, b_i+1\right) \end{aligned}$$

$\Rightarrow$ $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ is the Euler Beta–function.

Choice of Parameters

$$\boxed{x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x + \rho_i x^{\frac{1}{2}})}$$

Polarized densities to be fitted: Δu_v, Δd_v, $\Delta\bar{q}$, ΔG

$\Longrightarrow$ Follow the choice as done by BB

SU(3) flavour symmetry: $\eta_{\Delta u_v}$, $\eta_{\Delta d_v}$ fixed by F and D

Symmetric sea: $\Delta\bar{q} \equiv \Delta\bar{u}_s = \Delta\bar{d}_s = \Delta s = \Delta\bar{s}$

$\rho = 0$ fixed for all densities

$\gamma_{\Delta\bar{q}} = \gamma_{\Delta G} = 0$ fixed

$\Longrightarrow$ still 12 fit parameters

$\gamma_{\Delta u_v} = 24.62$, $\gamma_{\Delta d_v} = 16.84$ fixed after fit to data

$$\left.\begin{array}{l} a_{\Delta G} = a_{\Delta\bar{q}} + 1 \\ \left.\frac{b_{\Delta\bar{q}}}{b_{\Delta G}}\right|_{pol} = \left.\frac{b_{\bar{q}}}{b_G}\right|_{unpol} \end{array}\right\} \text{lead to positivity for } \Delta G$$

$b_{\Delta\bar{q}} = 8.08$, $b_{\Delta G} = 5.61$ fixed after fit to data

$\Longrightarrow$ 7 fit parameters

$\Lambda^{(4)}_{QCD} = 291 \pm 30\ MeV \Leftrightarrow \alpha_s(M_Z^2) = 0.117 \pm 0.002$

Input scale: $Q_0^2 = 4\ GeV^2$ $\Uparrow$

Data selection: $Q^2 \geq 1\ GeV^2$ fixed

α_s determined from data:

BB: $\alpha_s(M_Z^2) = 0.113{+}0.010{-}0.008$ [Nucl.Phys.**B636**(2002)225]

H1: $\alpha_s(M_Z^2) = 0.115 \pm 0.002 \pm 0.005$ [Eur.Phys.J.**C21**(2001)33]

The World Data on A_1 or g_1/F_1

Published Experimental Data with $Q^2 \geq 1.0\ GeV^2$

Exper.	x range	Q^2 (GeV^2)	type	# p	# d	# n
E142	0.035 - 0.466	1.10 - 6.10	A_1	-	-	**33**
E143	0.027 - 0.749	1.00 - 9.52	g_1/F_1	**82**	**82**	-
E154	0.017 - 0.564	1.20 - 15.0	A_1	-	-	**11**
E155	0.015 - 0.750	1.22 - 34.79	g_1/F_1	**24**	**24**	-
HERM.	0.028 - 0.660	1.13 - 7.36	A_1	**39**	-	**9**
			g_1/F_1	-	**39** *	-
SMC	0.004 - 0.484	1.14 - 72.10	A_1	**59**	**65**	-
EMC	0.015 - 0.466	3.5 - 29.5	A_1	**10**	-	-
Total: **477**				**214**	**210**	**53**

* g_1^d/F_1^d from HERMES still preliminary

$$\frac{g_1}{F_1} = \frac{1}{(1+\gamma^2)}\left[\left(\frac{A_{||}}{D} - \eta A_2\right) + \gamma A_2\right]$$

$$= \frac{1}{(1+\gamma^2)}\left[A_1 + \gamma A_2\right]$$

$$F_1 = \frac{(1+\gamma^2)}{2x(1+R)} F_2$$

$$A_2(x,Q^2) \overset{WW}{=} \frac{\gamma(x,Q^2)}{F_1(x,Q^2)} \int_x^1 \frac{dz}{z} F_1(z,Q^2) \frac{g_1}{F_1}(z,Q^2)$$

F_2–Parametrization: NMC, M. Arneodo et al., Phys. Lett. **B364** (1995) 107.

R–Parametrization: R1990, L. Withlow et al., Phys. Lett. **B250** (1990) 193.

g_1/F_1–Parametrization: E155, P.L. Anthony et al., Phys. Lett. **B493** (2000) 19.

χ^2 Calculation

$\Longrightarrow$ Problem: Systematic errors are known to be partly correlated which would lead to an overestimation of the errors when added in quadrature with the statistical ones.

- Statistical Errors:
  To treat all data sets on the same footing only the statistical errors are considered.

  $\Longrightarrow$ Only fits with a Positive Definite Covariance Matrix are accepted.

- Normalization Uncertainties:
  Allow for a Relative Normalization Shift N_i between the different data sets within the Normalization Uncertainties ΔN_i quoted by the experiments (fitted once and then fixed).

$\Longrightarrow$ The χ^2 Expression:

$$\chi^2 = \sum_{i=1}^{n^{exp}} \left[\frac{(N_i - 1)^2}{(\Delta N_i)^2} + \sum_{j=1}^{n^{data}} \frac{(N_i g_{1,j}^{data} - g_{1,j}^{theor})^2}{(\Delta g_{1,j}^{data})^2} \right]$$

NLO QCD Fit Results

- Parameter values at the input scale $Q_0^2 = 4.0\,GeV^2$

$$\boxed{x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1+\gamma_i x + \rho_i x^{\frac{1}{2}})}$$

Δu_v	η	0.926 (fixed)
	a	0.316 $\pm$ 0.035
	b	3.188 $\pm$ 0.201
	γ	24.62
Δd_v	η	-0.341 (fixed)
	a	0.268 $\pm$ 0.102
	b	3.592 $\pm$ 1.092
	γ	16.84
$\Delta\bar{q}$	η	-0.070 $\pm$ 0.028
	a	0.358 $\pm$ 0.235
	b	8.08
ΔG	η	0.616 $\pm$ 0.388
	a	$a_{\Delta\bar{q}} + 1$
	b	5.61
χ^2/NDF = 422/470 = 0.90		

$\Longrightarrow$ Positive Definite Covariance Matrix

Statistical Error Calculation

- The fully correlated 1σ statistical error for the polarized density Δf_q as given by Gaussian error propagation is

$$\sigma(\Delta f_q)^2 = \sum_{i,j=1}^{n_p} \left(\frac{\partial \Delta f_q}{\partial p_i} \frac{\partial \Delta f_q}{\partial p_j} \right) \mathrm{cov}(p_i, p_j) \ ,$$

  where $\mathrm{cov}(p_i, p_j)$ are the elements of the covariance matrix determined in the QCD fit.

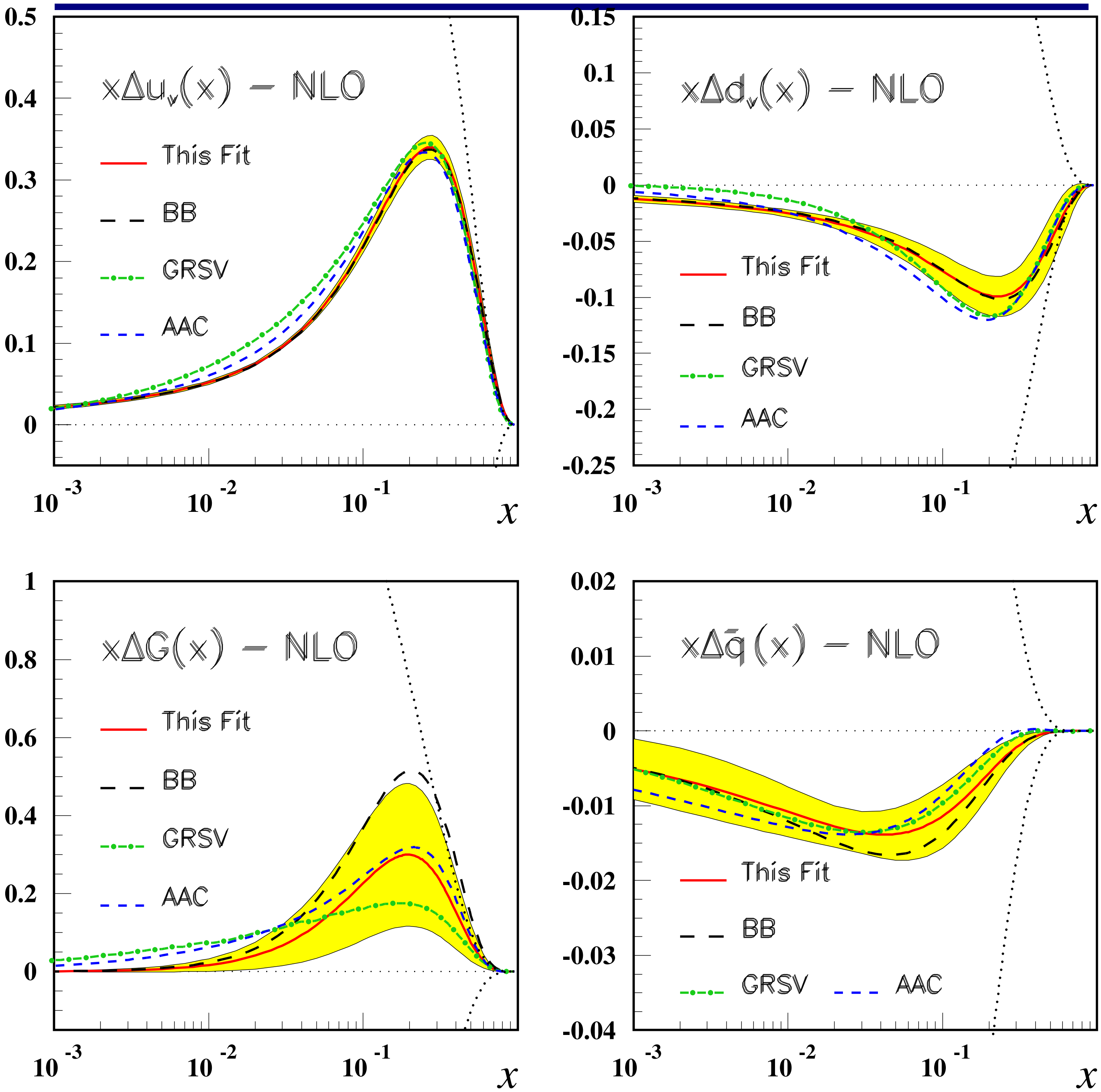
- The derivatives $\partial \Delta f_q / \partial p_i$ at the input scale Q_0^2 can be calculated analytically.

- Their values at Q^2 are calculated by evolution.

- The derivatives evolved in $\mathrm{MELLIN\text{-}N}$ space are transformed back to x–space and can then be used according to the error propagation equation.

$\Longrightarrow$ The expressions for all relevant derivatives, both in x and $\mathrm{MELLIN{-}N}$ space, are given in
Ref.: J.Blümlein, H.Böttcher, Nucl.Phys.**B636**(2002)225.

Pol. Parton Densities at $Q_0^2 = 4.0\,GeV^2$



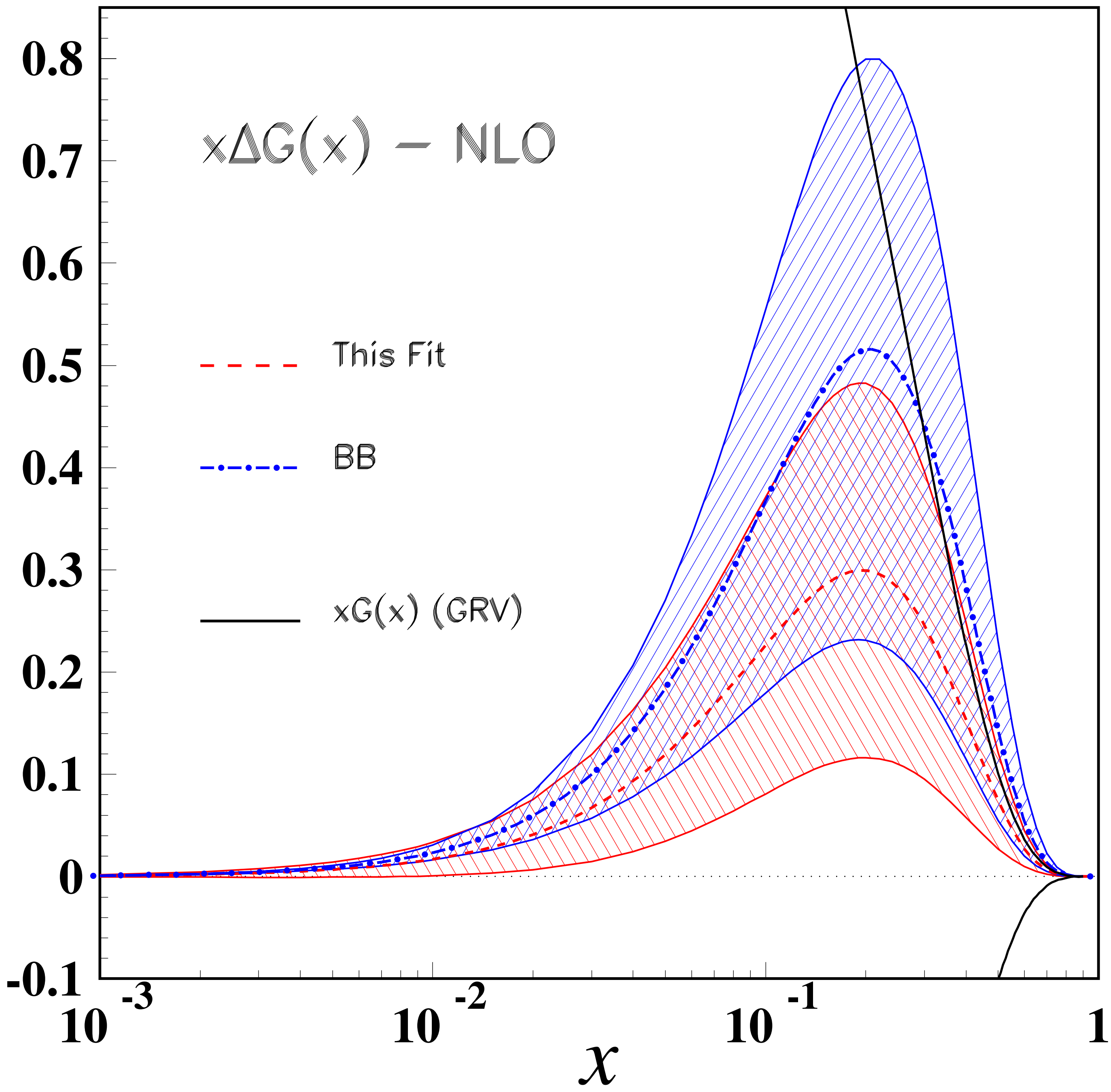
⇒ Yellow error band: Fully correlated 1σ statistical error band as given by Gaussian error propagation.

⇒ Dark dotted line: 'Positivity Limit' taken from GRV98.

BB: Nucl.Phys.**B636**(2002)225 , **GRSV**: Phys.Rev.**D63**(2001)094005,

AAC: Phys.Rev.**D62**(2000)034017

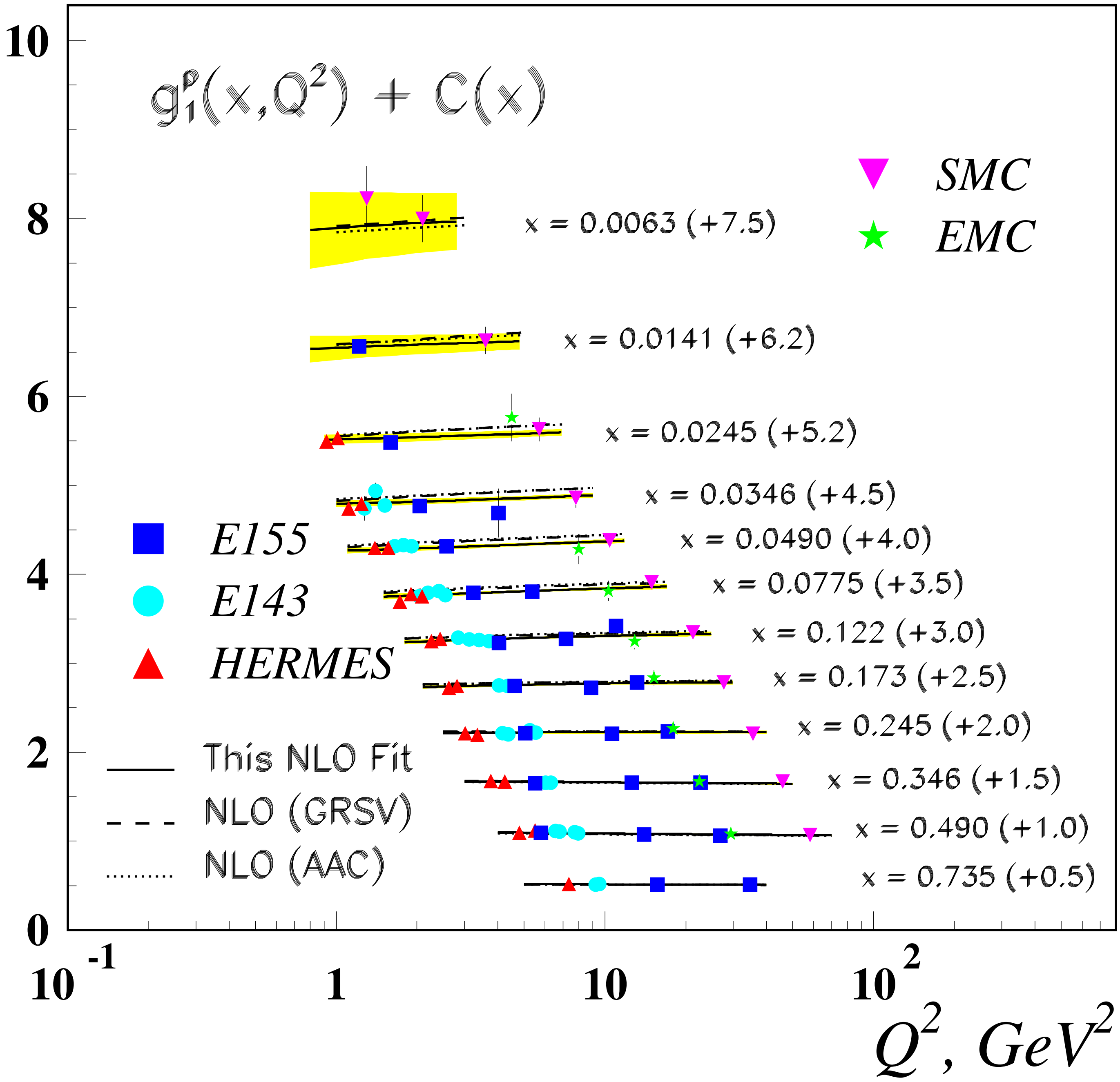
Pol. Gluon Densities at $Q_0^2 = 4.0\, GeV^2$



⇒ Hatched error bands: Fully correlated 1σ statistical error bands of the two analyses.

⇒ Solid line: 'Positivity Limit' taken from GRV98.

$g_1^p(x)$ **versus** Q^2



⇒ Yellow error band: Fully correlated 1σ statistical error band as given by Gaussian error propagation.

Systematic Studies

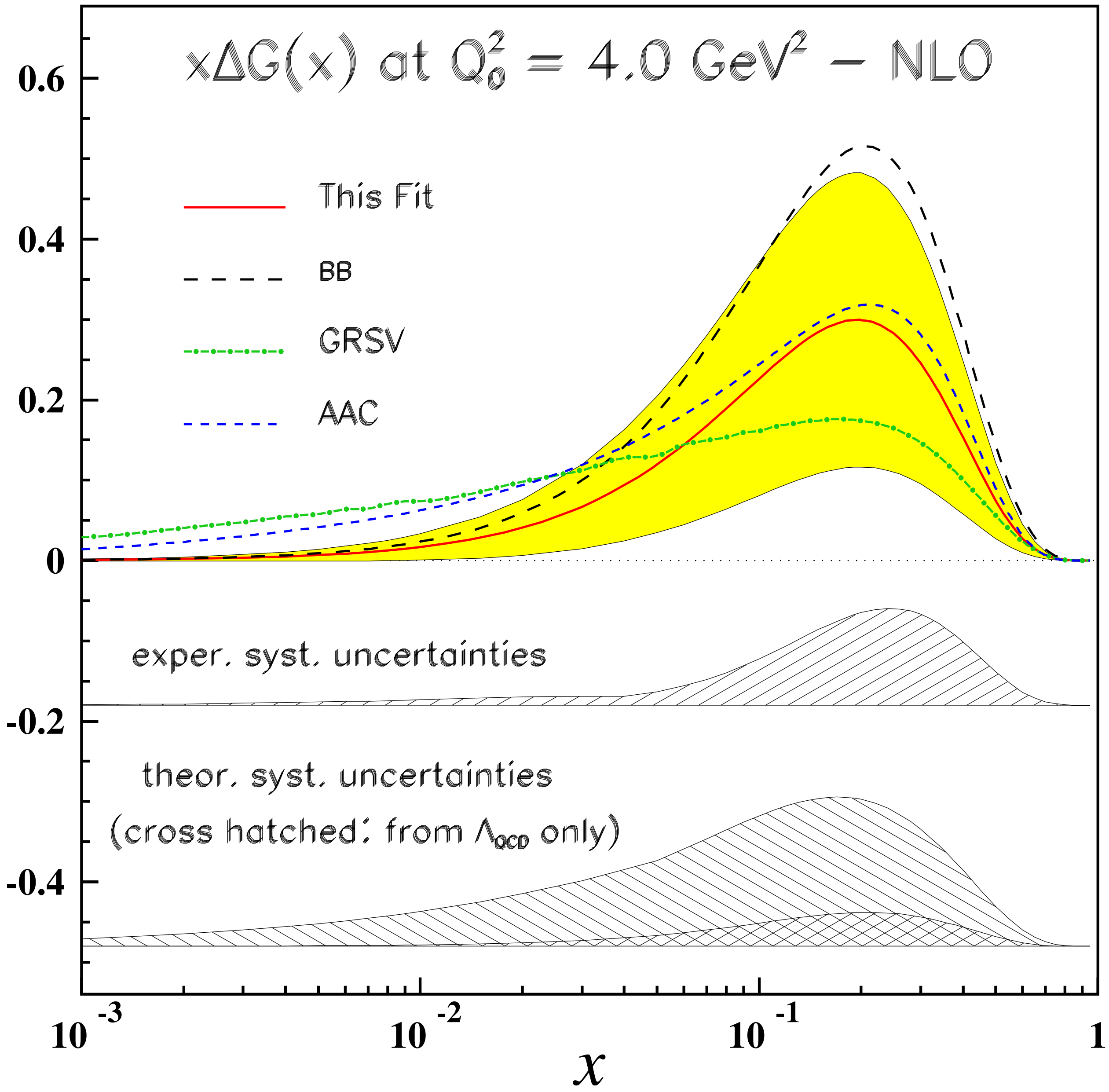
- Experimental Systematic Uncertainties:
  - modifying the data points by their Experimental Systematic Errors.
  - using the NMC F_2 Parameterization at its upper and lower limits.
  - using the SLAC R_{1990} Parameterization at its error limits.

- Theoretical Systematic Uncertainties:
  - varying the Factorization Scale by a factor of 2.
  - varying the Renormalization Scale by a factor of 2.
  - varying Λ_{QCD} by adding $\pm 30\ MeV$, which corresponds to varying α_s by the error of the world average of ± 0.002.
  - varying η_{u_v} and η_{d_v} within their errors while keeping g_A/g_V constant.
  - varying the parameterization of the input distributions.

$\Longrightarrow$ Always the extreme changes to the central curve are taken as systematic error.

$\Longrightarrow$ Finally the contribution from all items are added quadratically to get the total experimental and theoretical systematic error.

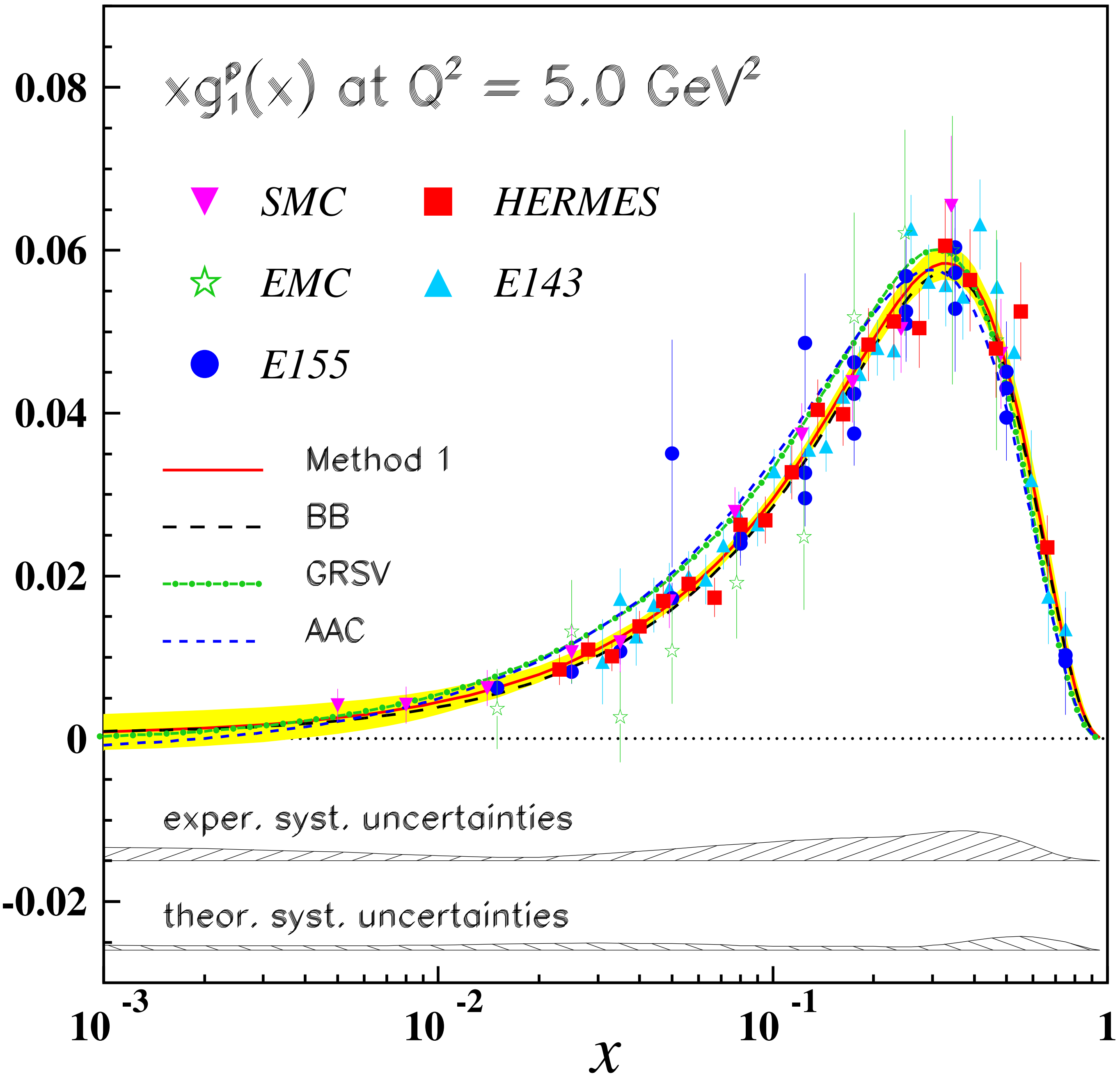
Systematics Uncertainties for ΔG



$\Rightarrow$ Yellow error band: Fully correlated 1σ statistical error band as given by Gaussian error propagation.

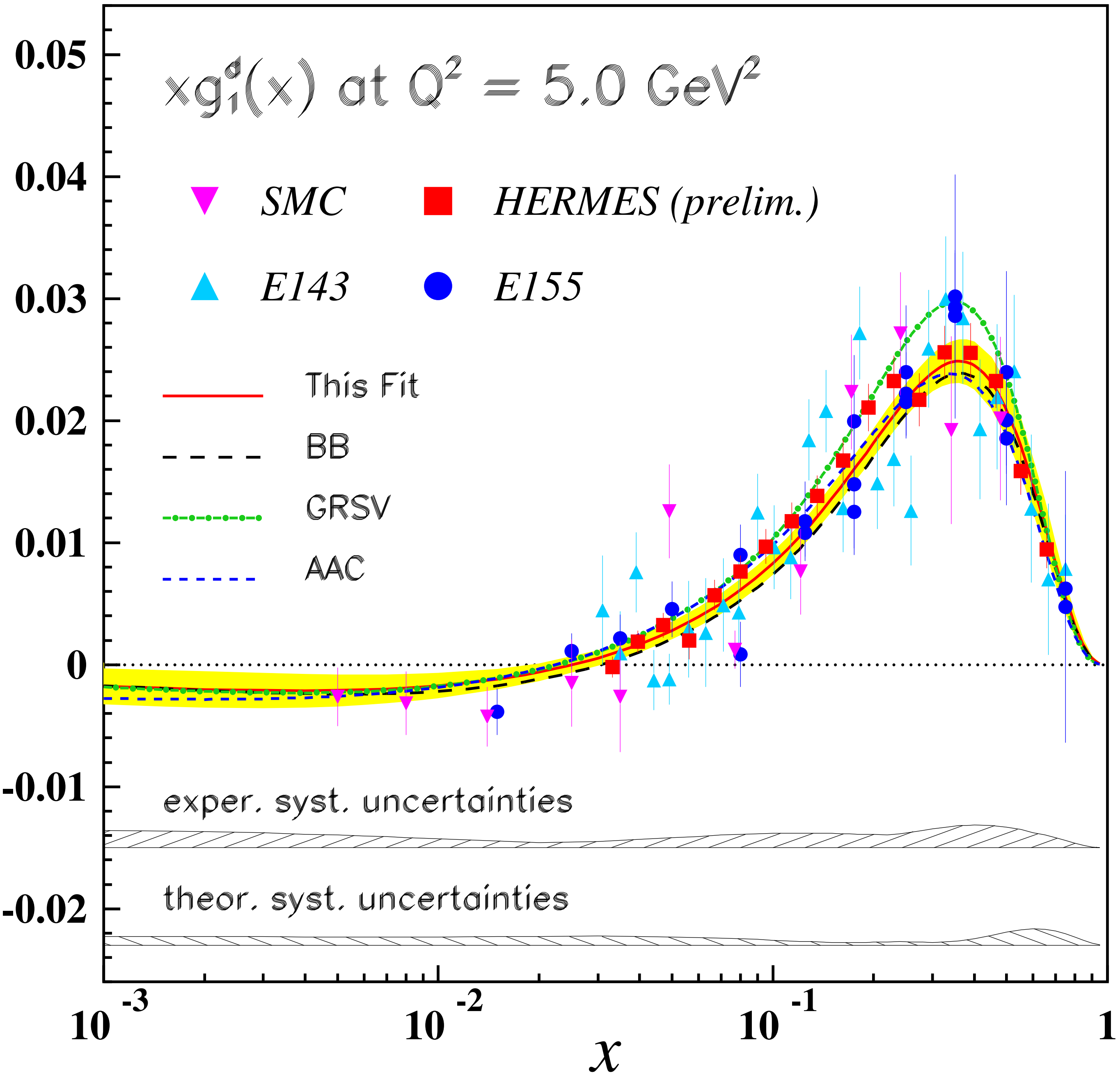
$$\Rightarrow \eta_{\Delta G} = 0.62 \pm 0.39\,(stat) \pm 0.17\,(exp) \pm 0.42\,(theo)$$

g_1^p world data



⇒ Yellow error band: Fully correlated 1σ statistical error band as given by Gaussian error propagation.

g_1^d world data



$\Rightarrow$ Yellow error band: Fully correlated 1σ statistical error band as given by Gaussian error propagation.

Summary

- An NLO QCD Analysis of the current World-Data of Polarized Structure Functions was performed.

- Parameterizations of the polarized parton densities including their fully correlated 1σ Error Bands were derived.

- While the polarized valence distributions Δu_v and Δd_v are well constrained by the present data, the polarized densities ΔG and $\Delta\bar{q}$ are only poorly determined. New precise data are needed both in the region of lower x and of higher Q^2.

- From the theoretical systematic uncertainties for ΔG it becomes clear that a refinement of the theoretical framework is required in the future ($\Rightarrow$NNLO) to cope with the ongoing improvement of the experimental accuracy.

- A very good description of the polarized structure function g_1 was achieved.