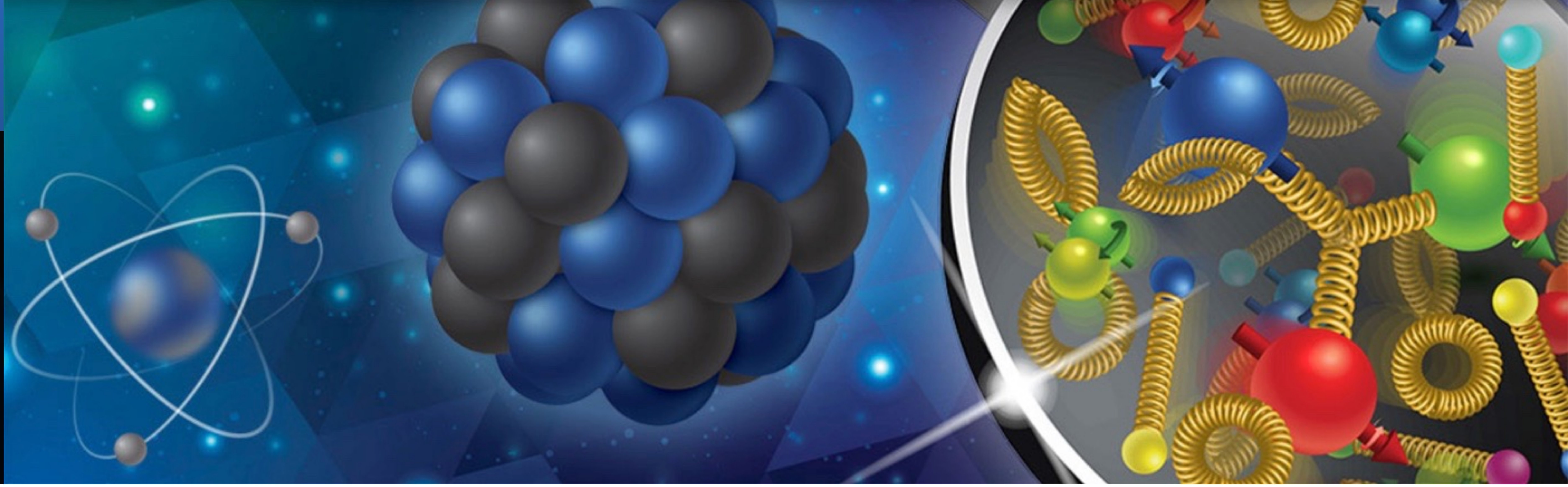
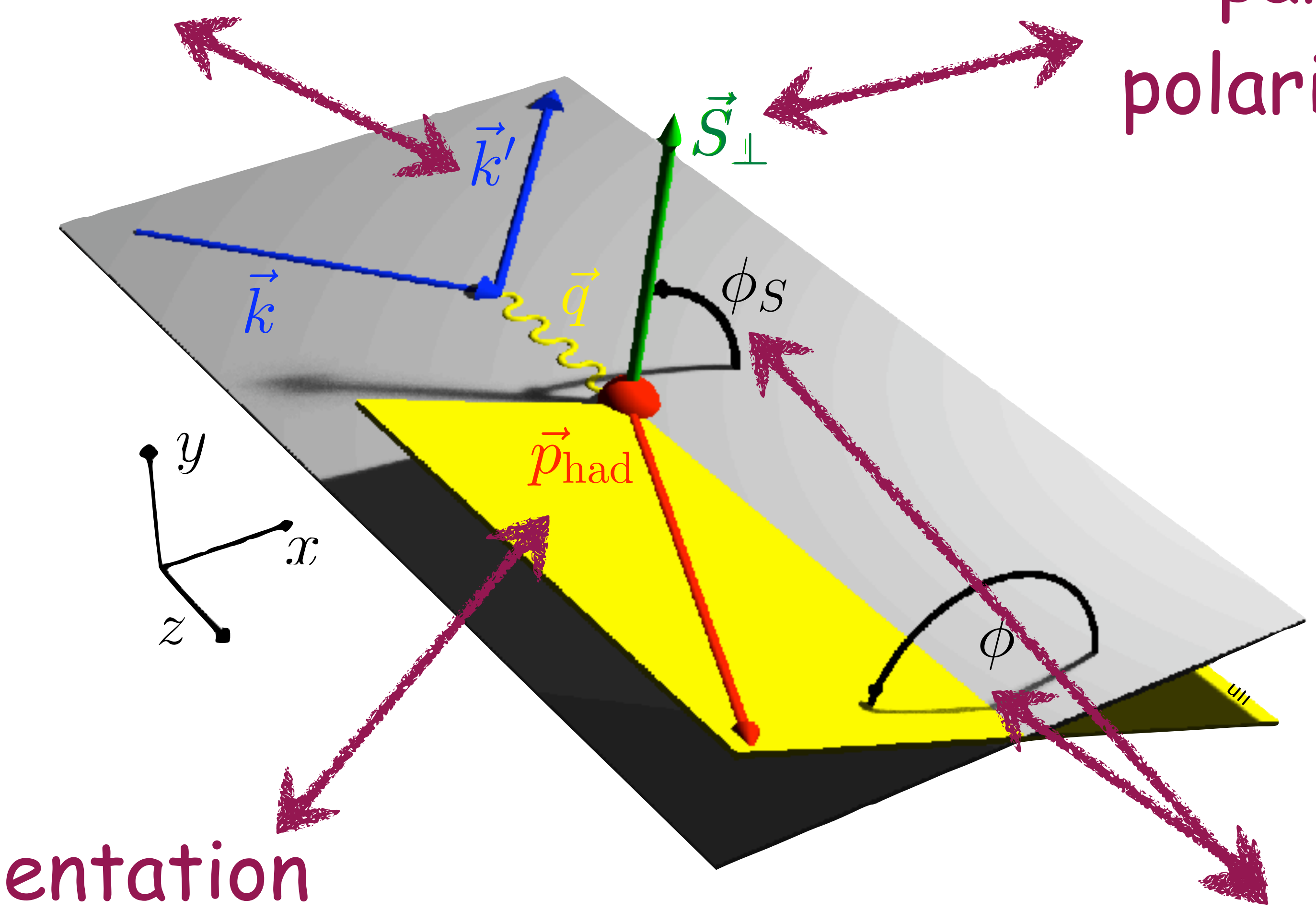
QCD EVOLUTION 2021

UCLA QCD EVOLUTION WORKSHOP 2021

May 10-14, 2021

UCLA





parton kinematics

parton polarization

fragmentation kinematics

FF selector

Overview of recent HERMES results on transverse-momentum dependent spin asymmetries

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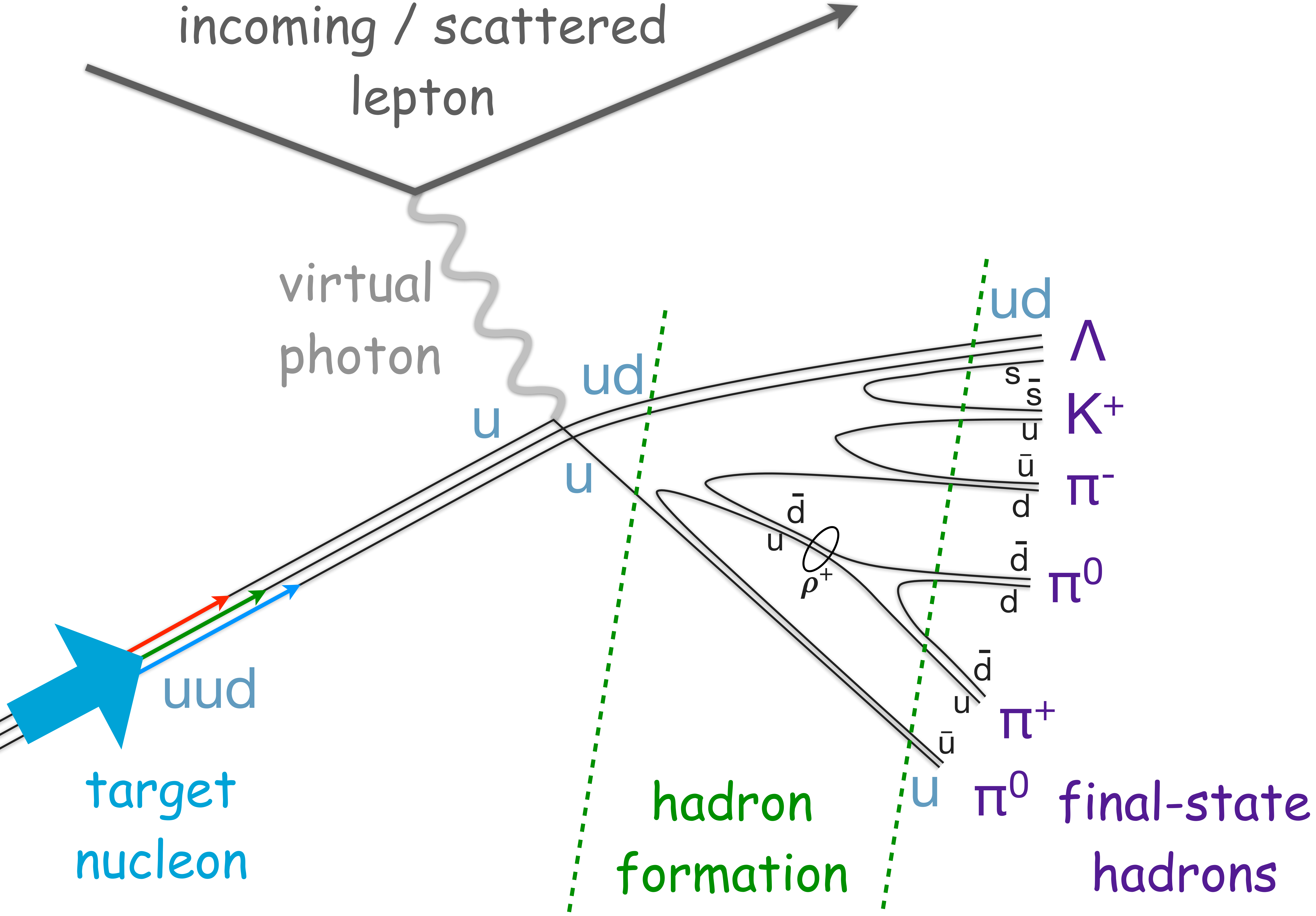
SIDIS: probing TMD PDFs through fragmentation



incoming / scattered lepton

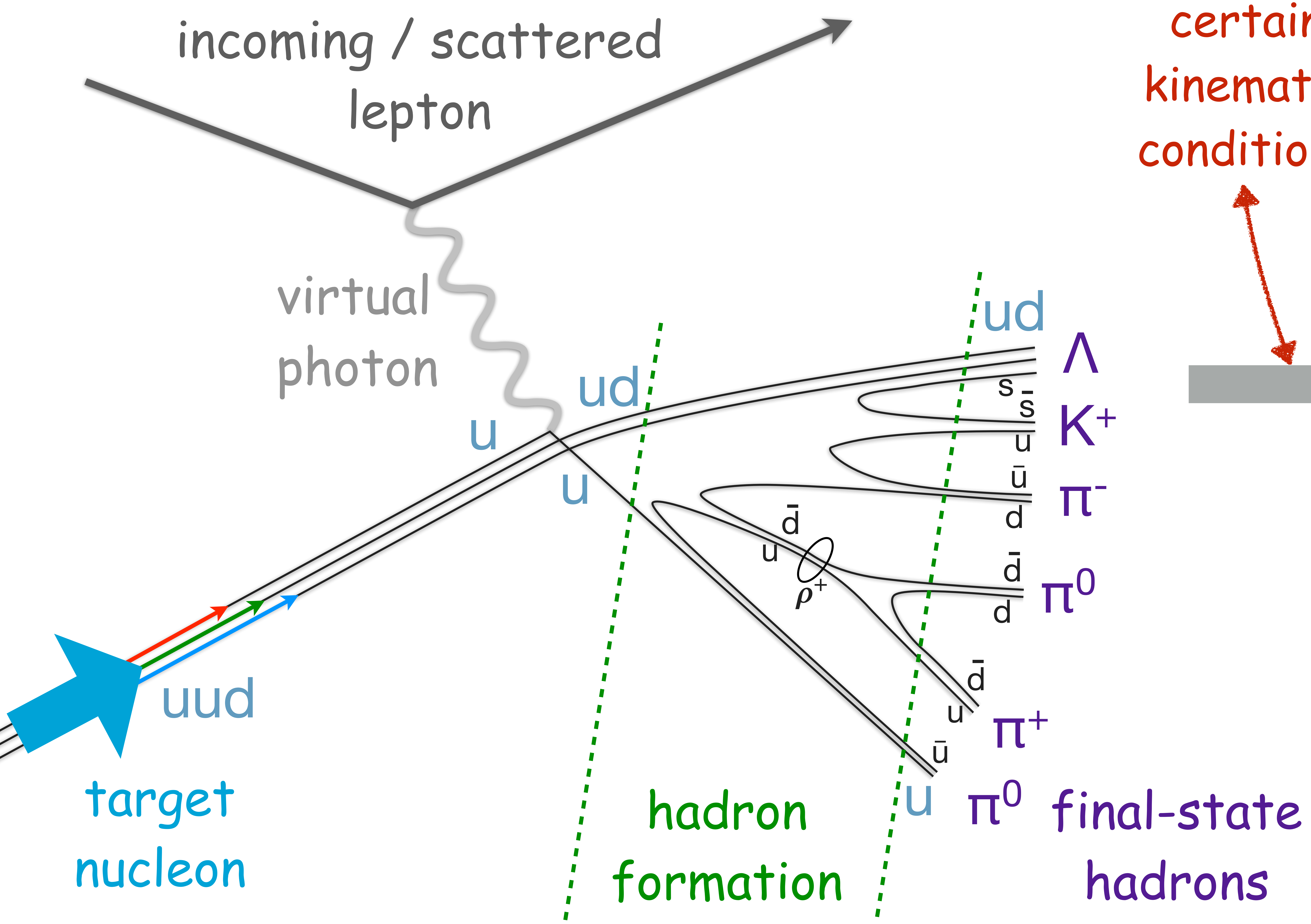
virtual photon

target nucleon

hadron formation

final-state hadrons

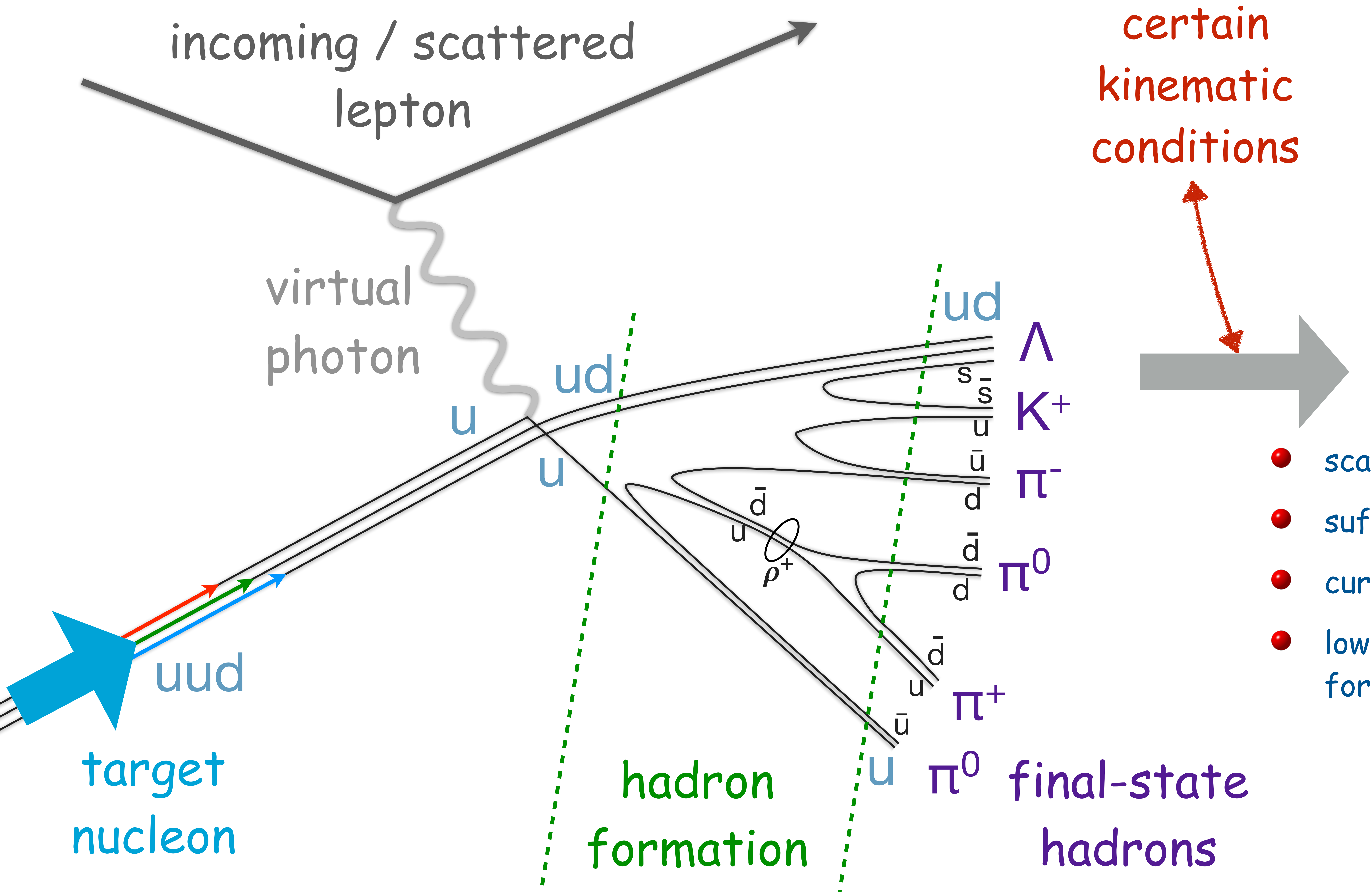
SIDIS: probing TMD PDFs through fragmentation



certain kinematic conditions

(TMD) PDF ⊗ (TMD) FF

SIDIS: probing TMD PDFs through fragmentation



(TMD) PDF ⊗ (TMD) FF

- scattering by partons
- sufficient energy for hadronization
- current vs. target fragmentation
- low enough transverse momentum for TMD factorization

Spin-momentum structure of the nucleon

$$\frac{1}{2}\mathrm{Tr}\Big[(\gamma^+ + \lambda\gamma^+\gamma_5)\,\Phi\Big] = \frac{1}{2}\Big[f_1 + S^i\epsilon^{ij}k^j\frac{1}{m}\,f_{1T}^\perp + \lambda\Lambda\,g_1 + \lambda\,S^ik^i\frac{1}{m}\,g_{1T}\Big]$$

$$\frac{1}{2}\mathrm{Tr}\Big[(\gamma^+ - s^j i\sigma^{+j}\gamma_5)\,\Phi\Big] = \frac{1}{2}\Big[f_1 + S^i\epsilon^{ij}k^j\frac{1}{m}\,f_{1T}^\perp + s^i\epsilon^{ij}k^j\frac{1}{m}\,h_1^\perp + s^iS^ih_1$$

$$+ s^i(2k^ik^j - \boldsymbol{k}^2\delta^{ij})S^j\frac{1}{2m^2}\,h_{1T}^\perp + \Lambda\,s^ik^i\frac{1}{m}\,h_{1L}^\perp\Big]$$

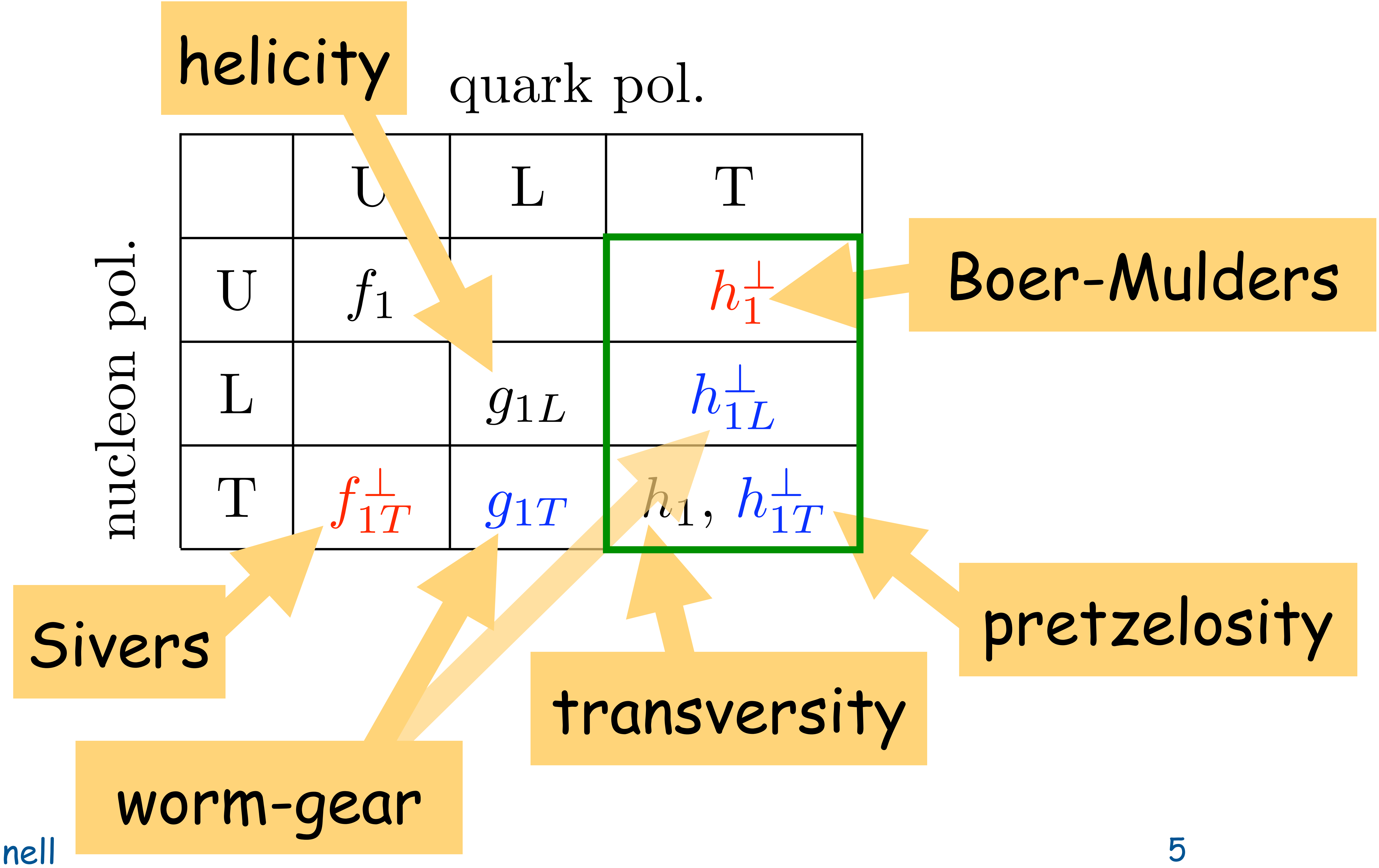
quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

- each TMD describes a particular spin-momentum correlation
- functions in black survive integration over transverse momentum
- functions in green box are chirally odd
- functions in red are naive T-odd

Spin-momentum structure of the nucleon

$$
\begin{aligned}
\frac{1}{2}\mathrm{Tr}\left[(\gamma^+ + \lambda\gamma^+\gamma_5)\,\Phi\right] &= \frac{1}{2}\left[f_1 + S^i\epsilon^{ij}k^j\,\frac{1}{m}\,f_{1T}^\perp + \lambda\Lambda\,g_1 + \lambda\,S^ik^i\,\frac{1}{m}\,g_{1T}\right]\\
\frac{1}{2}\mathrm{Tr}\left[(\gamma^+ - s^j i\sigma^{+j}\gamma_5)\,\Phi\right] &= \frac{1}{2}\Bigg[f_1 + S^i\epsilon^{ij}k^j\,\frac{1}{m}\,f_{1T}^\perp + s^i\epsilon^{ij}k^j\,\frac{1}{m}\,h_1^\perp + s^iS^ih_1\\
&\qquad + s^i(2k^ik^j - \boldsymbol{k}^2\delta^{ij})S^j\,\frac{1}{2m^2}\,h_{1T}^\perp + \Lambda\,s^ik^i\,\frac{1}{m}\,h_{1L}^\perp\Bigg]
\end{aligned}
$$



quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

- each TMD describes a particular spin-momentum correlation
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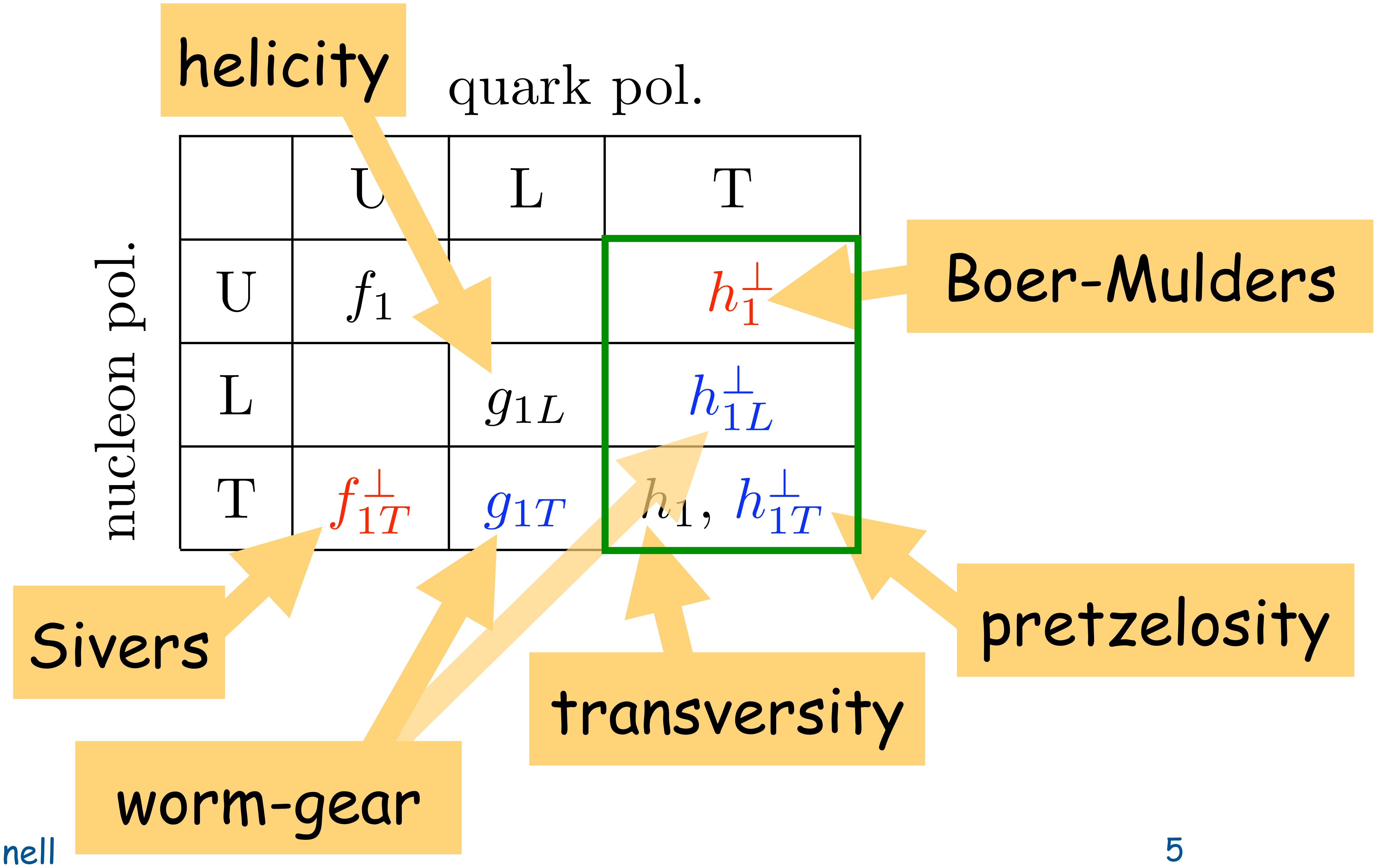
Spin-momentum structure of the nucleon

$$\frac{1}{2}\mathrm{Tr}\Big[(\gamma^+ + \lambda\gamma^+\gamma_5)\,\Phi\Big] = \frac{1}{2}\Big[f_1 + S^i\epsilon^{ij}k^j\,\frac{1}{m}\,f_{1T}^{\perp} + \lambda\Lambda\,g_1 + \lambda\,S^i k^i\,\frac{1}{m}\,g_{1T}\Big]$$

$$\begin{aligned}\frac{1}{2}\mathrm{Tr}\Big[(\gamma^+ - s^j i\sigma^{+j}\gamma_5)\,\Phi\Big] = \frac{1}{2}\Big[&f_1 + S^i\epsilon^{ij}k^j\,\frac{1}{m}\,f_{1T}^{\perp} + s^i\epsilon^{ij}k^j\,\frac{1}{m}\,h_1^{\perp} + s^iS^ih_1 \\ &+ s^i(2k^ik^j - \boldsymbol{k}^2\delta^{ij})S^j\,\frac{1}{2m^2}\,h_{1T}^{\perp} + \Lambda\,s^ik^i\,\frac{1}{m}\,h_{1L}^{\perp}\Big]\end{aligned}$$



quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1, $h_{1T}^{\perp}$

helicity

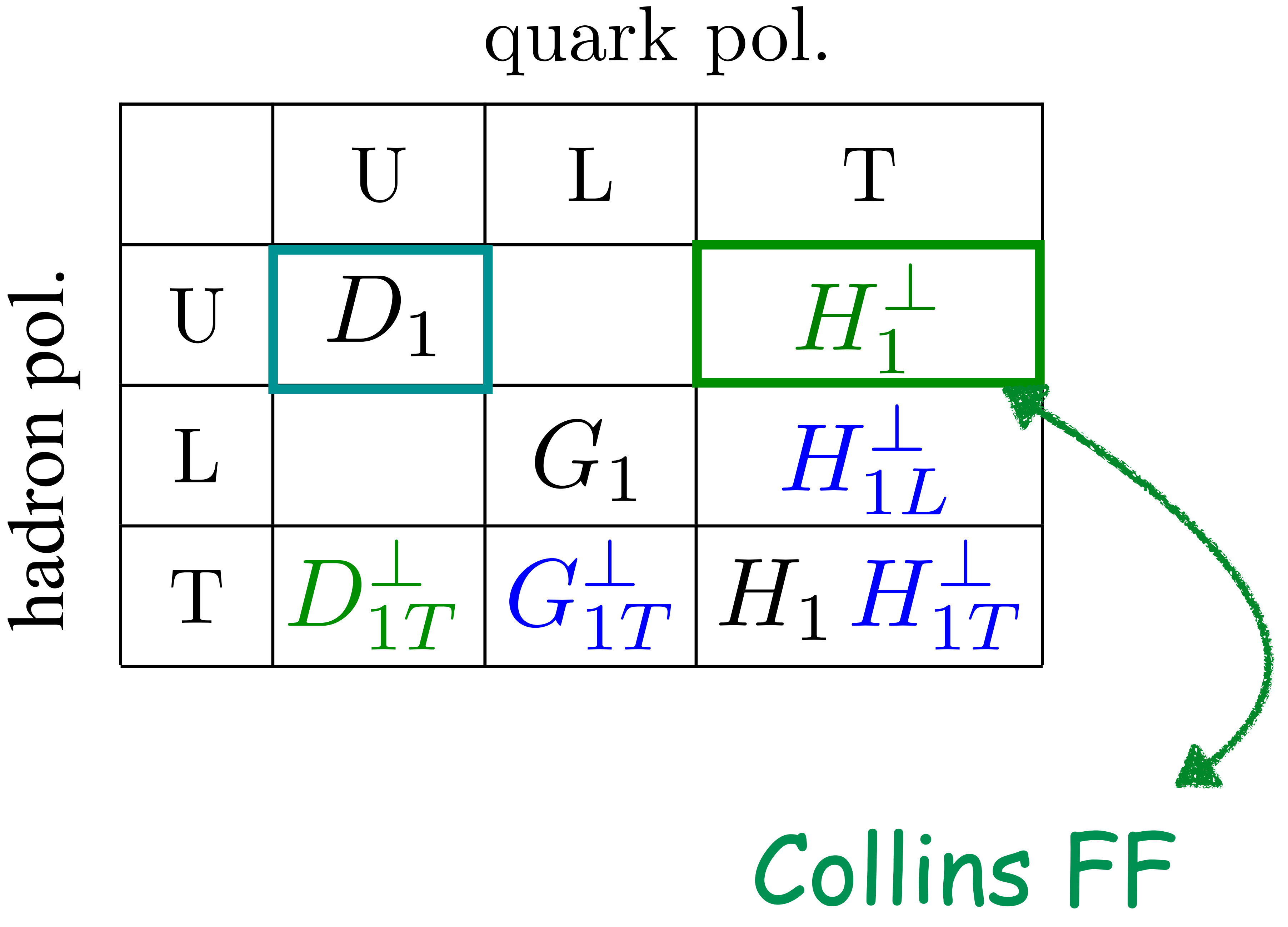
Boer-Mulders

Sivers

worm-gear

transversity

pretzelosity



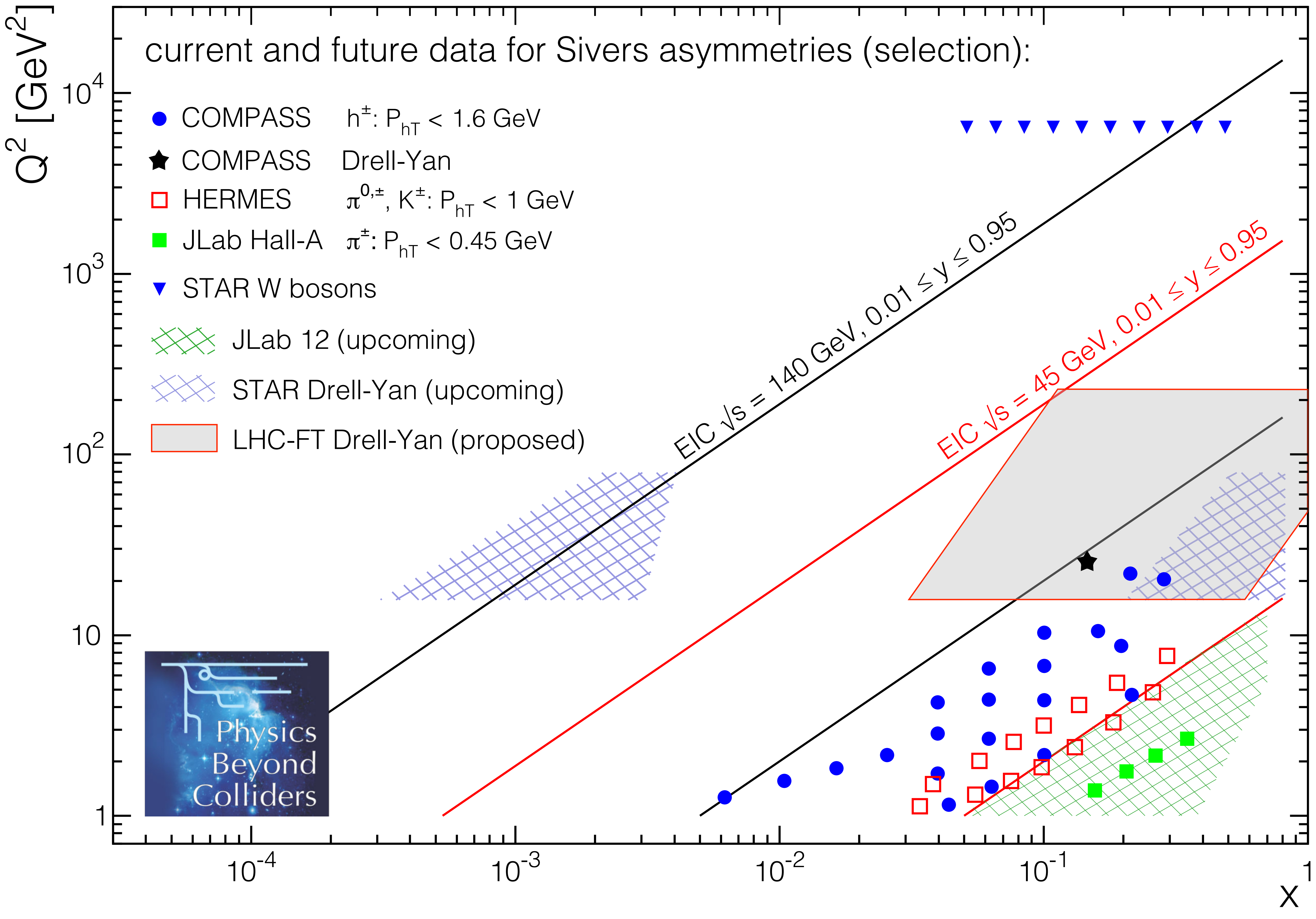
quark pol.

hadron pol.

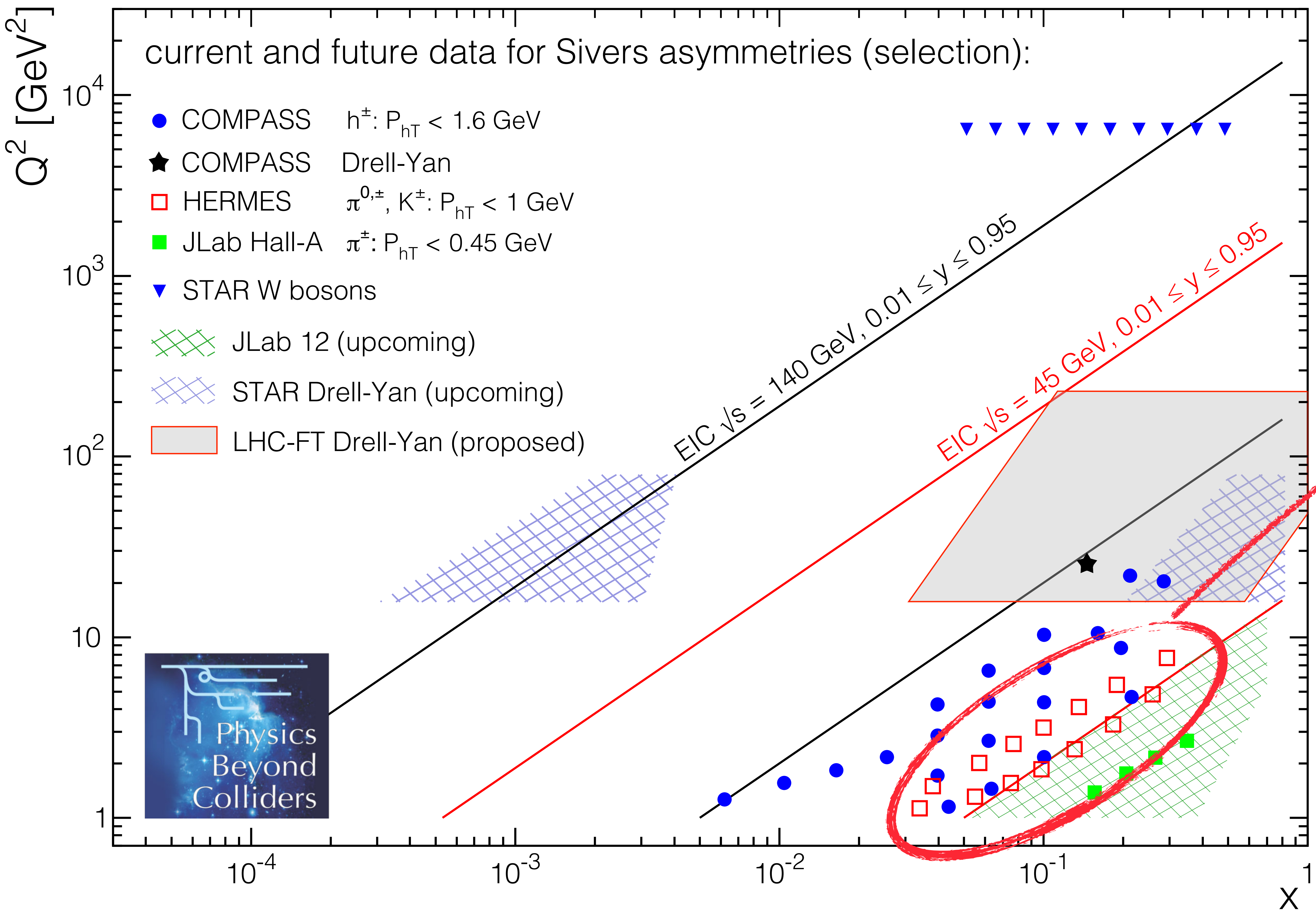
	U	L	T
U	D_1		$H_1^{\perp}$
L		G_1	$H_{1L}^{\perp}$
T	$D_{1T}^{\perp}$	$G_{1T}^{\perp}$	H_1 $H_{1T}^{\perp}$

Collins FF

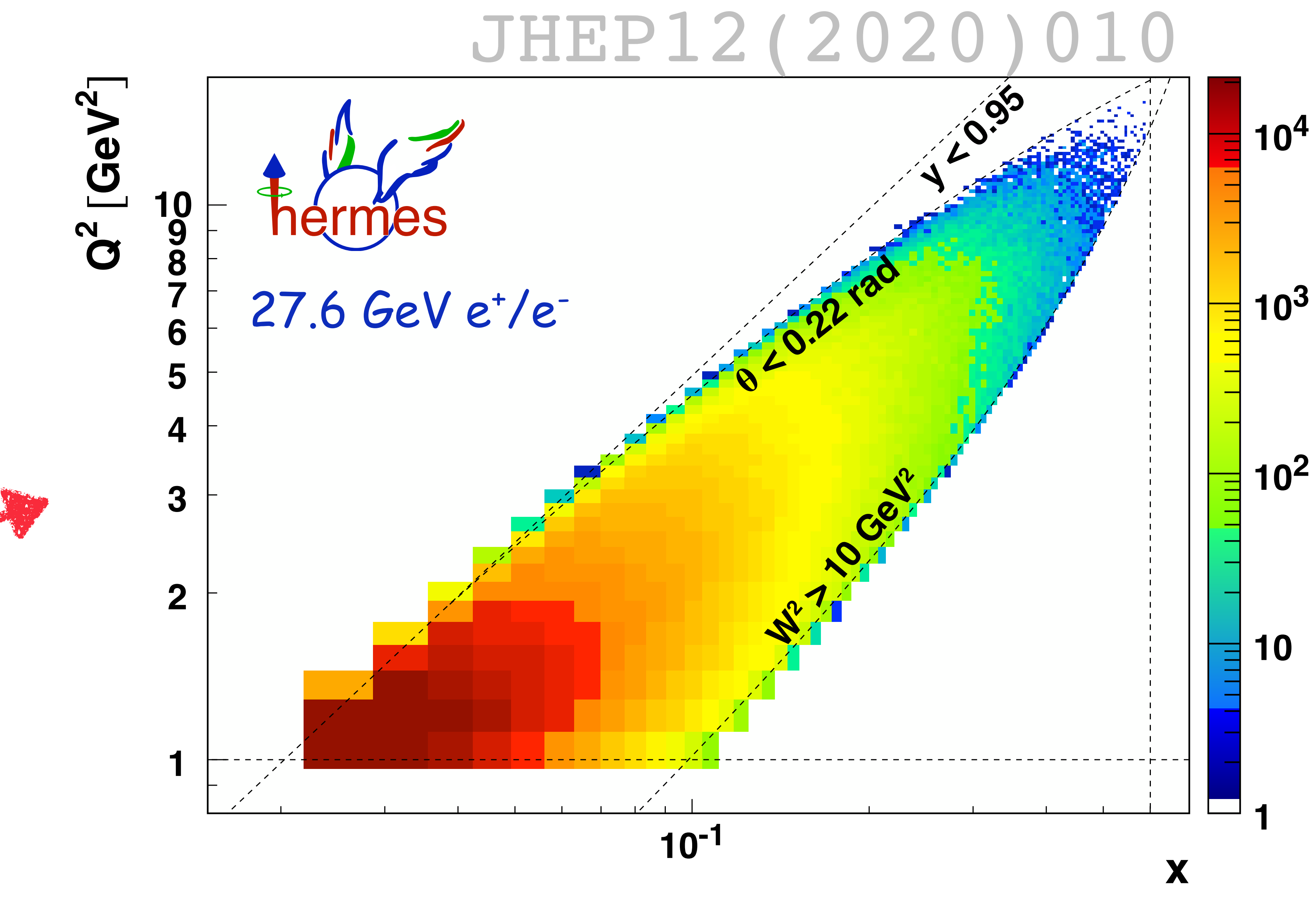
2d kinematic phase space



2d kinematic phase space



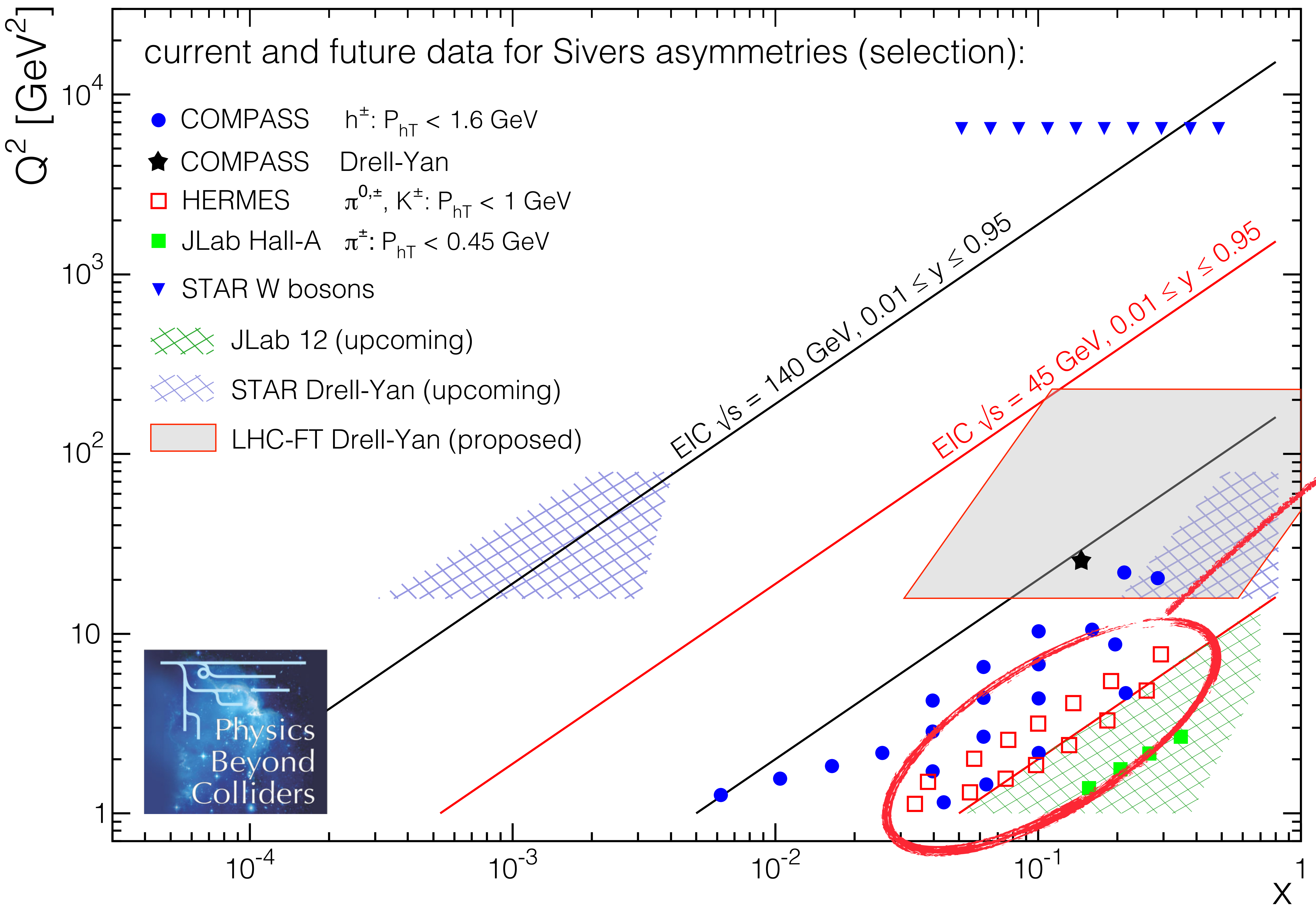
JHEP12(2020)010



Scattered lepton:		Q^2	$> 1\,\mathrm{GeV}^2$	
		W^2	$> 10\,\mathrm{GeV}^2$	
	$0.023 <$	x	< 0.6	
	$0.1 <$	y	< 0.95	
Detected hadrons:	$2\,\mathrm{GeV} <$	$\lvert \mathbf{P}_h \rvert$	$< 15\,\mathrm{GeV}$	charged mesons
	$4\,\mathrm{GeV} <$	$\lvert \mathbf{P}_h \rvert$	$< 15\,\mathrm{GeV}$	(anti)protons
		$\lvert \mathbf{P}_h \rvert$	$> 2\,\mathrm{GeV}$	neutral pions
		$P_{h\perp}$	$< 2\,\mathrm{GeV}$	
	$0.2 <$	z	< 0.7 (1.2 for the "semi-exclusive" region)	

Table 3. Restrictions on selected kinematics variables. The upper limit on z of 1.2 applies only to the analysis of the z dependence.

2d kinematic phase space



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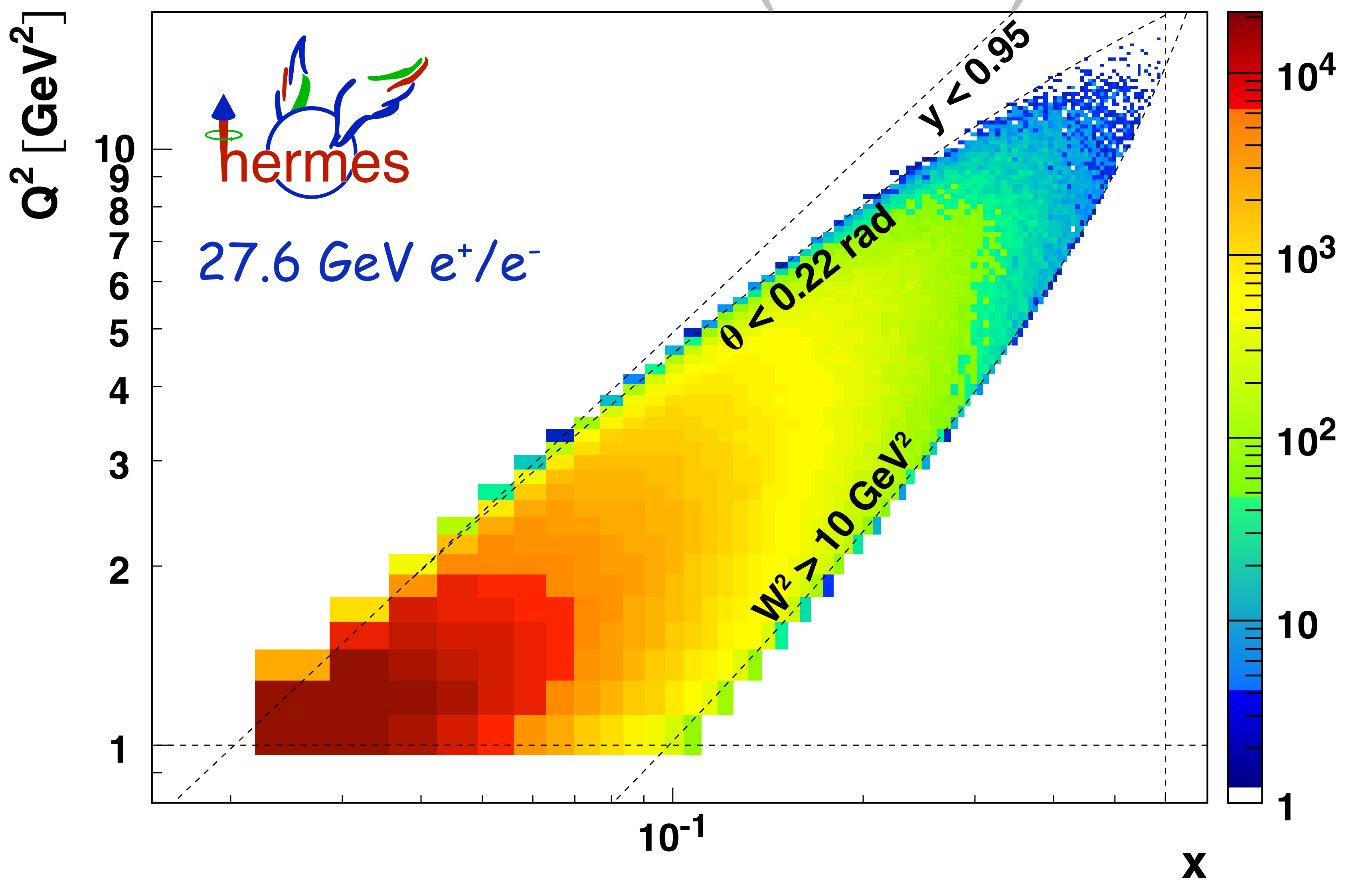
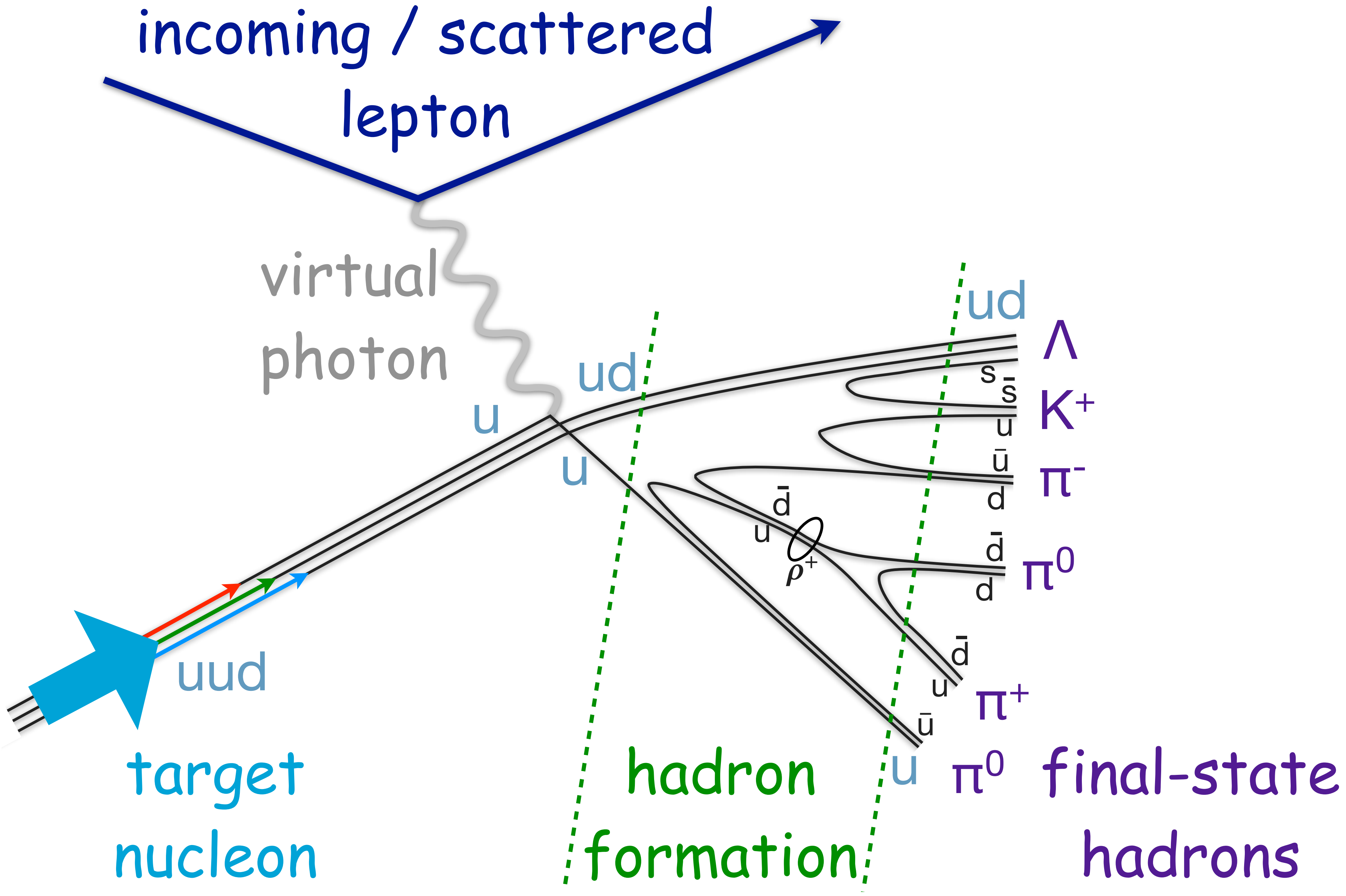
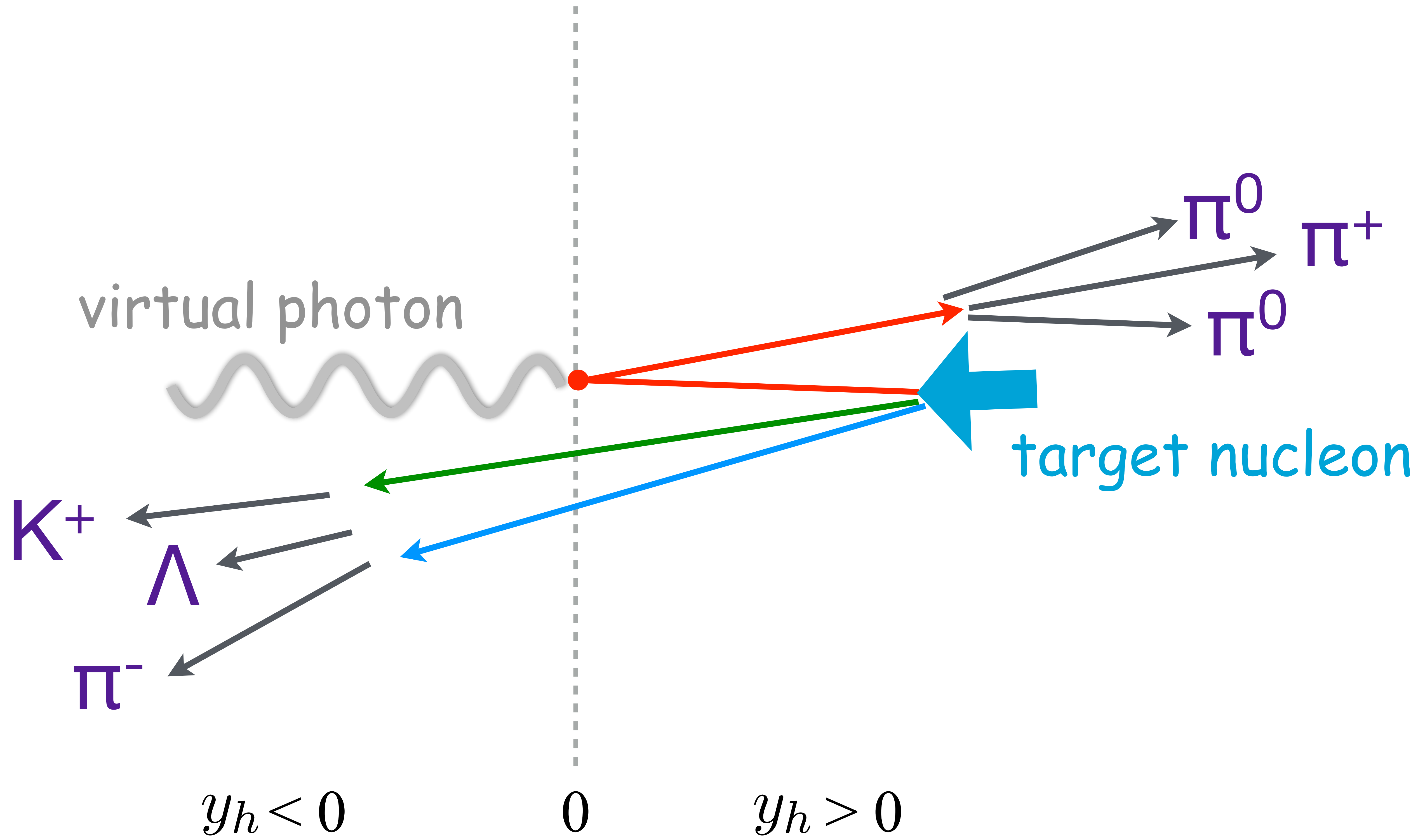


strongly correlated 2d (x-Q^2) kinematic space not favourable for evolution studies

current vs. target fragmentation



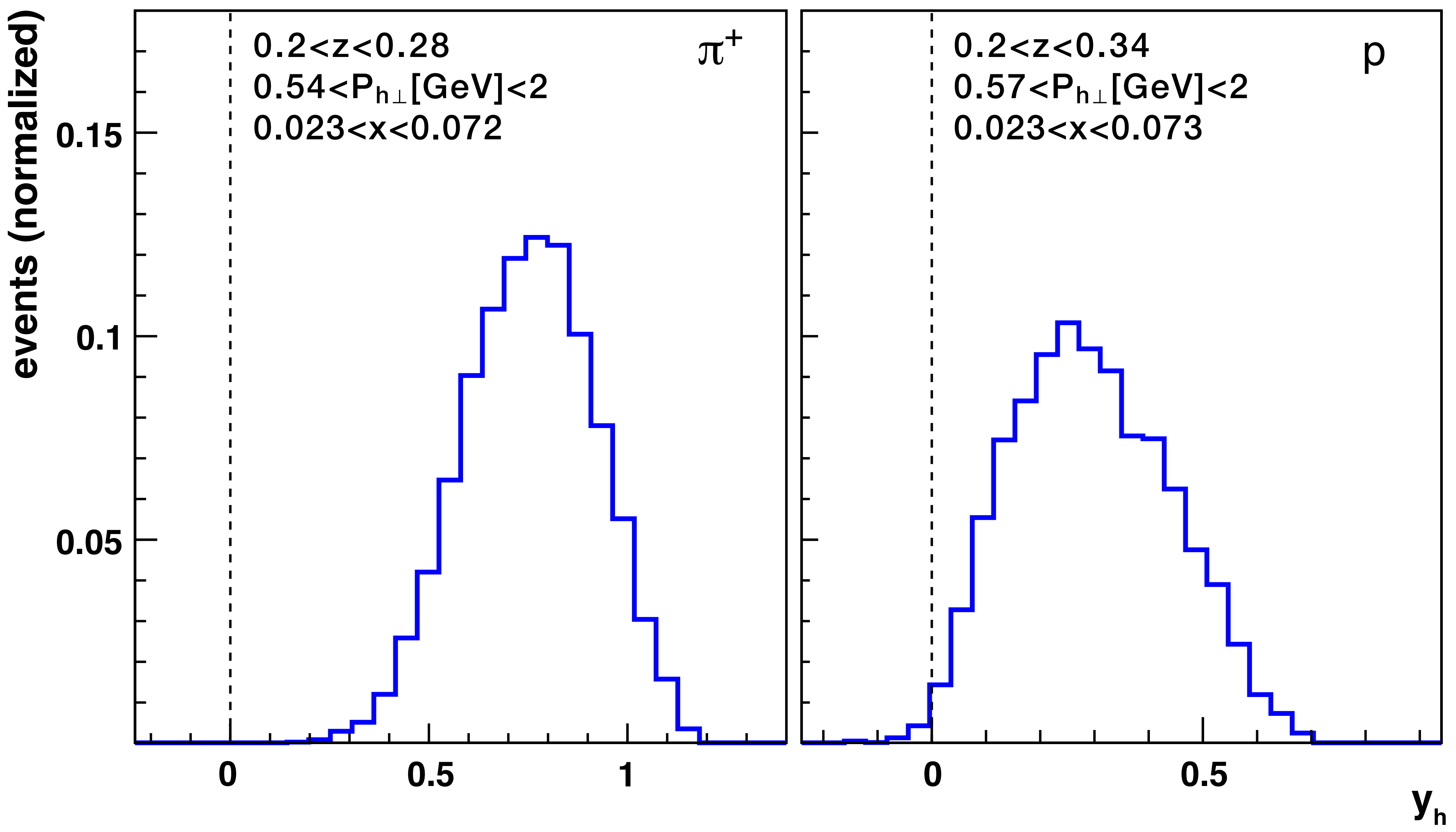
virtual-photon—nucleon c.m.s



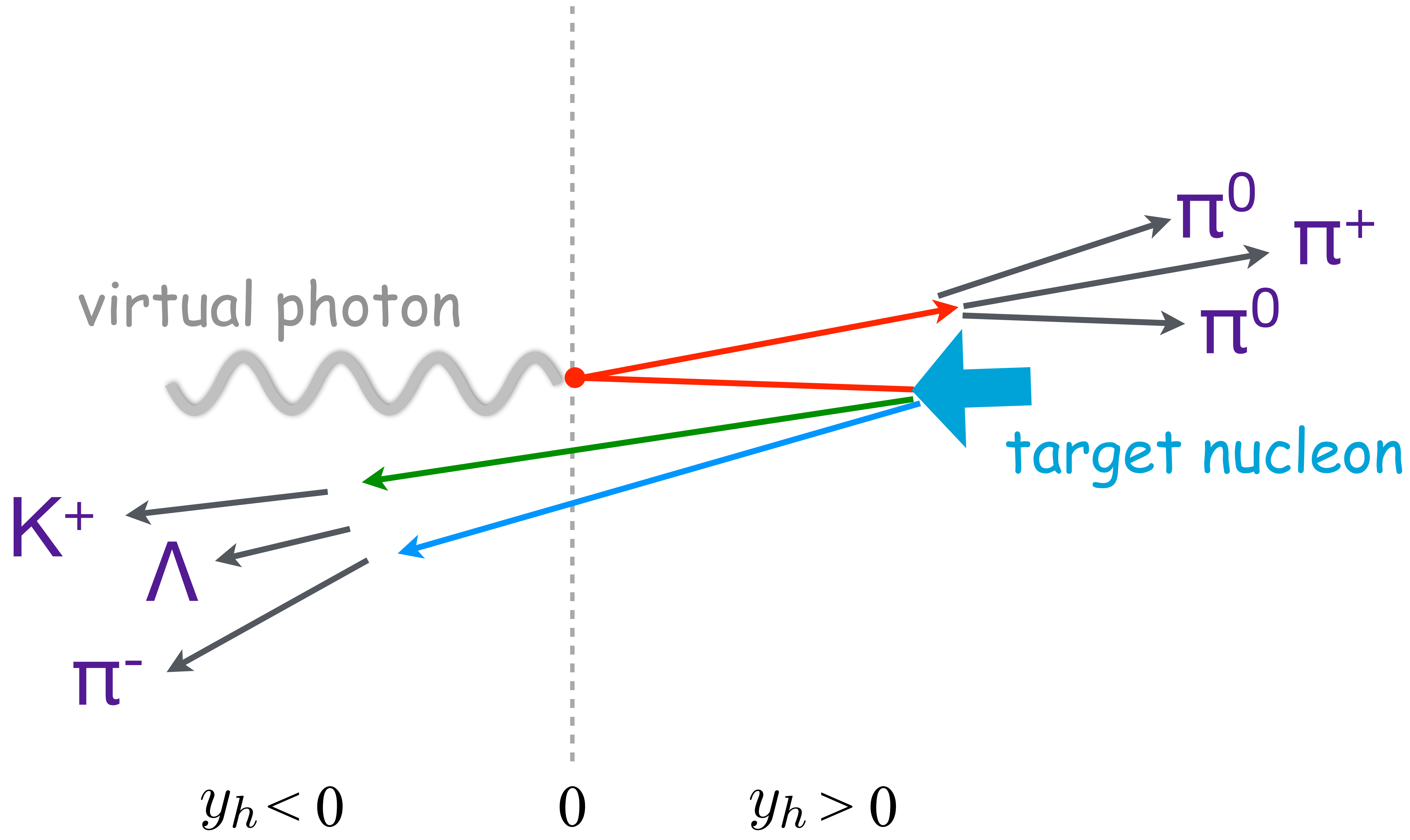
$$y_h \equiv \frac{1}{2} \ln \frac{P_h^+}{P_h^-}$$

$P_h^\pm$ … light-cone momenta

current vs. target fragmentation



virtual-photon—nucleon c.m.s



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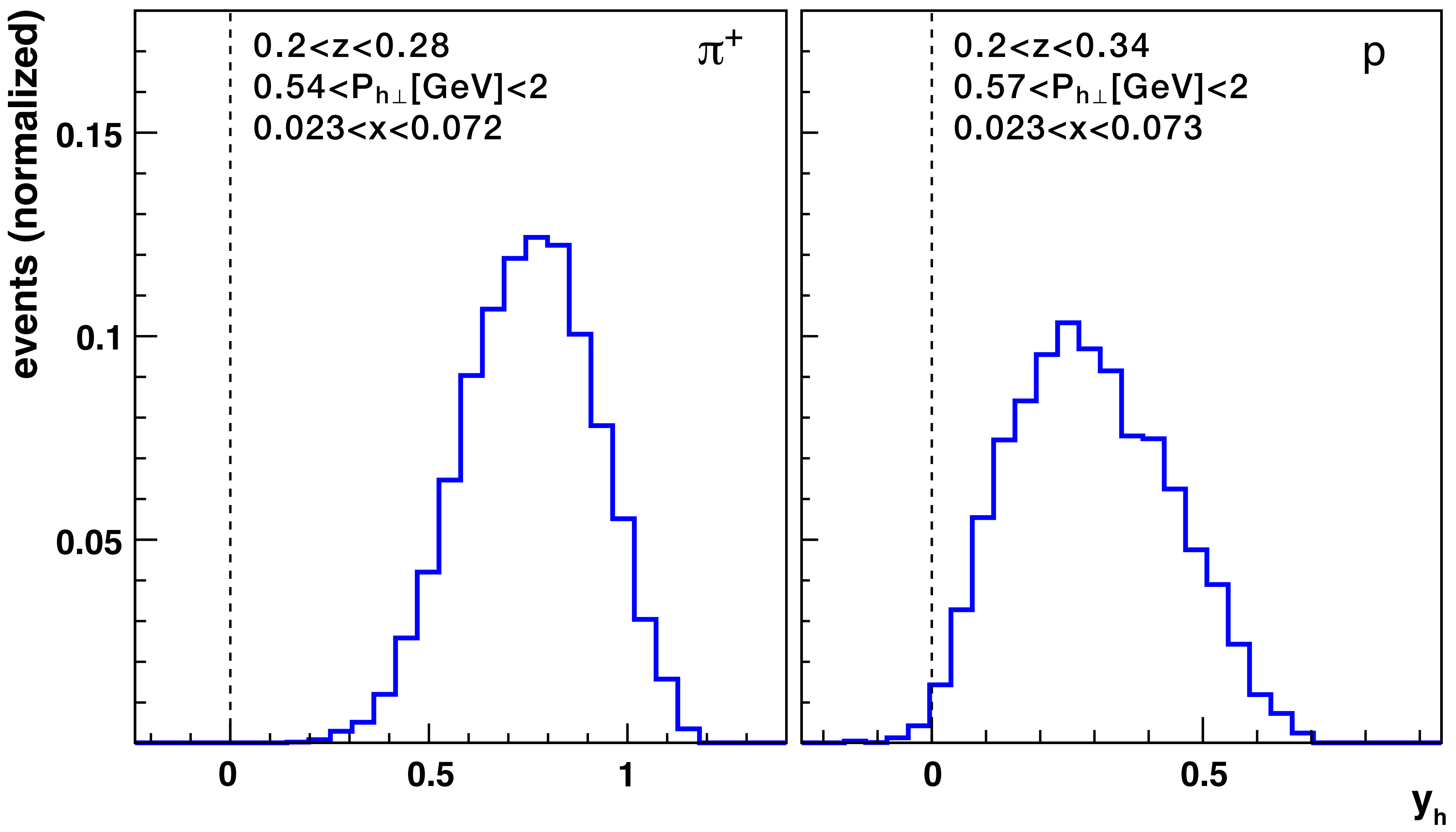
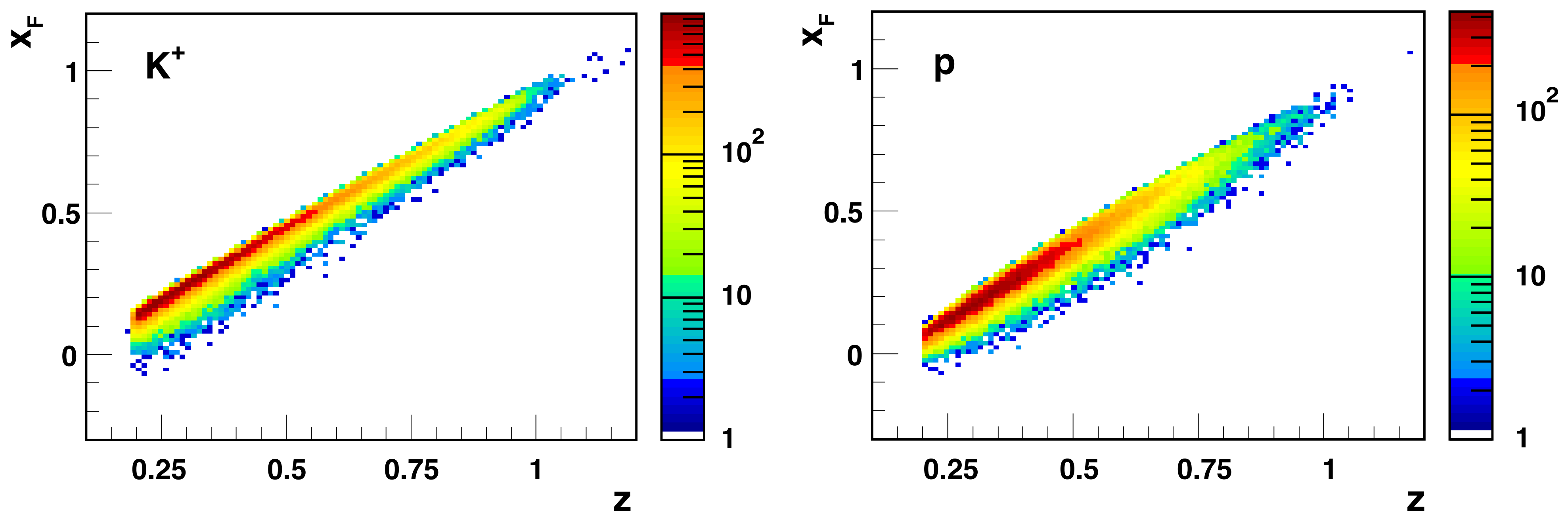
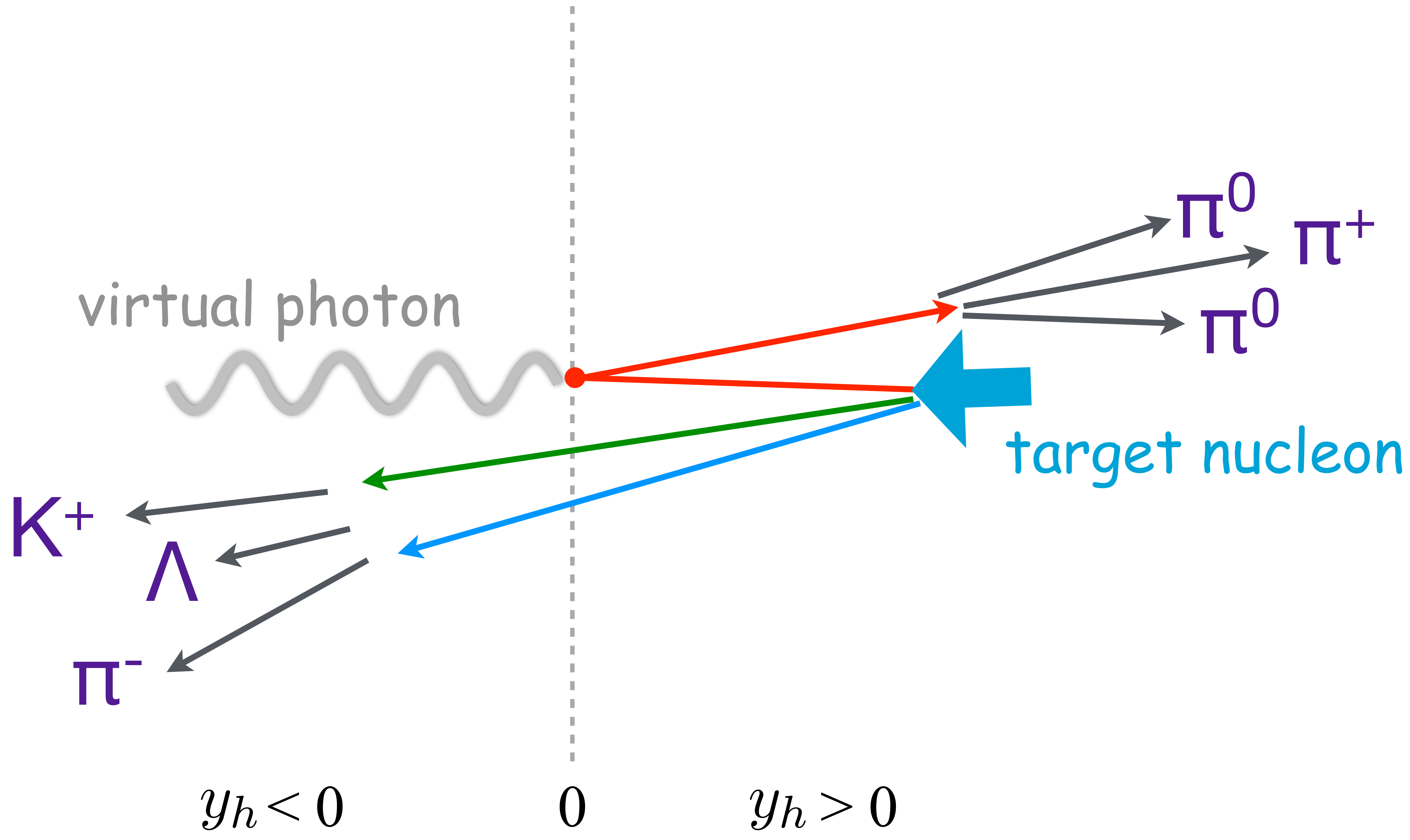




x_F … Feynman x

selected hadrons at HERMES mainly forward-going in photon-nucleon c.m.s.

virtual-photon—nucleon c.m.s

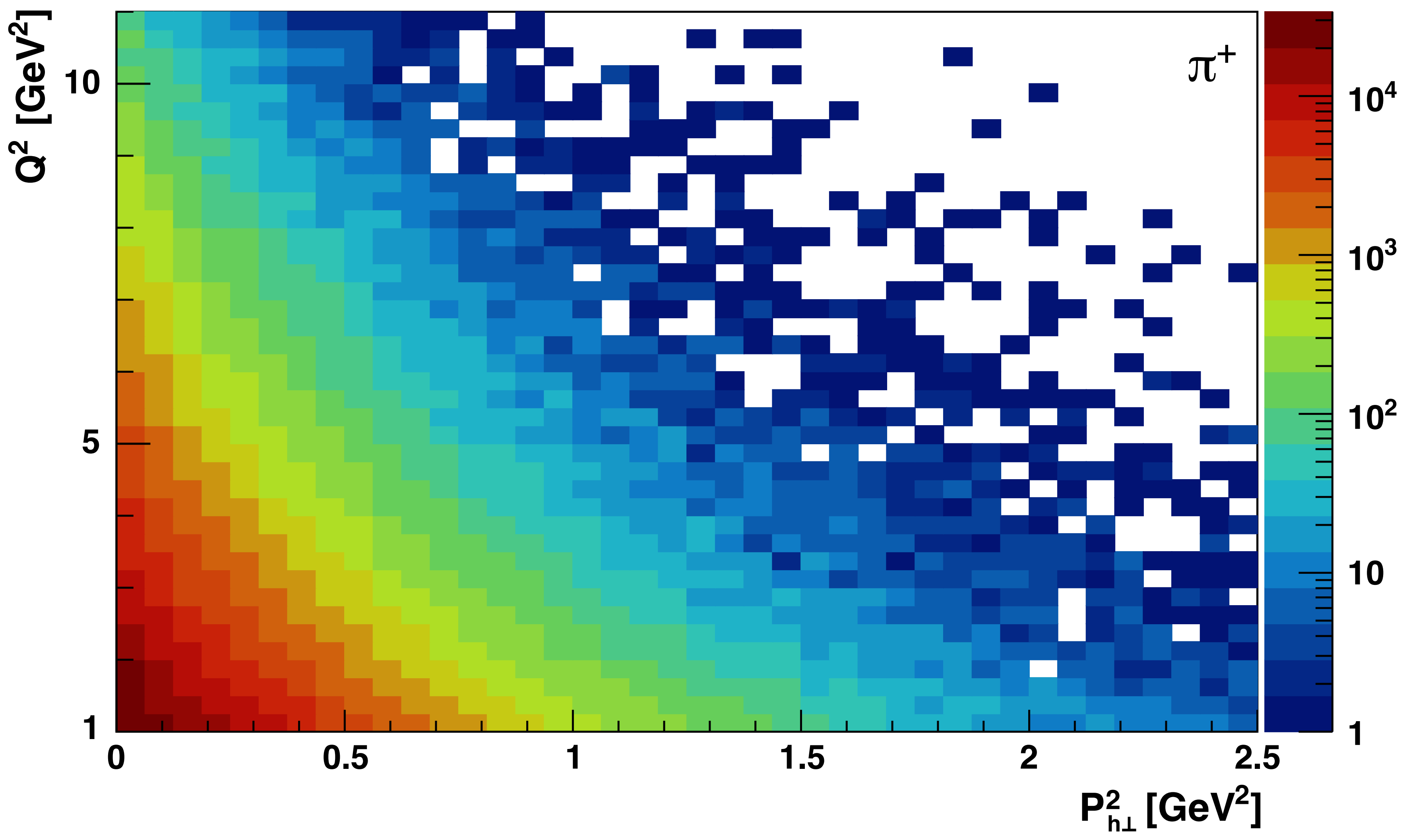


$$y_h \equiv \frac{1}{2} \ln \frac{P_h^+}{P_h^-}$$

$P_h^\pm$ … light-cone momenta

TMD factorization: a 2-scale problem

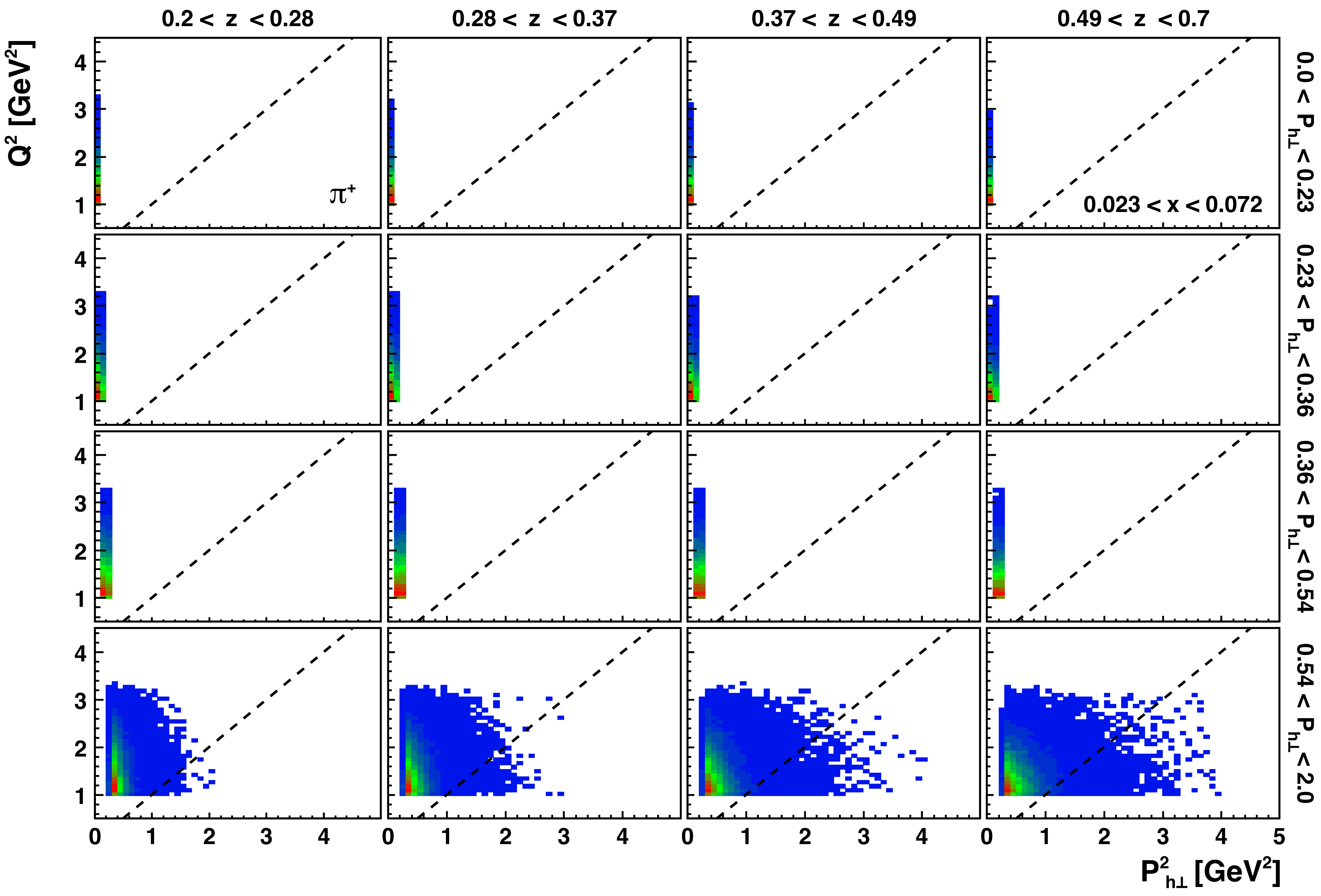
all events



- TMD factorization requires a large scale (Q^2) and small transverse momentum
- overall, Q mainly larger than $P_{h\perp}$
- not fulfilled in all kinematic bins
- more challenging, especially at low x (=low Q^2), for more stringent constraint of $zQ \gg P_{h\perp}$

TMD factorization: a 2-scale problem

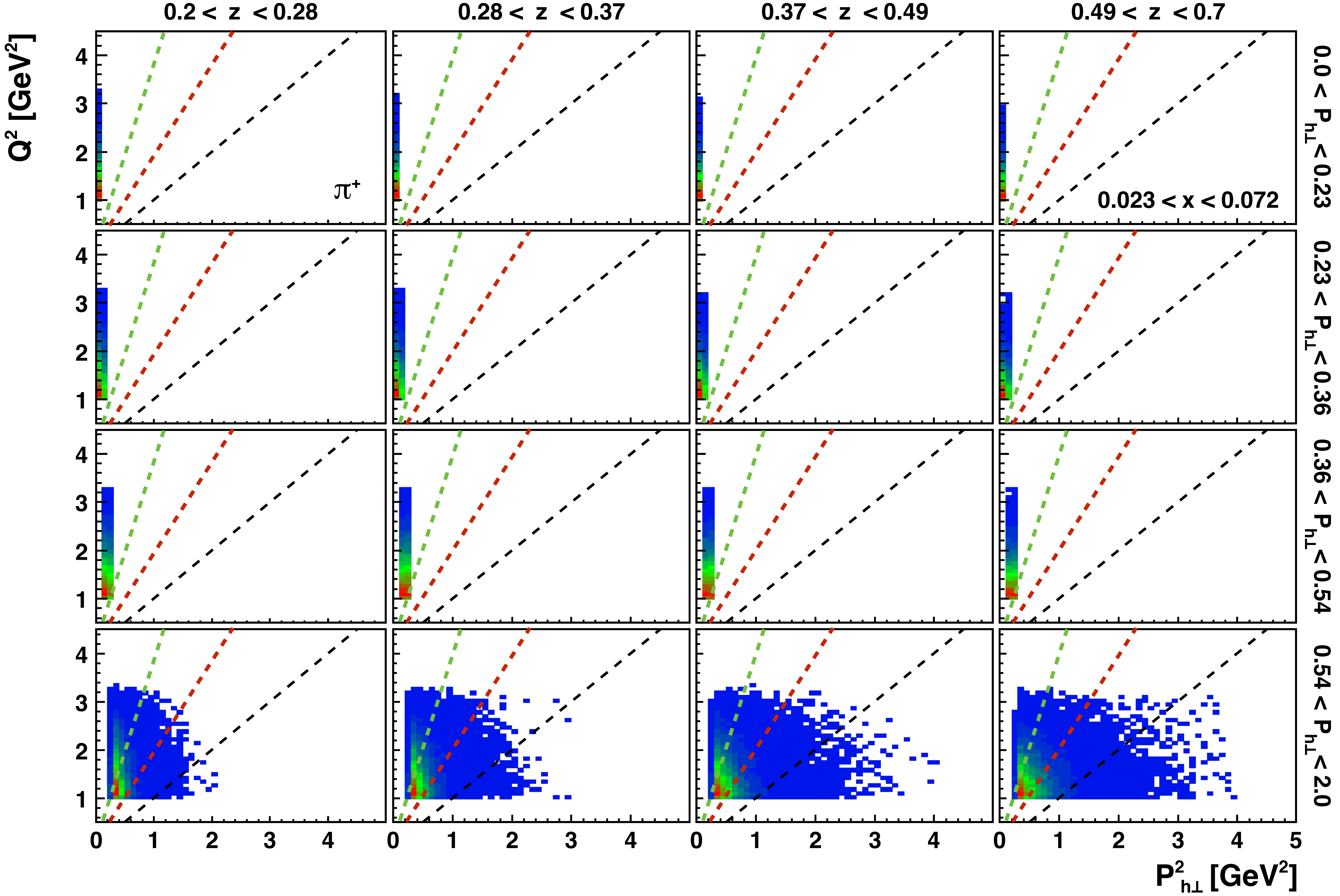
lowest x bin



- - - $Q^2 = P^2_{h\perp}$

TMD factorization: a 2-scale problem

lowest x bin



▬ ▬ ▬ $Q^2 = P^2_{h\perp}$

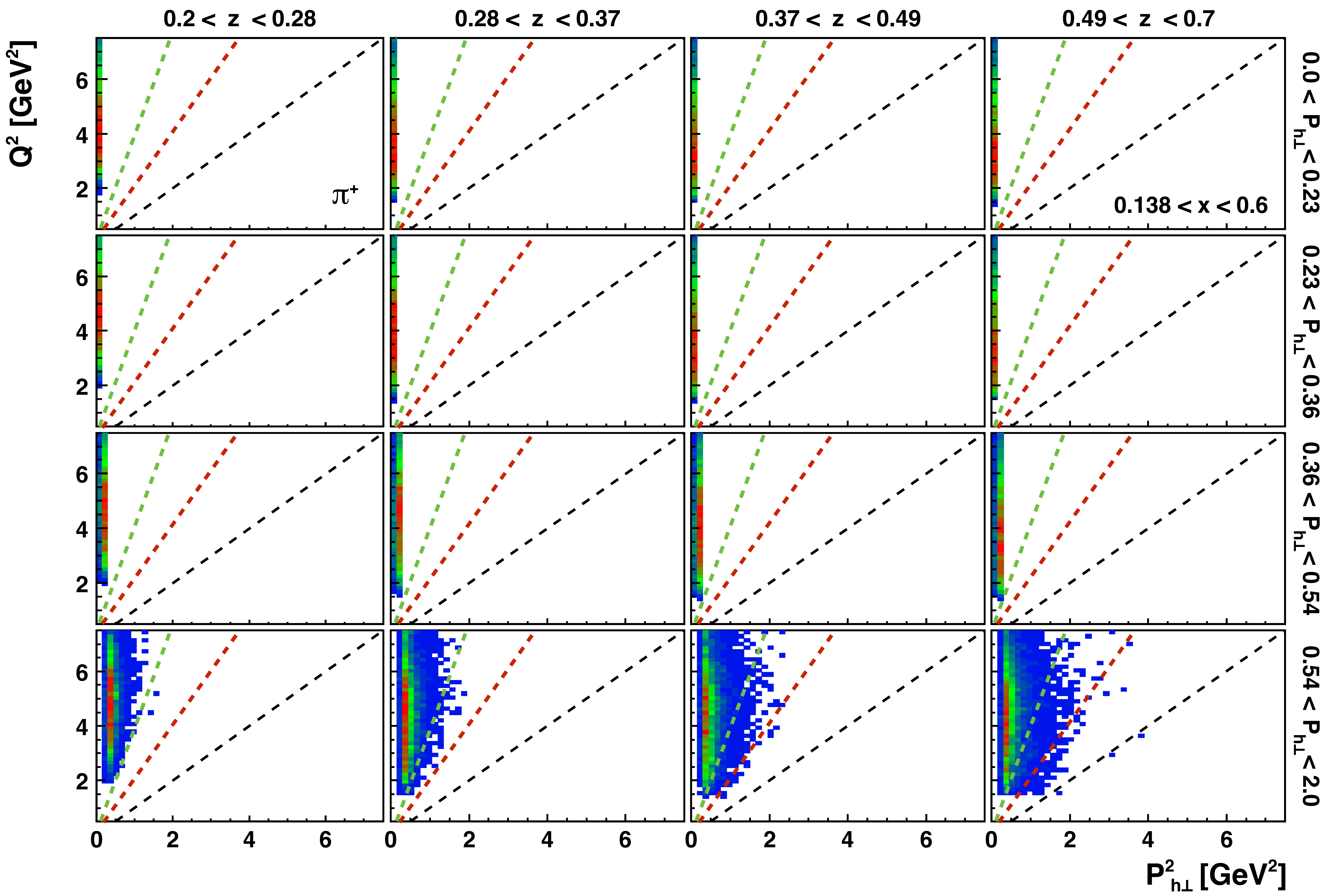
▬ ▬ ▬ $Q^2 = 2\ P^2_{h\perp}$

▬ ▬ ▬ $Q^2 = 4\ P^2_{h\perp}$

disclaimer: coloured lines drawn by hand

TMD factorization: a 2-scale problem

highest x bin



- - - $Q^2 = P^2_{h\perp}$

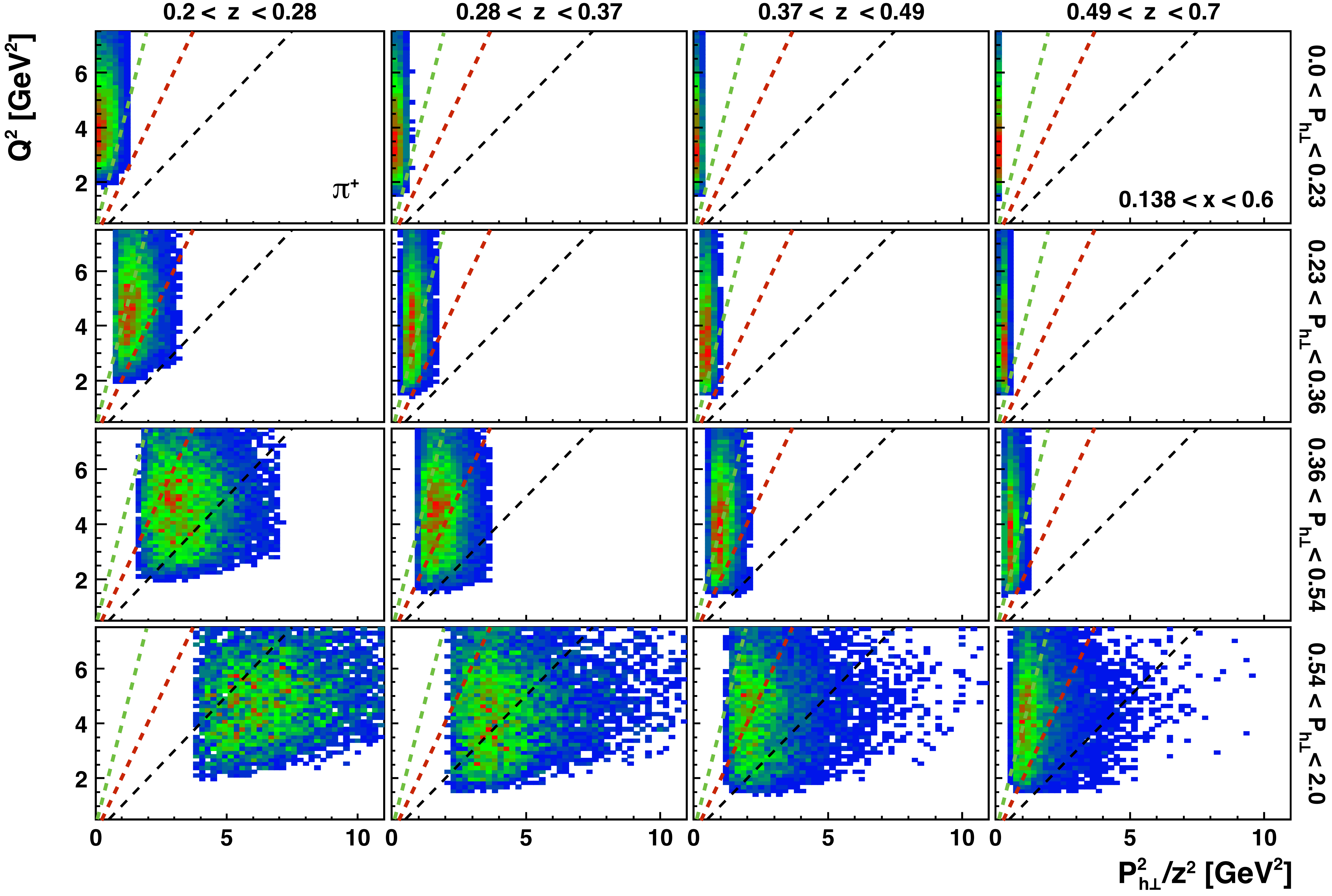
- - - $Q^2 = 2\ P^2_{h\perp}$

- - - $Q^2 = 4\ P^2_{h\perp}$

disclaimer: coloured lines drawn by hand

TMD factorization: a 2-scale problem

highest x bin



$Q^2 = P^2_{h\perp}/z^2$

$Q^2 = 2\ P^2_{h\perp}/z^2$

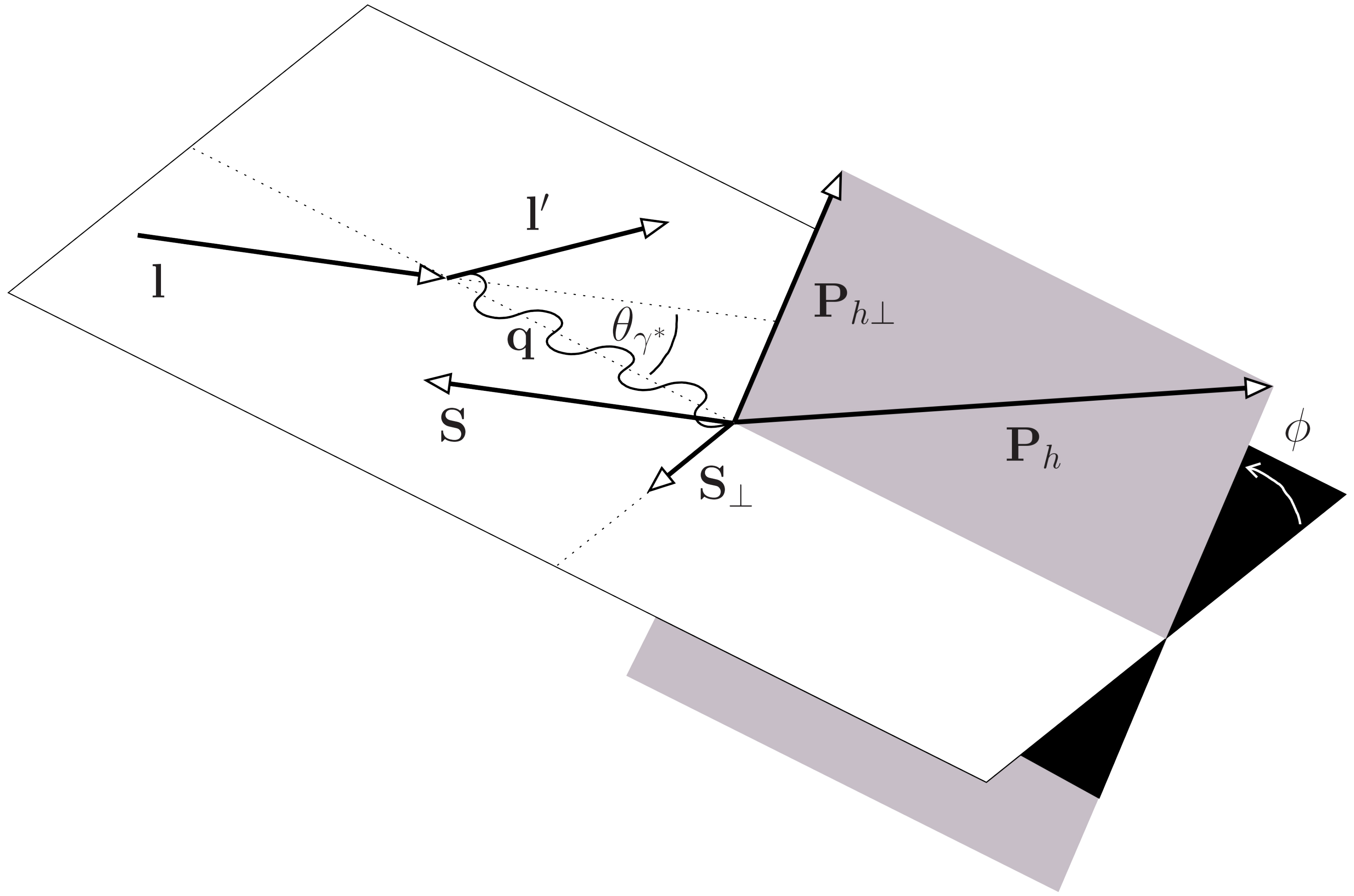
$Q^2 = 4\ P^2_{h\perp}/z^2$

all other kinematic bins included in the Supplemental Material of JHEP12(2020)010

some recent highlights

semi-inclusive DIS

- excluding transverse polarization:



$$\frac{d\sigma^h}{dx\,dy\,dz\,dP_{h\perp}^2\,d\phi} = \frac{2\pi\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

$$\Big\{ F_{UU,T}^h + \epsilon F_{UU,L}^h + \lambda\Lambda\sqrt{1-\epsilon^2}\,F_{LL}^h$$

$$+\sqrt{2\epsilon}\left[\lambda\sqrt{1-\epsilon}\,F_{LU}^{h,\sin\phi} + \Lambda\sqrt{1+\epsilon}\,F_{UL}^{h,\sin\phi}\right]\sin\phi$$

$$+\sqrt{2\epsilon}\left[\lambda\Lambda\sqrt{1-\epsilon}\,F_{LL}^{h,\cos\phi} + \sqrt{1+\epsilon}\,F_{UU}^{h,\cos\phi}\right]\cos\phi$$

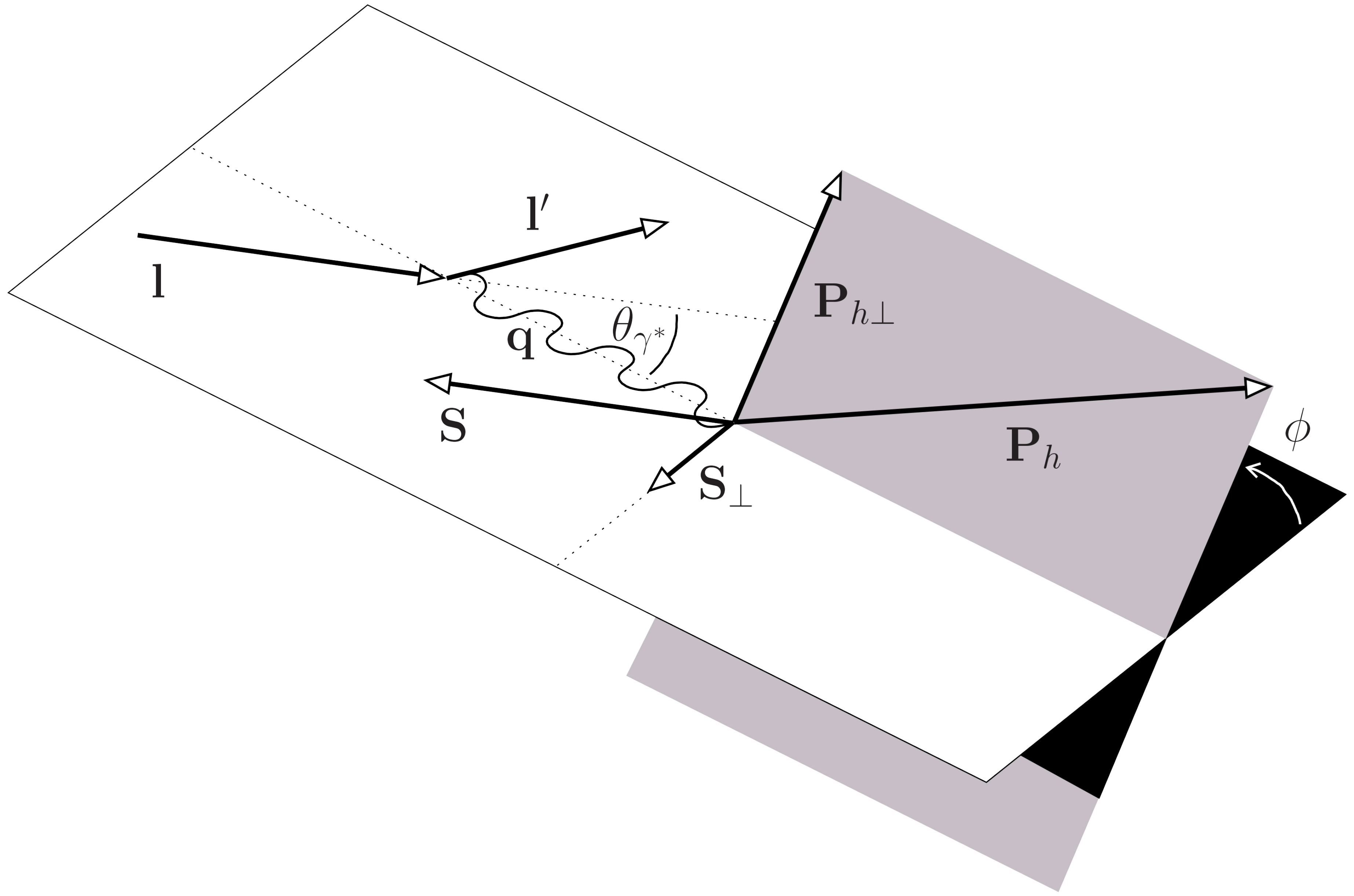
$$+\Lambda\epsilon\,F_{UL}^{h,\sin 2\phi}\sin 2\phi + \epsilon\,F_{UU}^{h,\cos 2\phi}\cos 2\phi\ \Big\}$$

$$F_{XY}^{h,\mathrm{mod}} = F_{XY}^{h,\mathrm{mod}}(x, Q^2, z, P_{h\perp})$$

Beam (λ) / Target (Λ)

helicities

semi-inclusive DIS



- excluding transverse polarization:

$$\frac{\mathrm{d}\sigma^h}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_{h\perp}^2\,\mathrm{d}\phi} = \frac{2\pi\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

$$\Big\{F_{UU,T}^h + \epsilon F_{UU,L}^h + \lambda\Lambda\sqrt{1-\epsilon^2}\,F_{LL}^h$$

$$+\sqrt{2\epsilon}\left[\lambda\sqrt{1-\epsilon}\,F_{LU}^{h,\sin\phi} + \Lambda\sqrt{1+\epsilon}\,F_{UL}^{h,\sin\phi}\right]\sin\phi$$

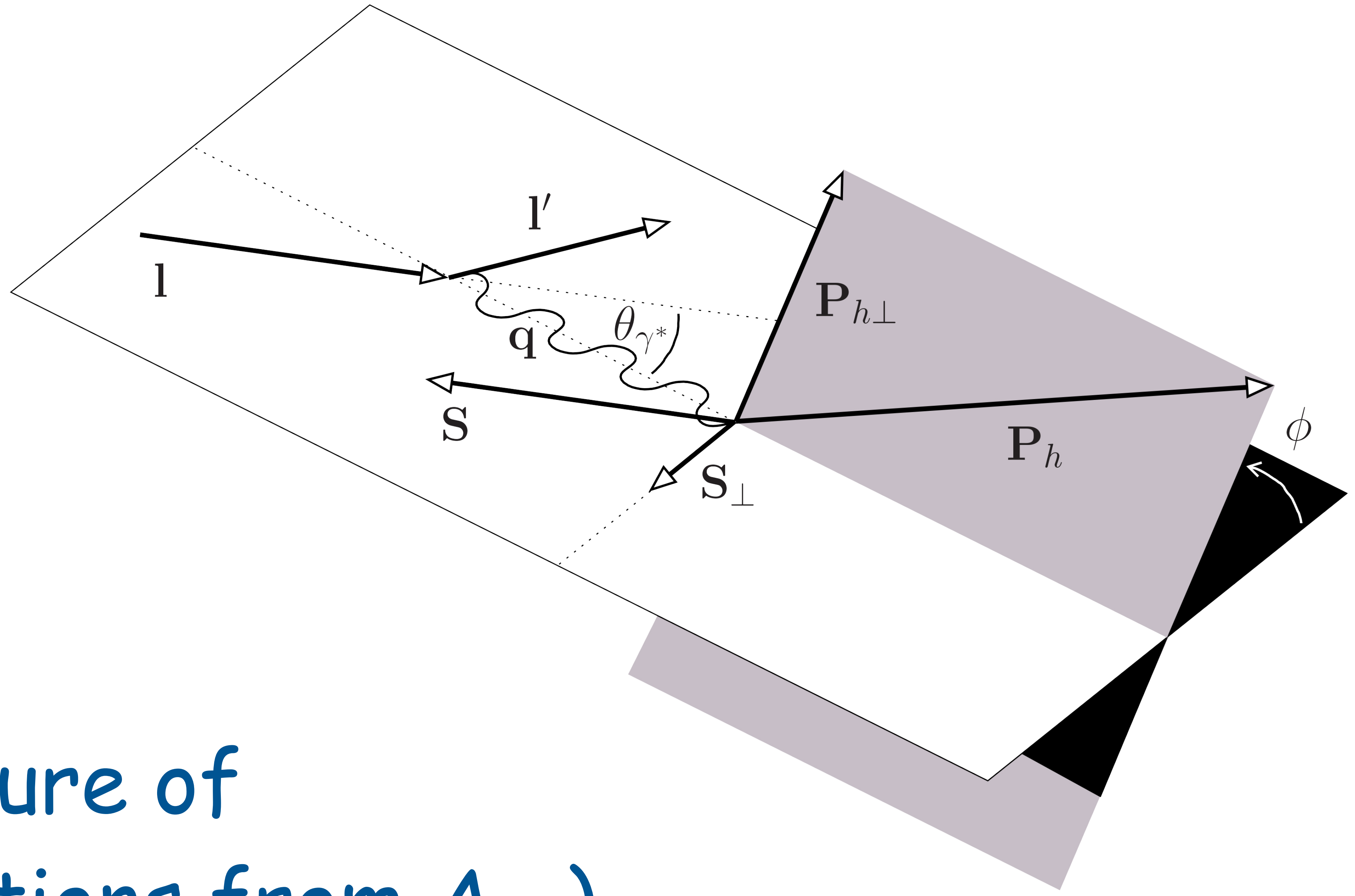
$$+\sqrt{2\epsilon}\left[\lambda\Lambda\sqrt{1-\epsilon}\,F_{LL}^{h,\cos\phi} + \sqrt{1+\epsilon}\,F_{UU}^{h,\cos\phi}\right]\cos\phi$$

$$+\Lambda\epsilon\,F_{UL}^{h,\sin 2\phi}\sin 2\phi + \epsilon\,F_{UU}^{h,\cos 2\phi}\cos 2\phi\Big\}$$

- double-spin asymmetry:

$$A_{LL}^h \equiv \frac{\sigma_{++}^h - \sigma_{+-}^h + \sigma_{--}^h - \sigma_{-+}^h}{\sigma_{++}^h + \sigma_{+-}^h + \sigma_{--}^h + \sigma_{-+}^h}$$

semi-inclusive DIS



- in experiment extract instead $A_{||}$ which differs from A_{LL} in the way the polarization is measured:
  - A_{LL}: along virtual-photon direction
  - $A_{||}$: along beam direction (results in small admixture of transverse target polarization and thus contributions from A_{LT})
- $A_{||}$ related to virtual-photon-nucleon asymmetry A_1

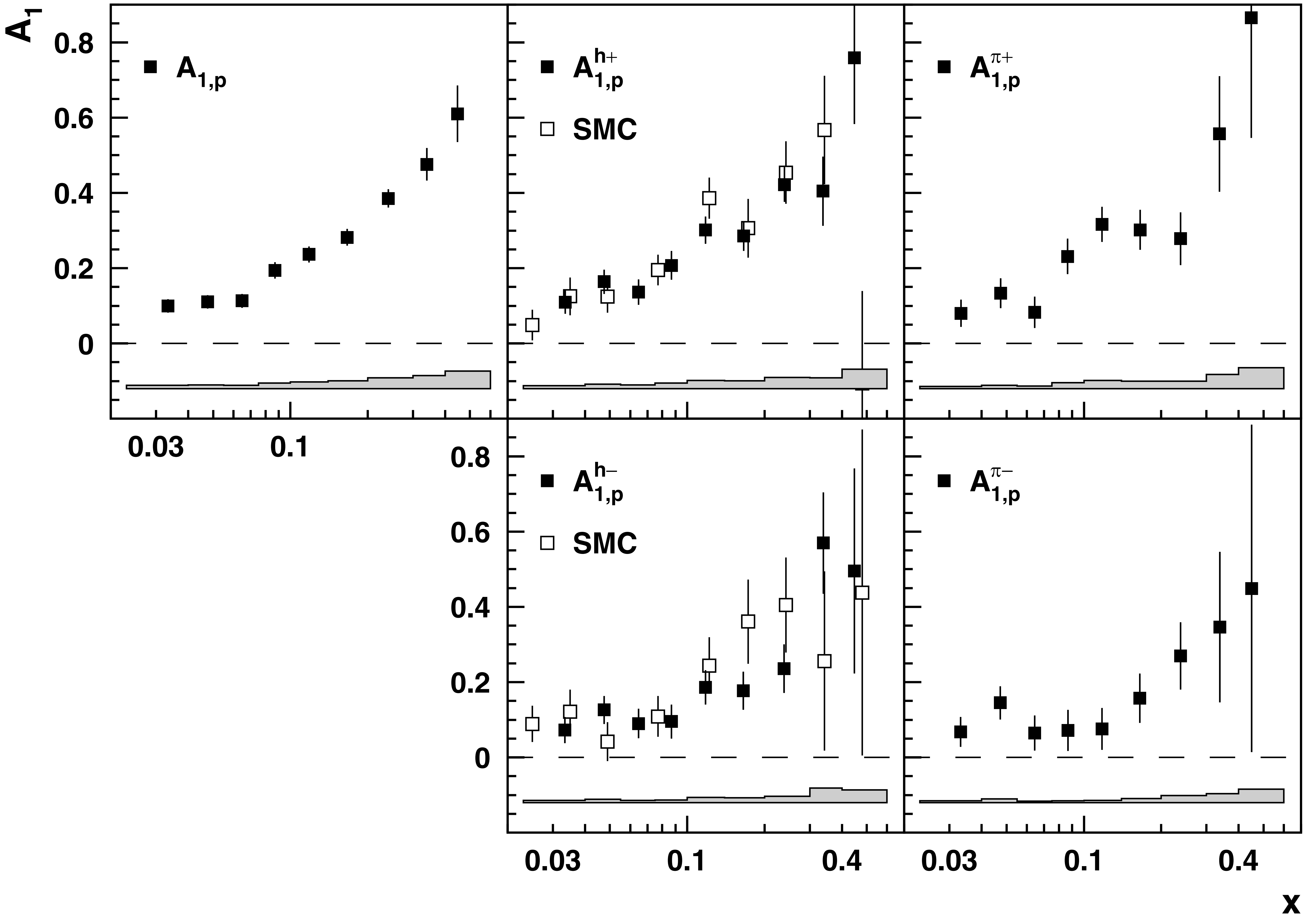
$$\boxed{A_1^h = \frac{1}{D(1+\eta\gamma)} A_{\|}^h}$$

$$D = \frac{1-(1-y)\epsilon}{1+\epsilon R}$$

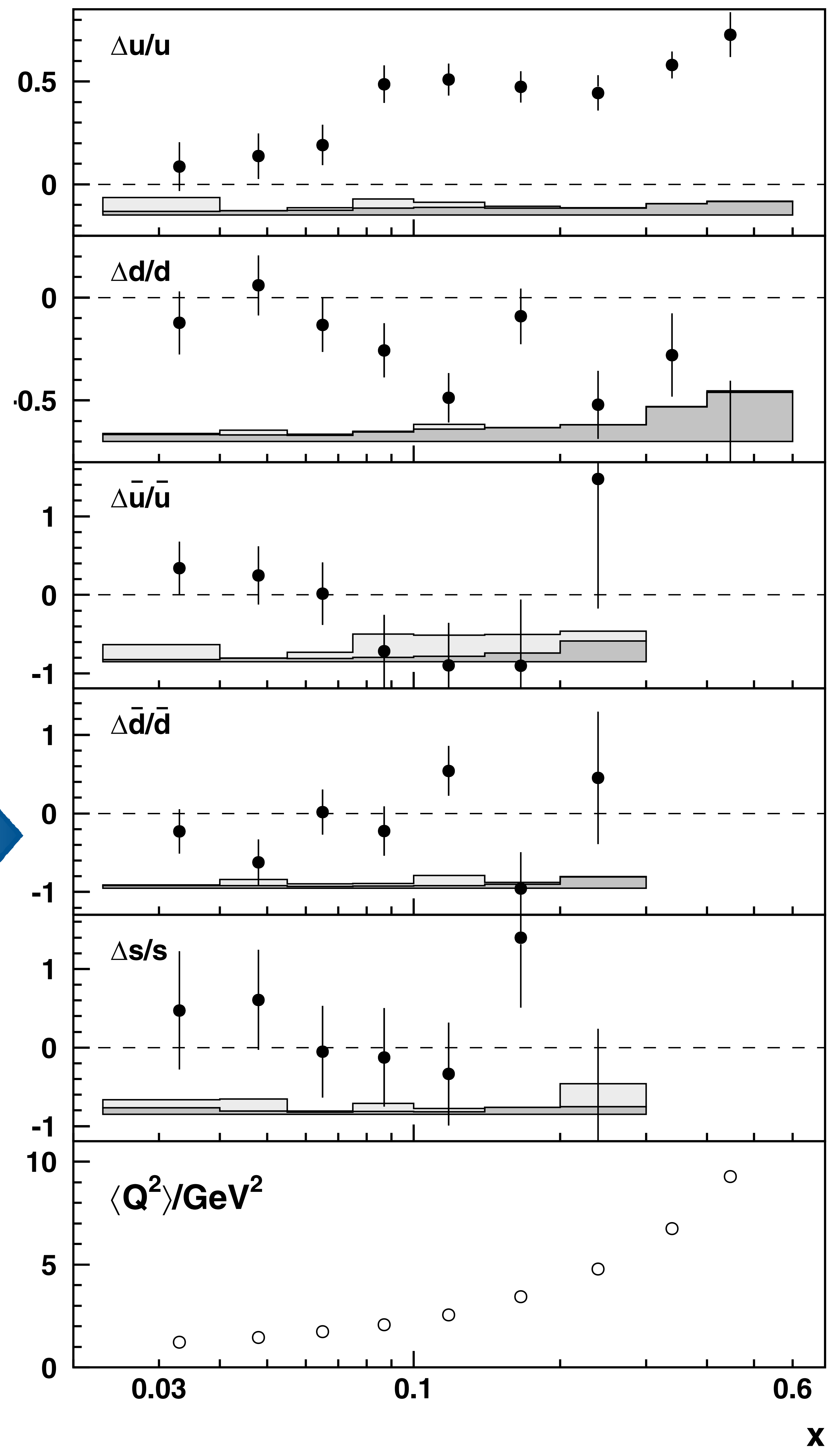
$$\eta = \frac{\epsilon\gamma y}{1-(1-y)\,\epsilon}$$

previous HERMES analysis

- (semi-) inclusive asymmetries used for LO extraction of helicity PDFs



PHYSICAL REVIEW D **71,** 012003 (2005)



re-analysis of double-spin asymmetries

- revisited [PRD 71 (2005) 012003] A_1 analysis at HERMES in order to
  - exploit slightly larger data set (less restrictive momentum range)
  - provide $A_{\parallel}$ in addition to A_1

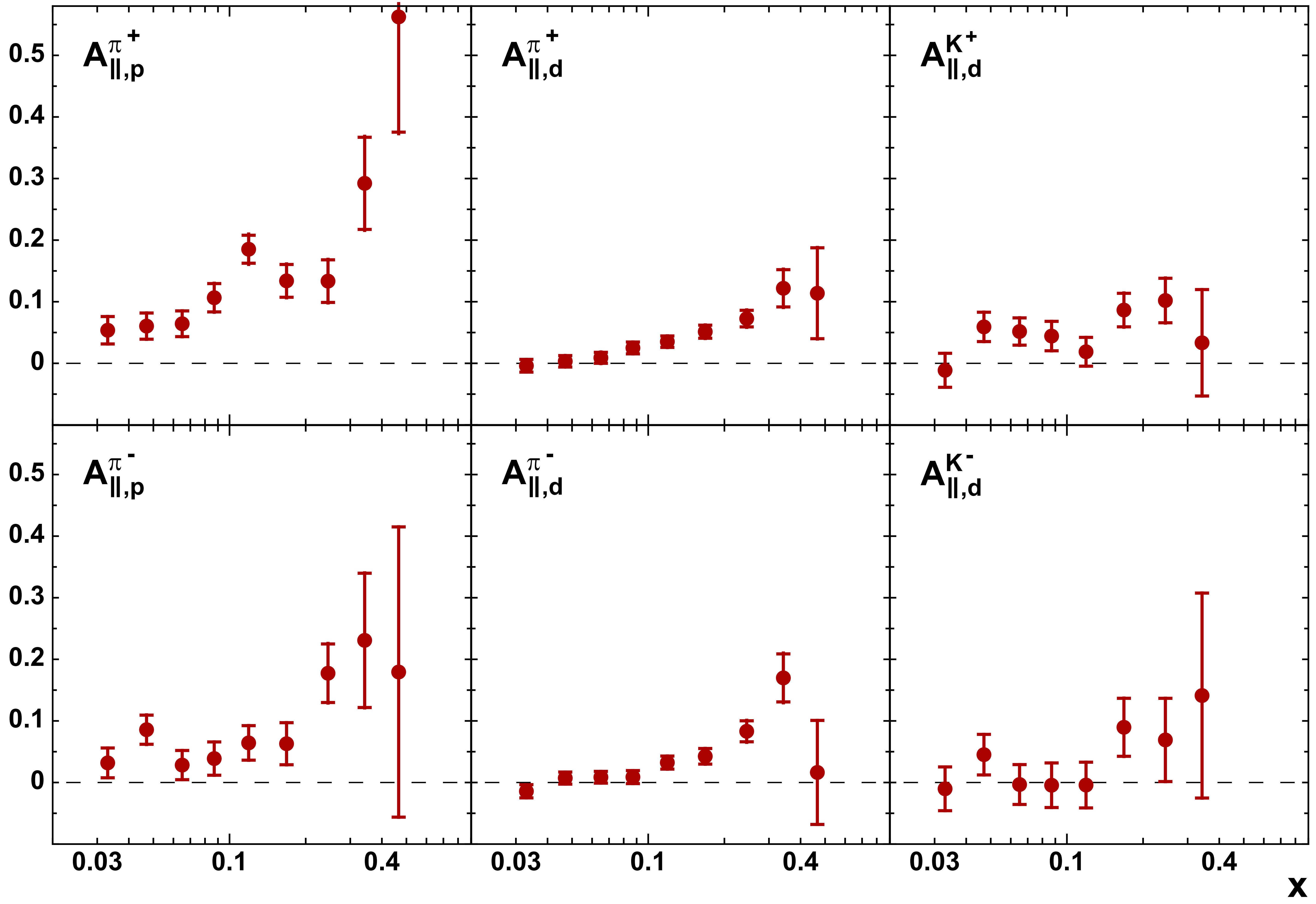
$$A_1^h = \frac{1}{D(1+\eta\gamma)} A_{\parallel}^h \qquad D = \frac{1-(1-y)\epsilon}{1+\epsilon R}$$

R (ratio of longitudinal-to-transverse cross-sec'n) still to be measured!
[only available for inclusive DIS data, e.g., used in g_1 SF measurements]

  - correct for D-state admixture (deuteron case) on asymmetry level
  - correct better for azimuthal asymmetries coupling to acceptance
  - **look at multi-dimensional (x, z, $P_{h\perp}$) dependences**
  - **extract twist-3 cosine modulations**

x dependence of $A_{||}$

[arXiv:1810.07054]

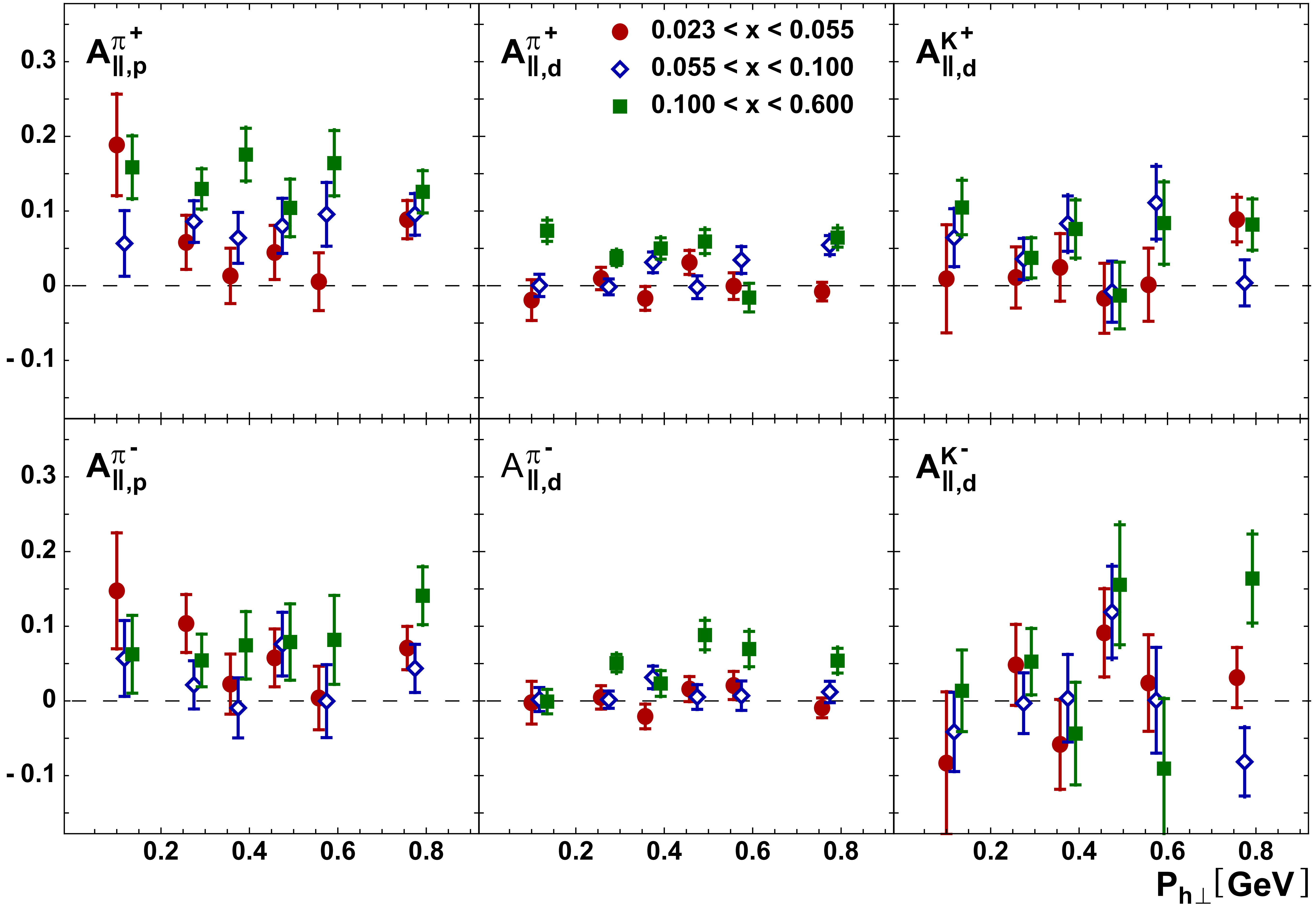


☑ fully consistent with previous HERMES publication [PRD 71 (2005) 012003]

$P_{h\perp}$ dependence of $A_{||}$ (three x ranges)

- no strong dependence (beyond on x)

[arXiv:1810.07054]



$P_{h\perp}$ dependence of $A_{||}$ (three x ranges)

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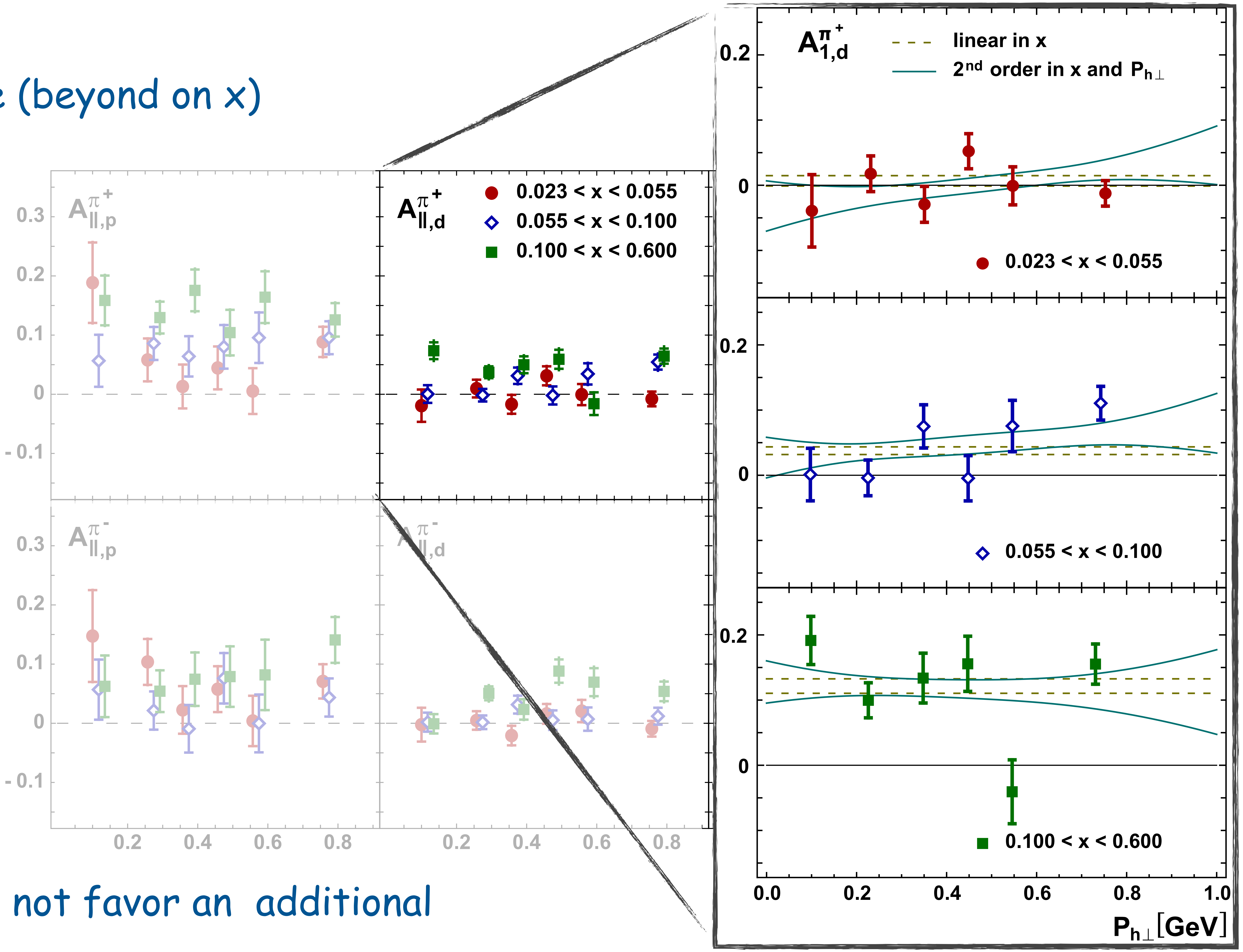
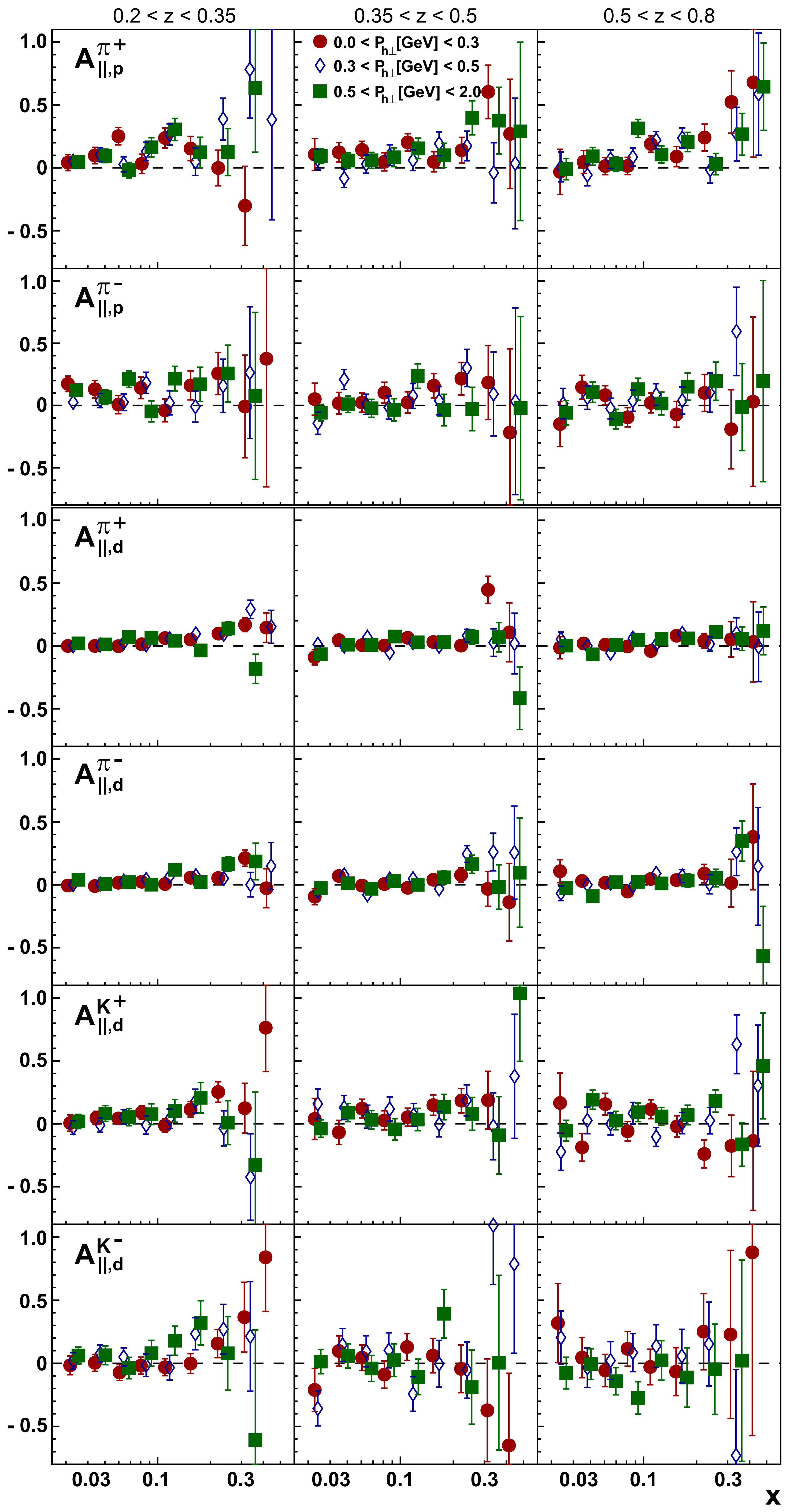


- also fit to A_1 fit does not favor an additional dependence on $P_{h\perp}$

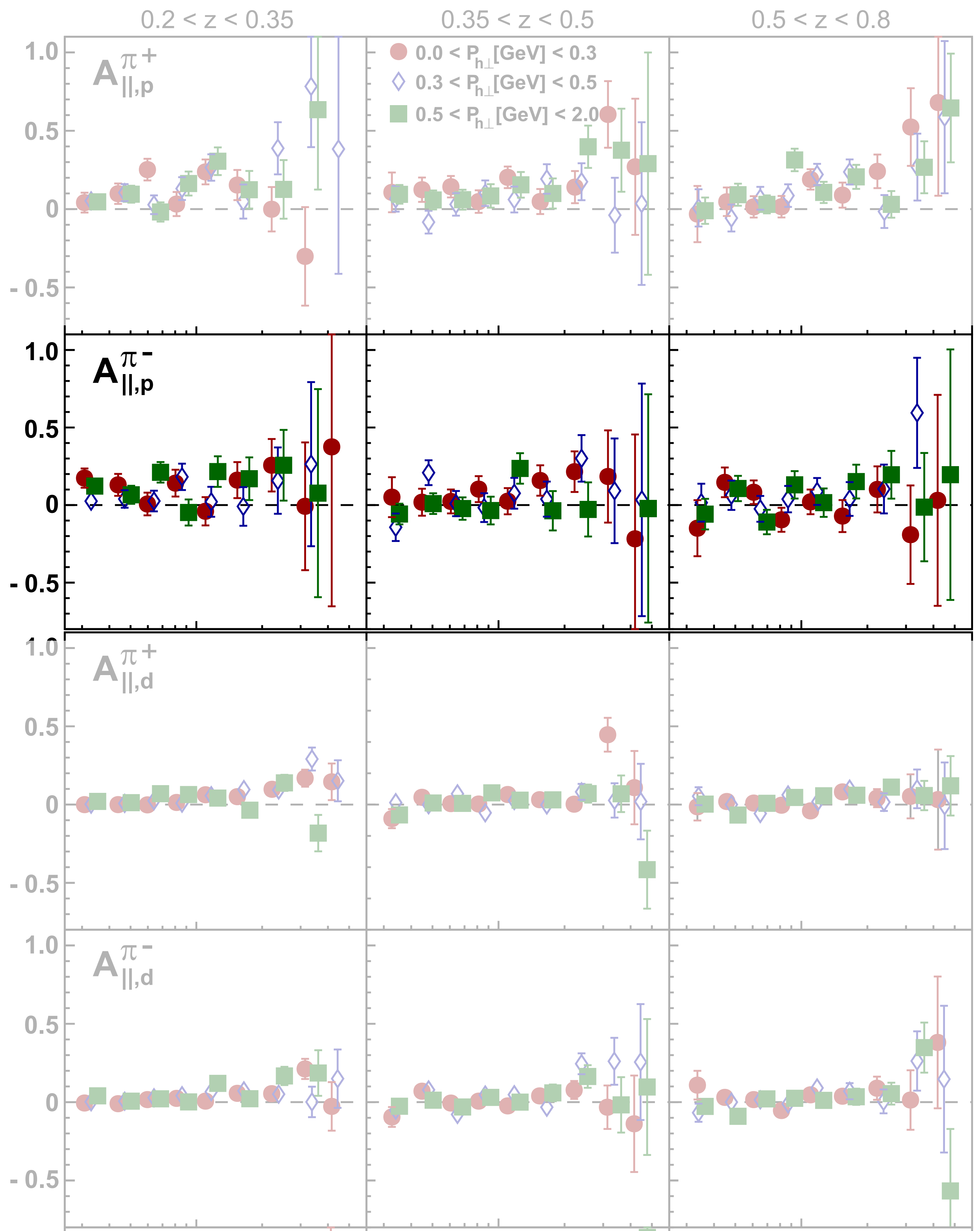
3-dimensional binning

- first-ever 3d binning provides transverse-momentum dependence

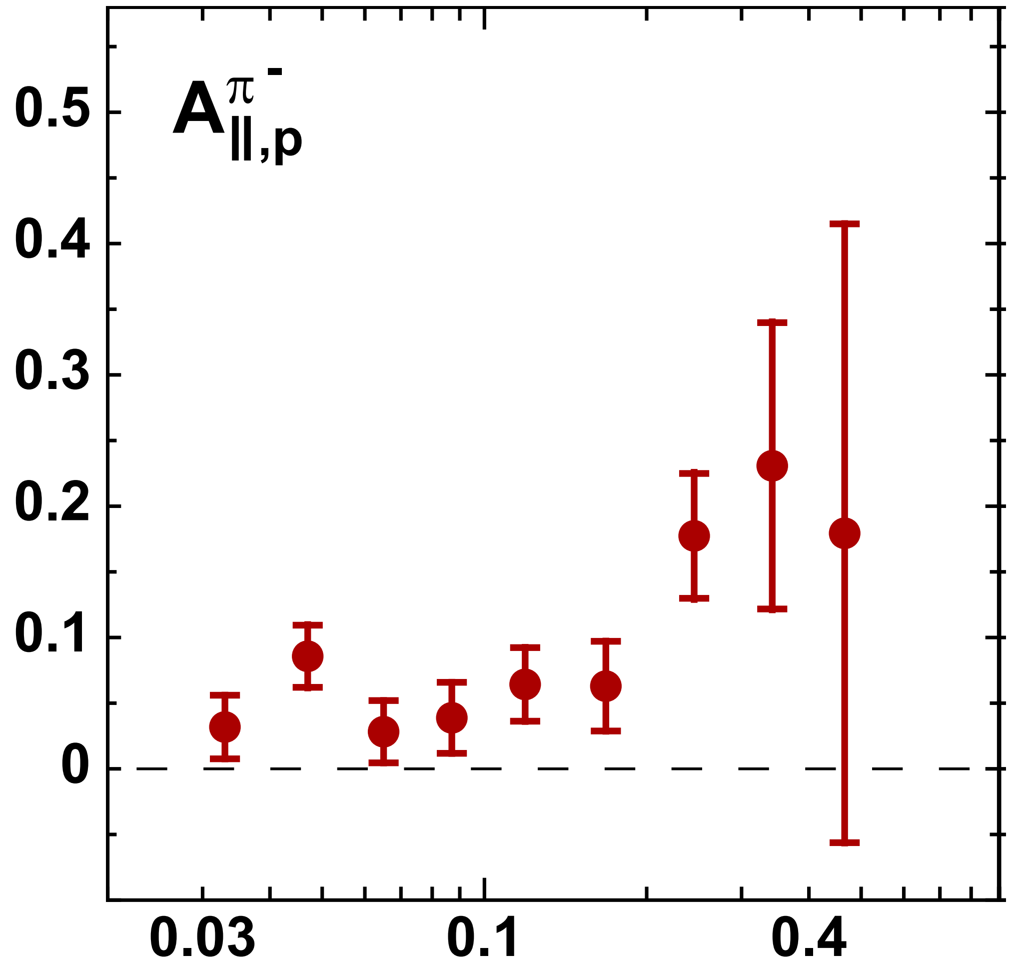


3-dimensional binning

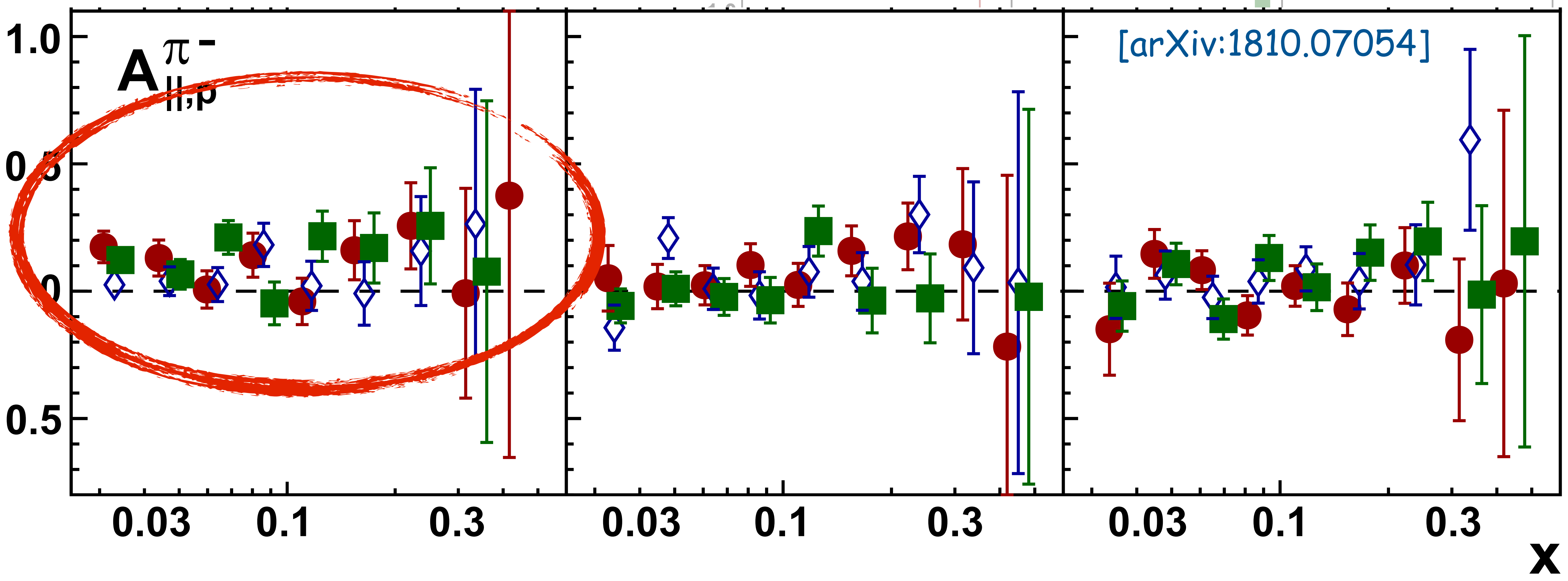
- first-ever 3d binning provides transverse-momentum dependence
- but also extra flavor sensitivity, e.g.,
  - π^- asymmetries mainly coming from **low-z** region where **disfavored fragmentation** large and thus **sensitivity** to the large **positive up-quark polarization**



1d



3d



[arXiv:1810.07054]

hadron-charge difference asymmetries

$$A_1^{h^+-h^-}(x) \equiv \frac{\left(\sigma_{1/2}^{h^+} - \sigma_{1/2}^{h^-}\right) - \left(\sigma_{3/2}^{h^+} - \sigma_{3/2}^{h^-}\right)}{\left(\sigma_{1/2}^{h^+} - \sigma_{1/2}^{h^-}\right) + \left(\sigma_{3/2}^{h^+} - \sigma_{3/2}^{h^-}\right)}$$

hadron-charge difference asymmetries

$$A_1^{h^+-h^-}(x) \equiv \frac{\left(\sigma_{1/2}^{h^+} - \sigma_{1/2}^{h^-}\right) - \left(\sigma_{3/2}^{h^+} - \sigma_{3/2}^{h^-}\right)}{\left(\sigma_{1/2}^{h^+} - \sigma_{1/2}^{h^-}\right) + \left(\sigma_{3/2}^{h^+} - \sigma_{3/2}^{h^-}\right)}$$

- at leading-order and leading-twist, assuming charge conjugation symmetry for fragmentation functions:

$$A_{1,d}^{h^+-h^-} \overset{\text{LO LT}}{=} \frac{g_1^{u_v} + g_1^{d_v}}{f_1^{u_v} + f_1^{d_v}}$$

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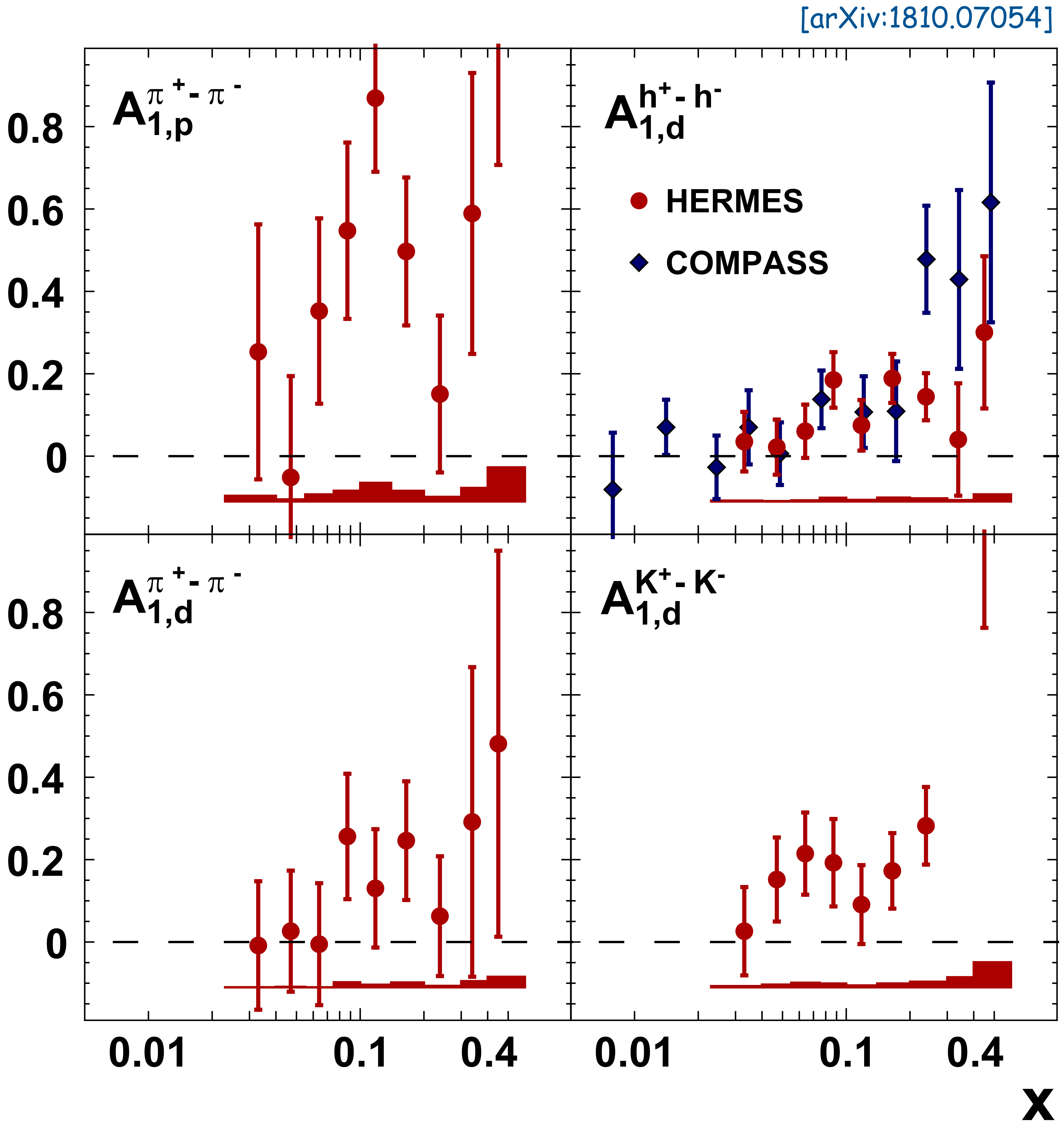
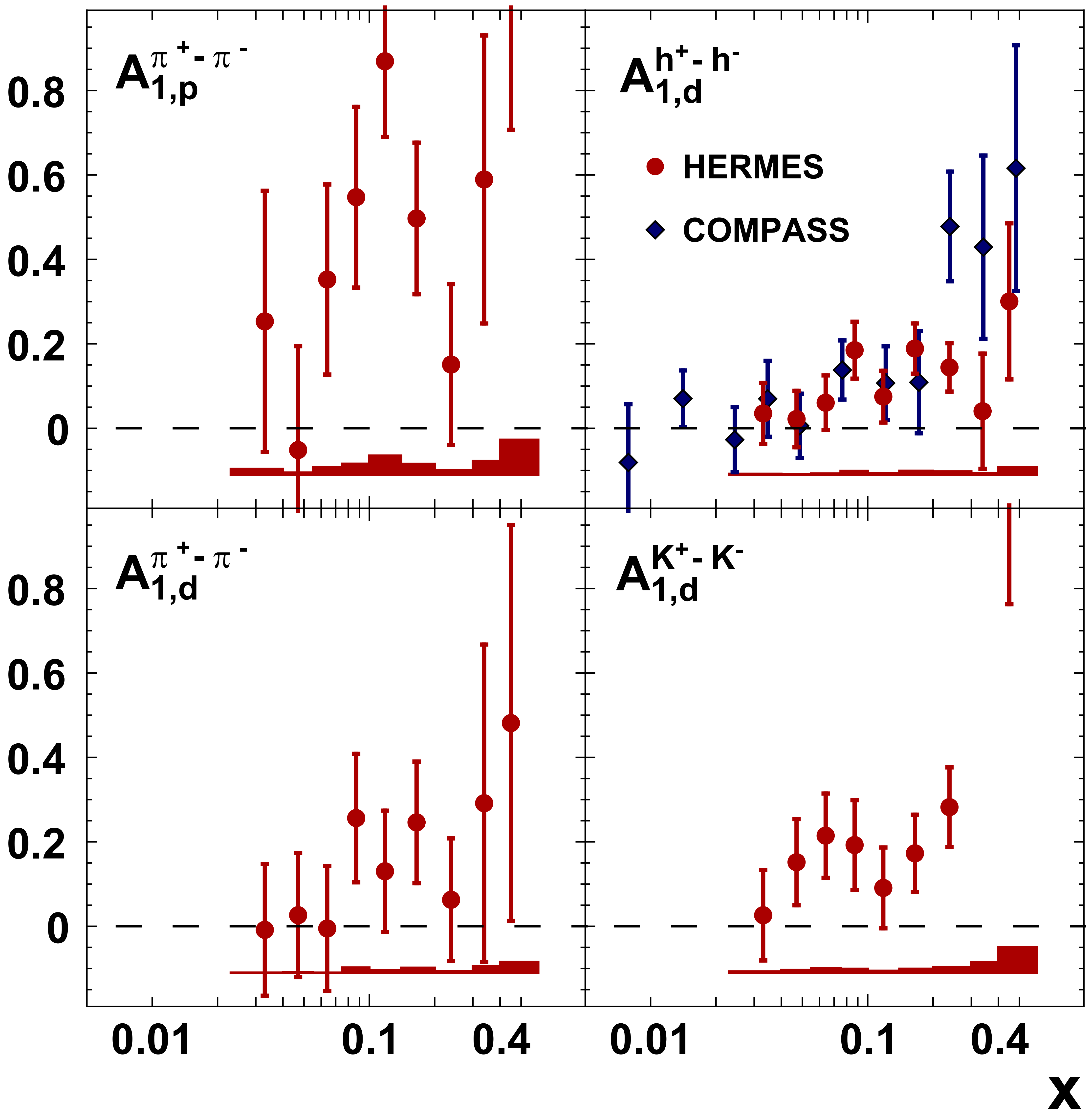
- can be used to extract valence helicity distributions

hadron-charge difference asymmetries

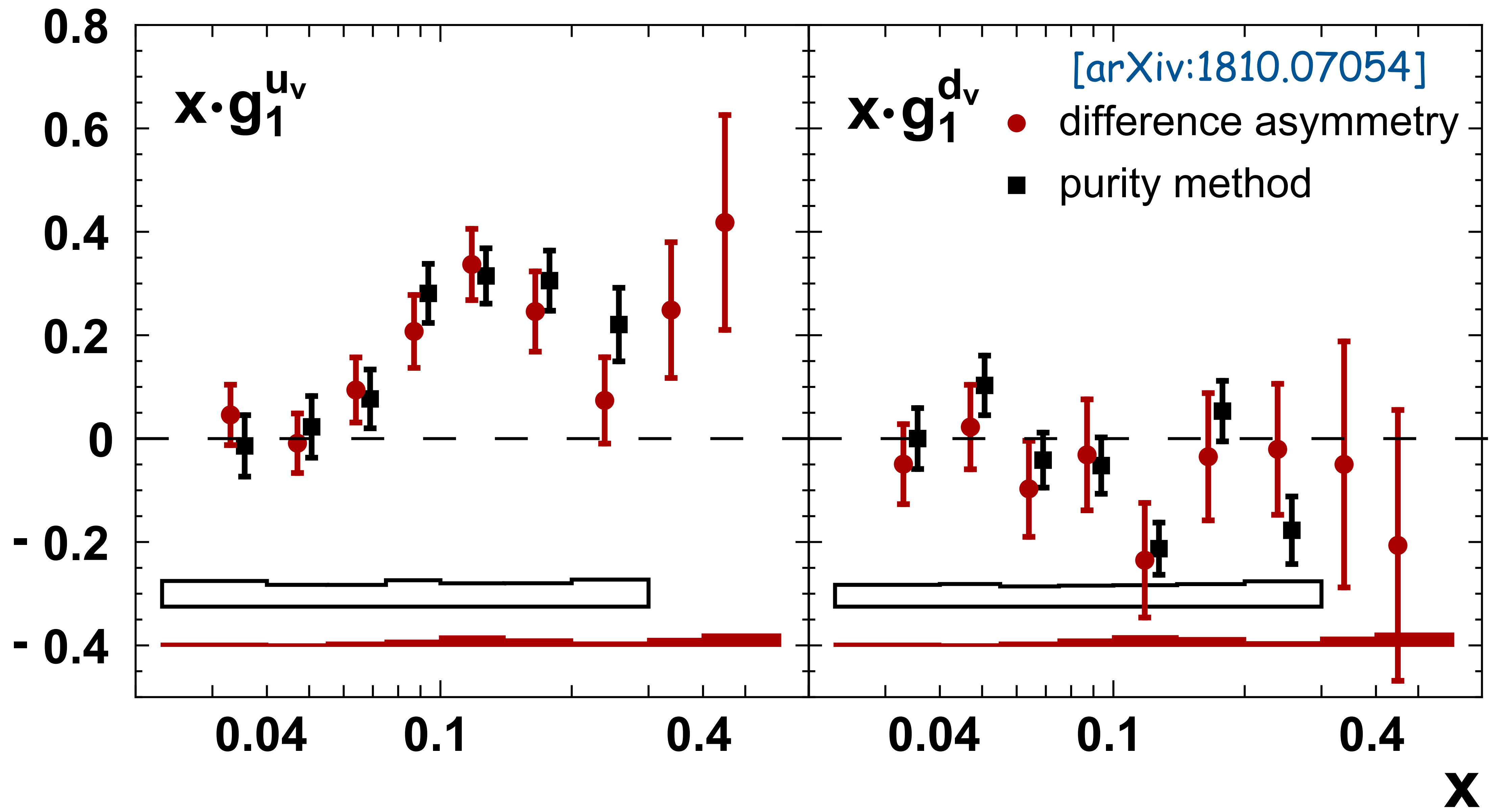


- no significant hadron-type dependence for deuterons
- deuteron results (unidentified hadrons) consistent with COMPASS

hadron-charge difference asymmetries

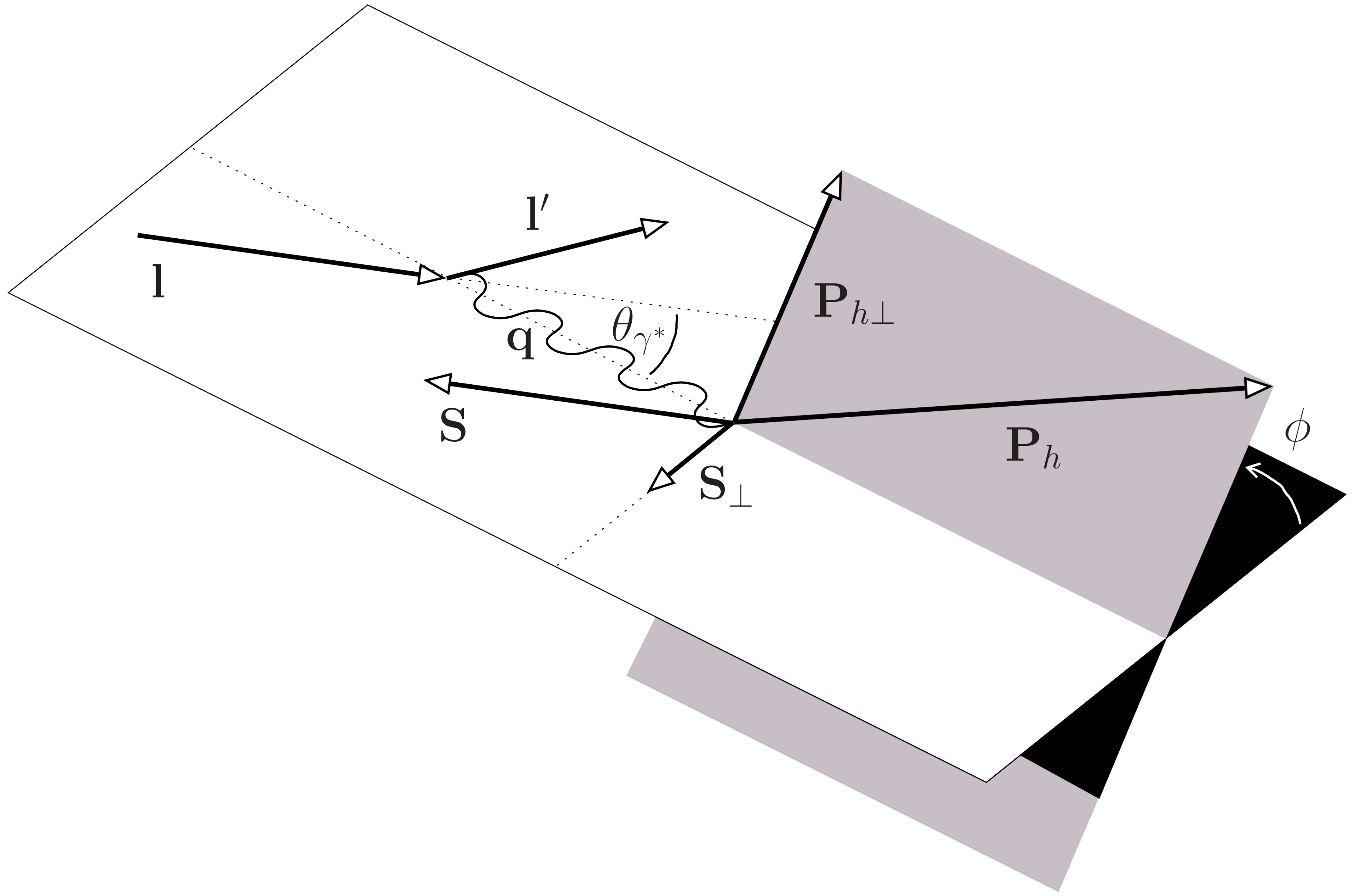

[arXiv:1810.07054]

- no significant hadron-type dependence for deuterons
- deuteron results (unidentified hadrons) consistent with COMPASS
- valence distributions consistent with JETSET-based extraction:


[arXiv:1810.07054]

semi-inclusive DIS

- excluding transverse polarization:



$$
\frac{\mathrm{d}\sigma^h}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_{h\perp}^2\,\mathrm{d}\phi} = \frac{2\pi\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)
$$

$$
\Big\{ F_{UU,T}^h + \epsilon F_{UU,L}^h + \lambda\Lambda\sqrt{1-\epsilon^2}F_{LL}^h
$$

$$
+\sqrt{2\epsilon}\left[\lambda\sqrt{1-\epsilon}\,F_{LU}^{h,\sin\phi} + \Lambda\sqrt{1+\epsilon}\,F_{UL}^{h,\sin\phi}\right]\sin\phi
$$

$$
+\sqrt{2\epsilon}\left[\lambda\Lambda\sqrt{1-\epsilon}\,F_{LL}^{h,\cos\phi} + \sqrt{1+\epsilon}\,F_{UU}^{h,\cos\phi}\right]\cos\phi
$$

$$
+\Lambda\epsilon\,F_{UL}^{h,\sin 2\phi}\sin 2\phi + \epsilon\,F_{UU}^{h,\cos 2\phi}\cos 2\phi\ \Big\}
$$

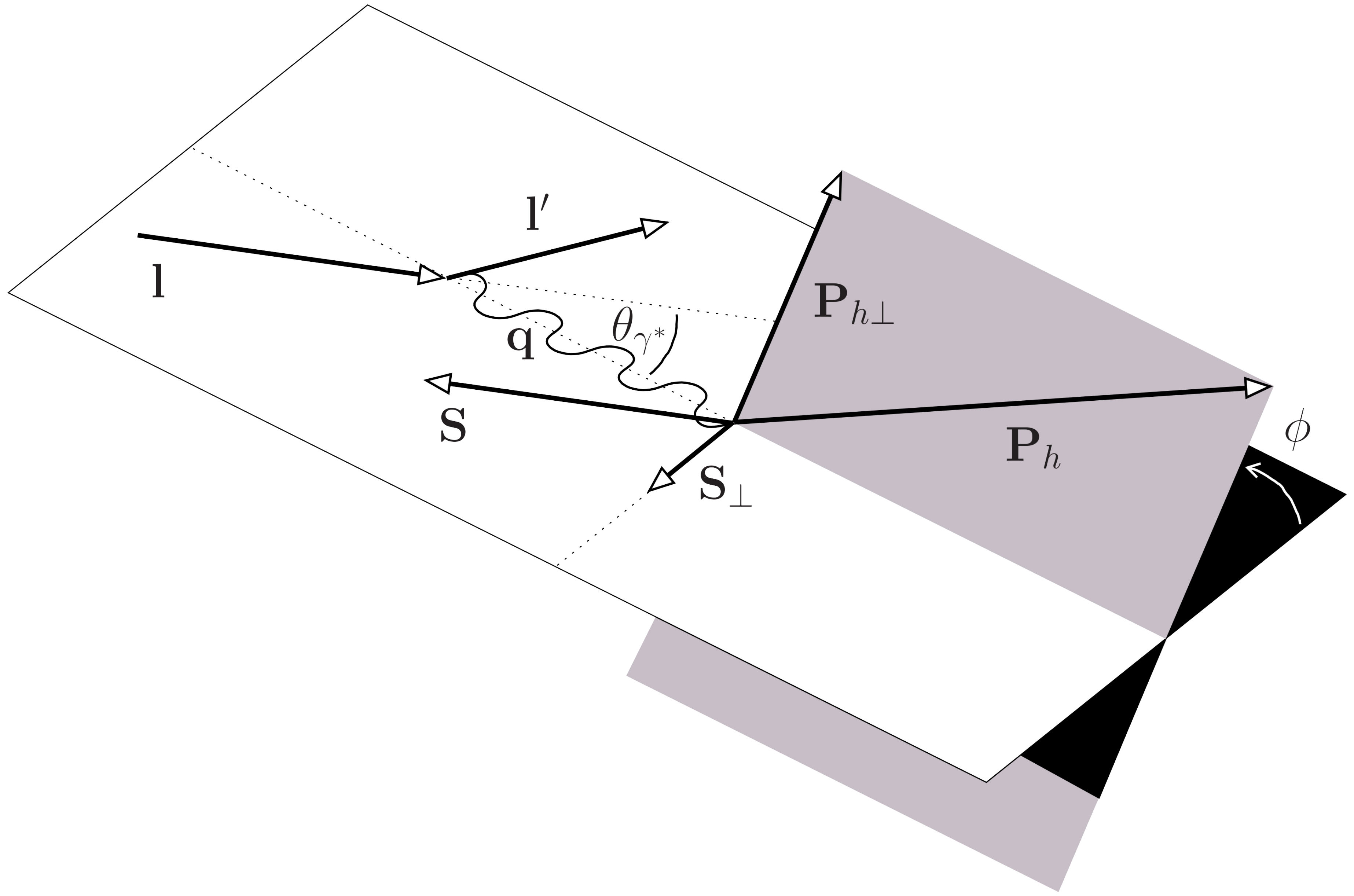
$$
F_{XY}^{h,\mathrm{mod}} = F_{XY}^{h,\mathrm{mod}}(x, Q^2, z, P_{h\perp})
$$

Beam (λ) / Target (Λ)

helicities

semi-inclusive DIS

- excluding transverse polarization:



$$\frac{d\sigma^h}{dx\,dy\,dz\,dP_{h\perp}^2\,d\phi} = \frac{2\pi\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

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$$+\Lambda\epsilon\,F_{UL}^{h,\sin 2\phi}\sin 2\phi + \epsilon\,F_{UU}^{h,\cos 2\phi}\cos 2\phi \Big\}$$

- single-spin asymmetry:

$$A_{LU}^h \equiv \frac{\sigma_{+-}^h + \sigma_{++}^h - \sigma_{-+}^h - \sigma_{--}^h}{\sigma_{+-}^h + \sigma_{++}^h + \sigma_{-+}^h + \sigma_{--}^h}$$

beam-helicity asymmetry

$$\frac{M_h}{Mz} h_1^{\perp} \tilde{E} \oplus x g^{\perp} D_1 \oplus \frac{M_h}{Mz} f_1 \tilde{G}^{\perp} \oplus x e H_1^{\perp}$$

- naive-T-odd Boer-Mulders (BM) function coupled to a twist-3 FF
  - signs of BM from unpolarized SIDIS
  - little known about interaction-dependent FF

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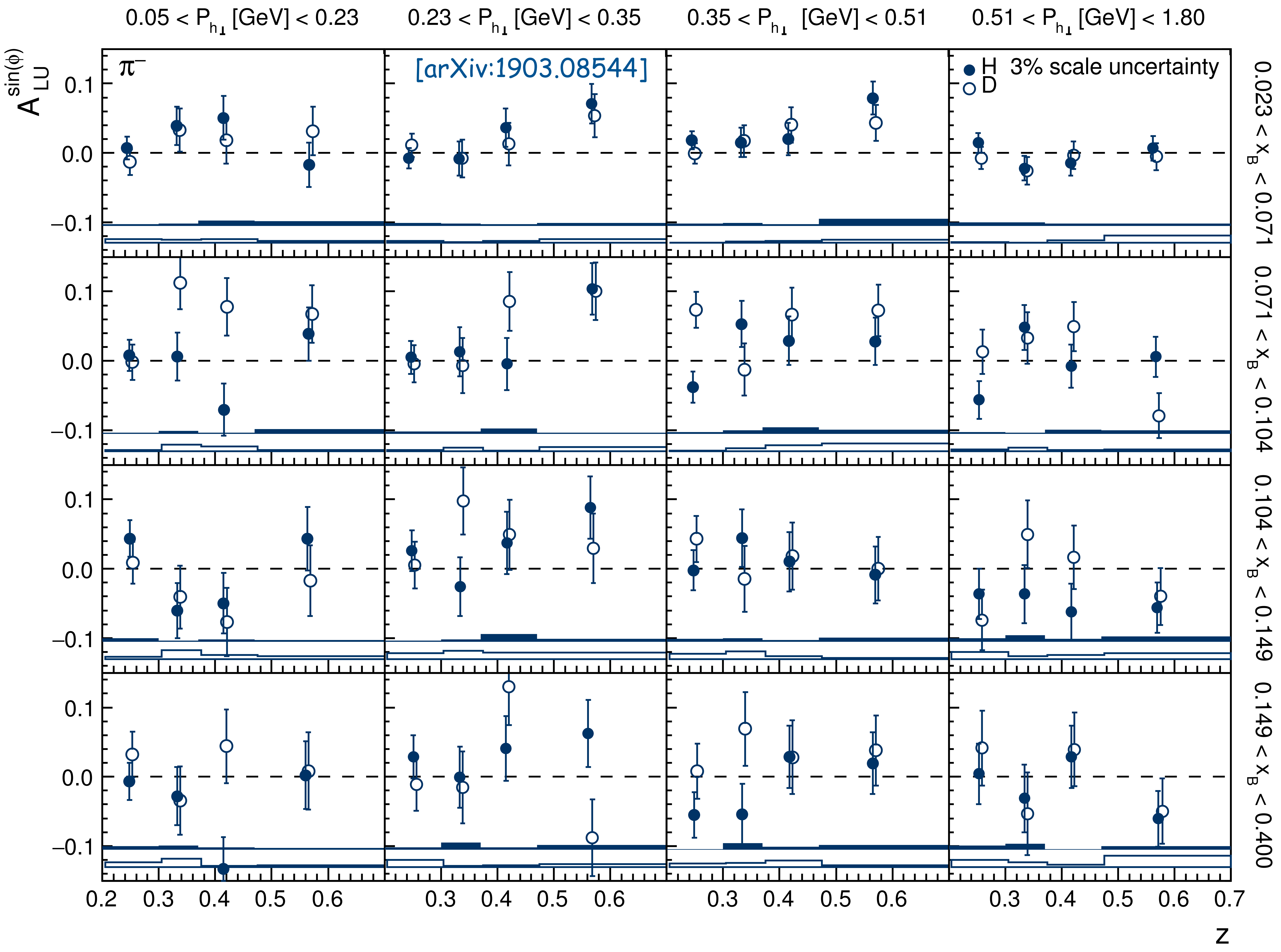
  - little known about interaction-dependent FF
- little known about naive-T-odd $g^\perp$; singled out in A_{LU} in jet production
- large unpolarized f_1, coupled to interaction-dependent FF
- twist-3 e survives integration over $P_{h\perp}$; here coupled to Collins FF
  - e linked to the pion–nucleon σ-term
  - interpreted as color force (from remnant) on transversely polarized quarks at the moment of being struck by virtual photon

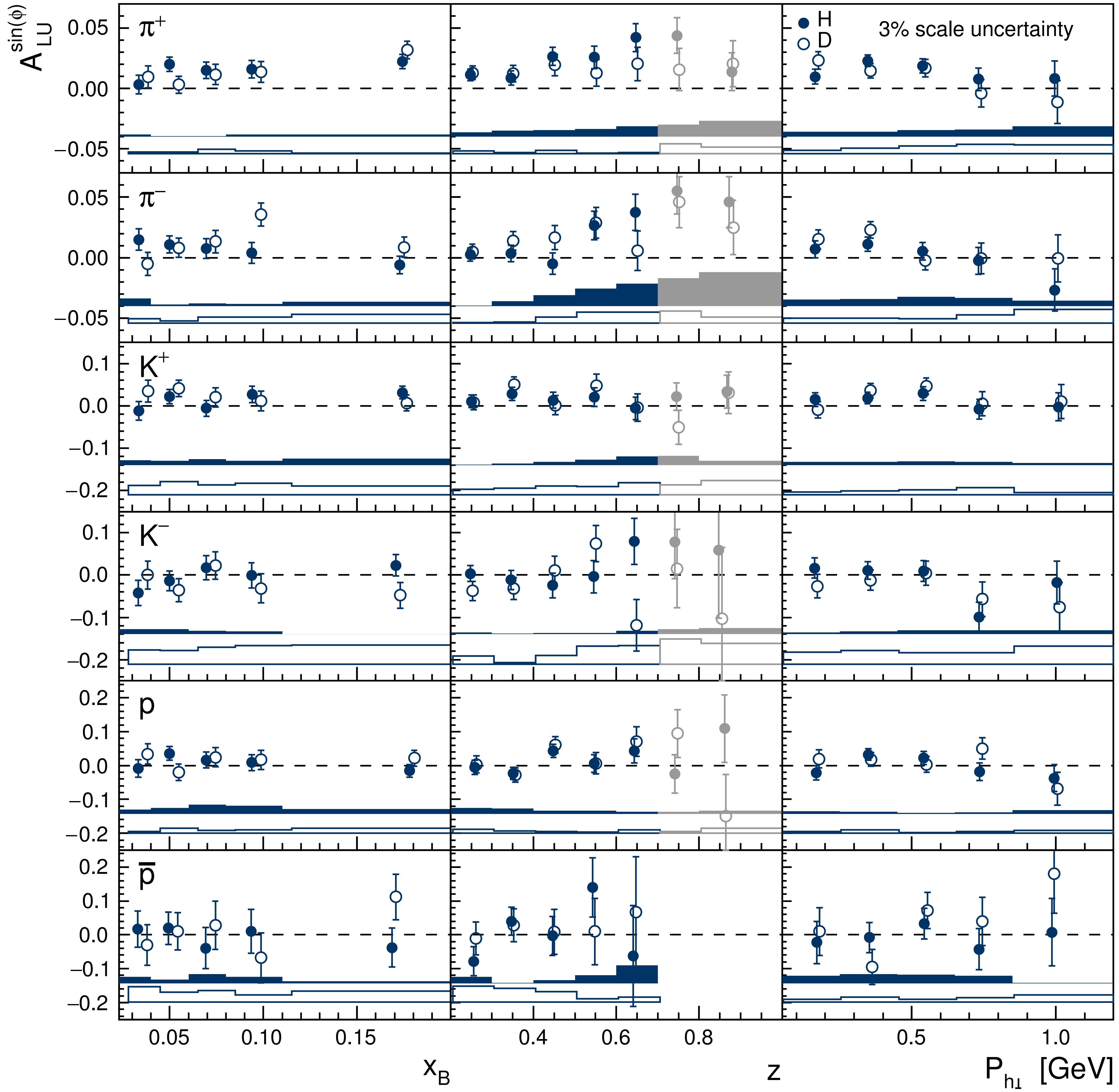
3d beam-helicity asymmetry for π^-



most comprehensive presentation, for discussion use 1d binning

$$\frac{M_h}{Mz} h_1^{\perp} \tilde{E} \oplus x g^{\perp} D_1 \oplus \frac{M_h}{Mz} f_1 \tilde{G}^{\perp} \oplus x e H_1^{\perp}$$

[arXiv:1903.08544]



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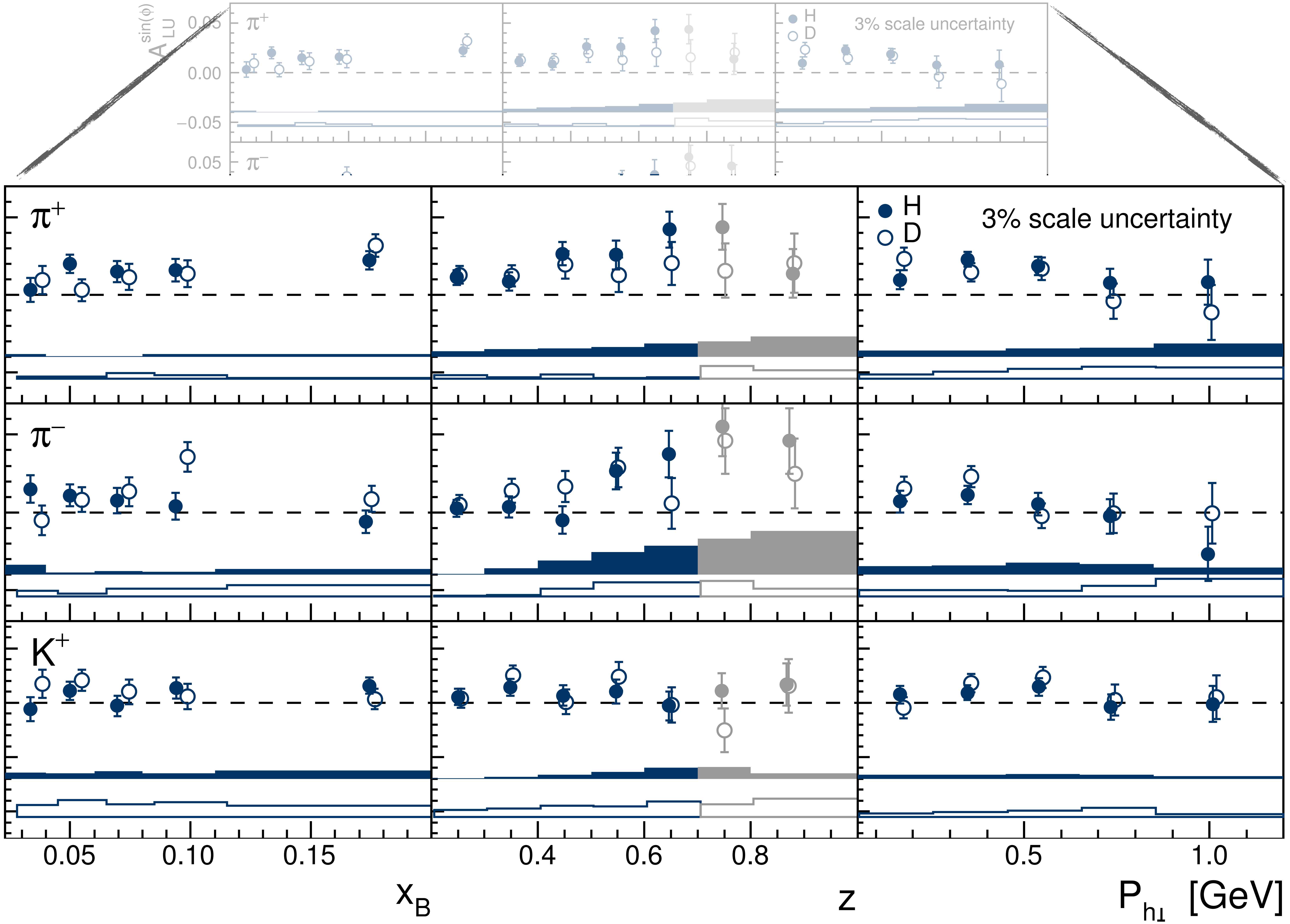

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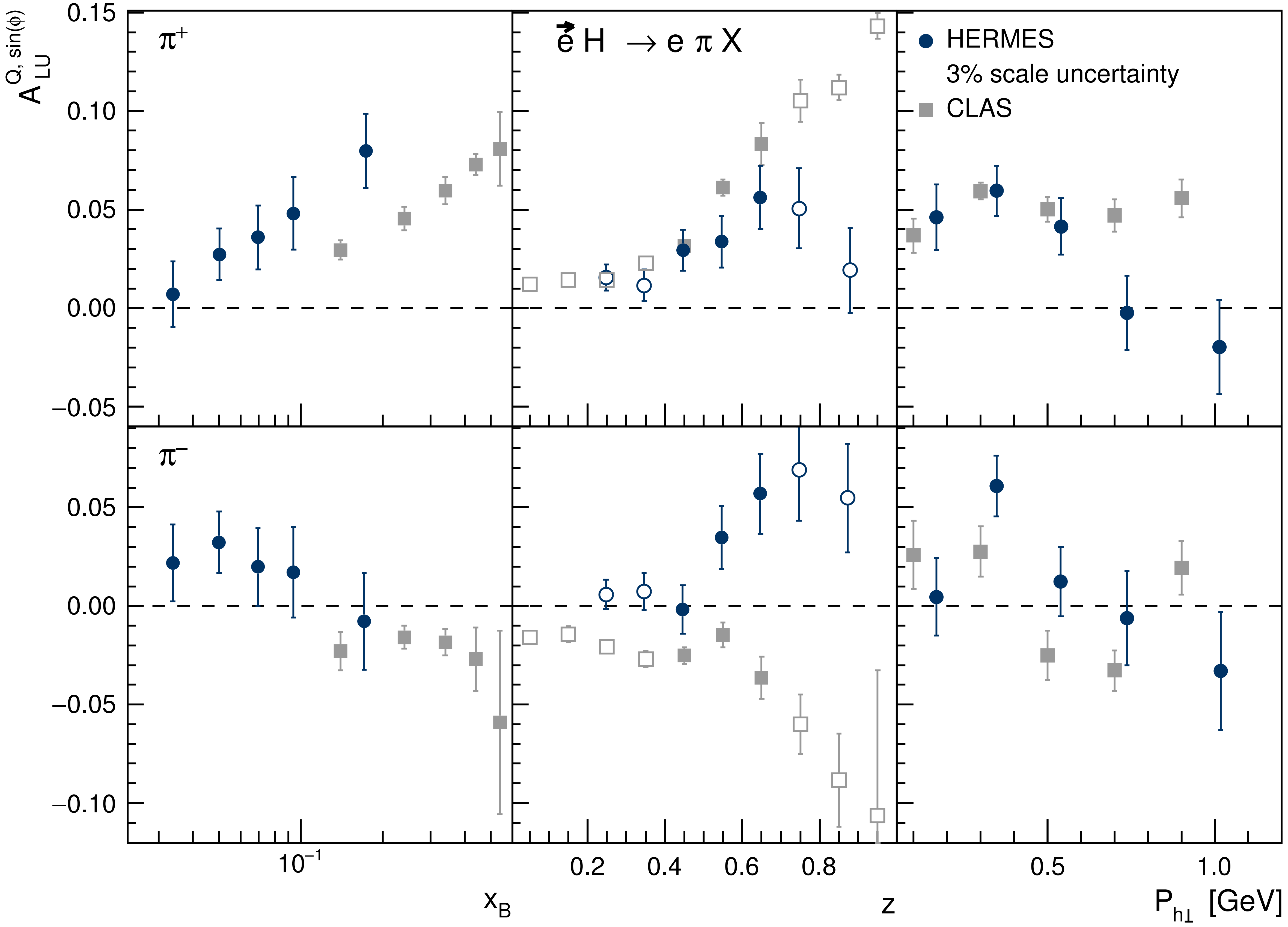
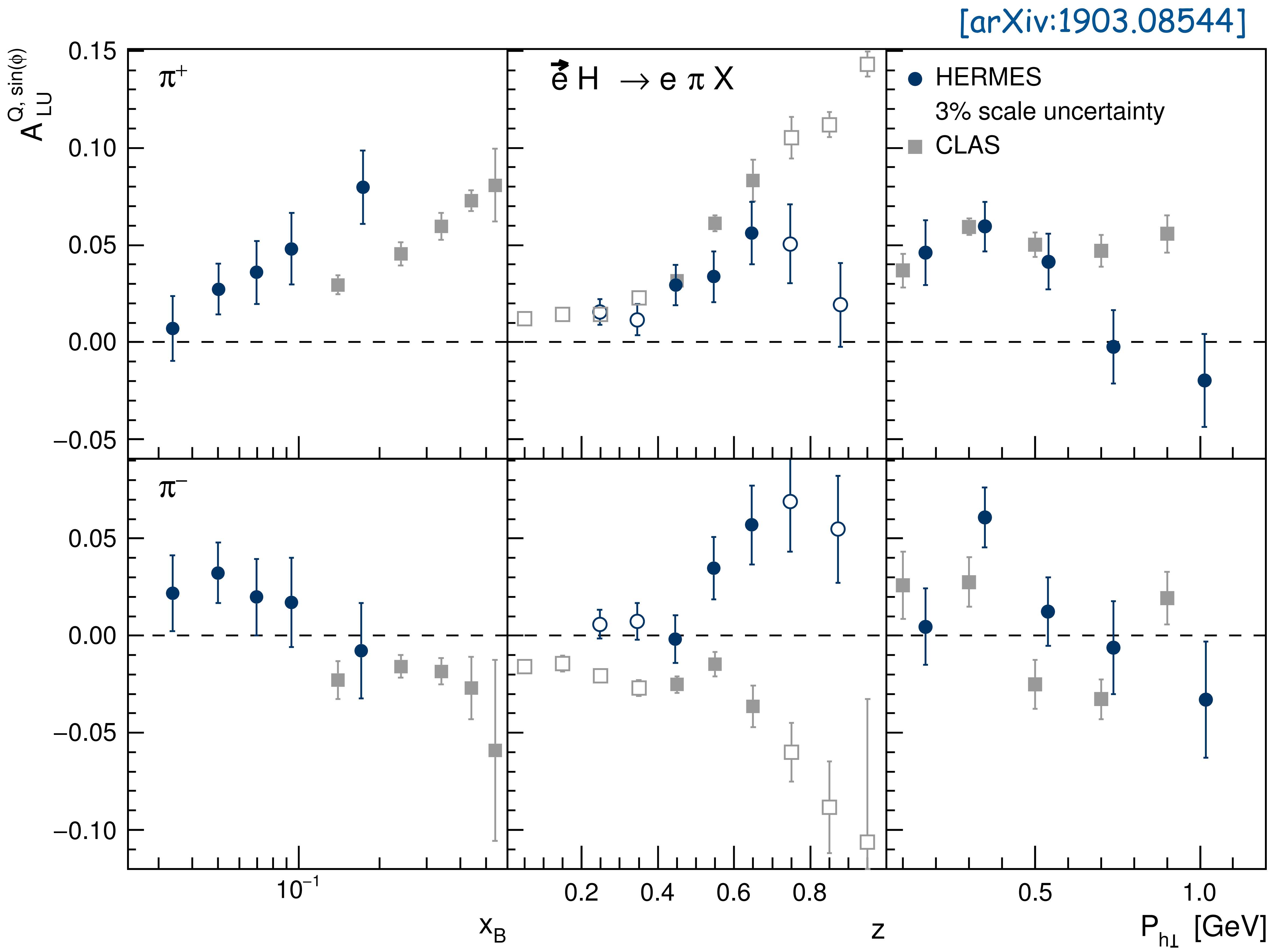
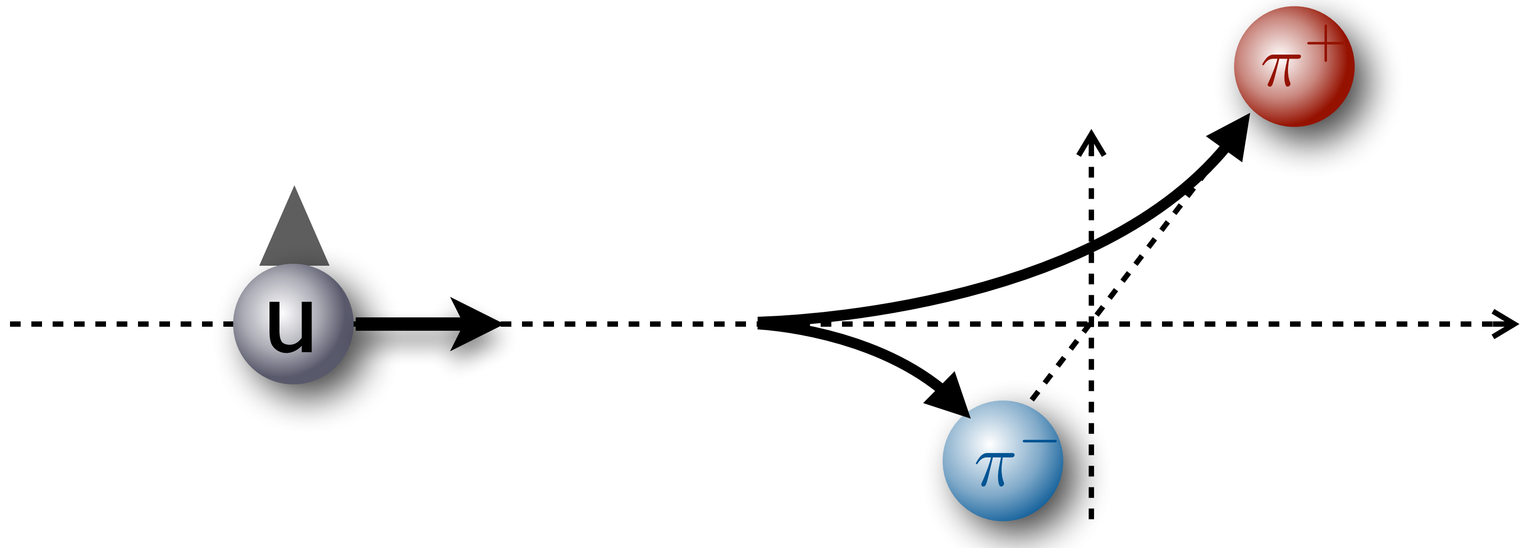


[arXiv:1903.08544]

- opposite behavior at HERMES/CLAS of negative pions in z projection due to different x-range probed

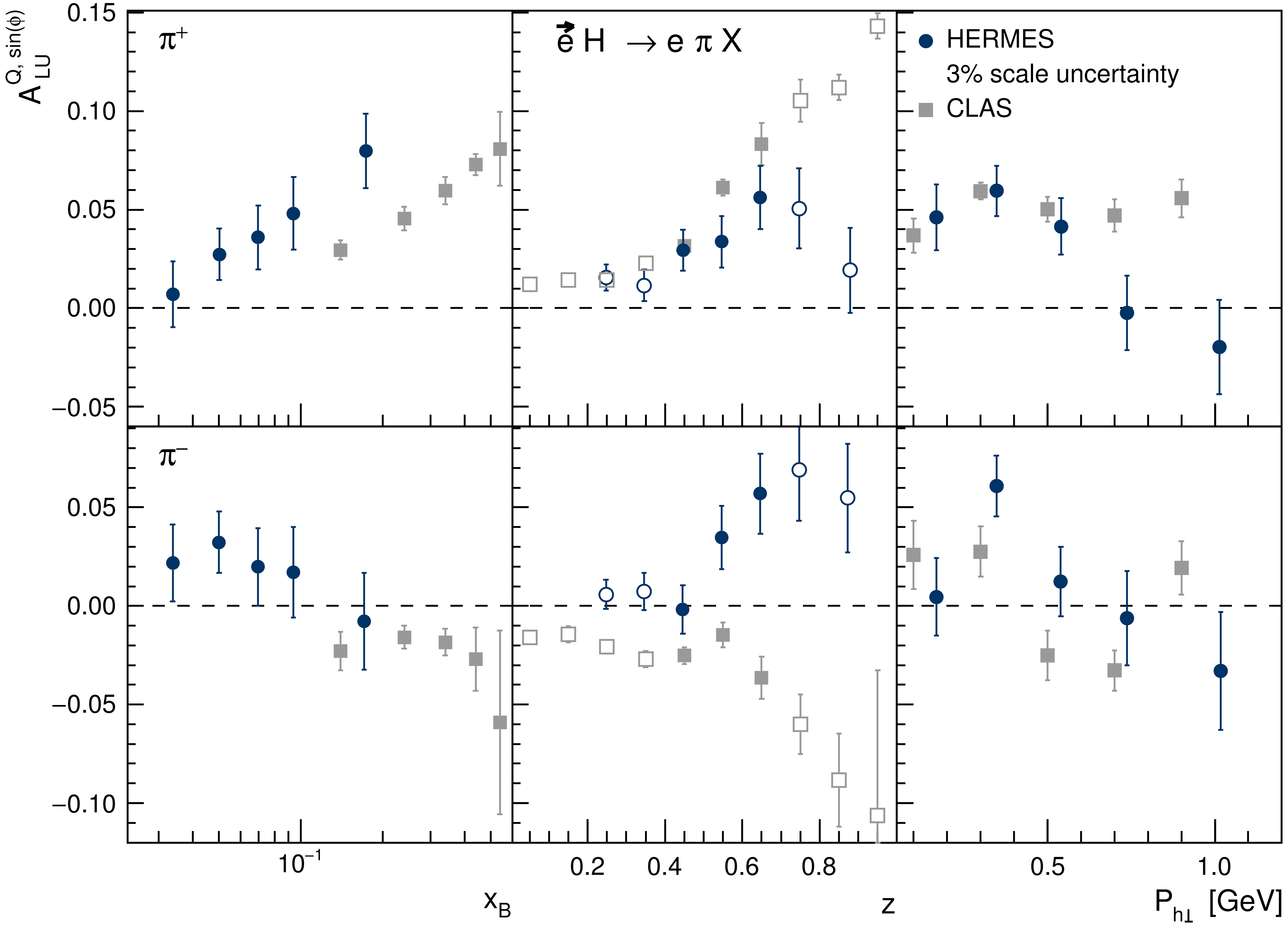
$$\frac{M_h}{Mz} h_1^\perp \tilde{E} \oplus x g^\perp D_1 \oplus \frac{M_h}{Mz} f_1 \tilde{G}^\perp \oplus xeH_1^\perp$$



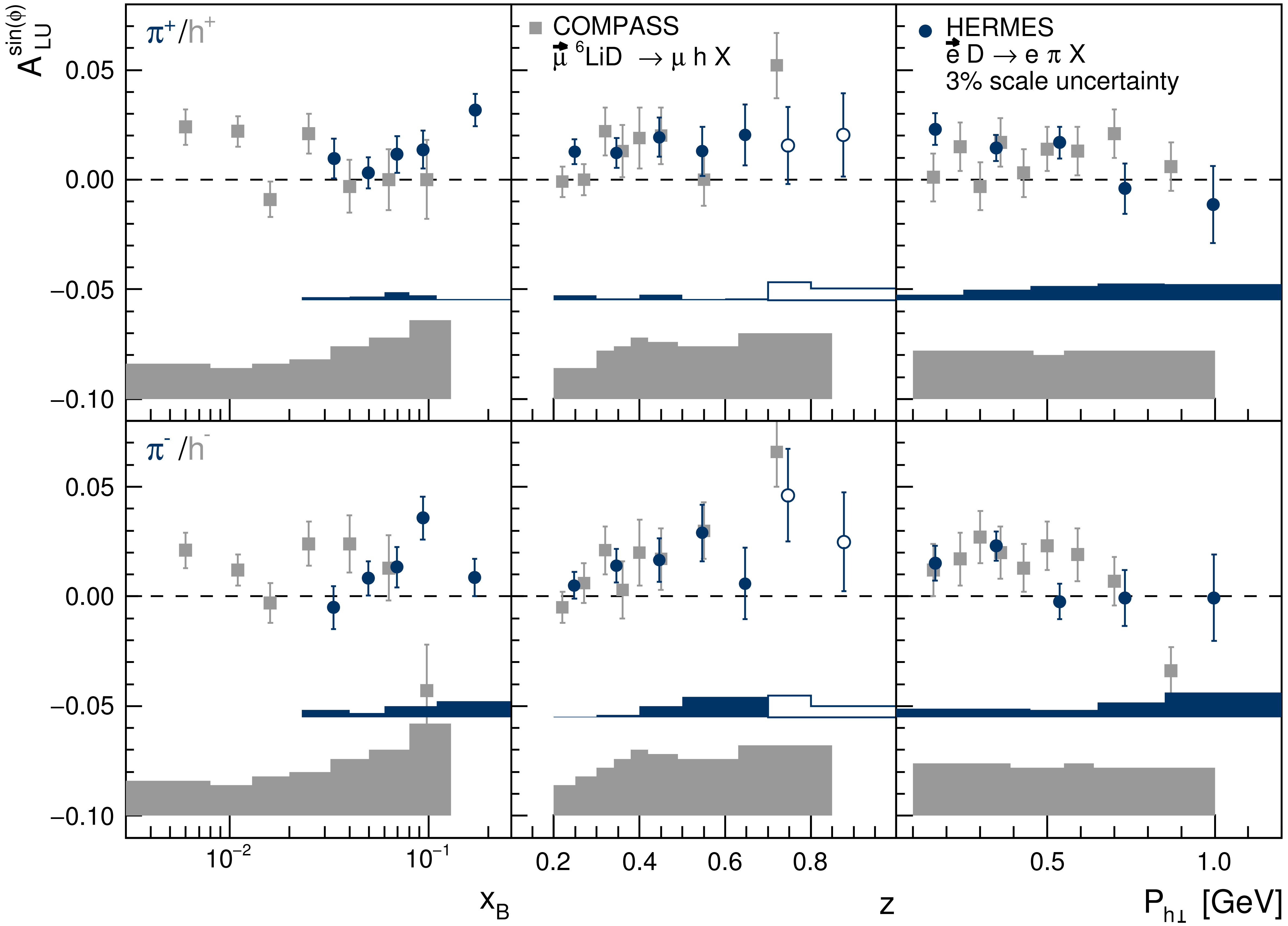


- opposite behavior at HERMES/CLAS of negative pions in z projection due to different x-range probed
- CLAS more sensitive to e(x) ⊗ Collins term due to higher x probed?

$$\frac{M_h}{Mz} h_1^{\perp} \tilde{E} \oplus x g^{\perp} D_1 \oplus \frac{M_h}{Mz} f_1 \tilde{G}^{\perp} \oplus x e H_1^{\perp}$$



[arXiv:1903.08544]

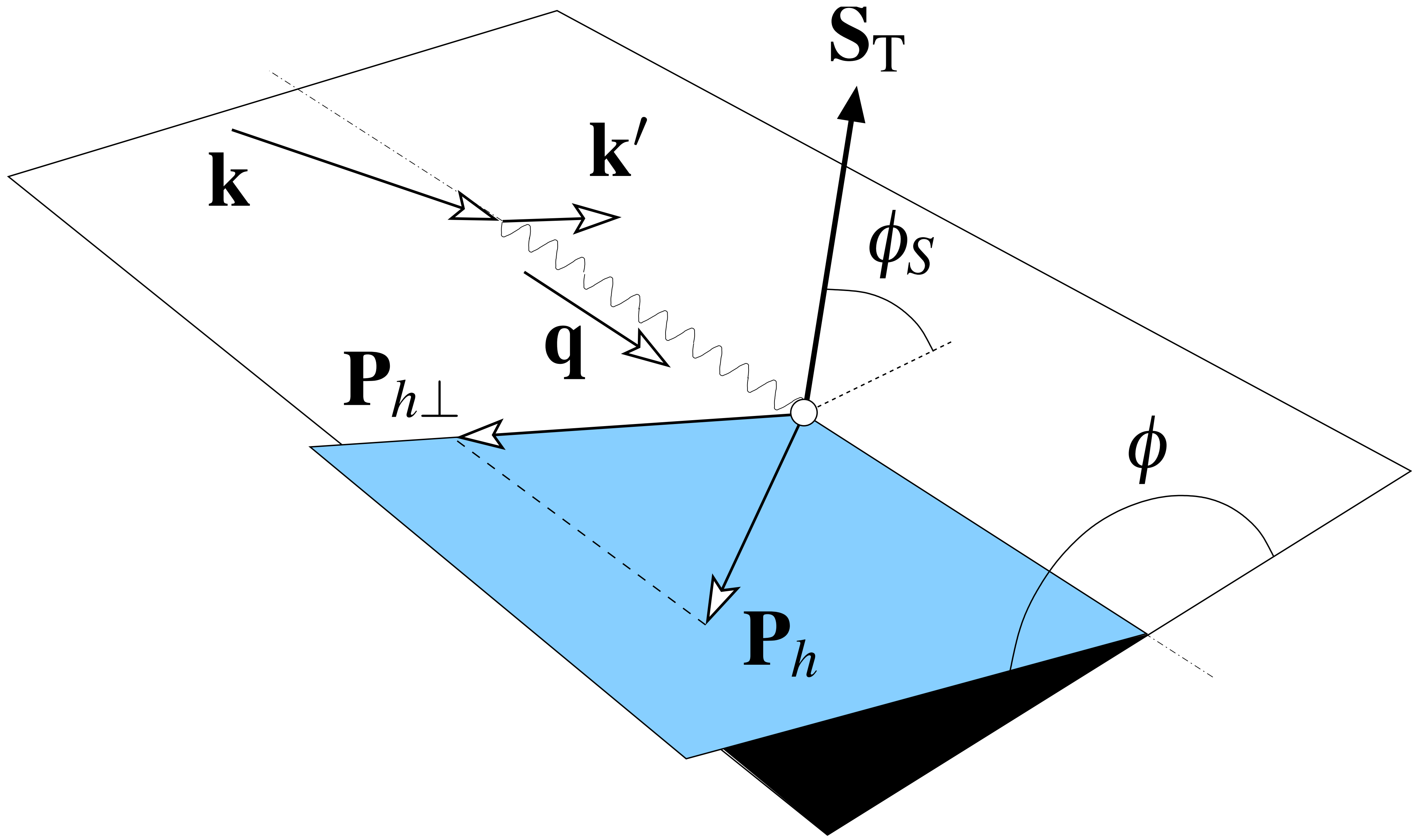


[arXiv:1903.08544]

- opposite behavior at HERMES/CLAS of negative pions in z projection due to different x-range probed
- CLAS more sensitive to e(x) ⊗ Collins term due to higher x probed?
- consistent behavior for charged pions / hadrons at HERMES / COMPASS for isoscalar targets

semi-inclusive DIS

- with transverse target polarization:



$$\frac{d\sigma^h}{dx\,dy\,dz\,dP_{h\perp}^2\,d\phi\,d\phi_s} = \frac{2\pi\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

$$\Big\{ F_{UU,T}^h + \epsilon F_{UU,L}^h + \text{terms not involving transv. polarization}$$

$$+ S_T \left[\left(F_{UT,T}^{h,\sin(\phi-\phi_s)} + \epsilon\, F_{UT,L}^{h,\sin(\phi-\phi_s)}\right)\sin(\phi-\phi_s)\right.$$

$$+ \epsilon\, F_{UT}^{h,\sin(\phi+\phi_s)}\sin(\phi+\phi_s) + \epsilon\, F_{UT}^{h,\sin(3\phi-\phi_s)}\sin(3\phi-\phi_s)$$

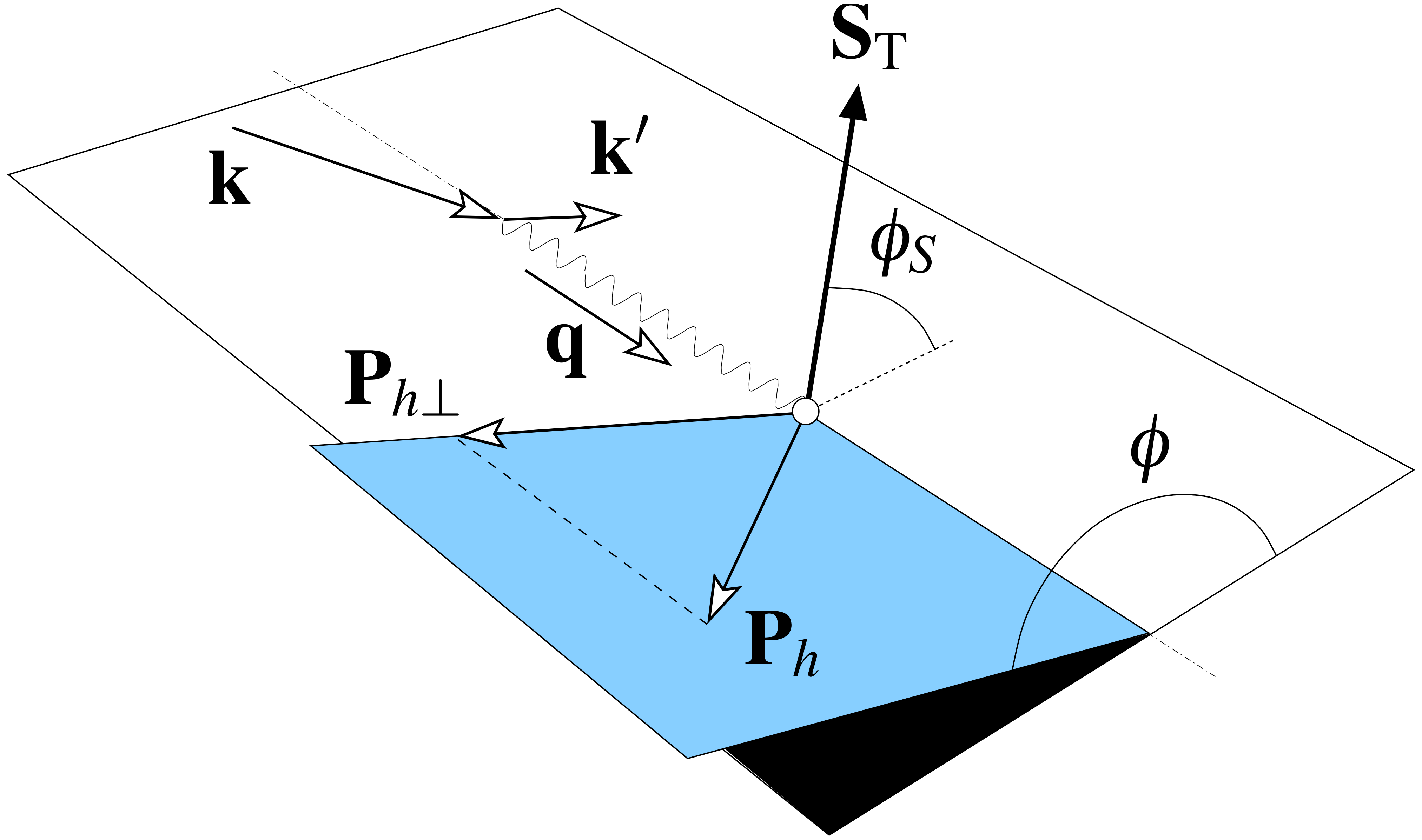
$$\left. + \sqrt{2\,\epsilon(1+\epsilon)}\,F_{UT}^{h,\sin\phi_s}\sin\phi_s + \sqrt{2\,\epsilon(1+\epsilon)}\,F_{UT}^{h,\sin(2\phi-\phi_s)}\sin(2\phi-\phi_s)\right]$$

$$+ S_T\lambda \left[\sqrt{1-\epsilon^2}\,F_{LT}^{h,\cos(\phi-\phi_s)}\cos(\phi-\phi_s)\right.$$

$$\left. + \sqrt{2\,\epsilon(1-\epsilon)}\,F_{LT}^{h,\cos\phi_s}\cos\phi_s + \sqrt{2\,\epsilon(1-\epsilon)}\,F_{LT}^{h,\cos(2\phi-\phi_s)}\cos(2\phi-\phi_s)\right]\Big\}$$

semi-inclusive DIS

- with transverse target polarization:



$$\frac{\mathrm{d}\sigma^h}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_{h\perp}^2\,\mathrm{d}\phi\,\mathrm{d}\phi_s} = \frac{2\pi\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$

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semi-inclusive DIS

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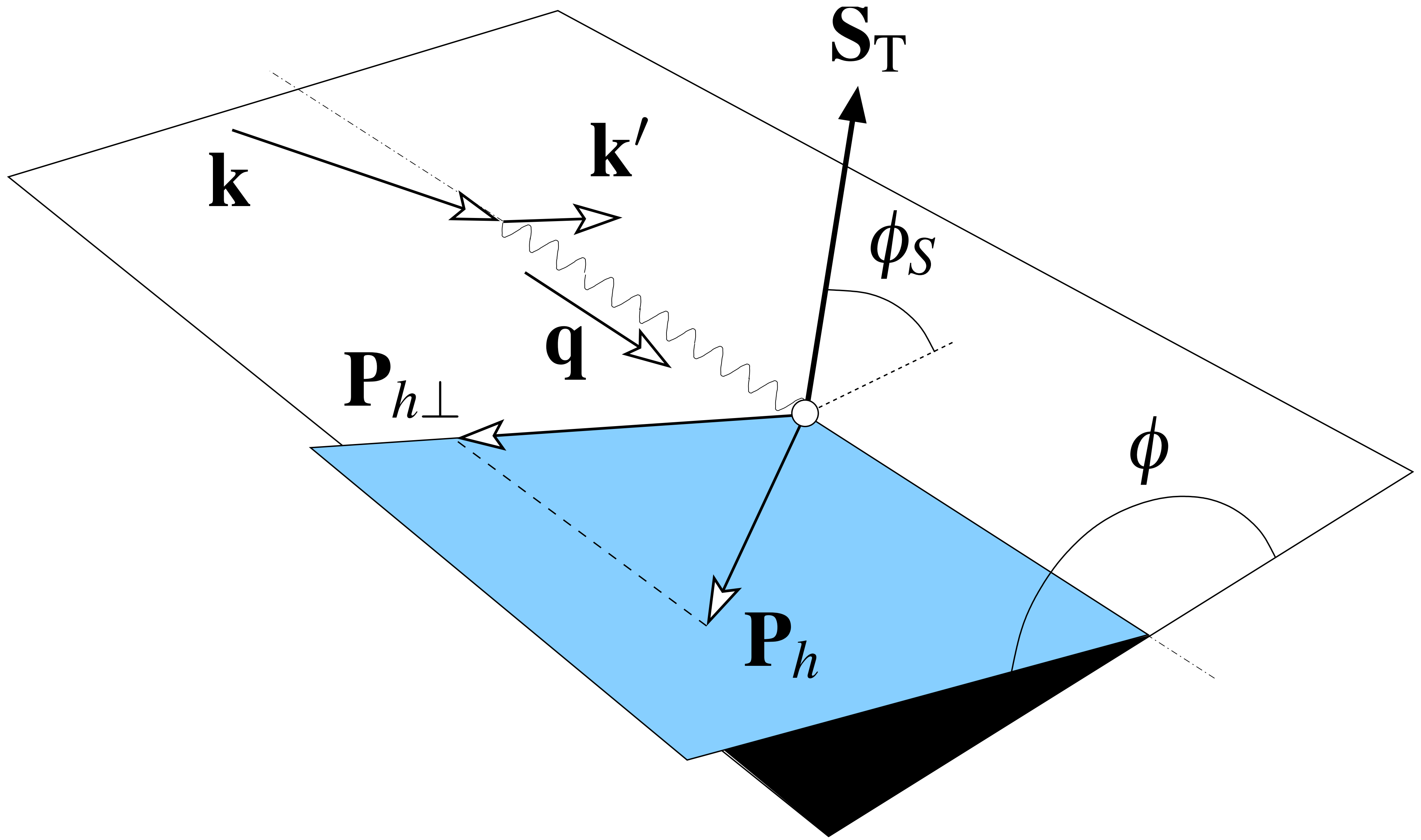

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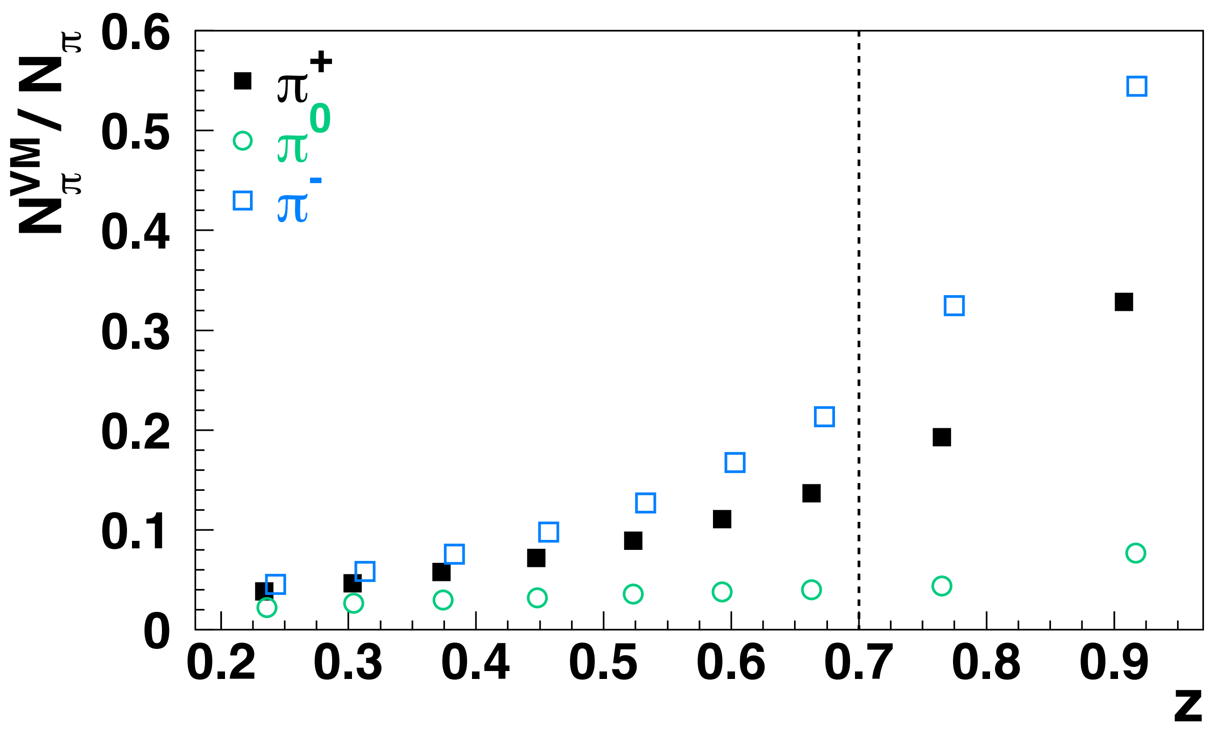
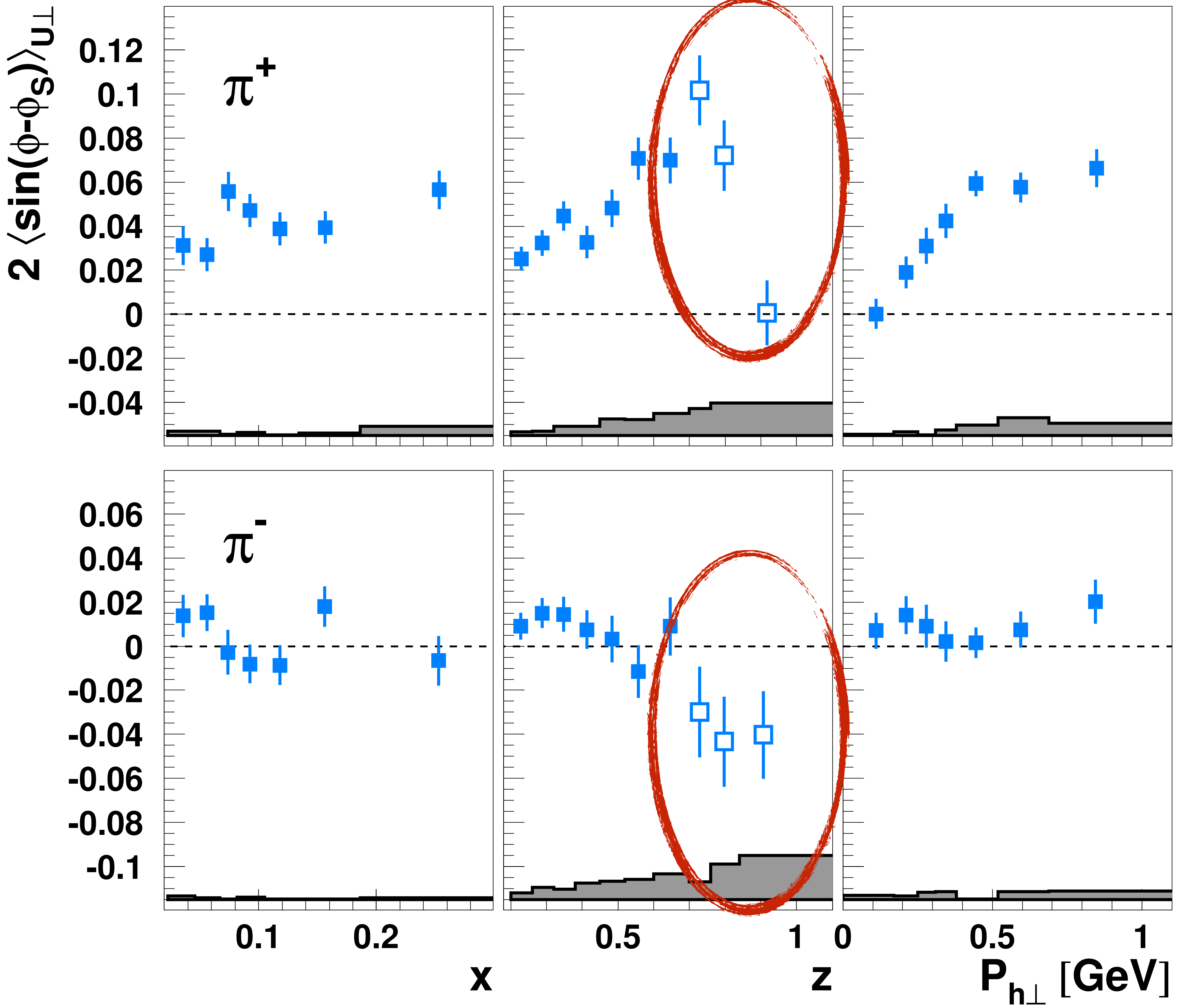
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

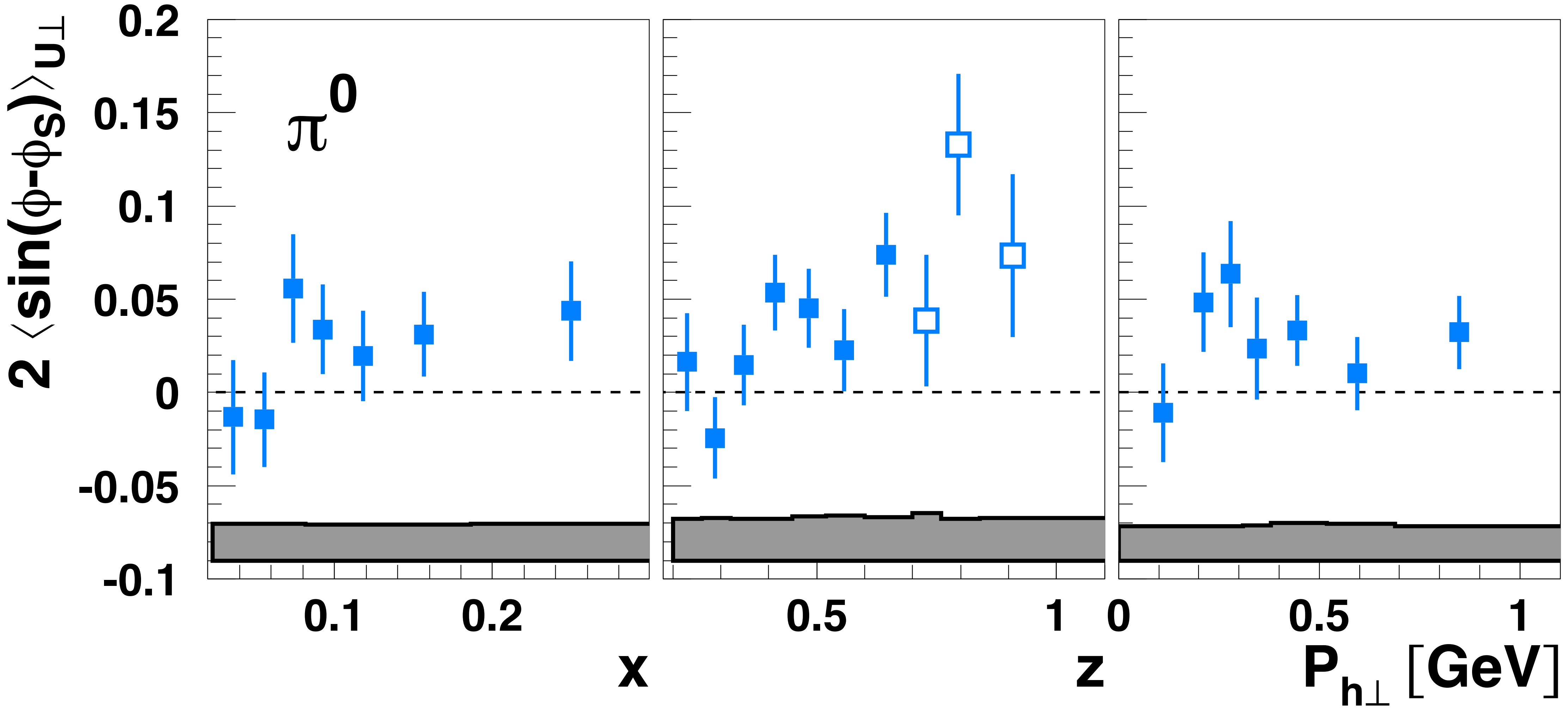


Sivers amplitudes for pions

- high-z data probes region where contributions from exclusive vector-meson production becomes significant
- only last z bin shows indication of sizable ρ^0 contribution (decaying into charged pions)

[A. Airapetian et al., JHEP12(2020)010]

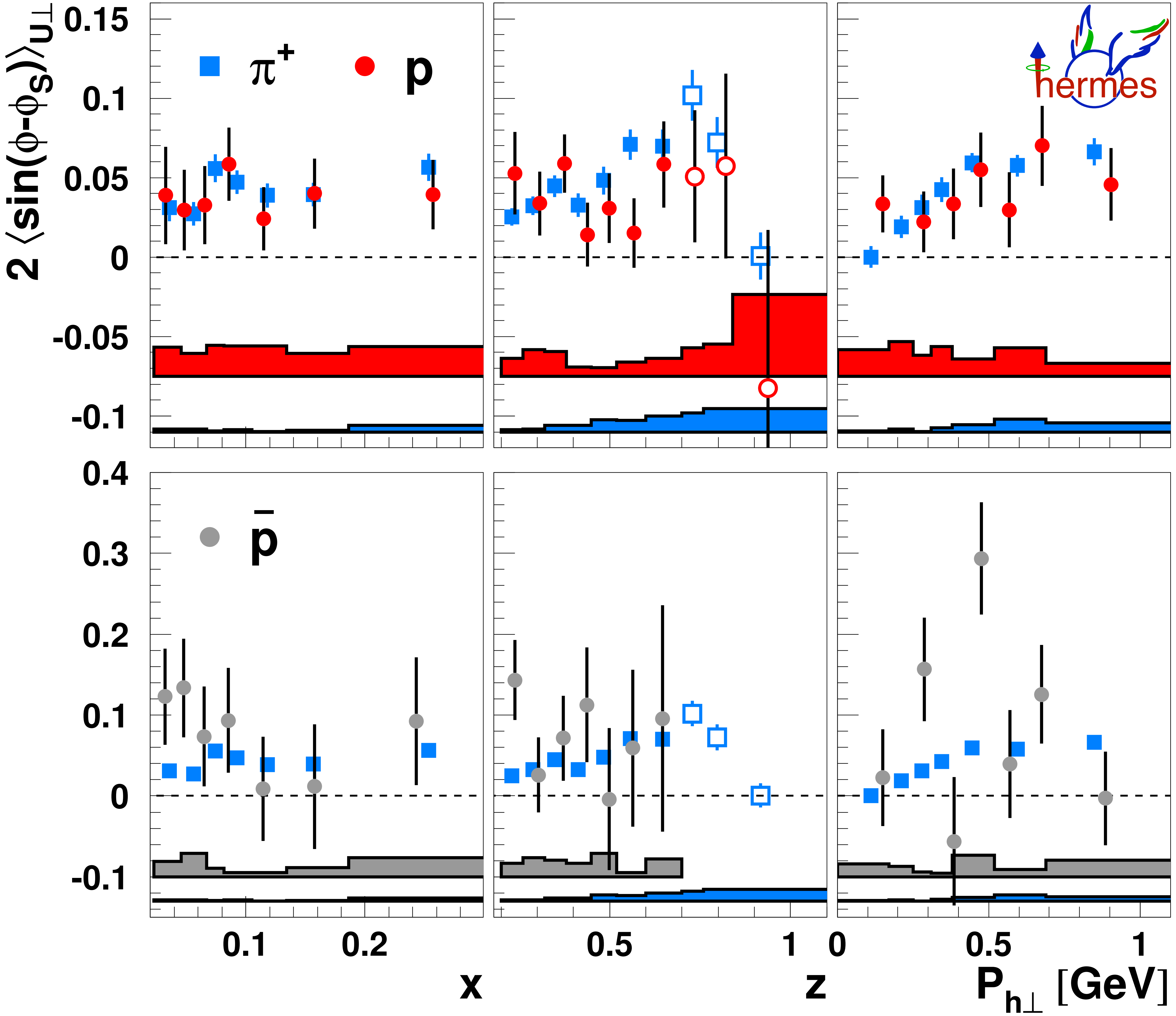




	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes

pions vs. (anti)protons



[A. Airapetian et al., JHEP12(2020)010]

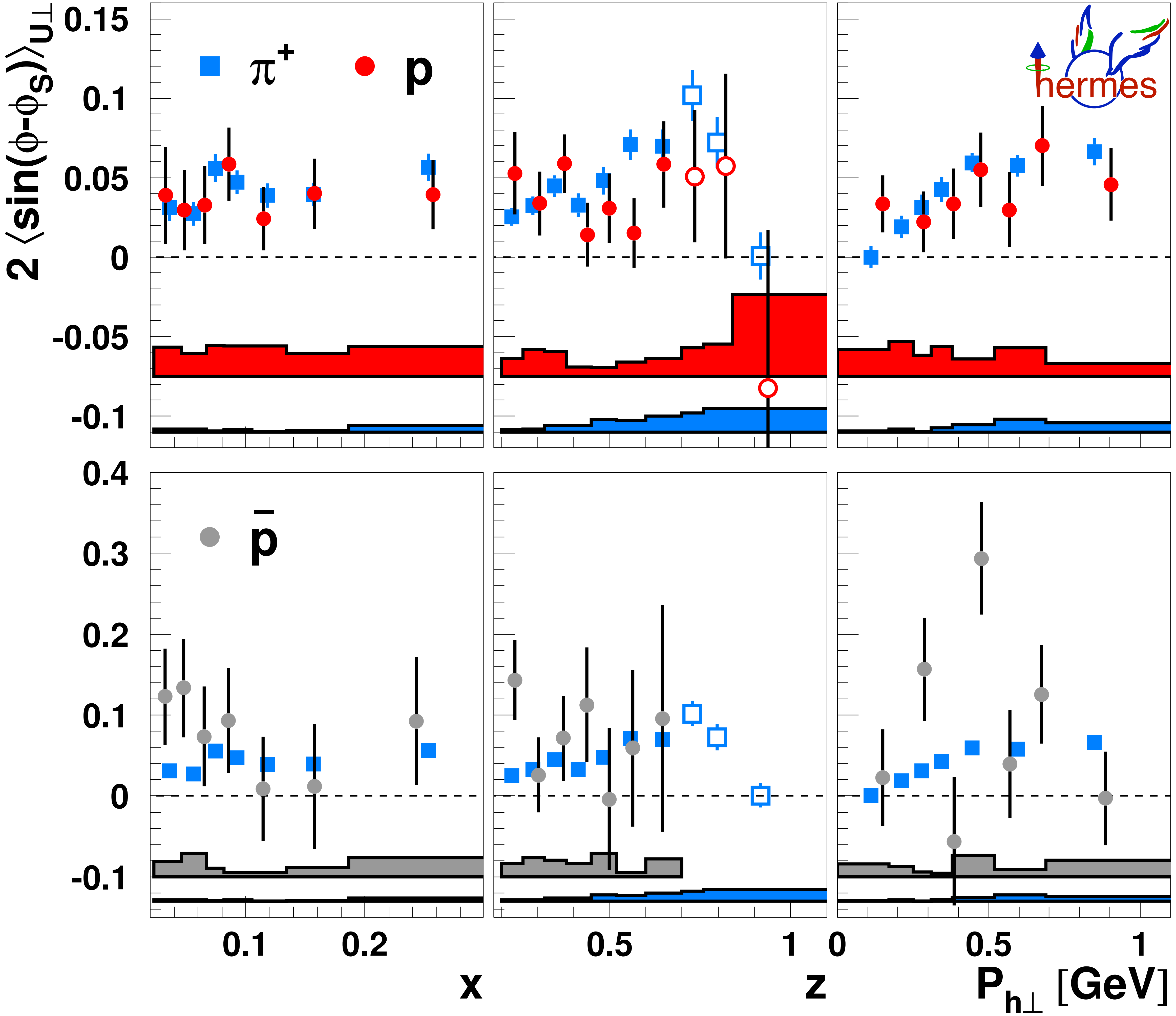
similar-magnitude asymmetries for (anti)protons and pions

➥consequence of u-quark dominance in both cases?

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes
pions vs. (anti)protons

[A. Airapetian et al., JHEP12(2020)010]



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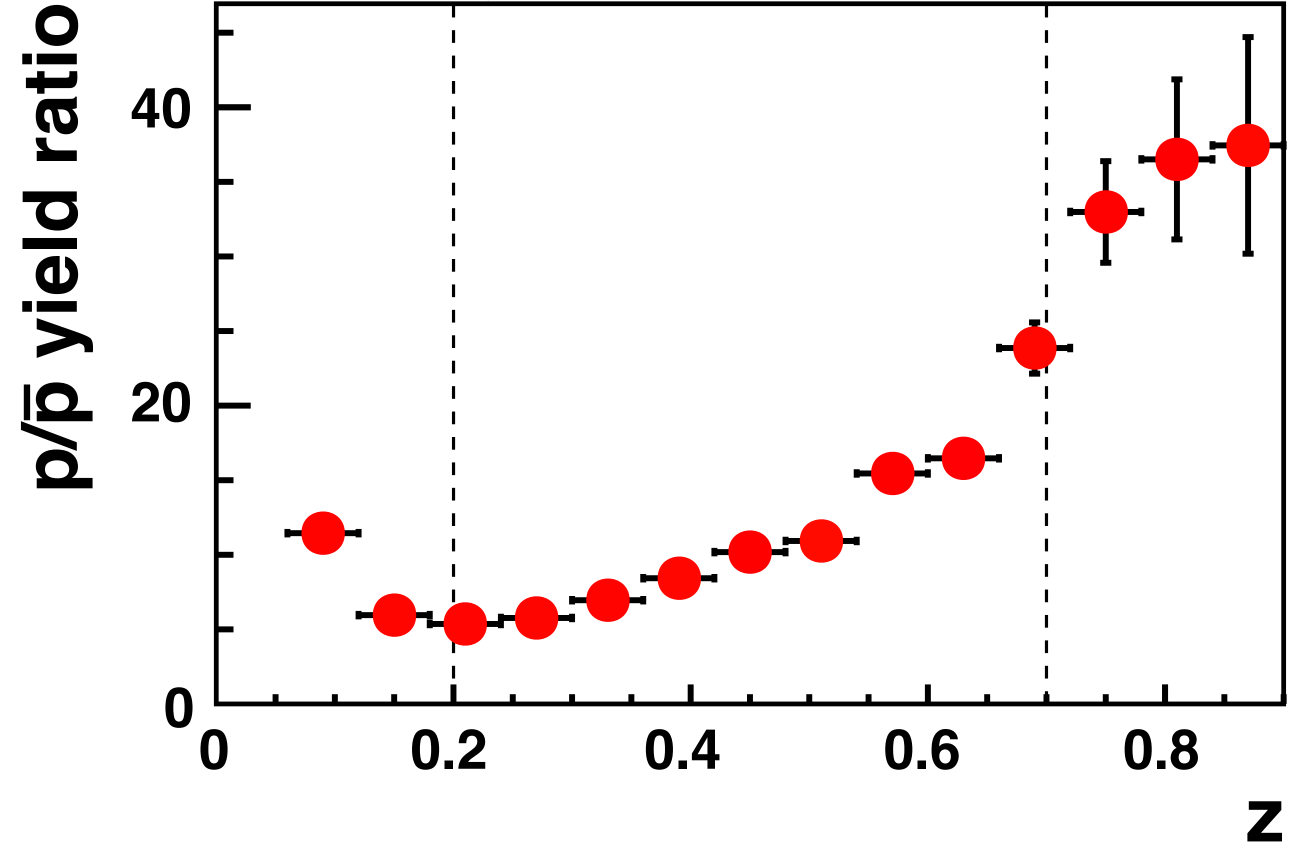
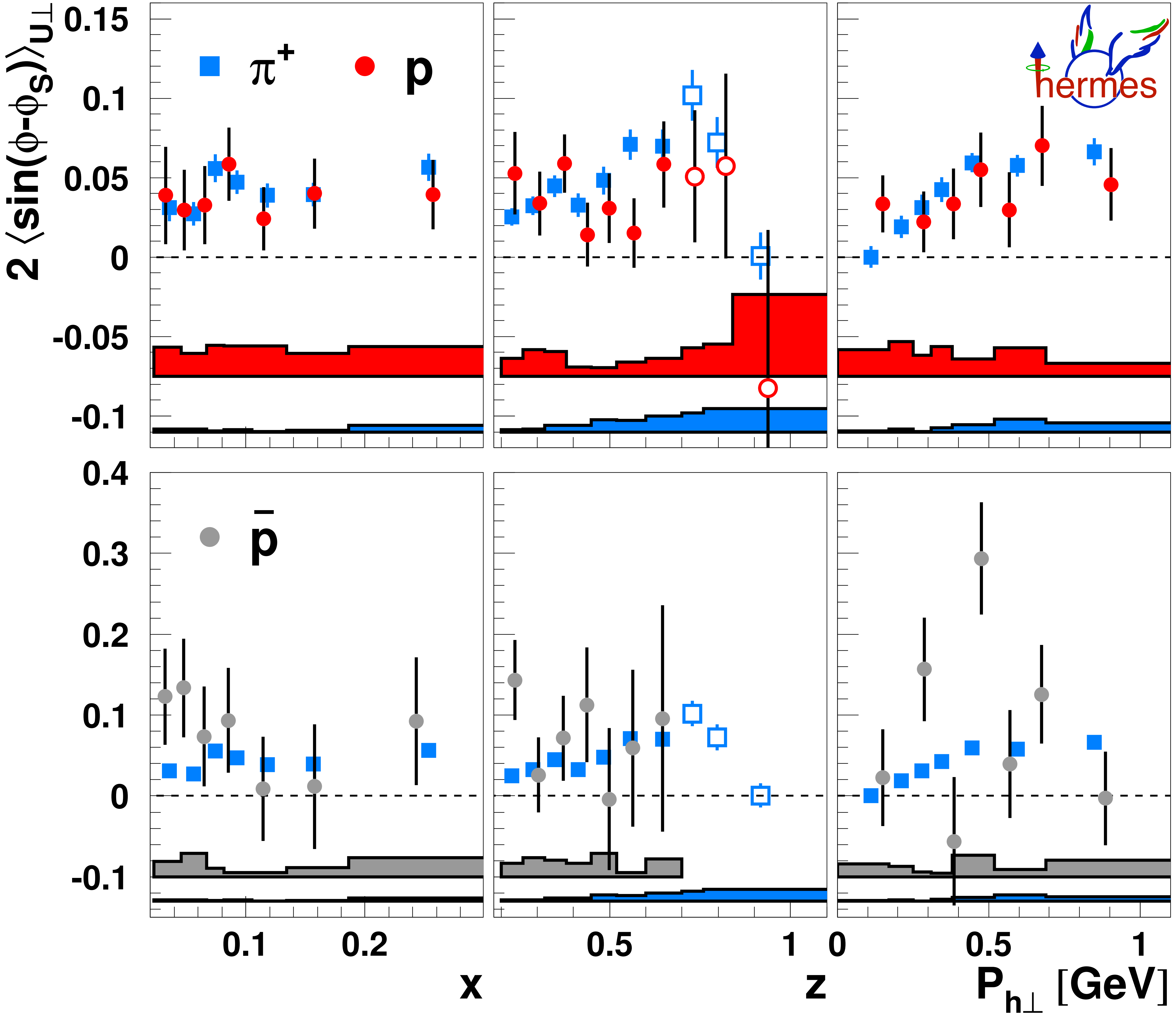


possibly, onset of target fragmentation only at lower z

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes
pions vs. (anti)protons

[A. Airapetian et al., JHEP12(2020)010]



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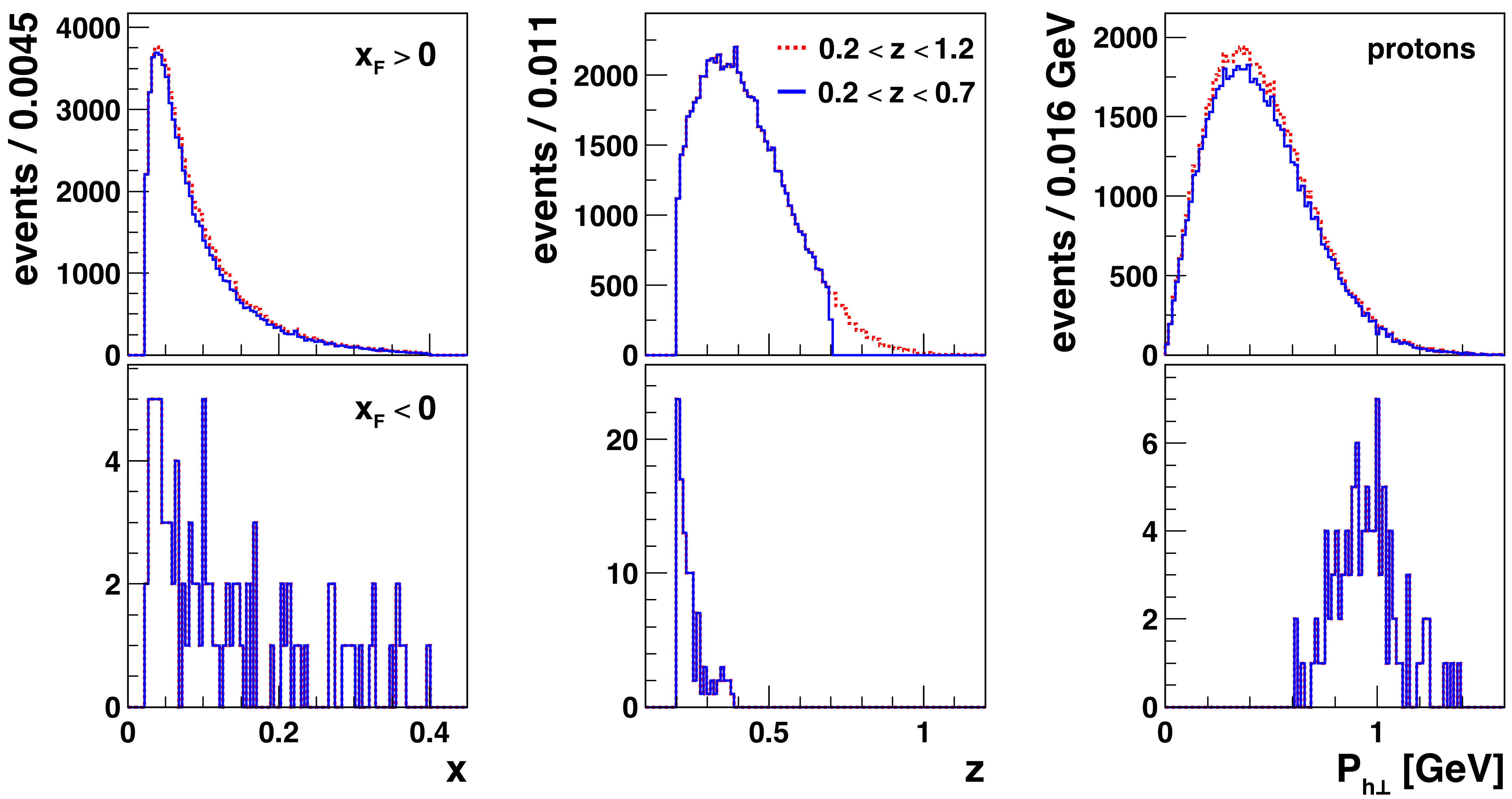

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L		g_{1L}	$h_{1L}^\perp$
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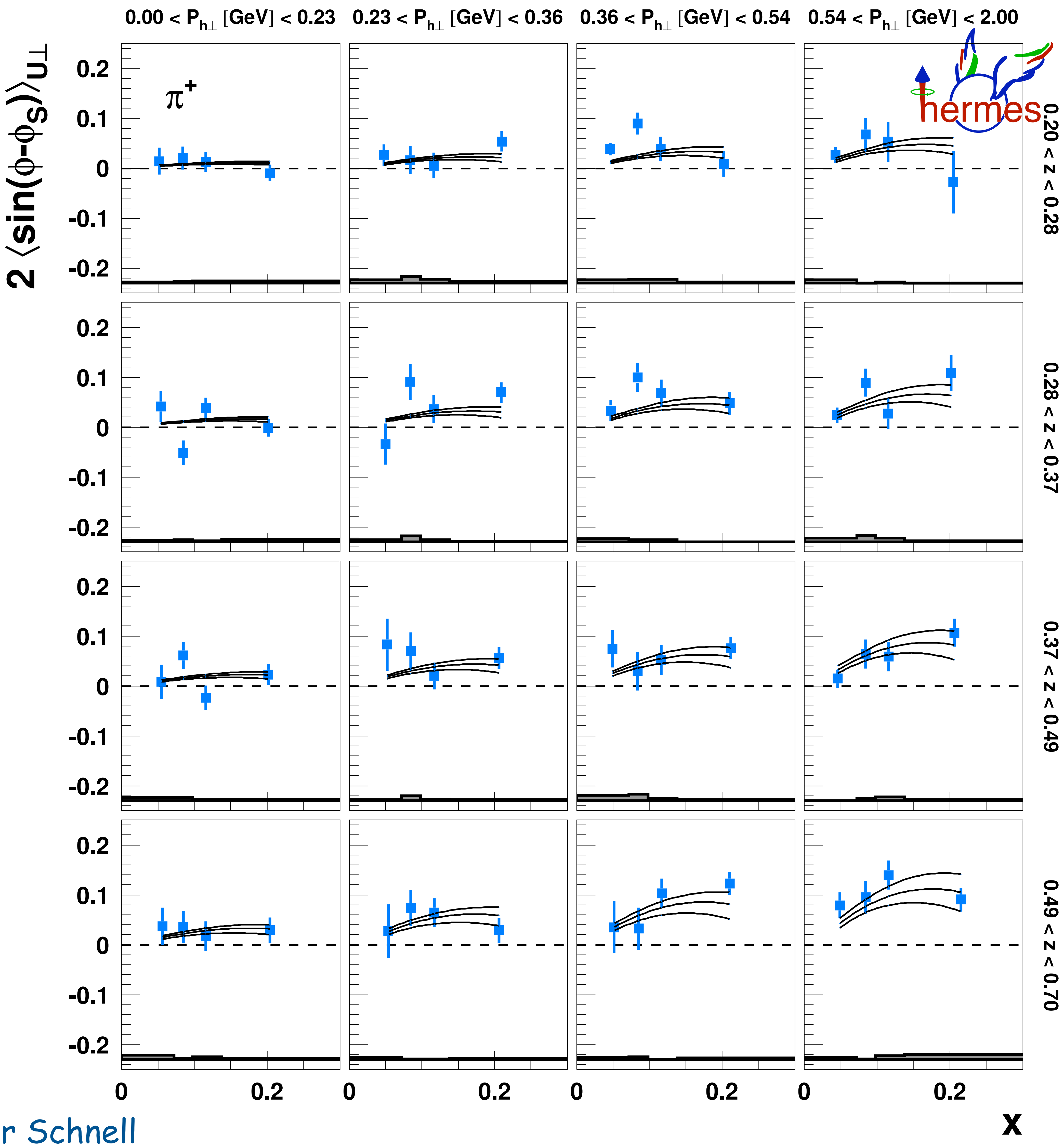
Sivers amplitudes
multi-dimensional analysis

[A. Airapetian et al., JHEP12(2020)010]

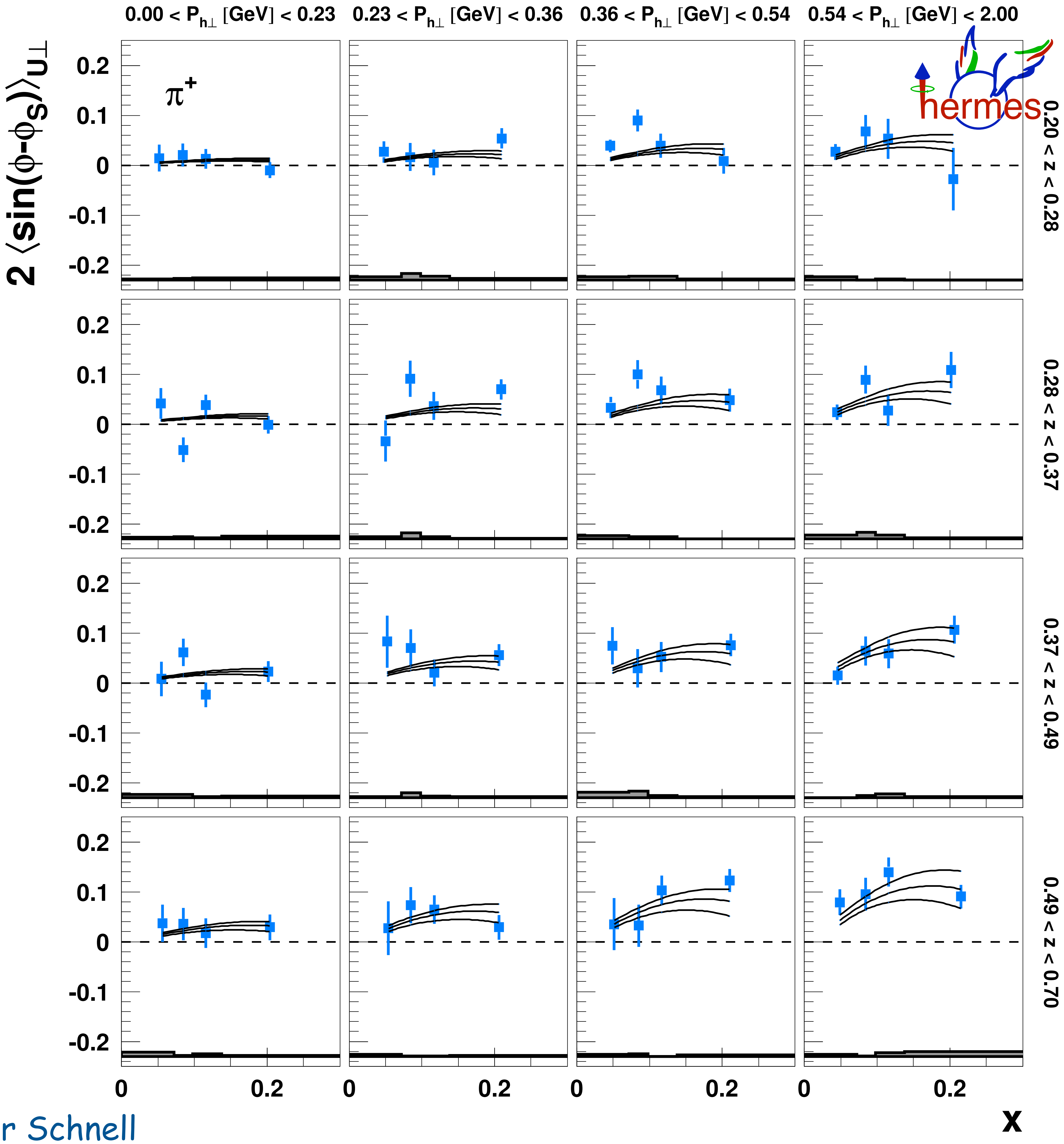


- 3d analysis: 4x4x4 bins in $(x,z,P_{h\perp})$
  - reduced systematics
  - disentangle correlations
  - isolate phase-space region with large signal strength

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\, h_{1T}^\perp$

Sivers amplitudes multi-dimensional analysis

[A. Airapetian et al., JHEP12(2020)010]

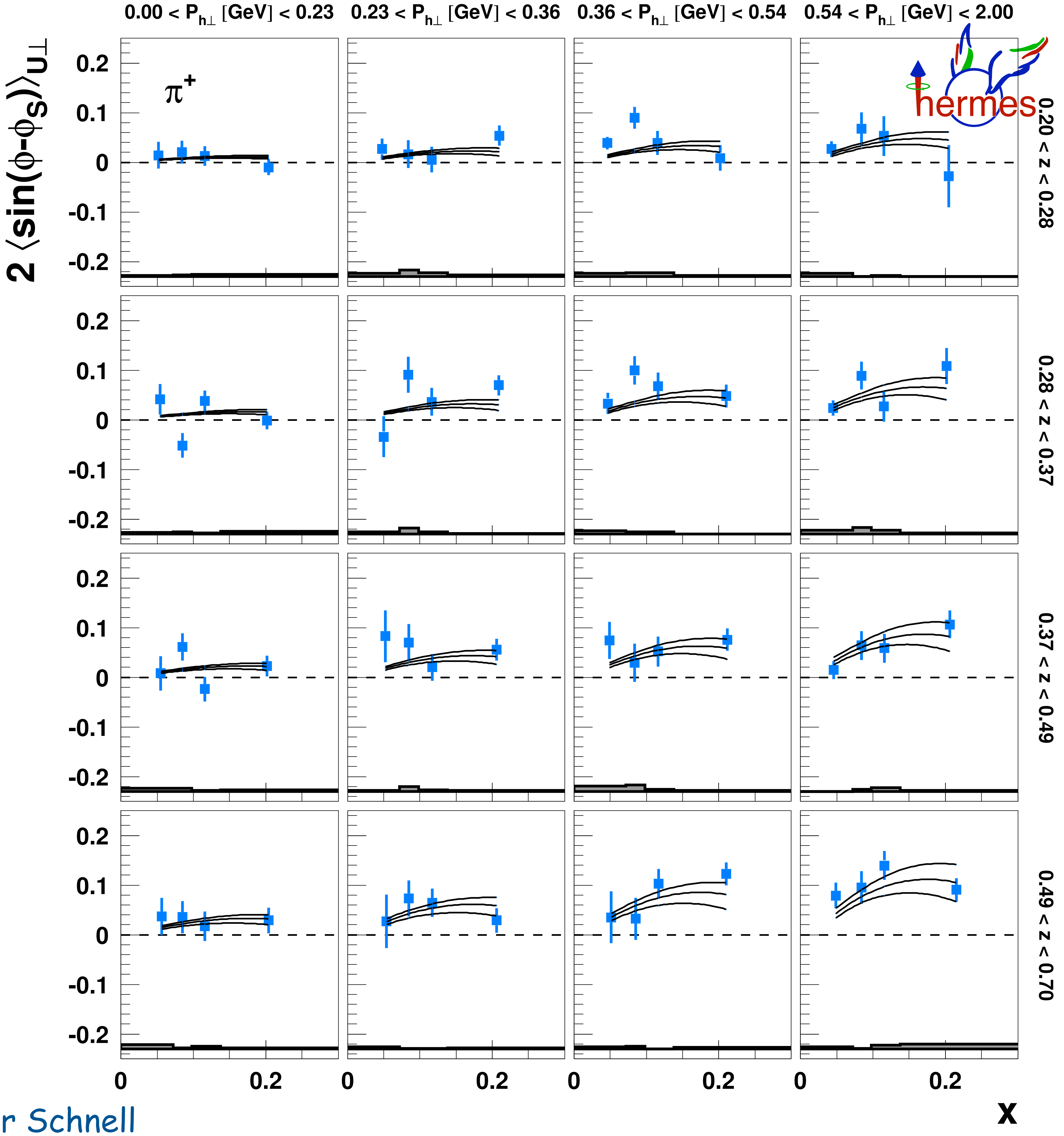


- 3d analysis: 4x4x4 bins in $(x, z, P_{h\perp})$
  - reduced systematics
  - disentangle correlations
  - isolate phase-space region with large signal strength
- allows more detailed comparison with calculations

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers amplitudes multi-dimensional analysis

[A. Airapetian et al., JHEP12(2020)010]

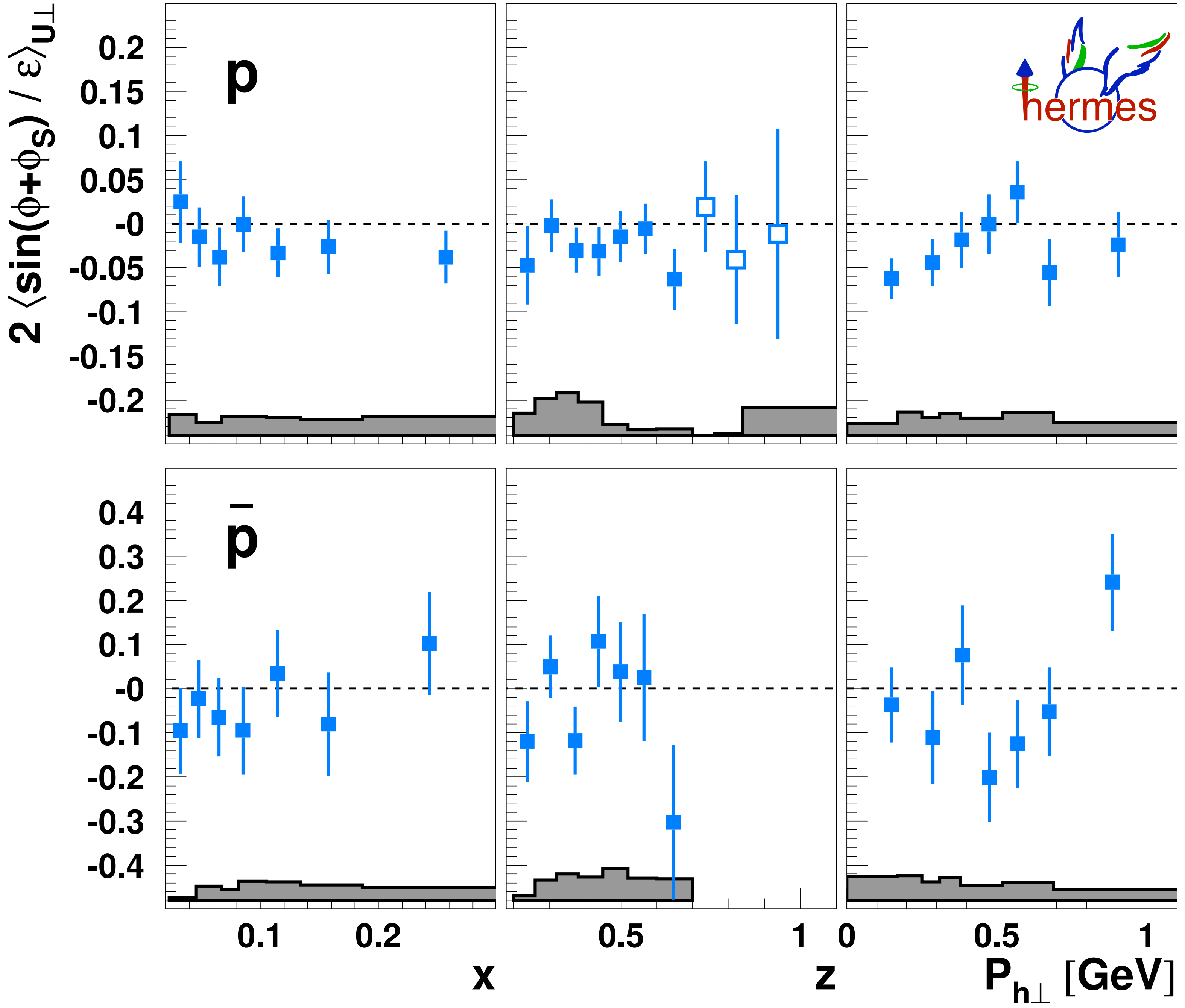


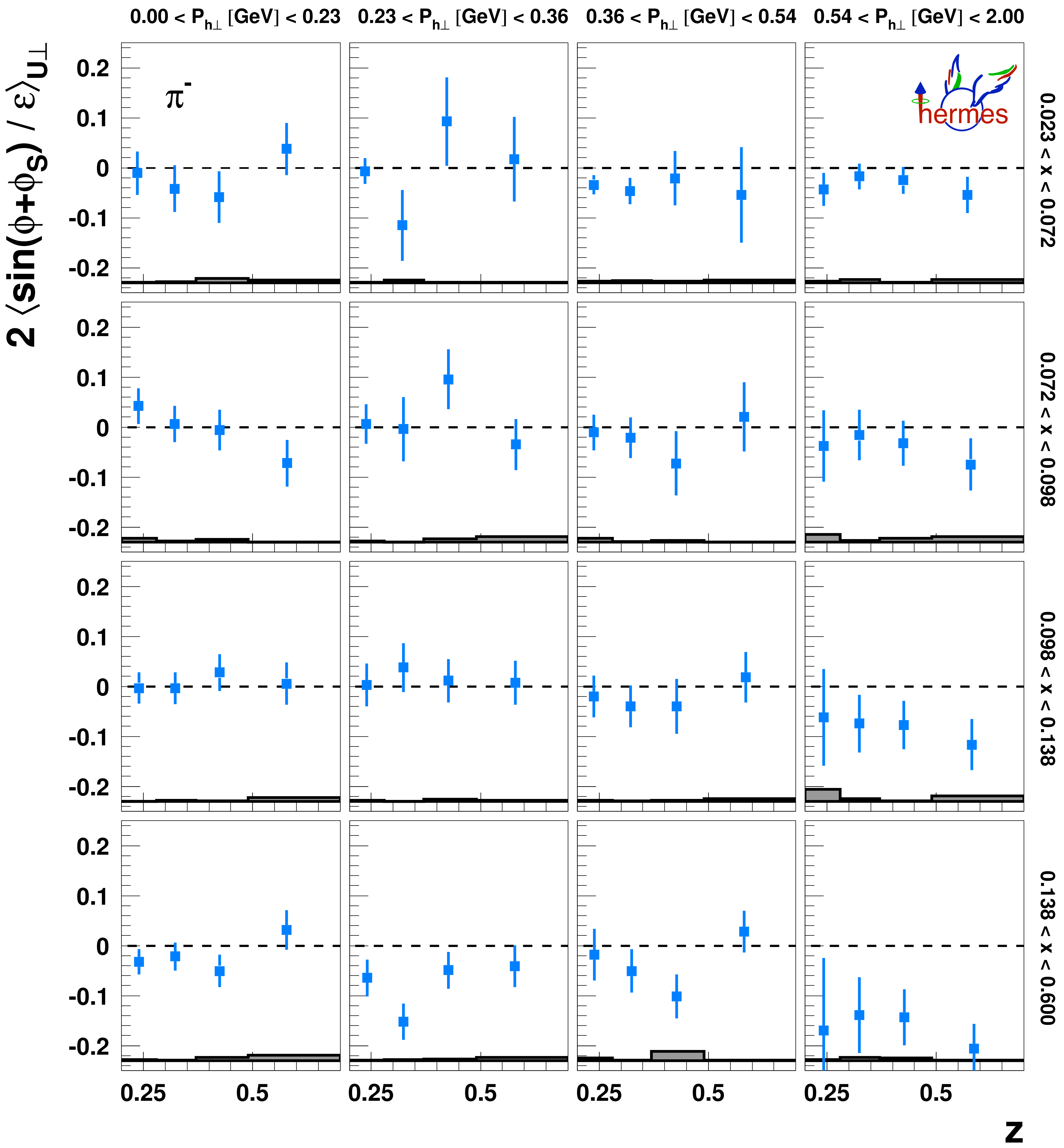
- 3d analysis: 4x4x4 bins in $(x, z, P_{h\perp})$
    - reduced systematics
    - disentangle correlations
    - isolate phase-space region with large signal strength
- allows more detailed comparison with calculations
- accompanied by kinematic distribution to guide phenomenology

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

new HERMES results on Collins amplitudes

[A. Airapetian et al., JHEP12(2020)010]





- results for **(anti-)protons** consistent with zero
  ➥ vanishing Collins effect for (spin-1/2) baryons?
- analysis now **performed in 3d**, both including or not including kinematic "depolarization" prefactor

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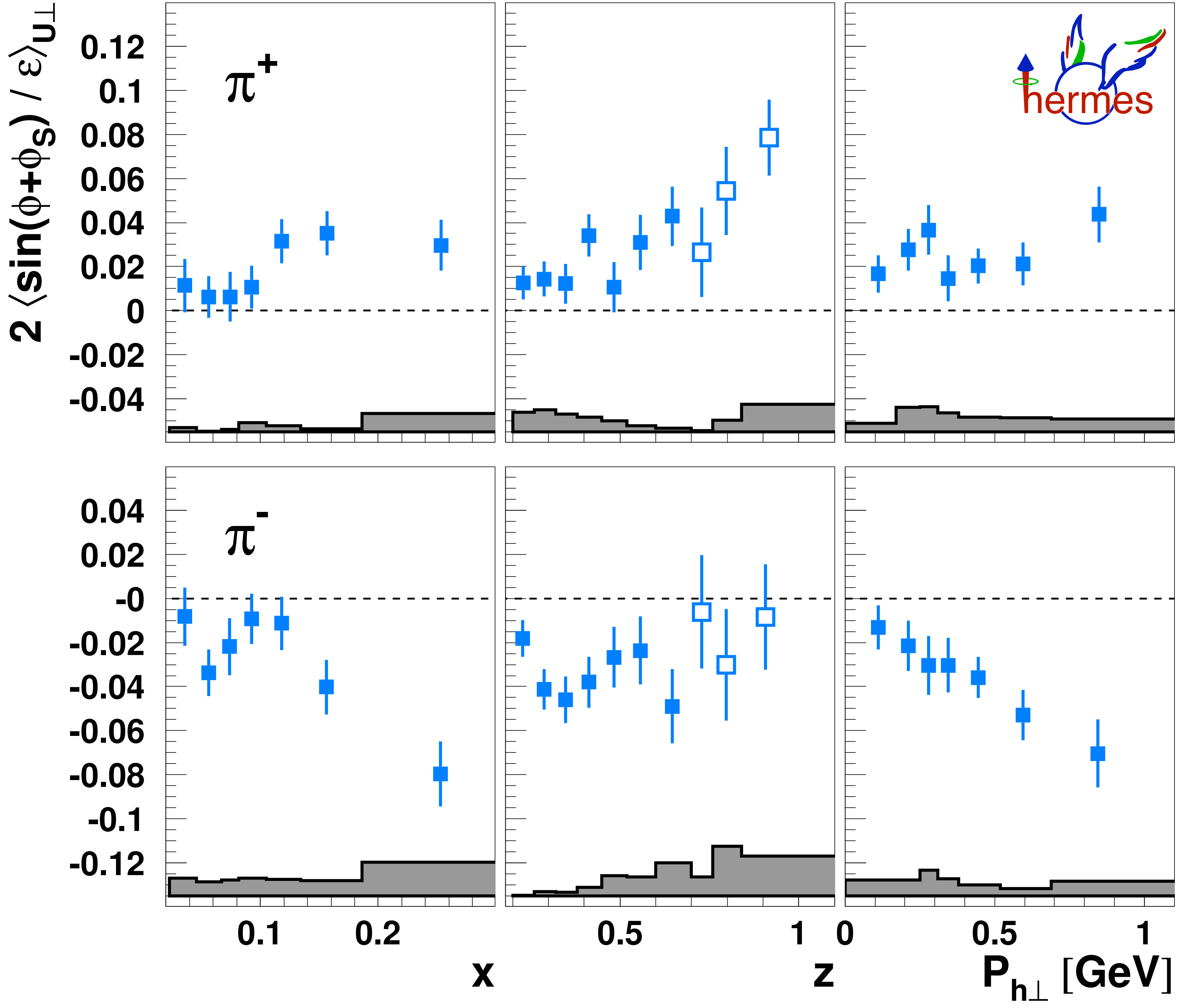
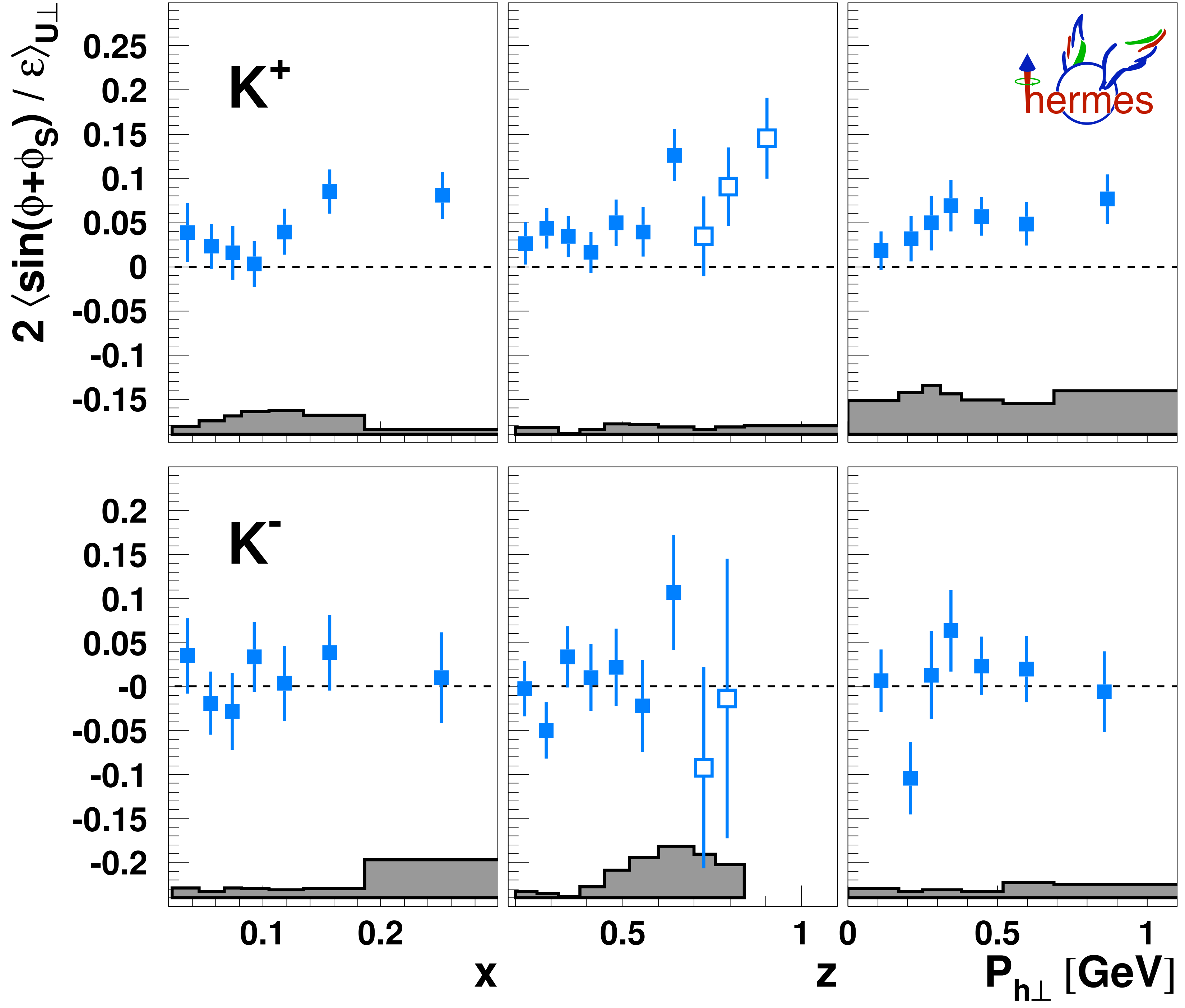
new HERMES results on Collins amplitudes


[A. Airapetian et al., JHEP12(2020)010]


[A. Airapetian et al., JHEP12(2020)010]

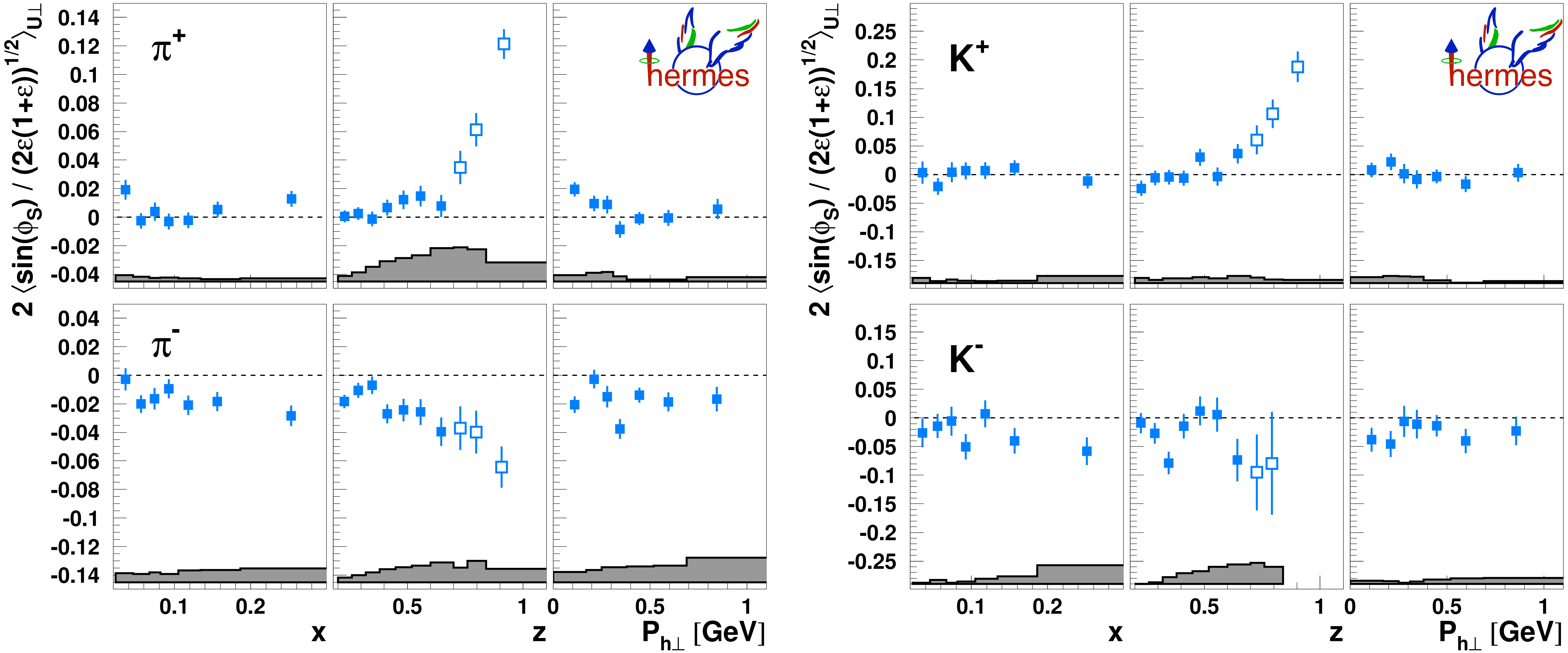
- results for **(anti-)protons** consistent with zero
  ➥ vanishing Collins effect for (spin-1/2) baryons?

- analysis now **performed in 3d**, both including or not including kinematic "depolarization" prefactor

- **high-z region** probes transition region to exclusive domain (with increasing amplitudes for positive pions and kaons)

subleading twist — $\langle\sin(\phi_s)\rangle_{UT}$

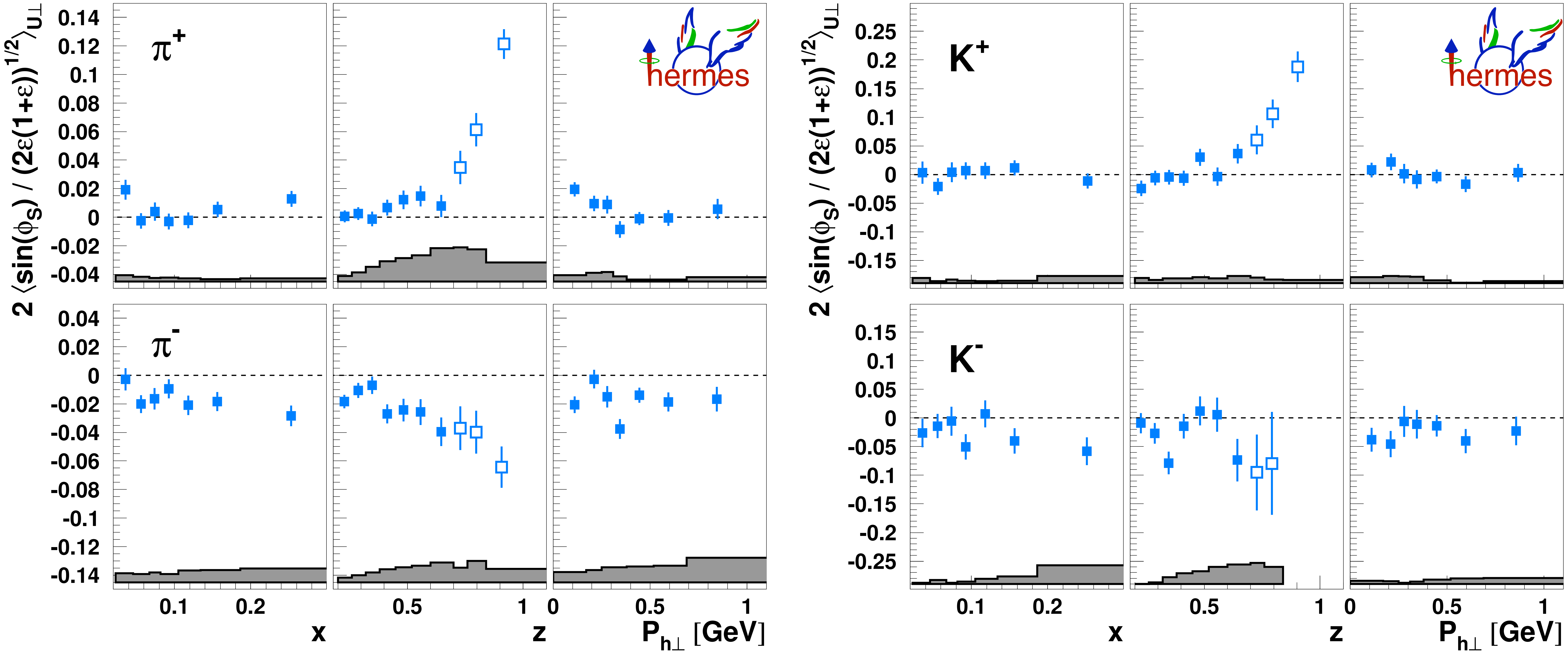
[A. Airapetian et al., JHEP12(2020)010]

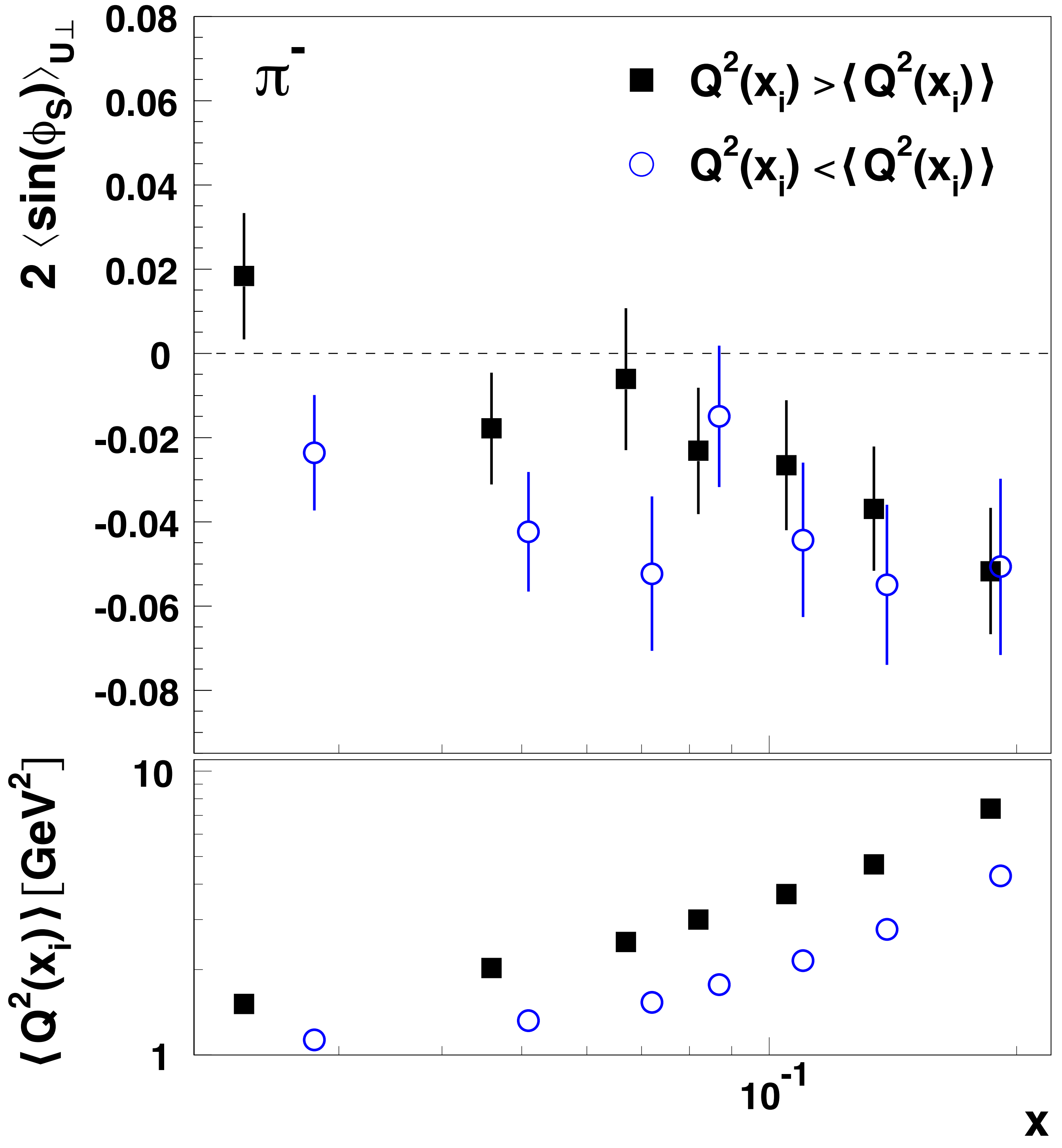


- clearly non-zero asymmetries with opposite sign for charged pions (Collins-like behavior)
- striking z dependence and in particular magnitude

subleading twist — $\langle \sin(\phi_S) \rangle_{UT}$

[A. Airapetian et al., JHEP12(2020)010]





- clearly non-zero asymmetries with opposite sign for charged pions (Collins-like behavior)
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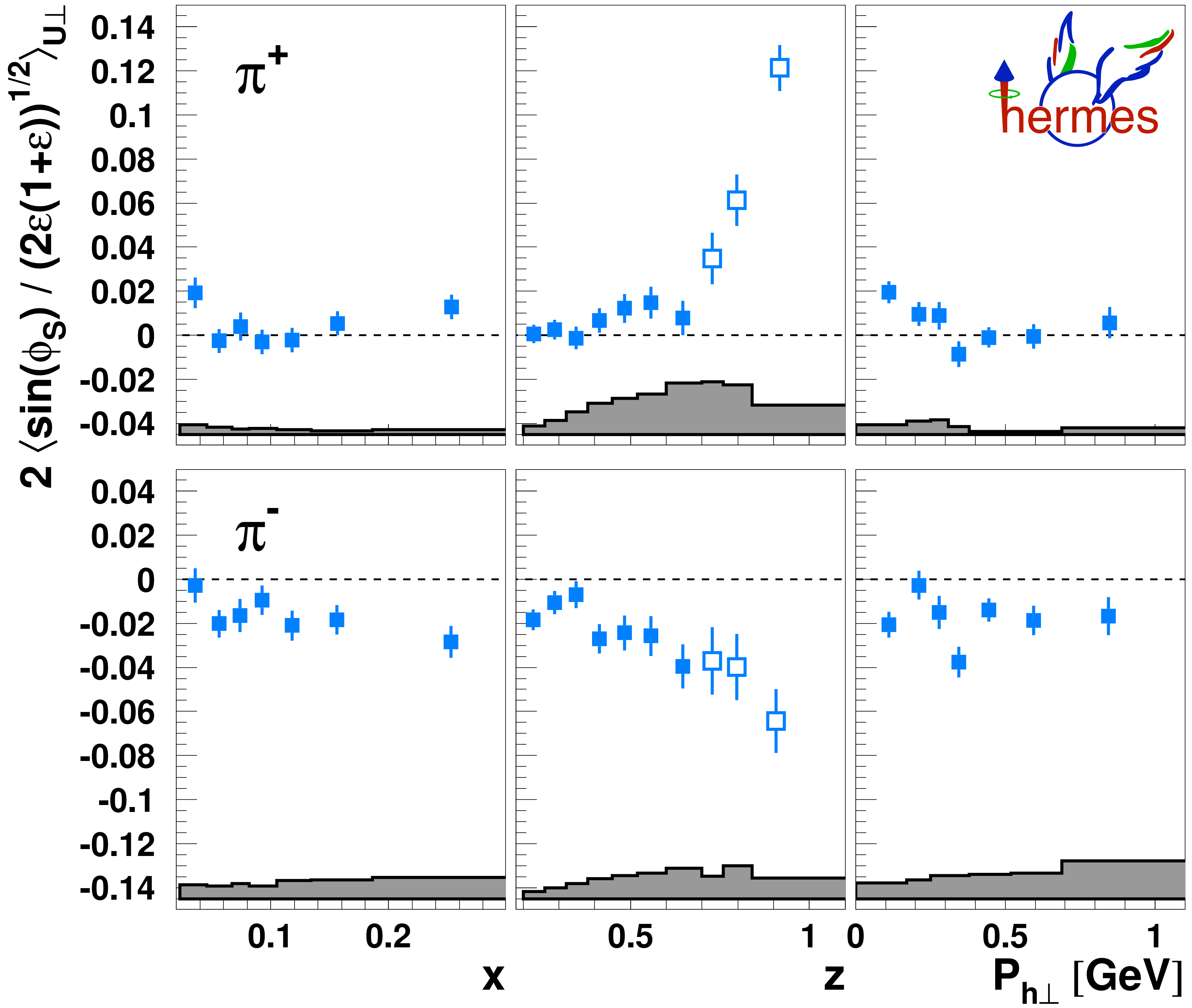
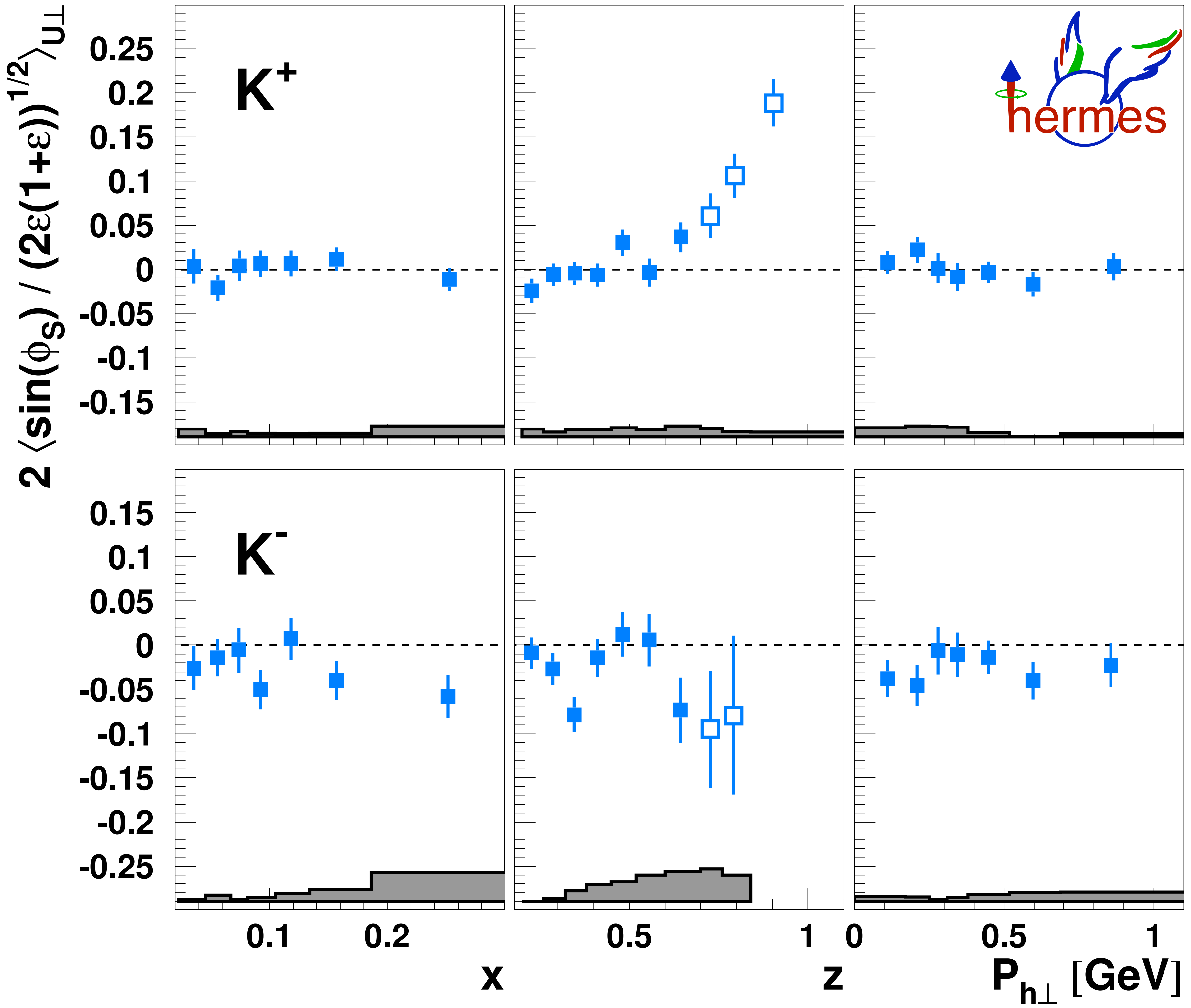
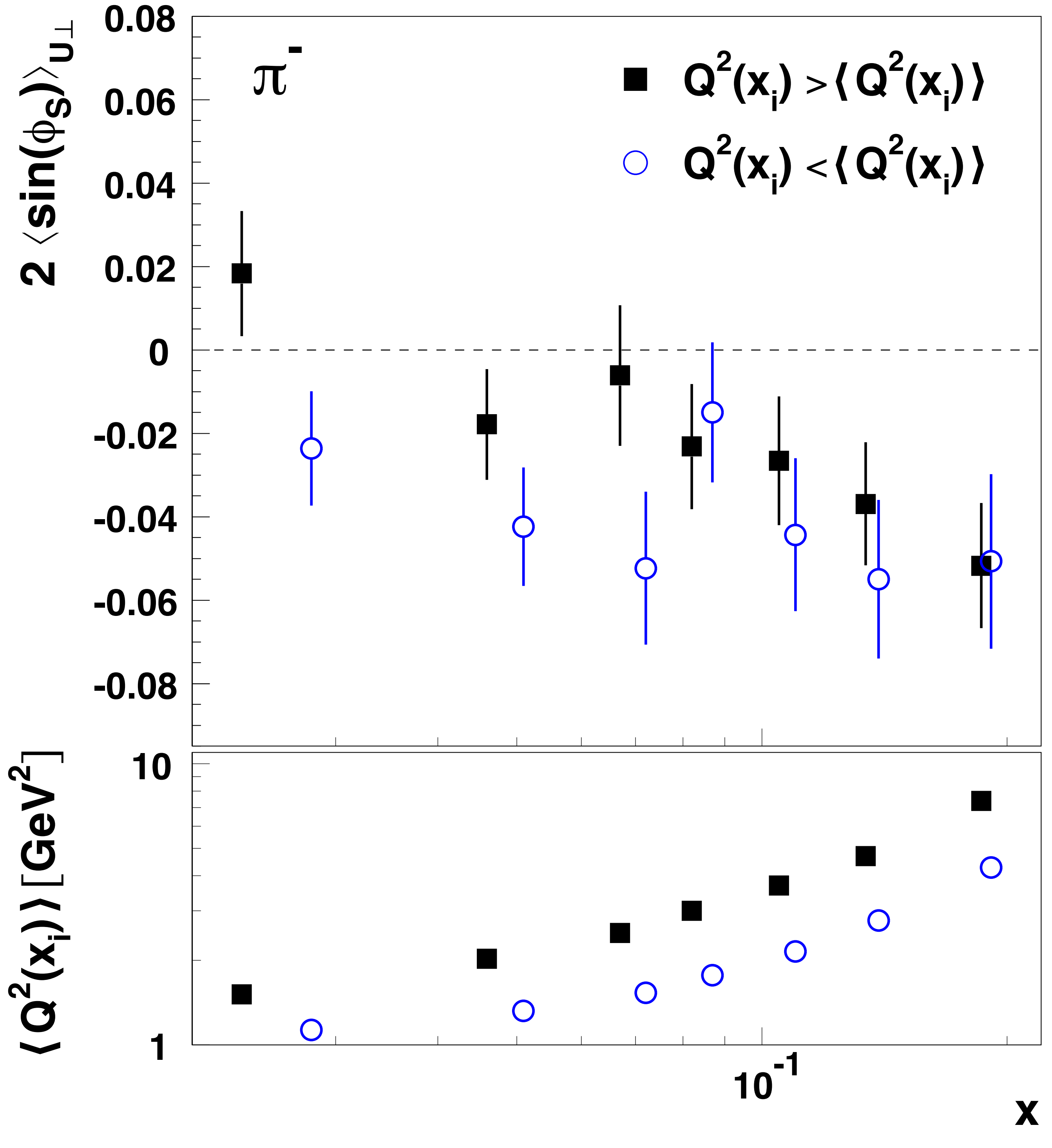
- hint of Q suppression

subleading twist — $\langle\sin(\phi_S)\rangle_{UT}$

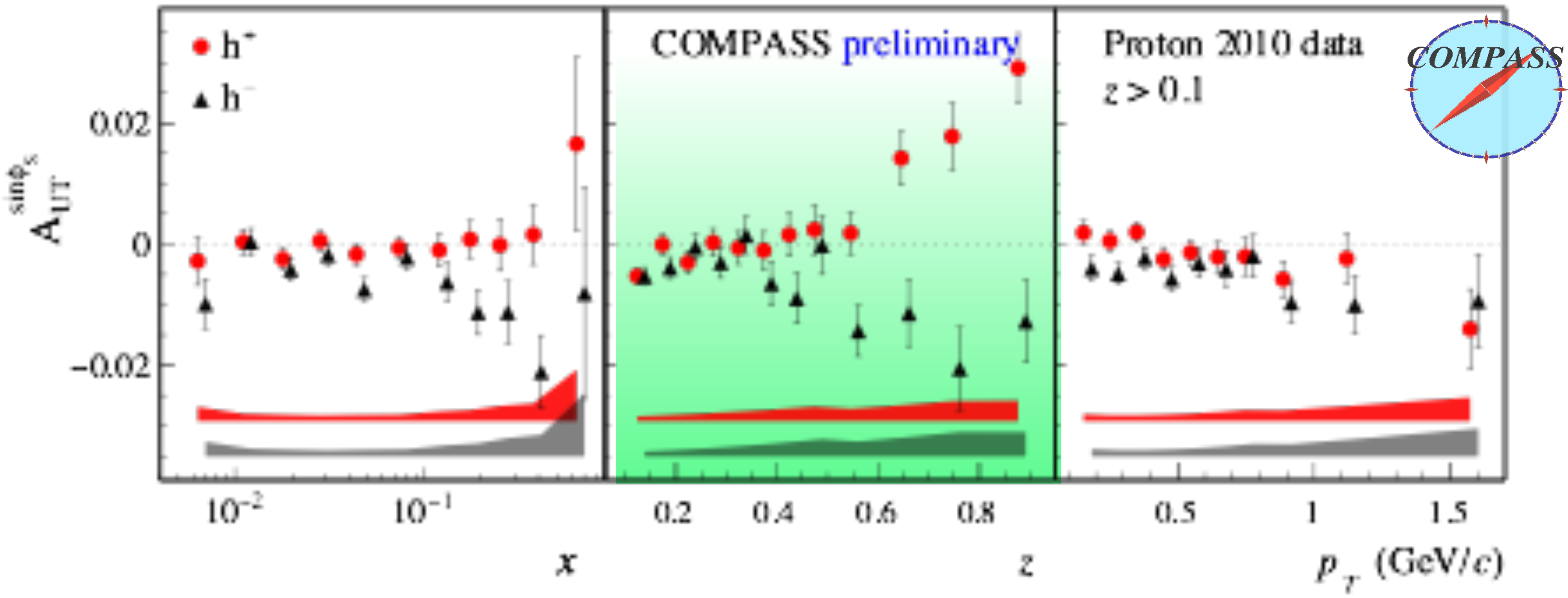




[A. Airapetian et al., JHEP12(2020)010]

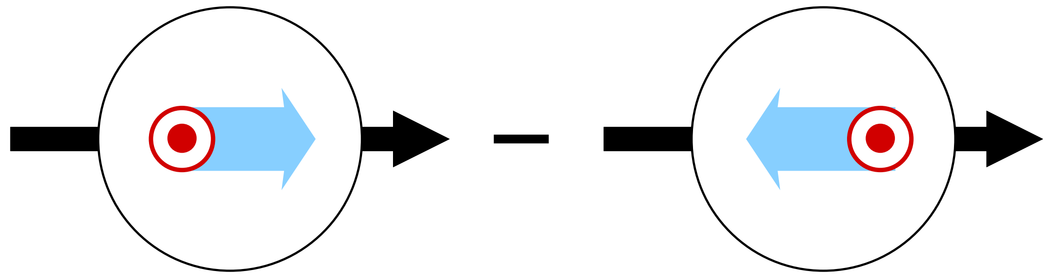


- clearly non-zero asymmetries with opposite sign for charged pions (Collins-like behavior)
- striking z dependence and in particular magnitude
- hint of Q suppression
- similar z behaviour seen at COMPASS

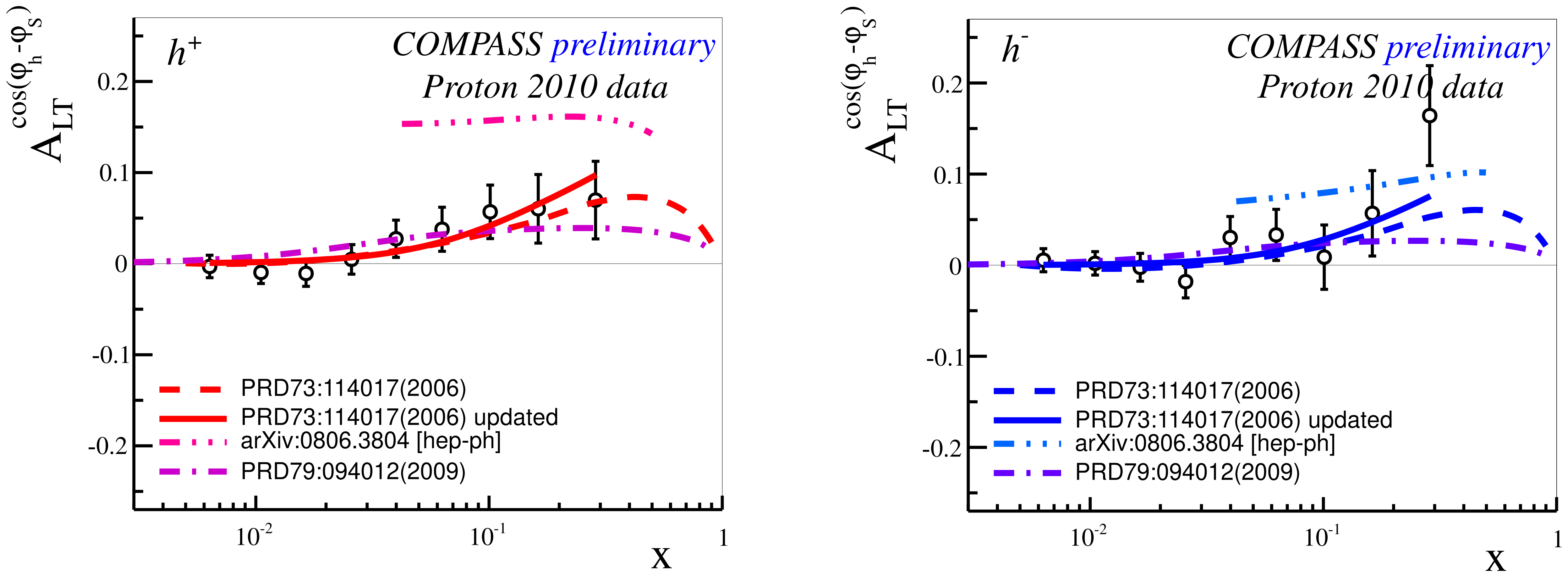


Worm-Gear II

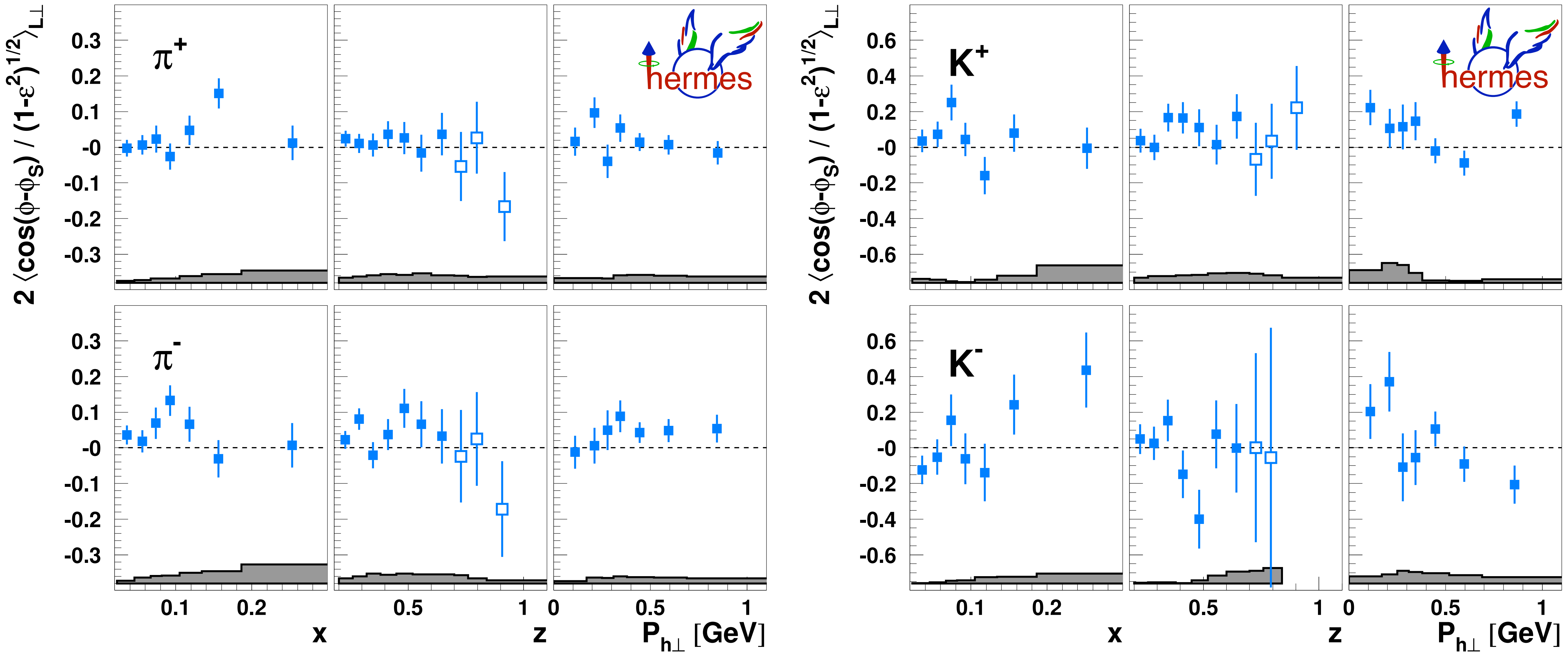
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$



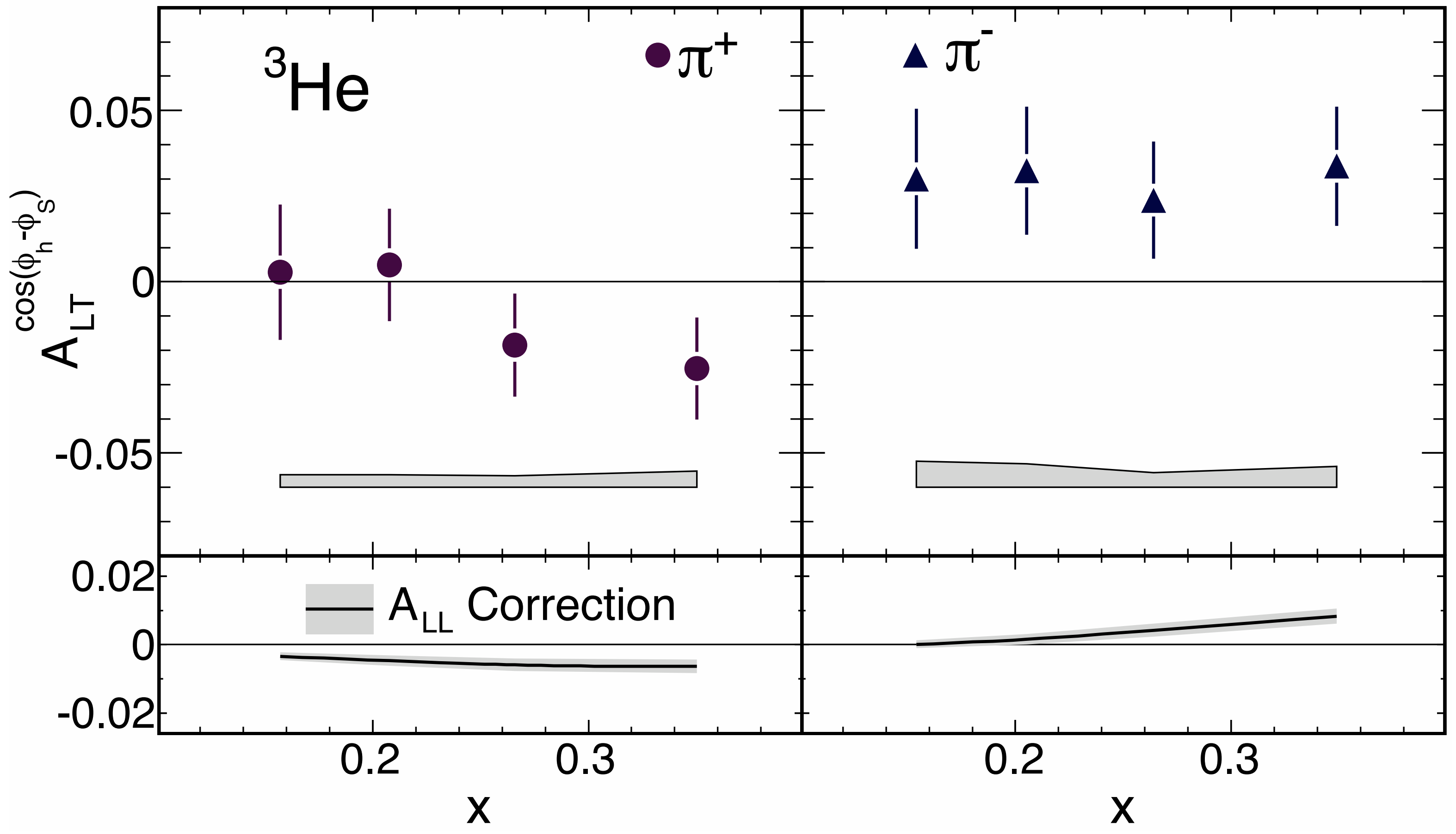
- chiral even, couples to D_1
- evidences from
  - ^{3}**He target at JLab**
  - **H target at COMPASS & HERMES**



[A. Airapetian et al., arXiv:2007.07755]



[PRL 108 (2012) 052001]



conclusions

- HERMES continues producing results
- latest three publications provide 3-dimensional presentations of longitudinal and transverse SSA and DSA
  - completes the TMD analysis of single-hadron production
  - by now, basically all but one (A_{UL}) asymmetries extracted in three or even four dimensions — a rich data set on transverse-momentum distributions
- multi-d analyses not only important to reduce experimental systematics but also to permit the isolation of the phase space of interest
- complementary to data from other facilities, and needed for evolution studies
- paves the way to high-statistics measurements of JLab12 and the future EIC

backup slides

double-spin asymmetry $A_{||}$

$$A_{\parallel}^{h} \equiv \frac{C_{\phi}^{h}}{f_{D}} \left[\frac{L_{\rightrightarrows} N^{h}_{\rightleftarrows} - L_{\rightleftarrows} N^{h}_{\rightrightarrows}}{L_{P,\rightrightarrows} N^{h}_{\rightleftarrows} + L_{P,\rightleftarrows} N^{h}_{\rightrightarrows}} \right]_{\mathrm{B}}$$

double-spin asymmetry $A_{||}$

azimuthal correction

$$A_{\parallel}^{h} \equiv \frac{C_{\phi}^{h}}{f_D} \left[\frac{L_{\rightrightarrows} N^{h}_{\rightleftarrows} - L_{\rightleftarrows} N^{h}_{\rightrightarrows}}{L_{P,\rightrightarrows} N^{h}_{\rightleftarrows} + L_{P,\rightleftarrows} N^{h}_{\rightrightarrows}} \right]_{\mathrm{B}}$$

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nucleon-in-nucleus **depolarization** factor (0.926 for deuteron due to D-state admixture)

double-spin asymmetry $A_{||}$

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luminosities

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nucleon-in-nucleus **depolarization** factor (0.926 for deuteron due to D-state admixture)

polarization-weighted luminosities

unfolded for QED radiation to Born level

double-spin asymmetry $A_{||}$

$$A_{\parallel}^{h} \equiv \frac{C_{\phi}^{h}}{f_D} \left[\frac{L_{\rightrightarrows} N_{\rightleftarrows}^{h} - L_{\rightleftarrows} N_{\rightrightarrows}^{h}}{L_{P,\rightrightarrows} N_{\rightleftarrows}^{h} + L_{P,\rightleftarrows} N_{\rightrightarrows}^{h}} \right]_{\mathrm{B}}$$

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- dominated by statistical uncertainties

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- dominated by statistical uncertainties
- main systematics arise from
  - polarization measurements [6.6% for hydrogen, 5.7% for deuterium)
  - azimuthal correction [O(few %)]

azimuthal-asymmetry corrections

measured

“polarized Cahn” effect etc.

$$\tilde{A}_{\|}^{h}(x, Q^2, z, P_{h\perp}) = \frac{\int \mathrm{d}\phi\ \sigma_{\|}^{h}\left(x, Q^2, z, P_{h\perp}, \phi\right)\ \xi(\phi)}{\int \mathrm{d}\phi\ \sigma_{UU}^{h}\left(x, Q^2, z, P_{h\perp}, \phi\right)\ \xi(\phi)}$$

Boer-Mulders and Cahn effects etc.

azimuthal
acceptance

- both numerator and in particular denominator ϕ dependent
  - in theory integrated out
  - in praxis, detector acceptance also ϕ dependent
  - convolution of physics & acceptance leads to bias in normalization of asymmetries

azimuthal-asymmetry corrections

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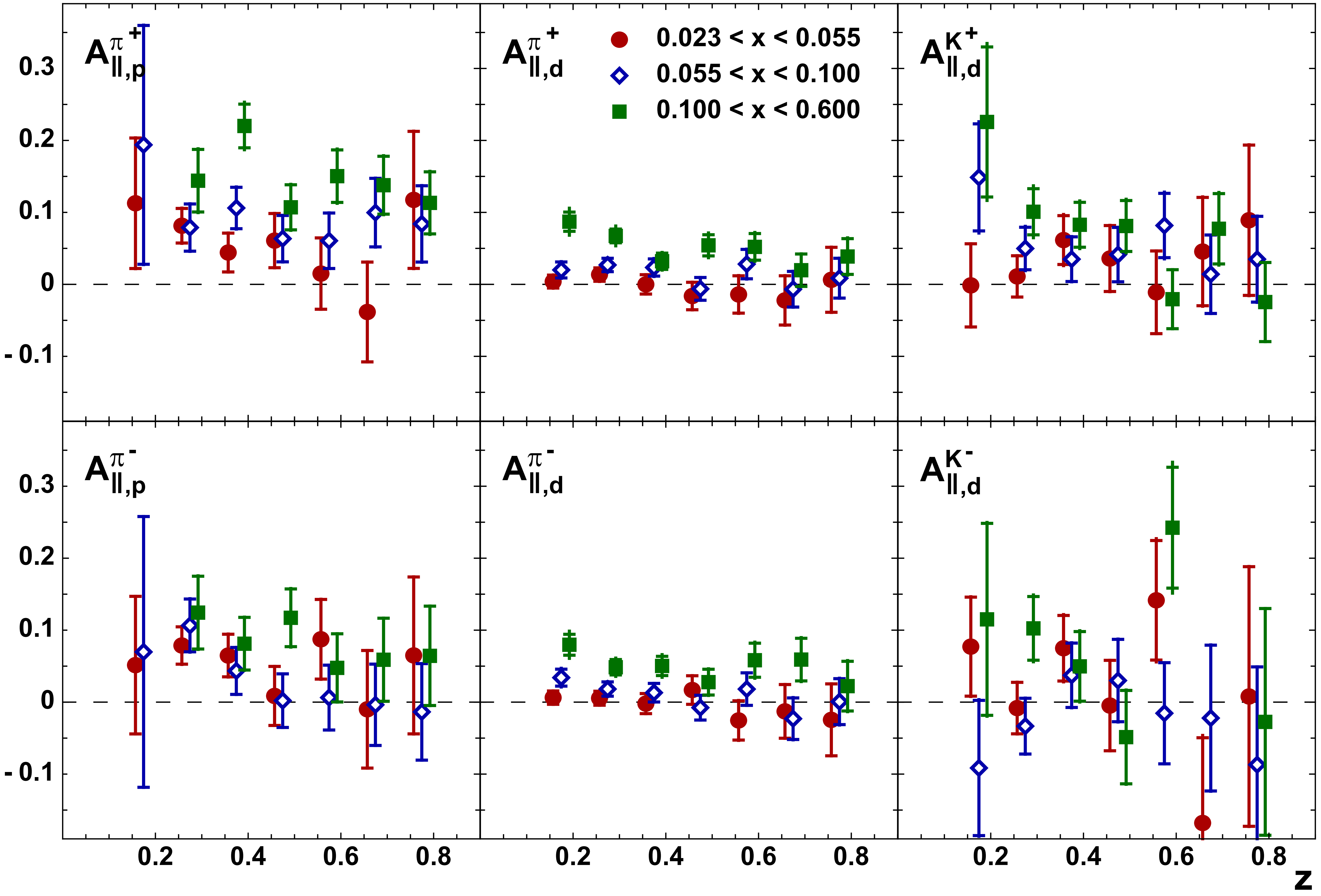
acceptance

- both numerator and in particular denominator ϕ dependent
  - in theory integrated out
  - in praxis, detector acceptance also ϕ dependent
  - convolution of physics & acceptance leads to bias in normalization of asymmetries
- implement data-driven model for azimuthal modulations [PRD 87 (2013) 012010] into MC ☛ extract correction factor & apply to data

z dependence of $A_{||}$ (three x ranges)

- in general, no strong z-dependence visible

[arXiv:1810.07054]



choice of fitting function

$$A_{LU}^{h} \simeq \sqrt{2\epsilon(1-\epsilon)}\,\frac{F_{LU}^{h,\sin\phi}}{F_{UU}^{h}}\,\sin\phi$$

$$A_{LU}^{h} \overset{\text{M.L. fit}}{\simeq} \sqrt{2\epsilon(1-\epsilon)}\,A_{LU}^{h,\sin\phi}\,\sin\phi$$

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- left one facilitates comparisons between experiments and simplifies kinematic dependences by removing known dependences
  - what about twist suppression and other kinematically suppressed contributions?

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- left one facilitates comparisons between experiments and simplifies kinematic dependences by removing known dependences
  - what about twist suppression and other kinematically suppressed contributions?

- asymmetry amplitudes extracted by minimizing

$$-\ln\mathbb{L} = -\sum_{i} w_i \,\ln\left[1 + P_{B,i}\,\sqrt{2\epsilon_i(1-\epsilon_i)}\,A_{LU}^{h,\sin(\phi)}\,\sin(\phi_i)\right]$$

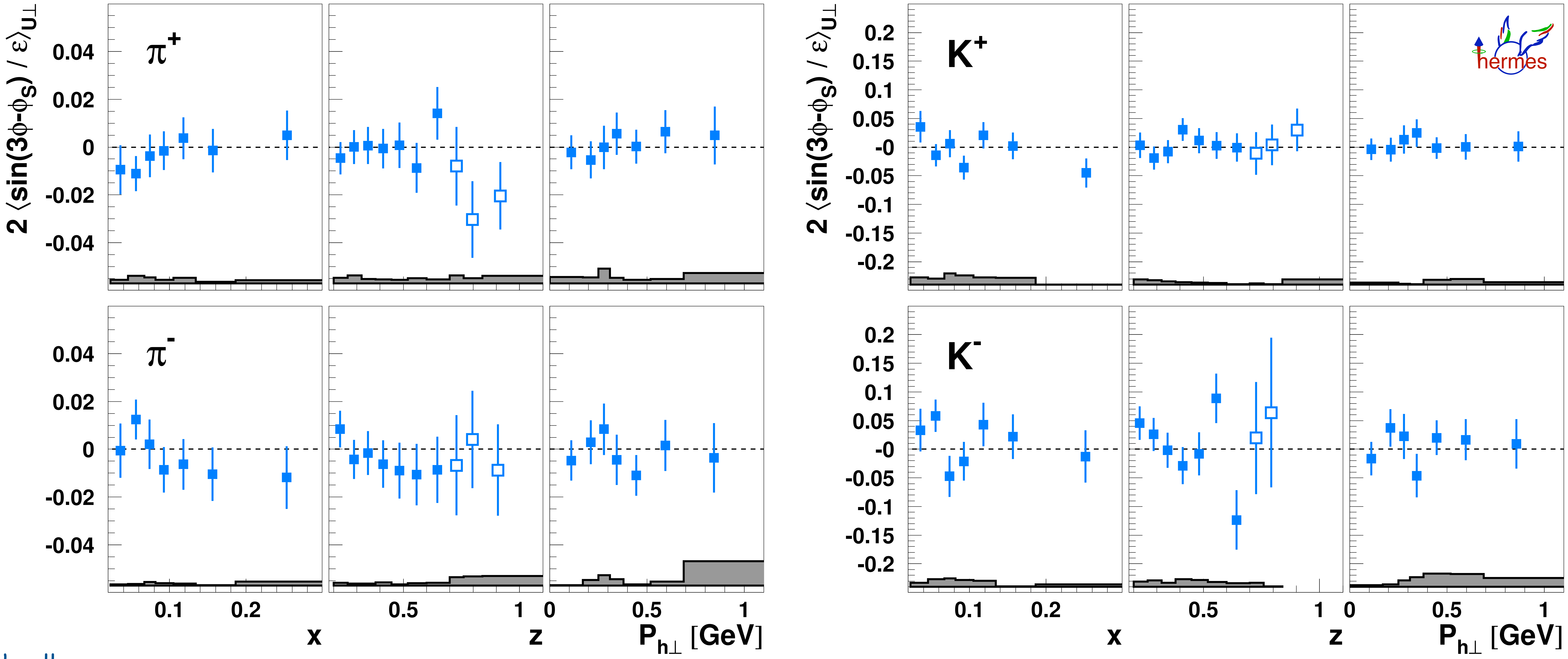
where w_i is event weight from hadron-ID, charge-symmetric BG etc.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Pretzelosity

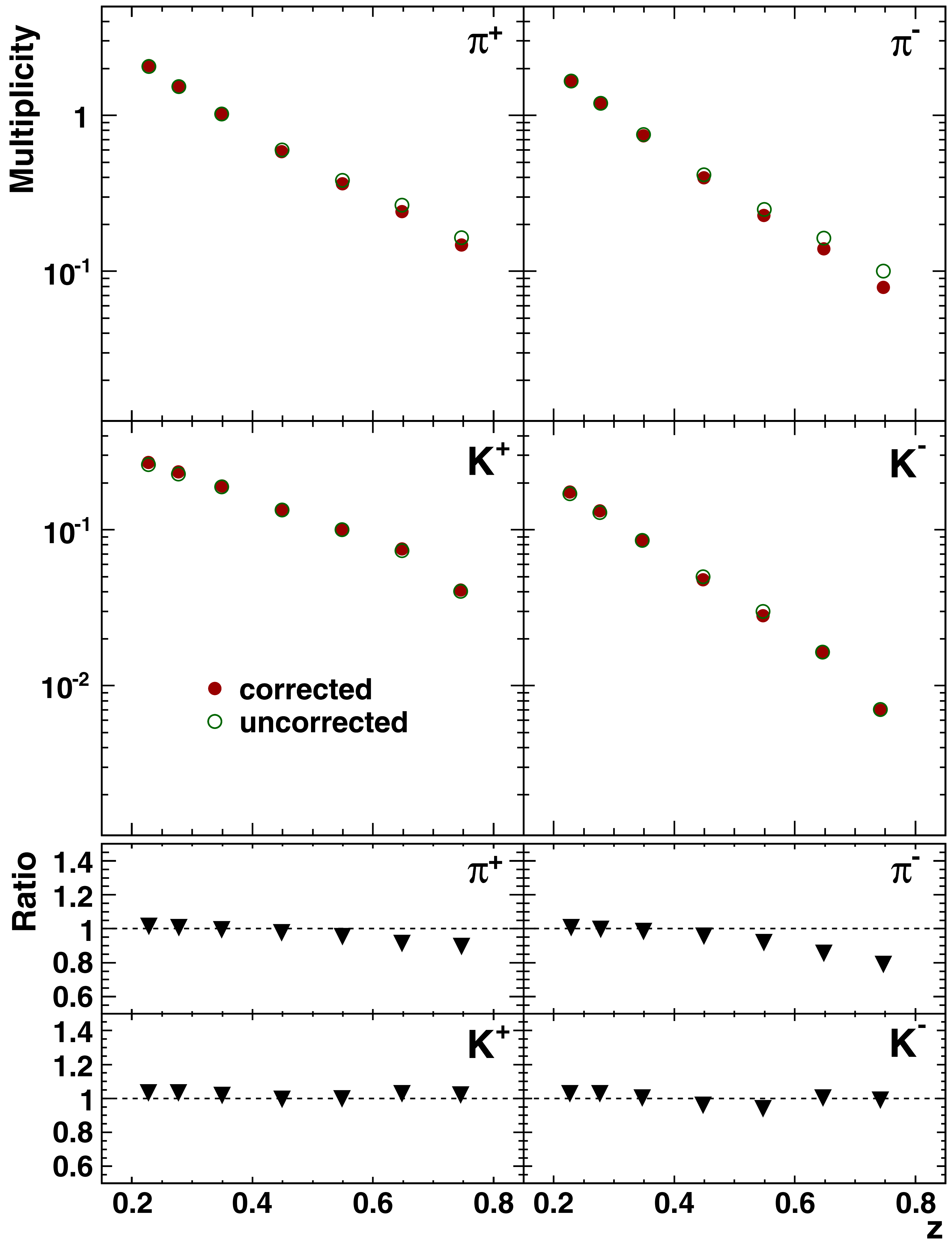
- chiral-odd ➥ needs Collins FF (or similar)
- ^{1}H, ^{2}H & ^{3}He data consistently small
- cancelations? pretzelosity=zero? or just the additional suppression by two powers of $P_{h\perp}$

[A. Airapetian et al., JHEP12(2020)010]



Subtracting mesons from decay of excl. VM production

PHYSICAL REVIEW D **87,** 074029 (2013)



- "corrected" should better read "VM subtracted"

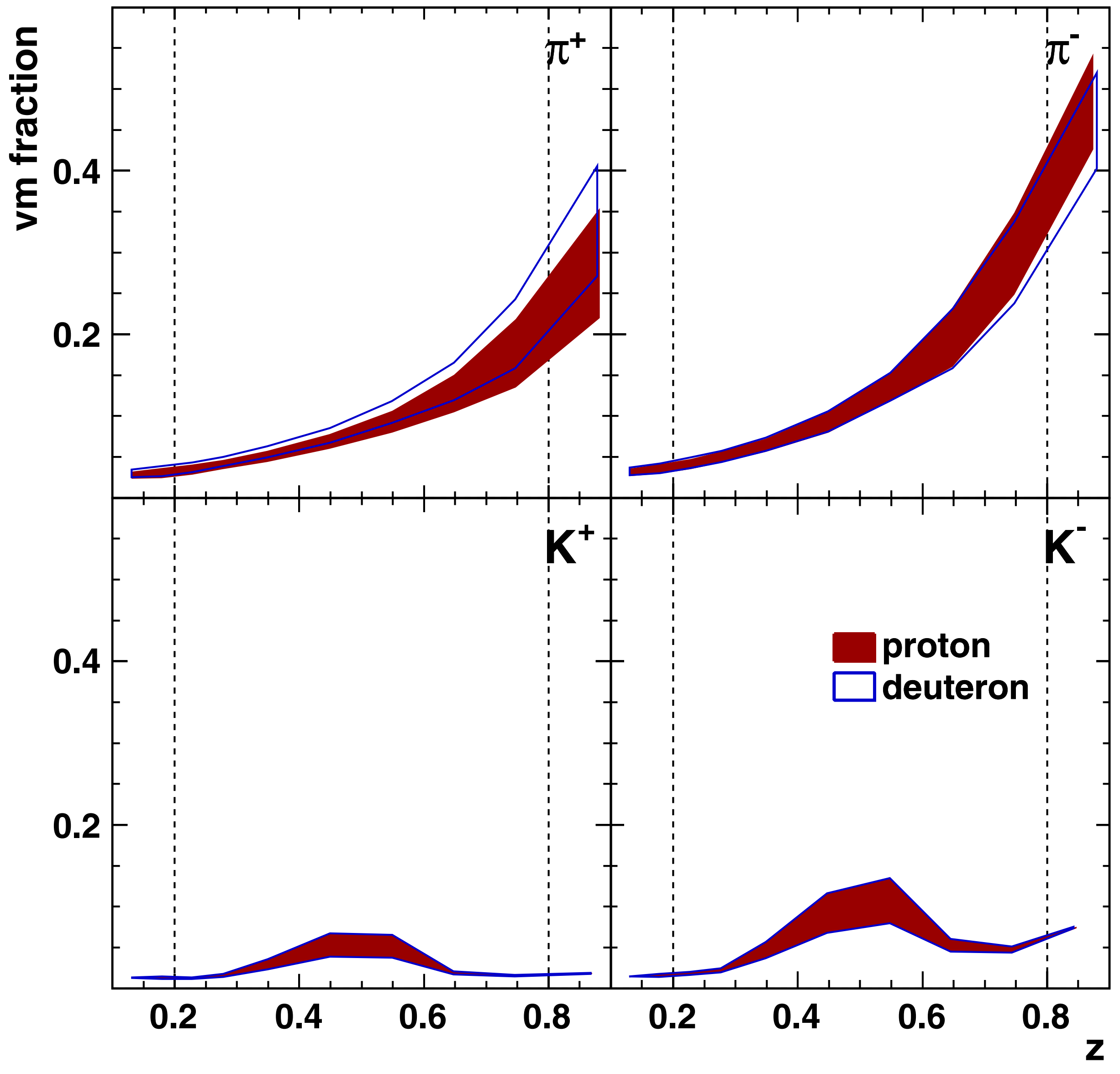
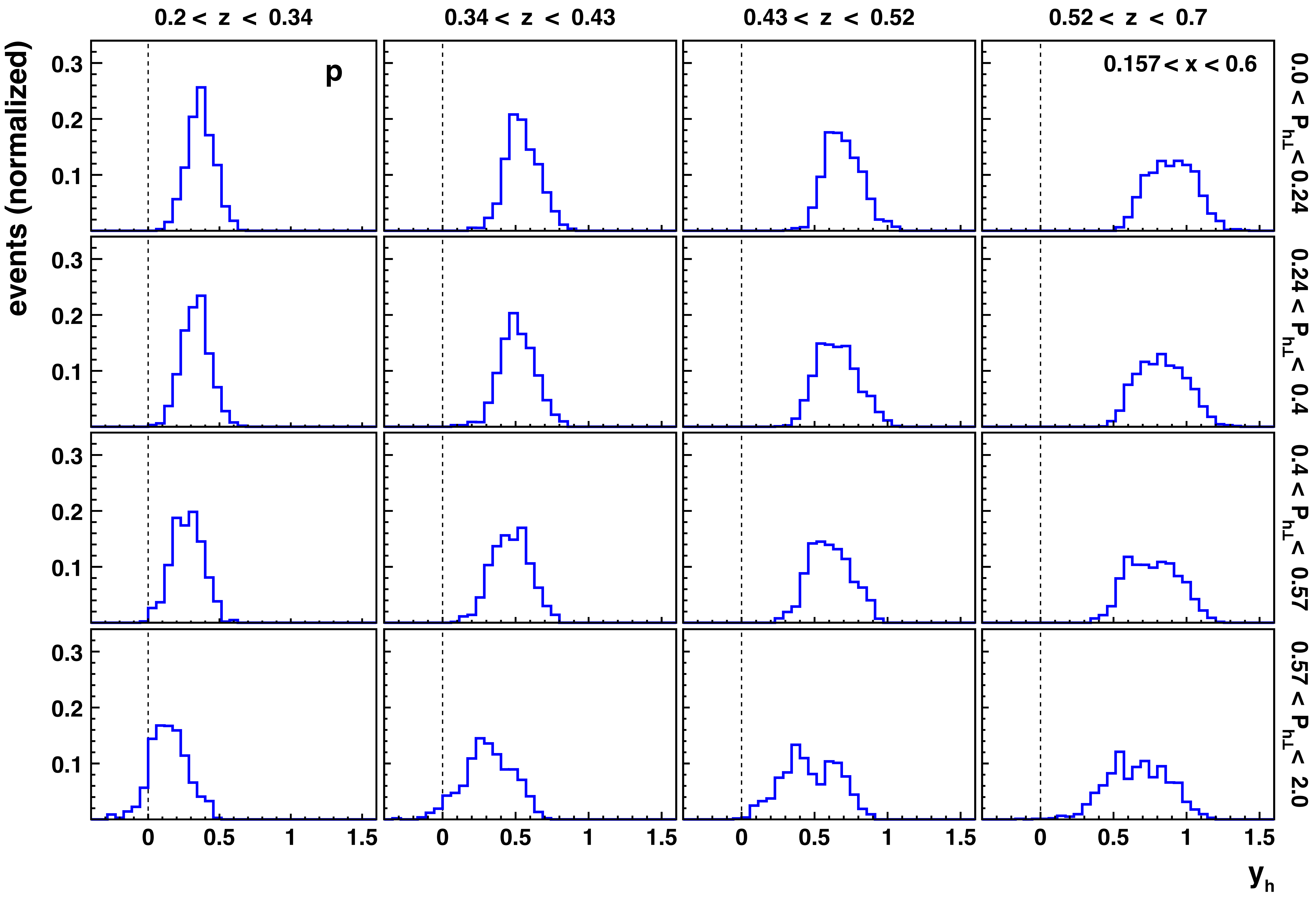


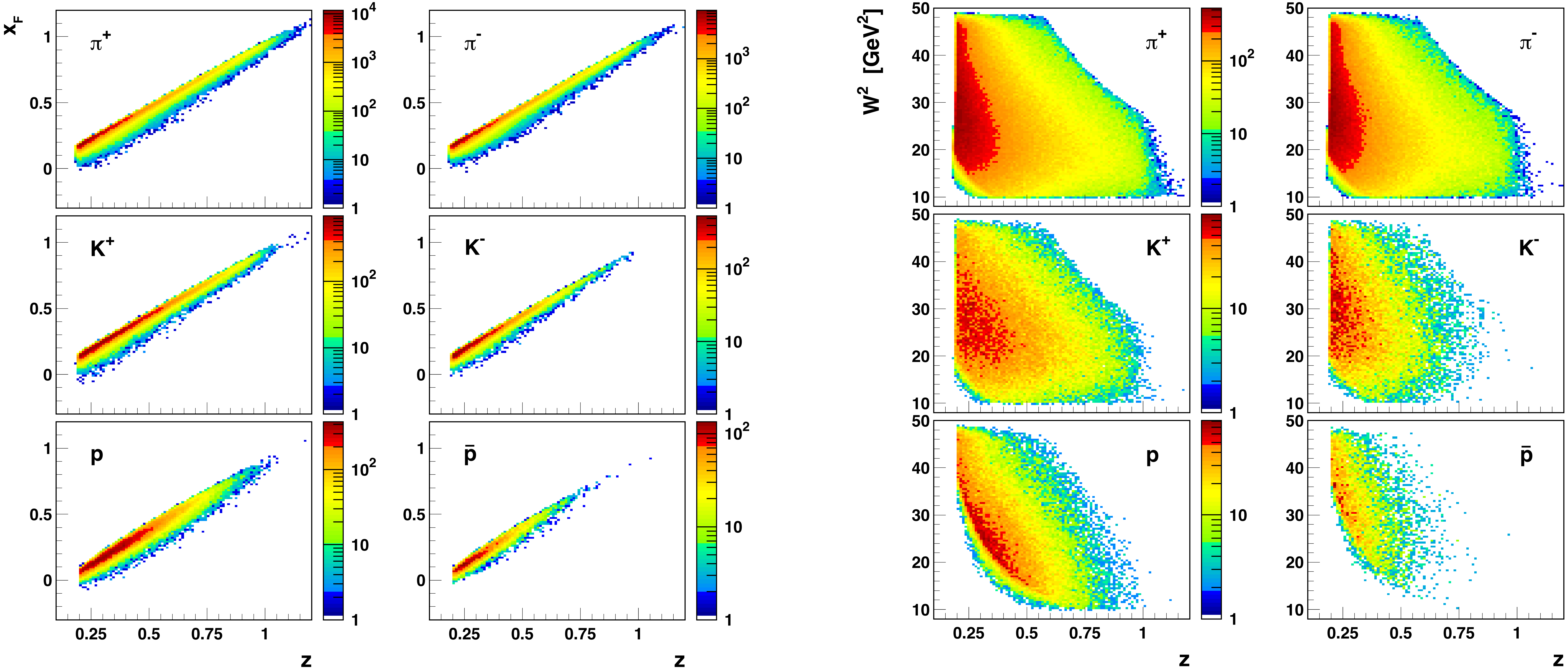
FIG. 3 (color online). Fraction of mesons generated by the decay of exclusive vector mesons as a function of z, from PYTHIA (see text). The widths of the bands indicate the uncertainty in the corresponding fractions. The vertical dashed lines are the limits in z used in the multiplicity extractions.

hadron production at HERMES



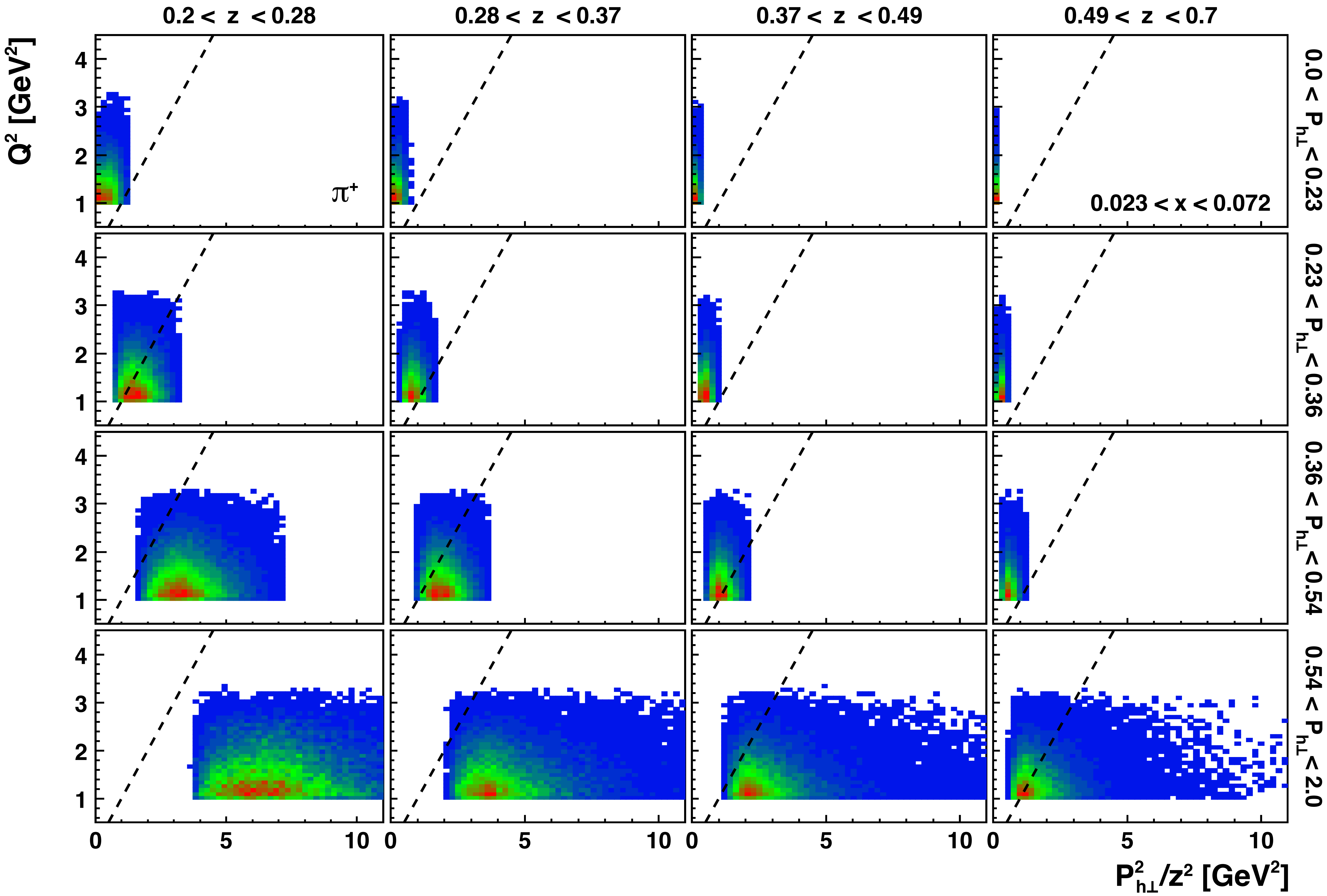
- forward-acceptance favors current fragmentation
- backward rapidity populates large-$P_{h\perp}$ region [as expected]
- rapidity distributions available for all kinematic bins (e.g., highest-x bin protons)

current vs. target fragmentation



TMD factorization: a 2-scale problem

lowest x bin

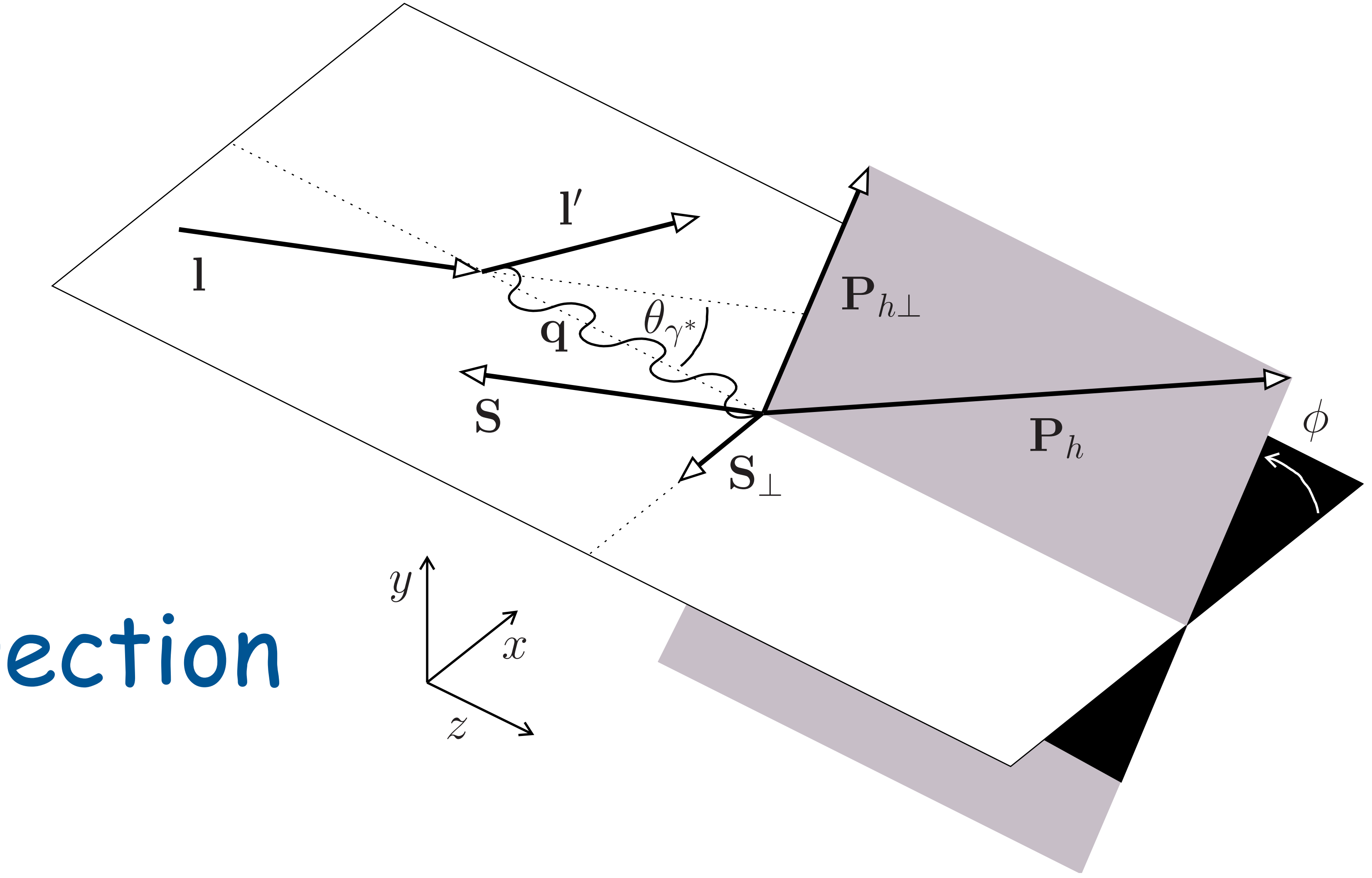


--- $Q^2 = P^2_{h\perp}/z^2$

all other x-bins included in the Supplemental Material of JHEP12(2020)010

Mixing of target polarizations

- theory done w.r.t. virtual-photon direction
- experiments use targets polarized w.r.t. lepton-beam direction



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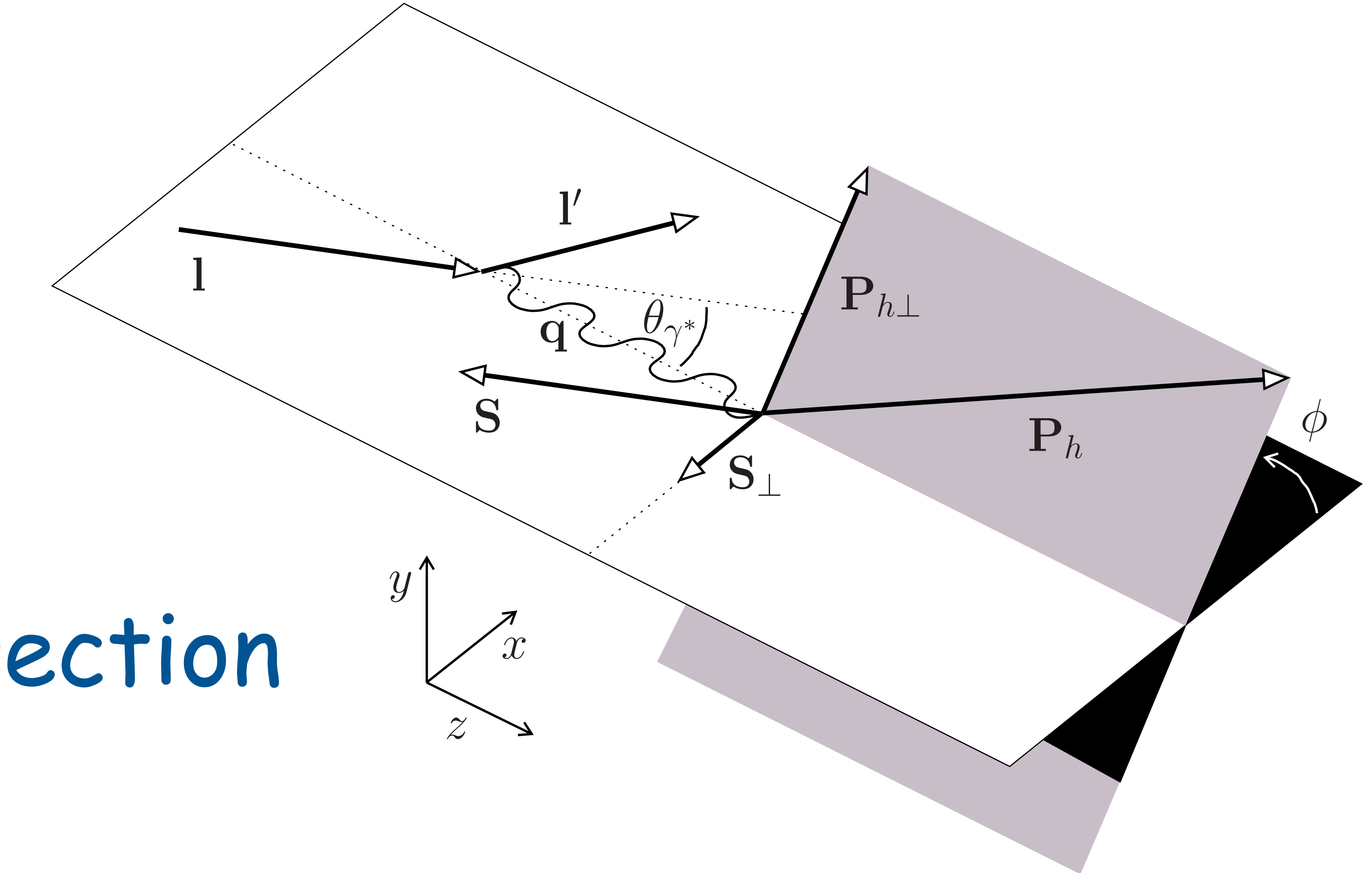

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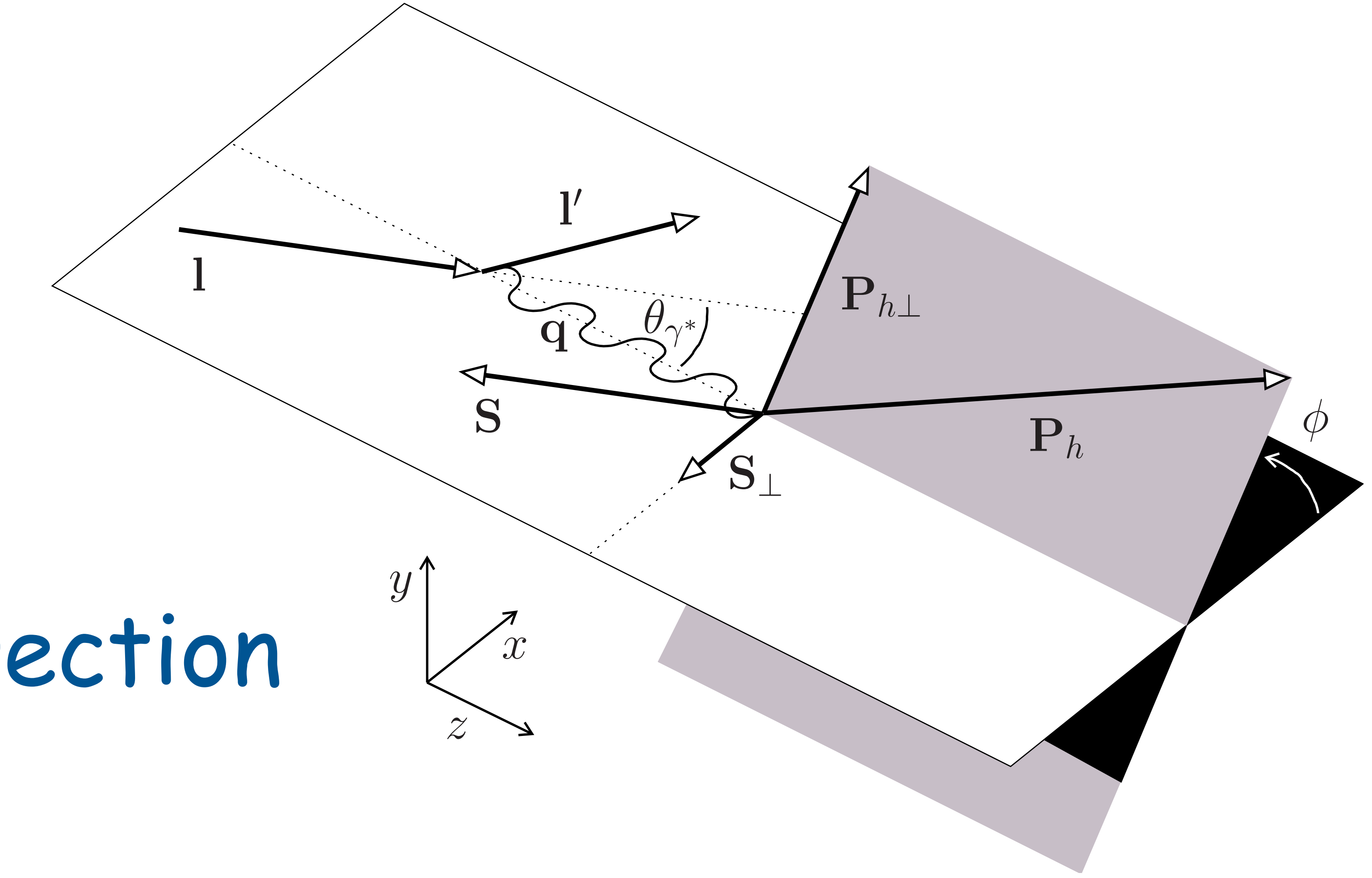
➡ mixing of longitudinal and transverse polarization effects [Diehl & Sapeta, EPJ C 41 (2005) 515], e.g.,

$$\begin{pmatrix} \left\langle \sin\phi \right\rangle_{UL}^{l} \\ \left\langle \sin(\phi-\phi_S) \right\rangle_{UT}^{l} \\ \left\langle \sin(\phi+\phi_S) \right\rangle_{UT}^{l} \end{pmatrix} = \begin{pmatrix} \cos\theta_{\gamma^*} & -\sin\theta_{\gamma^*} & -\sin\theta_{\gamma^*} \\ \frac{1}{2}\sin\theta_{\gamma^*} & \cos\theta_{\gamma^*} & 0 \\ \frac{1}{2}\sin\theta_{\gamma^*} & 0 & \cos\theta_{\gamma^*} \end{pmatrix} \begin{pmatrix} \left\langle \sin\phi \right\rangle_{UL}^{q} \\ \left\langle \sin(\phi-\phi_S) \right\rangle_{UT} \\ \left\langle \sin(\phi+\phi_S) \right\rangle_{UT} \end{pmatrix}$$

Mixing of target polarizations



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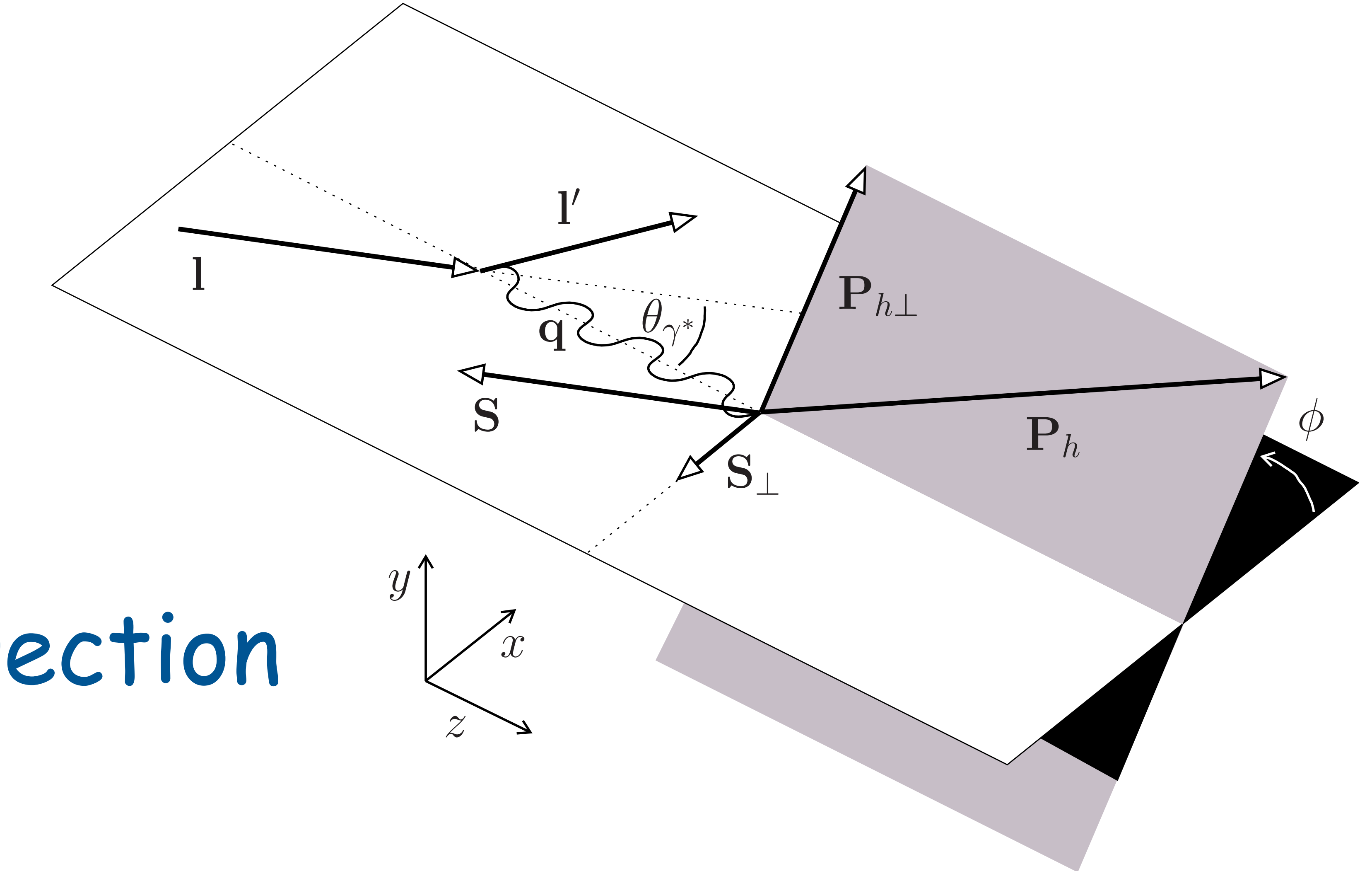
➡ need data on same target for both polarization orientations!

Mixing of target polarizations



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