

Transverse Spin Physics at HERMES

G. Schnell

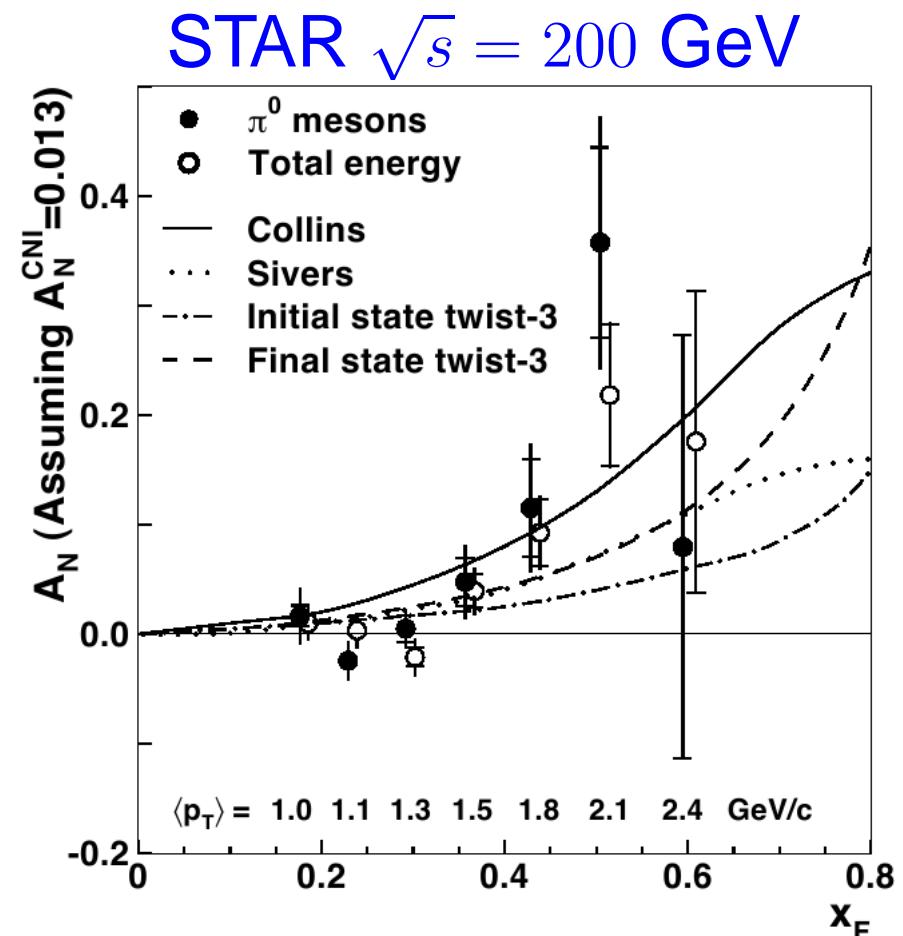
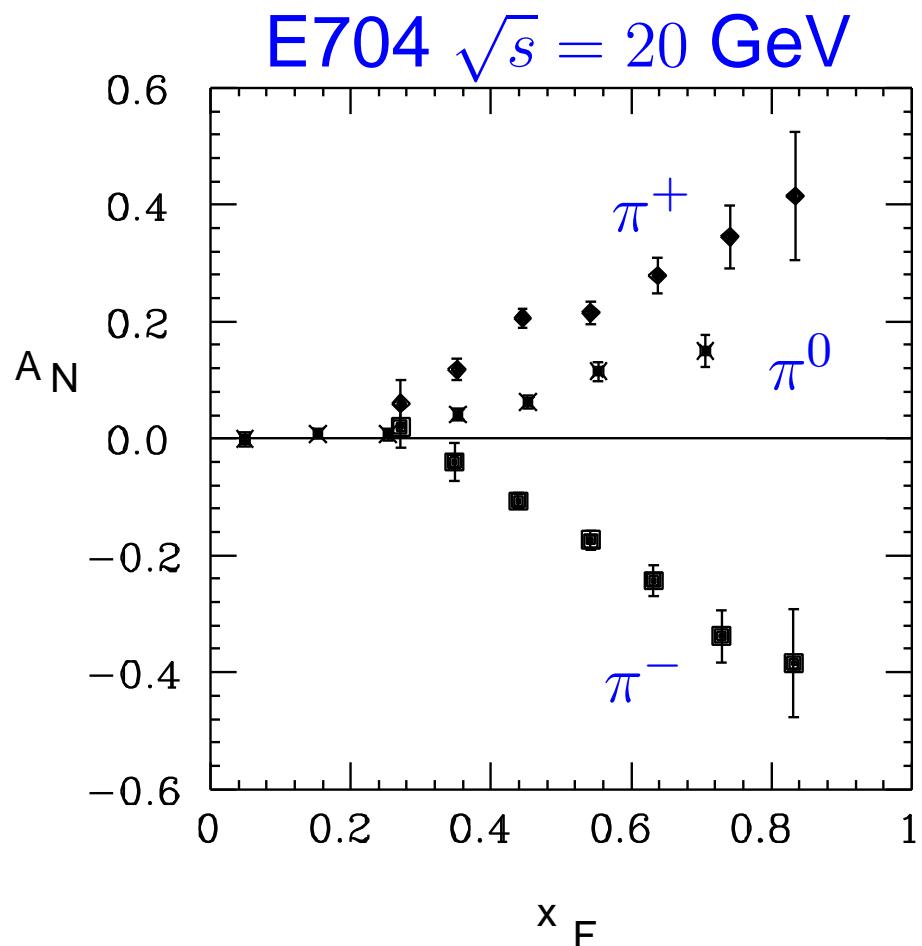
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Transverse Single-Spin Asymmetries in pp Collisions

$$p^\uparrow p \rightarrow \pi X$$



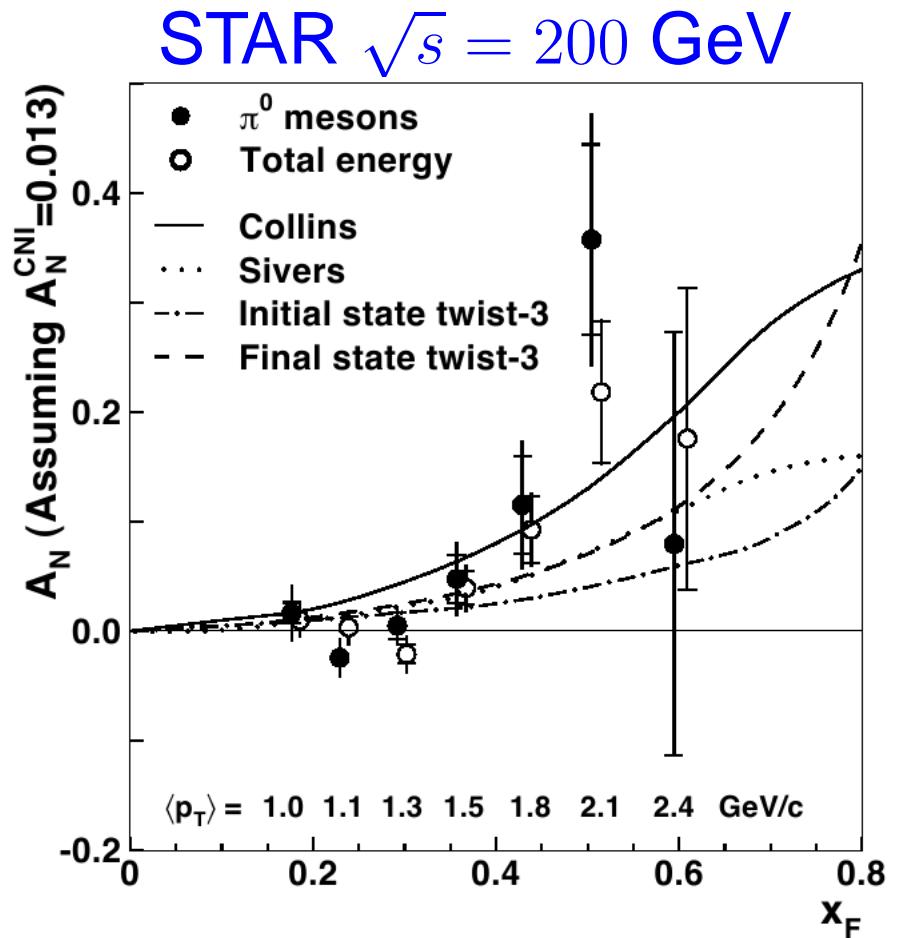
left-right-asymmetry w.r.t. incoming proton's spin

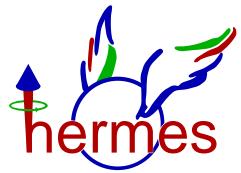
Transverse Single-Spin Asymmetries in pp Collisions

$$p^\uparrow p \rightarrow \pi X$$

SSAs persist even at high energies! (despite being suppressed in pQCD)

- asymmetry in quark fragmentation (Collins)
- asymmetry in quark distribution (Sivers)
- subleading-twist effects

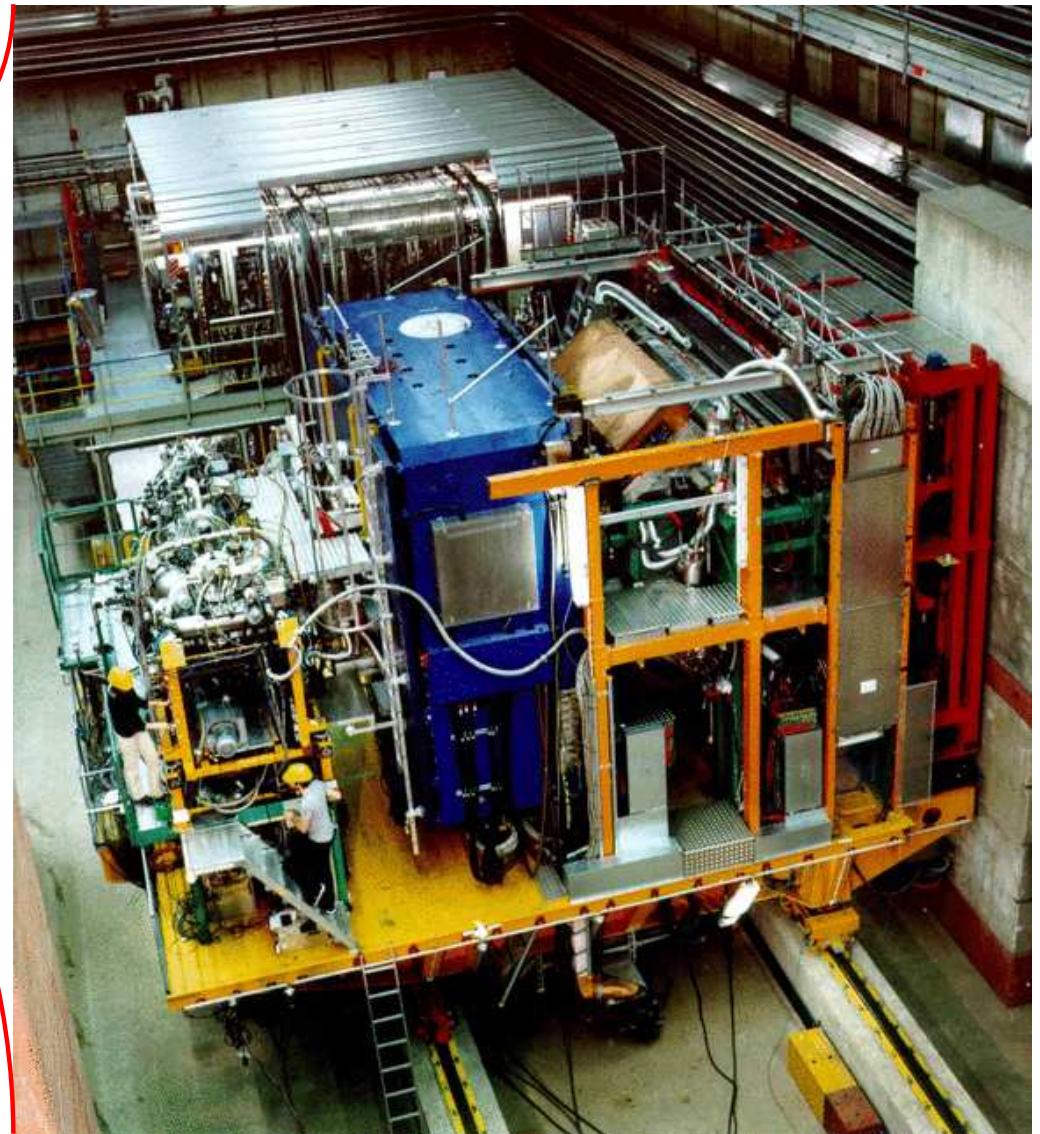




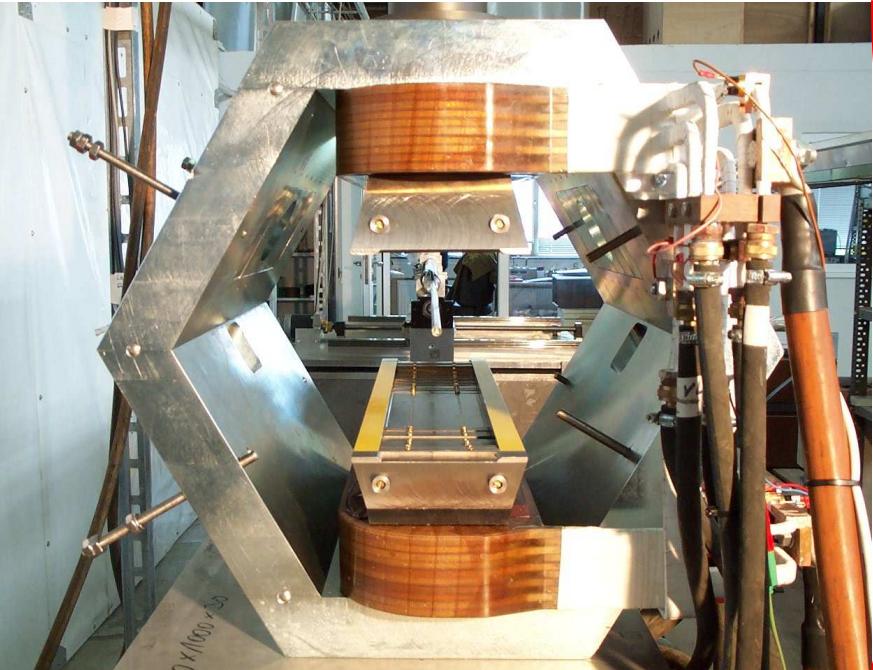
HERa MEasurement of Spin

a DIS experiment to study the nucleon structure

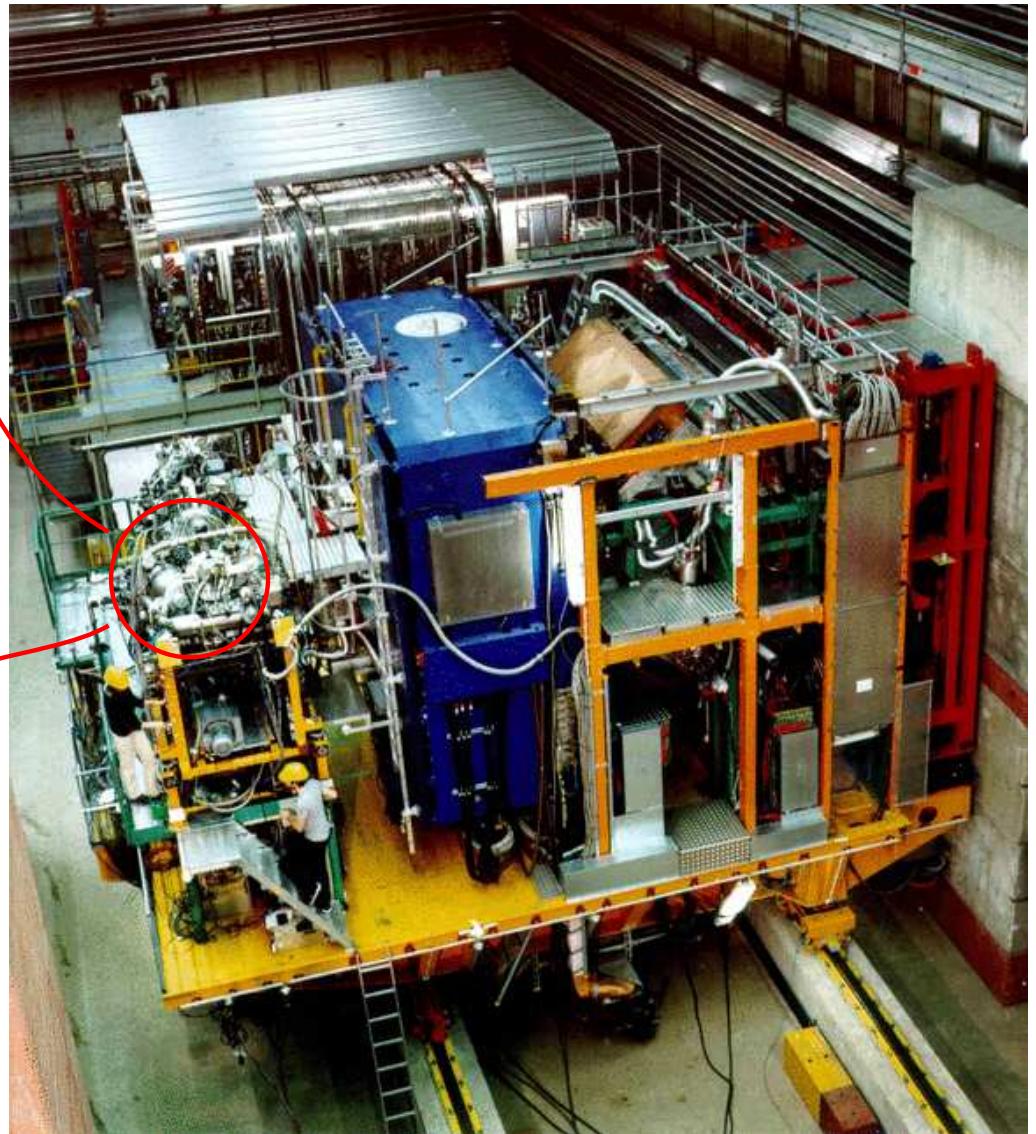
27.5 GeV e^+ / e^- beam of HERA

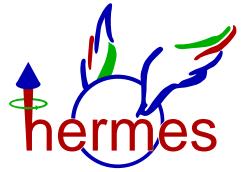


- forward-acceptance spectrometer
- ⇒ $40\text{mrad} < \theta < 220\text{mrad}$
- high lepton ID efficiency and purity
- excellent hadron ID thanks to dual-radiator RICH



- atomic beam source
- ⇒ pure gas target, no dilution
- transversely pol. hydrogen
- polarization $\sim 75\%$
- 90s flipping time ⇒ small systematics





Transverse Spin of the Nucleon

Quark Distribution Functions

$$f_1^q = \text{circle with dot}$$



$$g_1^q = \text{two circles with dots and arrows}$$



$$h_1^q = \text{two circles with dots and arrows}$$



**Unpolarized quarks
and nucleons**

$f_1^q(x)$: spin averaged
(well known)

⇒ Vector Charge

$$\langle PS | \bar{\Psi} \gamma^\mu \Psi | PS \rangle = \int dx (f_1^q(x) - f_1^{\bar{q}}(x))$$

**Longitudinally
polarized quarks
and nucleons**

$g_1^q(x)$: helicity
difference (known)

⇒ Axial Charge

$$\langle PS | \bar{\Psi} \gamma^\mu \gamma_5 \Psi | PS \rangle = \int dx (g_1^q(x) + g_1^{\bar{q}}(x))$$

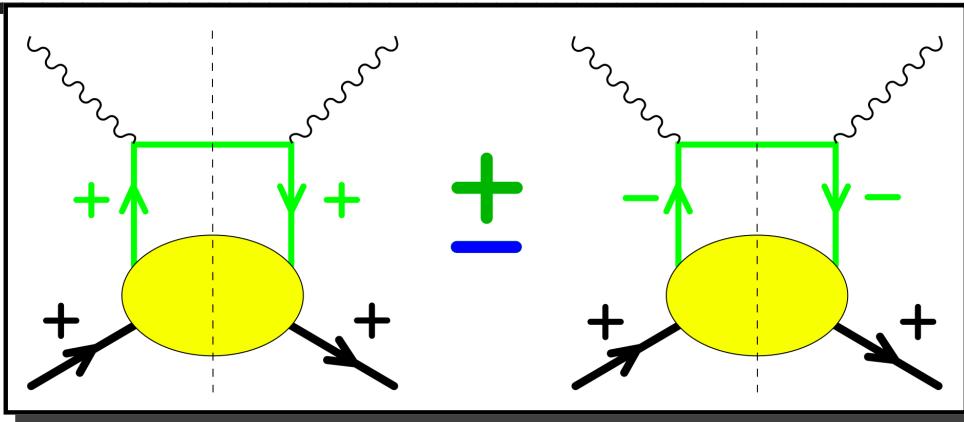
**Transversely
polarized quarks
and nucleons**

$h_1^q(x)$: transversity
(hardly known!)

⇒ Tensor Charge

$$\langle PS | \bar{\Psi} \sigma^{\mu\nu} \gamma_5 \Psi | PS \rangle = \int dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

Quark Distribution Functions



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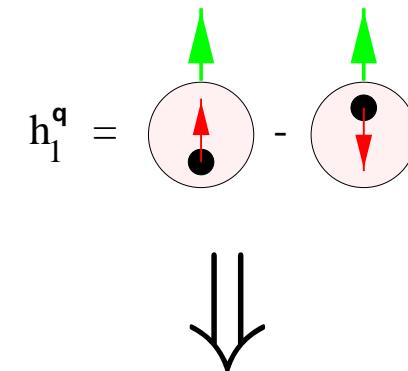
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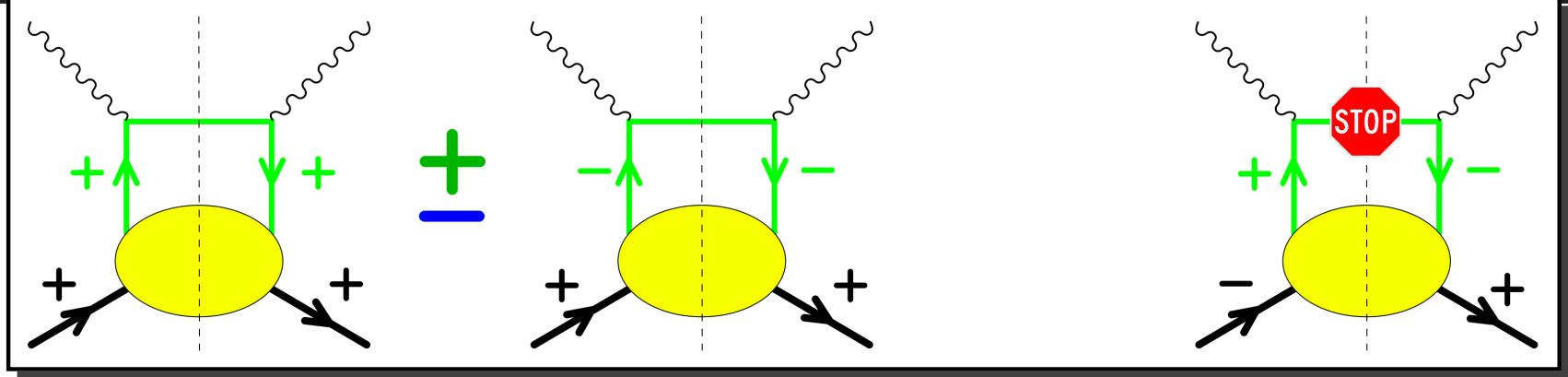
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Transverse
polarized q
and nucleons

$h_1^q(x)$: transversity
(hardly known!)

⇒ Transversity Charge

$$\langle PS | \bar{\Psi} \gamma^\mu \gamma_5 \gamma^\nu \gamma_5 \Psi | PS \rangle = \int dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

CHIRAL ODD!

- transverse spin eigenstates related to helicity eigenstates via $|\perp\tau\rangle = \frac{1}{2}(|+\rangle \pm i|-\rangle) \Rightarrow$ transversity ($\langle\perp|\hat{O}|\perp\rangle - \langle\tau|\hat{O}|\tau\rangle$)
flips helicity of quark and nucleon $\Rightarrow h_1^q$ chiral odd
- No Access In Inclusive DIS!

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- no “gluon transversity”
 \Rightarrow different Q^2 -evolution than for $f_1^q(x)$ and $g_1^q(x)$

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- positivity bounds: $|h_1^q(x)| \leq f_1^q(x)$
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- first moment \rightarrow tensor charge calculable in **lattice QCD**



Transversity Measurements

How can one measure transversity?

Need another chiral-odd object!

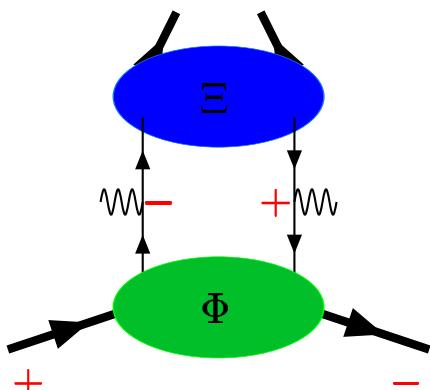
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$$\sigma^{ep \rightarrow ehX} = \sum_q h_1^q \otimes \sigma^{eq \rightarrow eq} \otimes FF^{q \rightarrow h}$$



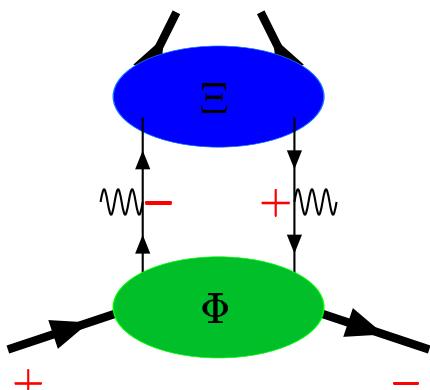
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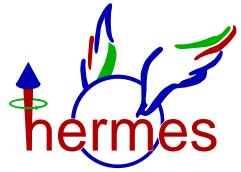


chiral-odd
DF

chiral-odd
FF

CHIRAL EVEN

→ chiral-odd FF as a **polarimeter** of transv. quark polarization



Semi-Inclusive 2-Hadron Production

2-Hadron Fragmentation in Semi-Inclusive DIS

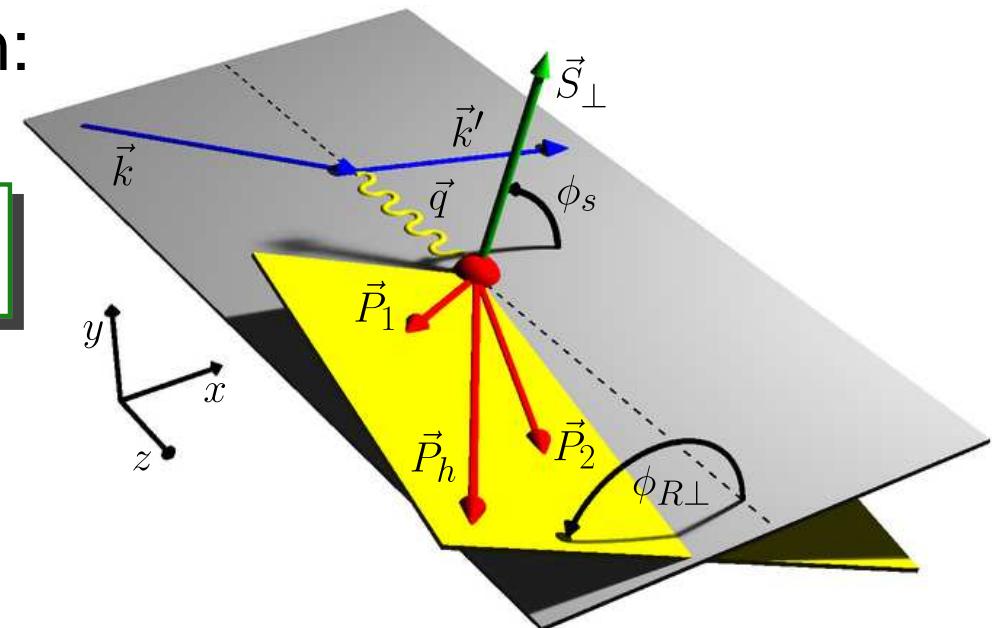
polarized 2-hadron cross section:

(Unpolarized beam, Transversely pol. target)

$$\sigma_{UT} \sim \sin(\phi_{R\perp} + \phi_s) \sum e_q^2 h_1^q H_1^\triangleleft$$

$$H_1^\triangleleft = H_1^\triangleleft(z, \zeta, M_{\pi\pi}^2)$$

$$(\zeta \sim z_1/(z_1 + z_2))$$



2-Hadron Fragmentation in Semi-Inclusive DIS

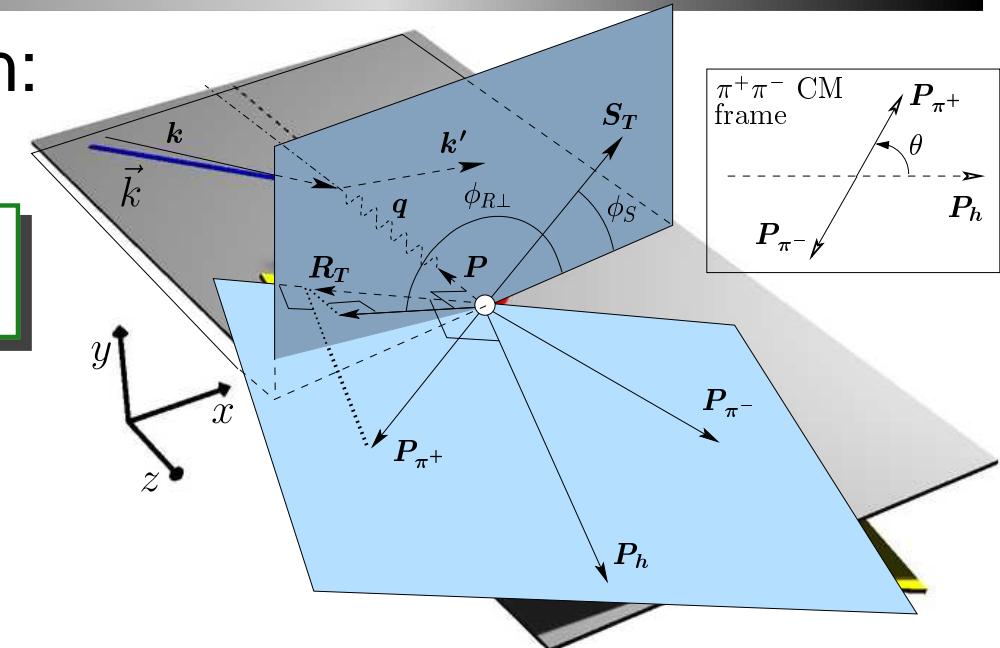
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- only *relative* momentum of hadron pair relevant
- ⇒ integration over transverse momentum of hadron pair simplifies factorization and Q^2 evolution
- however, cross section becomes more complex (differential in 9 variables)

2-Hadron Fragmentation in Semi-Inclusive DIS

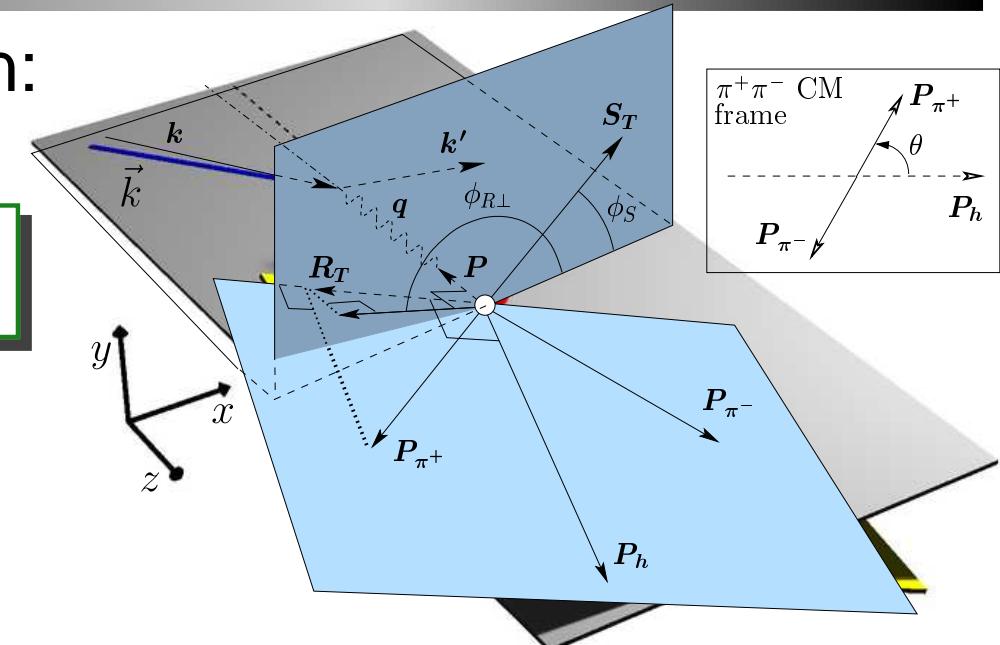
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difficult to measure directly $\sigma_{UT} \equiv \sigma_{U\uparrow} - \sigma_{U\downarrow}$

⇒ measure **cross section asymmetry** A_{UT} :

$$A_{UT} \equiv \frac{1}{\langle |S_T| \rangle} \frac{N_{2\pi}^\uparrow(\phi_{R\perp}, \phi_S, \theta) - N_{2\pi}^\downarrow(\phi_{R\perp}, \phi_S, \theta)}{N_{2\pi}^\uparrow(\phi_{R\perp}, \phi_S, \theta) + N_{2\pi}^\downarrow(\phi_{R\perp}, \phi_S, \theta)}$$

$\uparrow\downarrow \dots$ target spin states
 $N_{2\pi} \dots$ (norm.) 2π yield
 $S_T \dots$ target polarization

2-Hadron Fragmentation in Semi-Inclusive DIS

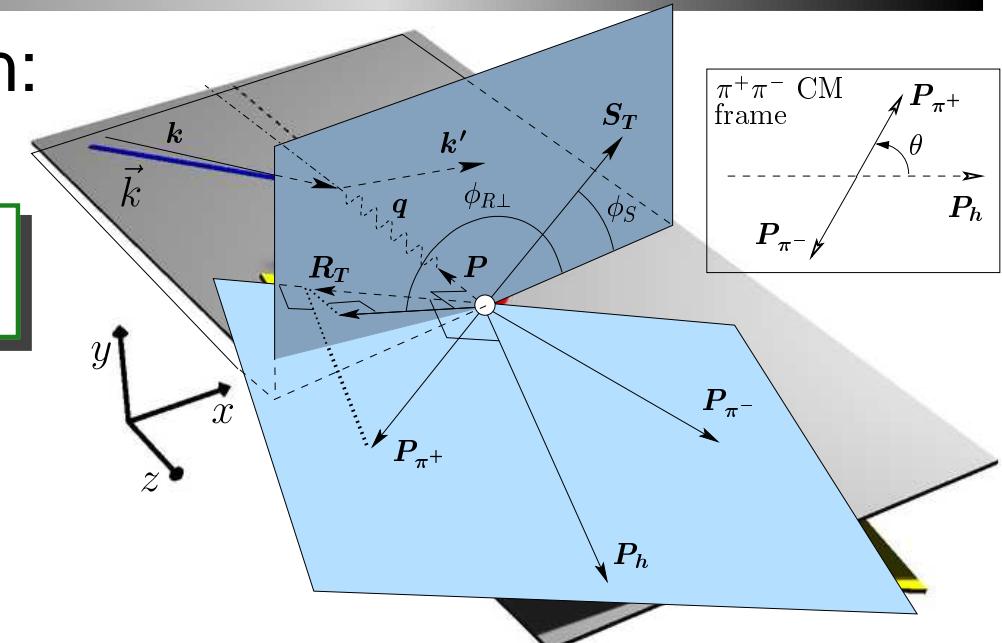
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But: asymmetry involves unknown unpolarized 2π cross section

Interference Fragmentation – Models

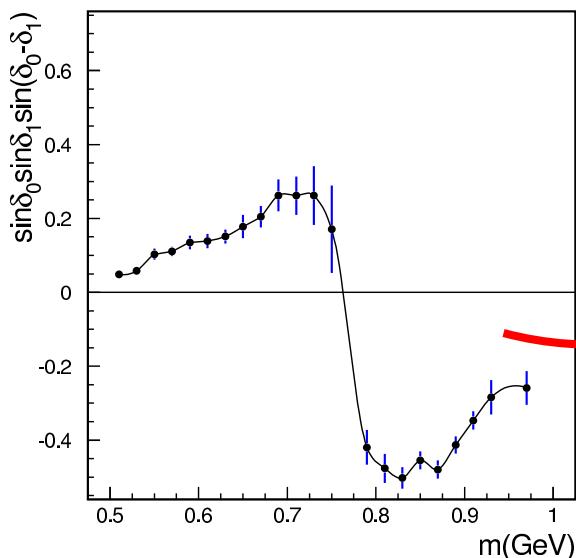
$$A_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sin \theta h_1 H_1^\triangleleft$$

Expansion of H_1^\triangleleft in Legendre moments:

$$H_1^\triangleleft(z, \cos \theta, M_{\pi\pi}^2) = H_1^{\triangleleft, sp}(z, M_{\pi\pi}^2) + \cos \theta H_1^{\triangleleft, pp}(z, M_{\pi\pi}^2)$$

describe interference between 2 pion pairs
coming from different production channels.

about $H_1^{\triangleleft, sp}$:



Jaffe et al. [[hep-ph/9709322](#)]:

$$\begin{aligned} H_1^{\triangleleft, sp}(z, M_{\pi\pi}^2) &= \sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1) H_1^{\triangleleft, sp'}(z) \\ &\quad \delta_0 (\delta_1) \rightarrow \text{S(P)-wave phase shifts} \\ &= \mathcal{P}(M_{\pi\pi}^2) H_1^{\triangleleft, sp'}(z) \end{aligned}$$

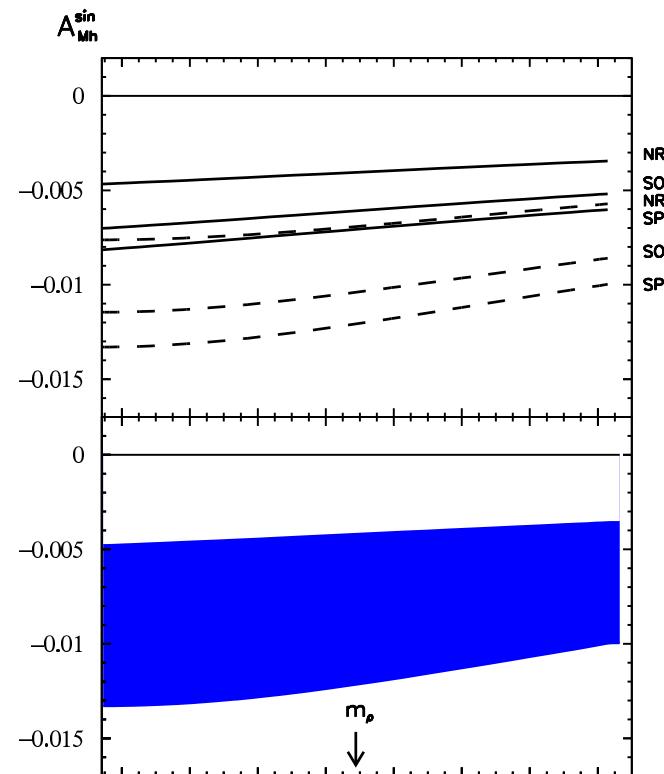
$\Rightarrow A_{UT}$ might depend strongly on $M_{\pi\pi}$

Interference Fragmentation – Models

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Expansion of H_1^\triangleleft in Legendre moments:

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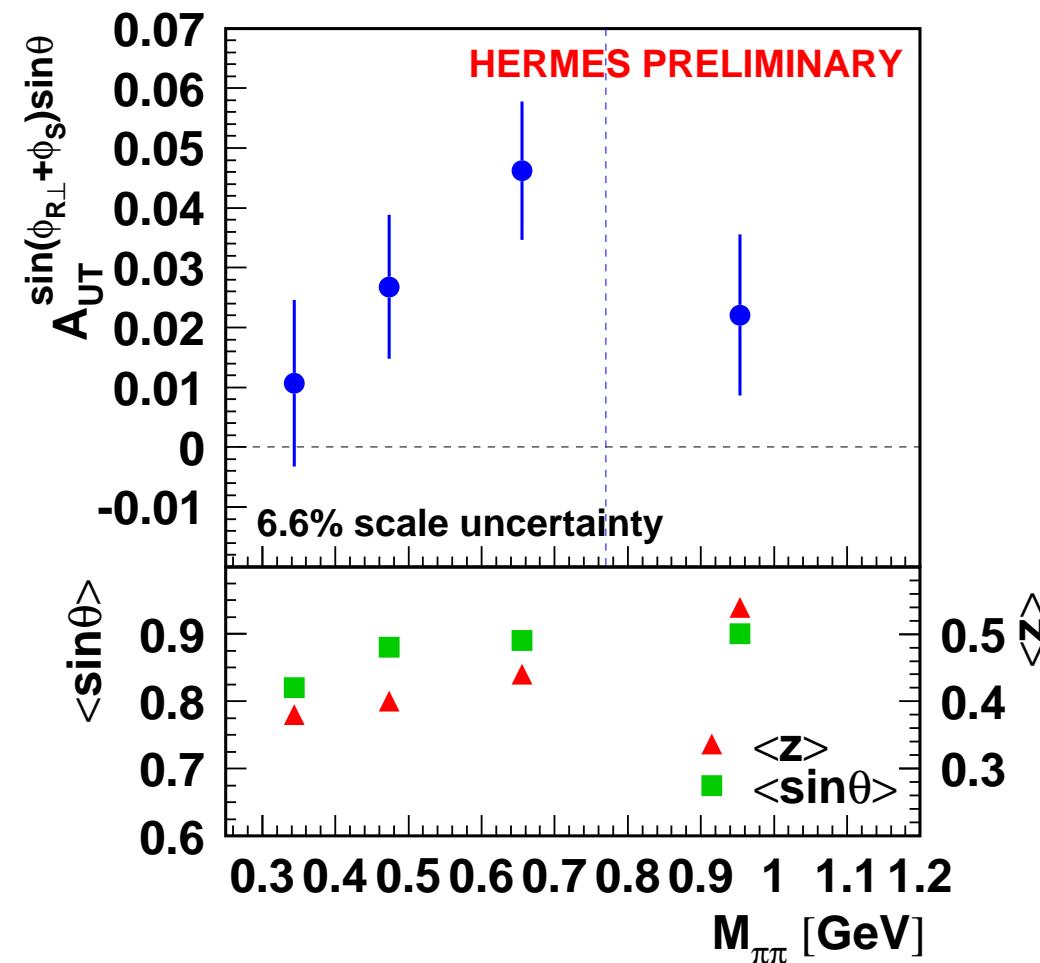


Radici et al. [hep-ph/0110252]:

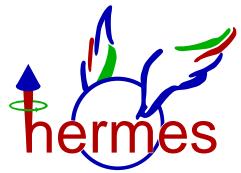
- completely different model, not predicting a sign change of the asymmetry

Mass Dependence of A_{UT}

2002-04 Data



- 2-hadron (aka Interference) FF is not zero!
- asymmetry grows with $M_{\pi\pi}$ below ρ^0 mass
- positive asymmetries in all invariant mass bins
- rules out predicted sign change at ρ^0 mass (Jaffe et al.)
- to extract transversity (h_1) need IFF from Belle (or BaBar etc.)
- non-zero IFF shows feasibility of using it at, e.g., RHIC for transversity measurements



Semi-Inclusive 1-Hadron Production

Leading-Twist Distribution Functions

$$f_1 = \text{○}$$

$$g_1 = \text{○} \rightarrow - \text{○} \rightarrow$$

$$h_1 = \text{○} \uparrow - \text{○} \downarrow$$

$$g_{1T} = \text{○} \uparrow - \text{○} \uparrow$$

Fragmentation Functions

$$D_1 = \text{○}$$

$$G_1 = \text{○} \rightarrow - \text{○} \rightarrow$$

$$H_1 = \text{○} \uparrow - \text{○} \downarrow$$

$$G_{1T} = \text{○} \uparrow - \text{○} \uparrow$$

$$f_{1T}^\perp = \text{○} \uparrow - \text{○} \downarrow$$

$$h_1^\perp = \text{○} \downarrow - \text{○} \uparrow$$

$$h_{1L}^\perp = \text{○} \rightarrow - \text{○} \rightarrow$$

$$h_{1T}^\perp = \text{○} \uparrow - \text{○} \uparrow$$

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$$H_{1L}^\perp = \text{○} \rightarrow - \text{○} \rightarrow$$

$$H_{1T}^\perp = \text{○} \uparrow - \text{○} \uparrow$$

Chiral-odd **transversity** h_1 must couple to chiral-odd FF

Leading-Twist Distribution Functions

$$\begin{aligned} f_1 &= \text{Diagram of a single quark loop} \\ g_1 &= \text{Diagram of two quark loops connected by a minus sign} \\ h_1 &= \text{Diagram of two quark loops with arrows, enclosed in a red box} \end{aligned}$$

$$g_{1T} = \text{Diagram of two quark loops with arrows, enclosed in a blue box}$$

Fragmentation Functions

$$\begin{aligned} D_1 &= \text{Diagram of a single quark loop} \\ G_1 &= \text{Diagram of two quark loops connected by a minus sign} \\ H_1 &= \text{Diagram of two quark loops with arrows, enclosed in a green box} \end{aligned}$$

$$G_{1T} = \text{Diagram of two quark loops with arrows}$$

$$\begin{aligned} f_{1T}^\perp &= \text{Diagram of two quark loops with arrows, one up and one down} \\ h_1^\perp &= \text{Diagram of two quark loops with arrows, one up and one down} \\ h_{1L}^\perp &= \text{Diagram of two quark loops with arrows, both horizontal} \end{aligned}$$

$$h_{1T}^\perp = \text{Diagram of two quark loops with arrows, both vertical}$$

$$\begin{aligned} D_{1T}^\perp &= \text{Diagram of two quark loops with arrows, one up and one down} \\ H_1^\perp &= \text{Diagram of two quark loops with arrows, one up and one down} \\ H_{1L}^\perp &= \text{Diagram of two quark loops with arrows, both horizontal} \\ H_{1T}^\perp &= \text{Diagram of two quark loops with arrows, both vertical} \end{aligned}$$

Chiral-odd **transversity** h_1 must couple to chiral-odd FF
 $\Rightarrow H_1$ is the only k_T -integrated chiral-odd FF \Rightarrow DSA
 (Example: transverse-spin transfer in Λ -production)

Leading-Twist Distribution Functions

$$f_1 = \text{Diagram}$$

$$g_1 = \text{Diagram} - \text{Diagram}$$

$$h_1 = \boxed{\text{Diagram} - \text{Diagram}}$$

$$f_{1T}^\perp = \text{Diagram} - \text{Diagram}$$

$$h_1^\perp = \text{Diagram} - \text{Diagram}$$

$$h_{1L}^\perp = \text{Diagram} - \text{Diagram}$$

$$g_{1T} = \text{Diagram} - \text{Diagram}$$

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Chiral-odd **transversity** h_1 must couple to chiral-odd FF
 can use k_T -unintegrated chiral-odd FF \Rightarrow T-odd Collins FF
 \Rightarrow leads to Single-Spin Asymmetrie (SSA)

Leading-Twist Distribution Functions

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T-odd

Fragmentation Functions

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SSAs require one and only one T-odd function

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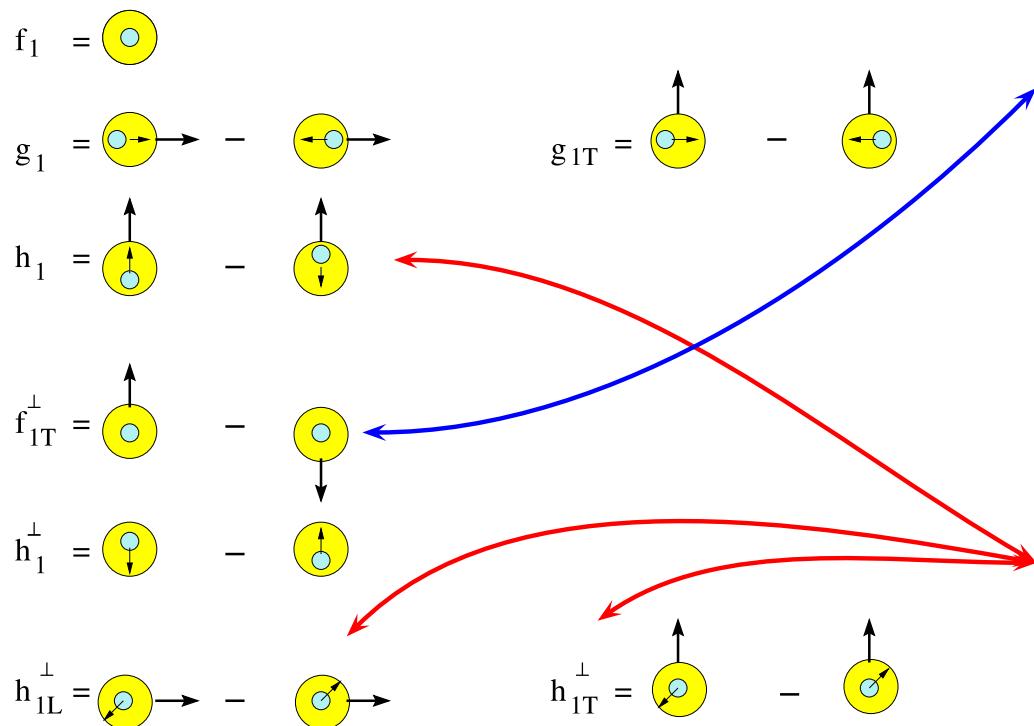
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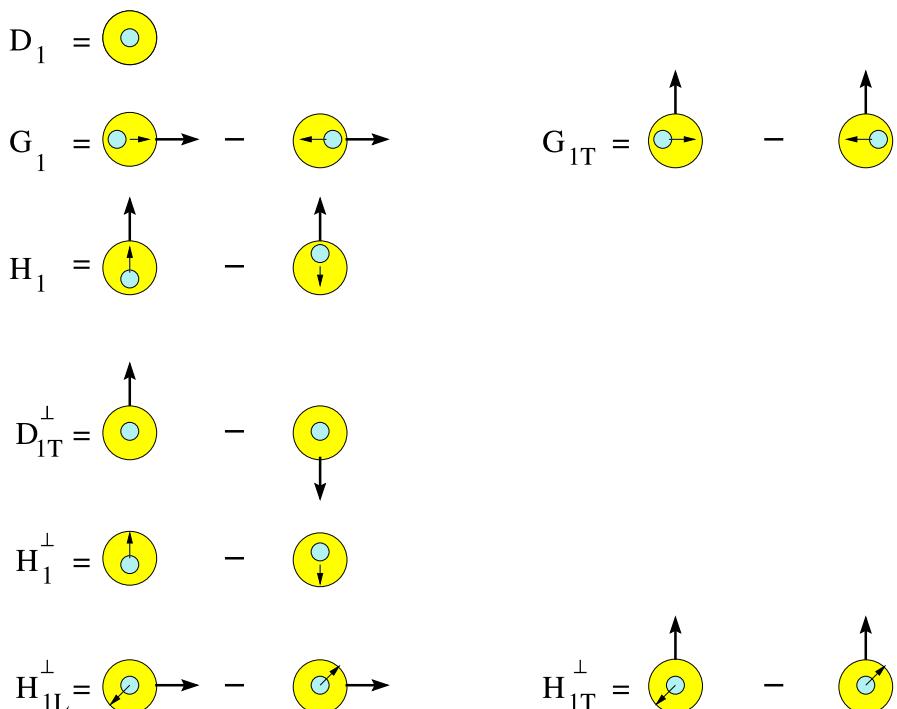
$$H_{1T}^\perp = \text{○} \uparrow - \text{○} \uparrow$$

SSAs require one and only one T-odd function
 \Rightarrow SSAs through Collins function

Leading-Twist Distribution Functions



Fragmentation Functions



SSAs require one and only one T-odd function

⇒ SSAs through **Collins function** or **Sivers function**

(Boer-Mulders DF couples to H_1 , but SSA requires polarization of final state!)

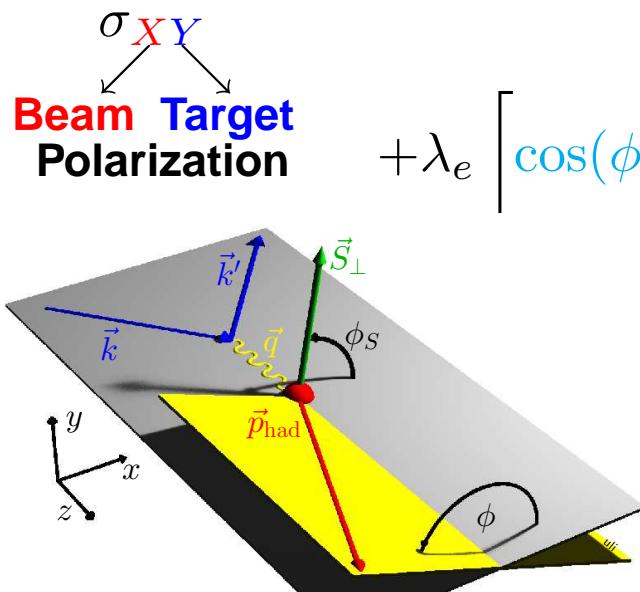
$$d\sigma = d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3$$

$$+ S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\}$$

$$+ S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \right.$$

$$\left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right)$$

$$+ \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}$$



Mulders and Tangermann, Nucl. Phys. B 461 (1996) 197

Boer and Mulders, Phys. Rev. D 57 (1998) 5780

Bacchetta et al., Phys. Lett. B 595 (2004) 309

Bacchetta et al., JHEP 0702 (2007) 093

“Trento Conventions”, Phys. Rev. D 70 (2004) 117504



SIDIS Cross Section (up to subleading order in $1/Q$)

$$d\sigma = d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3$$

$$+ S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\}$$

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$$\left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right)$$

$$+ \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}$$

σ_{XY}
 Beam Target
 Polarization

<u>This talk:</u>	$\sin \phi d\sigma_{UL}^5$...	Subleading Twist
	$\sin(\phi - \phi_S) d\sigma_{UT}^8$...	Sivers Effect
	$\sin(\phi + \phi_S) d\sigma_{UT}^9$...	Collins Effect

$$d\sigma = d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3$$

$$+ S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\}$$

$$+ S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \right.$$

$$\left. + \frac{1}{Q} (\sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12}) \right)$$

$$+ \lambda_e \left[\cos(\phi - \phi_S) d\sigma_{LT}^{13} + \frac{1}{Q} (\cos \phi_S d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) d\sigma_{LT}^{15}) \right] \right\}$$

σ_{XY}
 Beam Target
 Polarization

Also Interesting: $\sin \phi_S d\sigma_{UT}^{12}$, $\cos \phi_S d\sigma_{LT}^{14} \dots \Rightarrow$ Transversity, g_2
 (and under study!) $\cos \phi d\sigma_{UU}^2$... Cahn Effect
 $\cos 2\phi d\sigma_{UU}^1$... Boer-Mulders Effect

Azimuthal Single-Spin Asymmetries

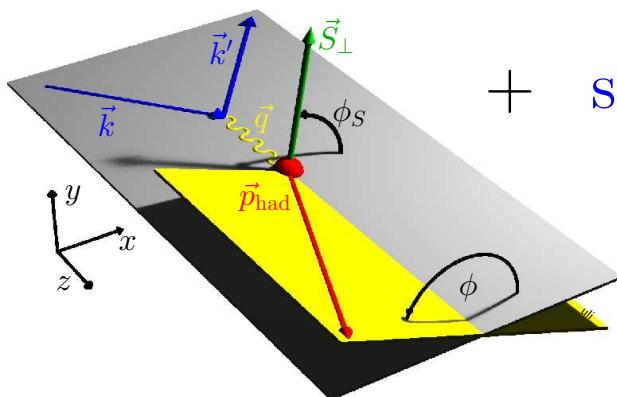
$$A_{UT}(\phi, \phi_S) = \frac{1}{\langle |S_\perp| \rangle} \frac{N_h^\uparrow(\phi, \phi_S) - N_h^\downarrow(\phi, \phi_S)}{N_h^\uparrow(\phi, \phi_S) + N_h^\downarrow(\phi, \phi_S)}$$

$$\sim \sin(\phi + \phi_S) \sum_q e_q^2 \mathcal{I} \left[\frac{k_T \hat{P}_{h\perp}}{M_h} h_1^q(x, p_T^2) H_1^{\perp, q}(z, k_T^2) \right]$$

$$+ \sin(\phi - \phi_S) \sum_q e_q^2 \mathcal{I} \left[\frac{p_T \hat{P}_{h\perp}}{M} f_{1T}^{\perp, q}(x, p_T^2) D_1^q(z, k_T^2) \right]$$

+ ...

$\mathcal{I}[\dots]$: convolution integral over initial (p_T) and final (k_T) quark transverse momenta



⇒ 2D Max.Likelihd. fit of to get Collins and Sivers amplitudes:

$$PDF(2\langle \sin(\phi \pm \phi_S) \rangle_{UT}, \dots, \phi, \phi_S) = \frac{1}{2} \{ 1 + P_T(2\langle \sin(\phi \pm \phi_S) \rangle_{UT} \sin(\phi \pm \phi_s) + \dots) \}$$

Weight with transverse hadron momentum $P_{h\perp}$ to resolve convolution:

$$\tilde{A}_{UT}(\phi, \phi_S) = \frac{1}{\langle S_\perp \rangle} \frac{\sum_{i=1}^{N^+} P_{h\perp,i} - \sum_{i=1}^{N^-} P_{h\perp,i}}{N^+ + N^-}$$

$$\sim \sin(\phi + \phi_C) \cdot \sum_q e_q^2 \ h_1^q(x) \ z \ H_1^{\perp(1),q}(z)$$

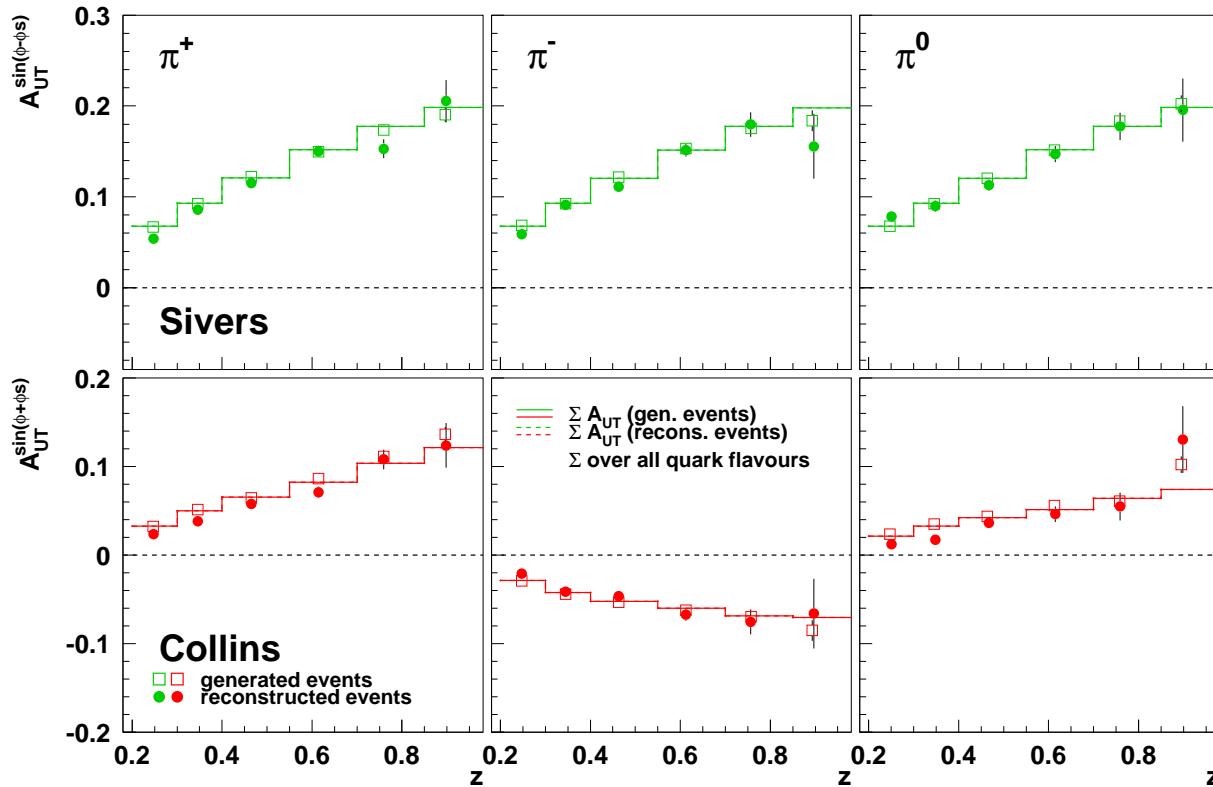
$$- \sin(\phi - \phi_S) \cdot \sum_q e_q^2 \ f_{1T}^{\perp(1),q}(x) \ z \ D_1^q(z)$$

+ ...

(1): p_T^2/k_T^2 -moment of distribution / fragmentation function

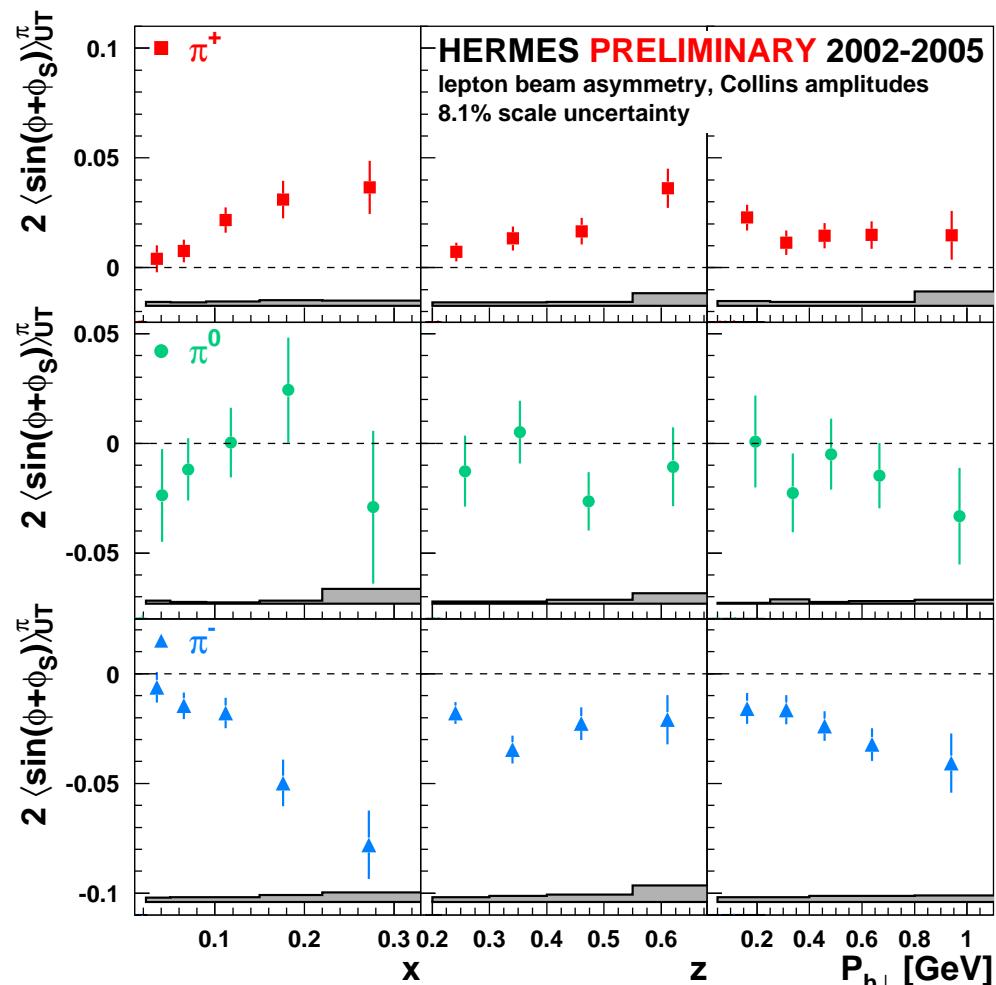
Monte Carlo Test of the Extraction Method

- generate Collins and Sivers asymmetries (Gaussian Ansatz in p_T^2)
- analyze MC data like experimental data and extract amplitudes:



- Collins-Sivers cross contamination negligible
- insensitive to $\cos(2\phi)$ moments in unpolarized cross section
- insensitive to transverse target tracking corrections

Collins Amplitudes 2002-2005



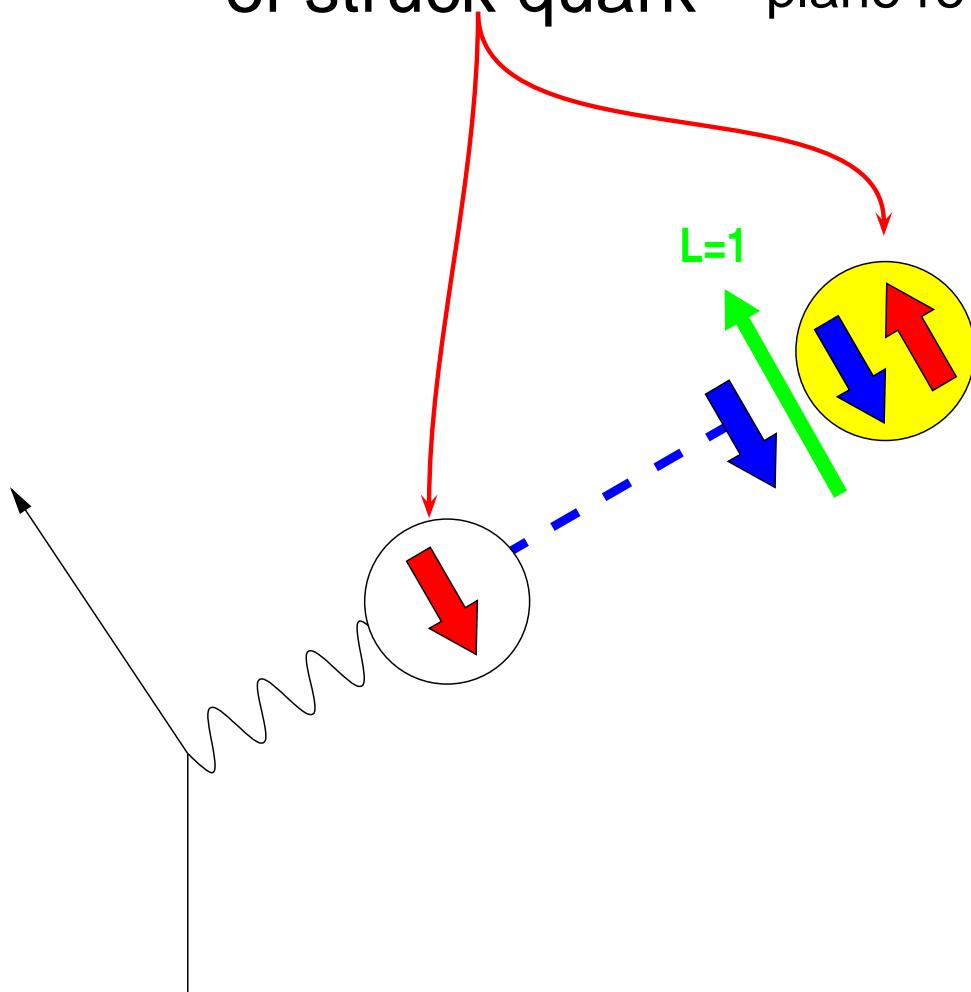
- published[†] results confirmed with much higher statistical precision
- overall scale uncertainty of 8.1%
- positive for π^+ and negative for π^- as maybe expected ($\delta u > 0$
 $\delta d < 0$)
- unexpected large π^- asymmetry
 \Rightarrow role of disfavored Collins FF
most likely: $H_1^{\perp, disf} \approx -H_1^{\perp, fav}$
- isospin symmetry among charged and neutral pions fulfilled

[†] [A. Airapetian et al, Phys. Rev. Lett. 94 (2005)
012002]

Understanding the Collins FF - String Model Interpretation (Artru)

transverse spin
of struck quark

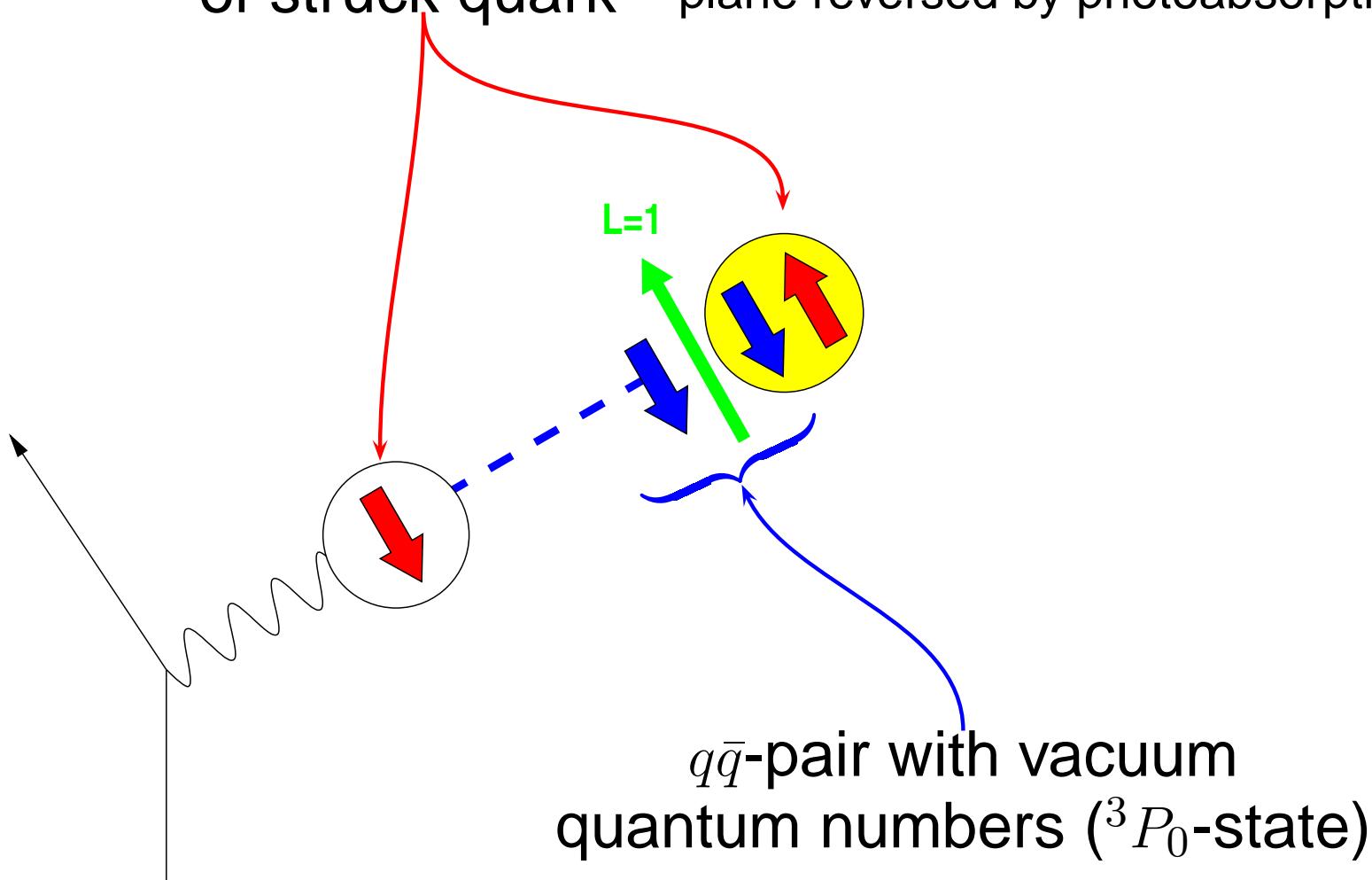
(polarization component in lepton scattering
plane reversed by photoabsorption)



Understanding the Collins FF - String Model Interpretation (Artru)

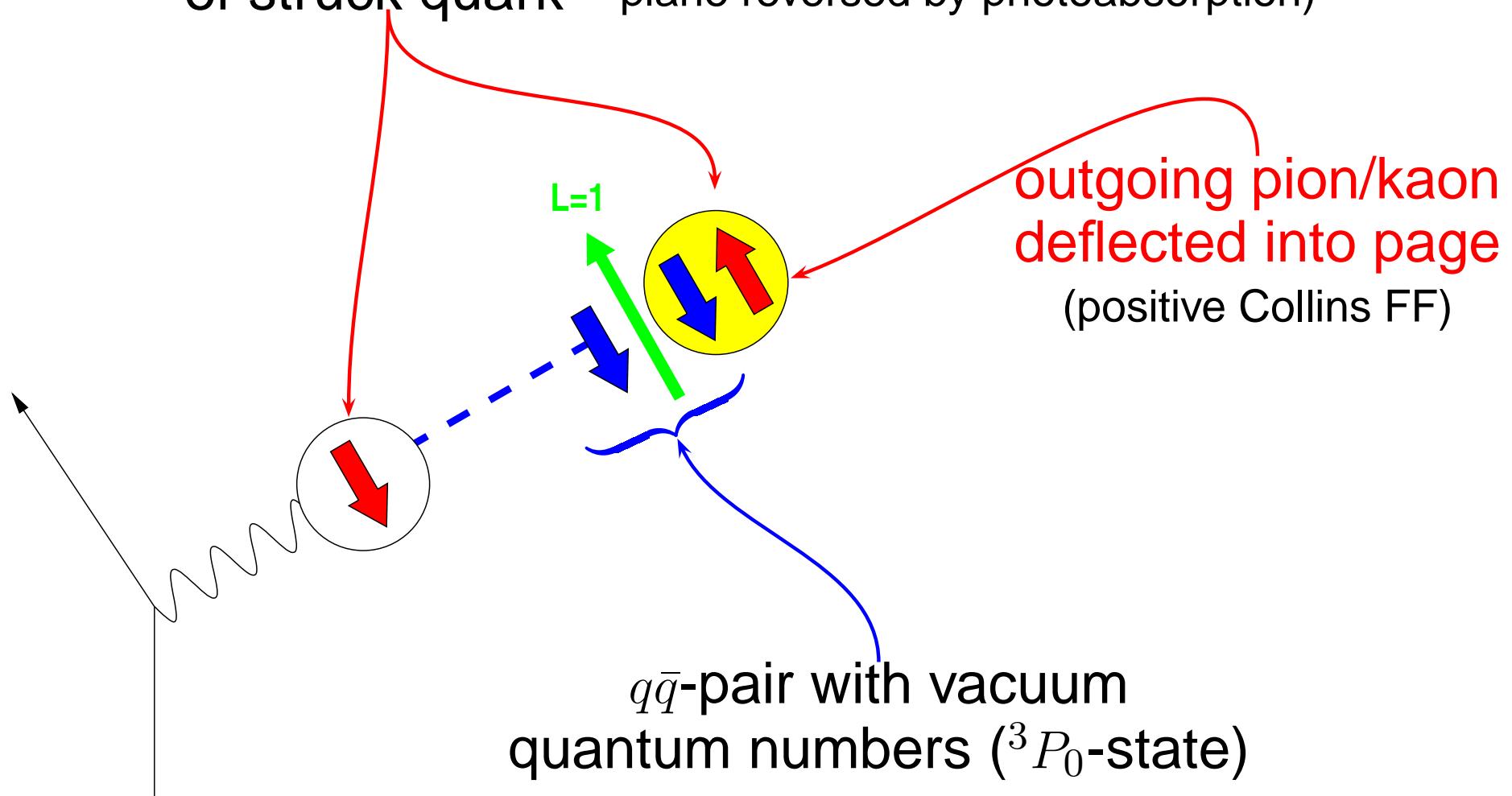
transverse spin
of struck quark

(polarization component in lepton scattering
plane reversed by photoabsorption)



Understanding the Collins FF - String Model Interpretation (Artru)

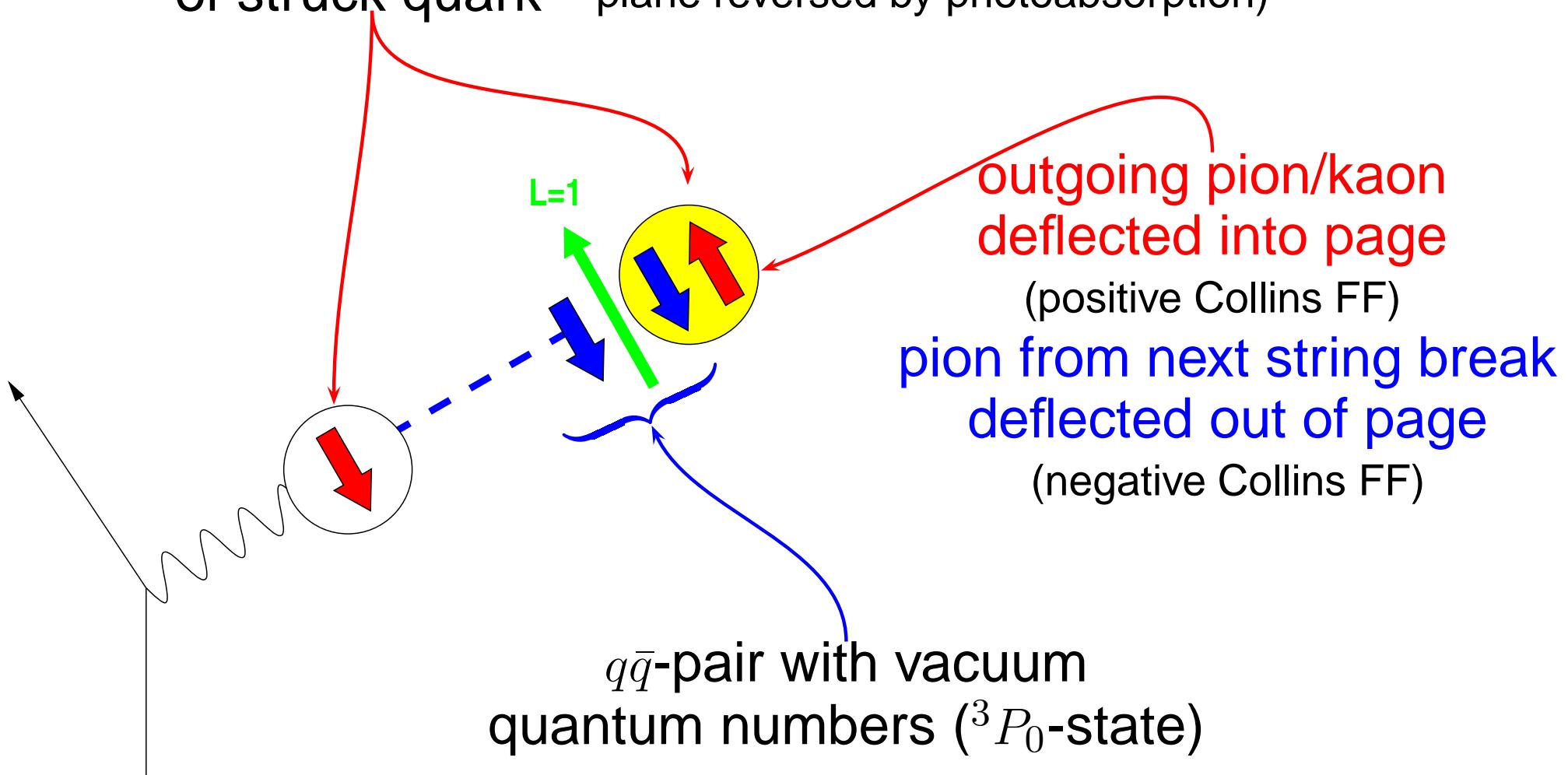
transverse spin
of struck quark (polarization component in lepton scattering
plane reversed by photoabsorption)

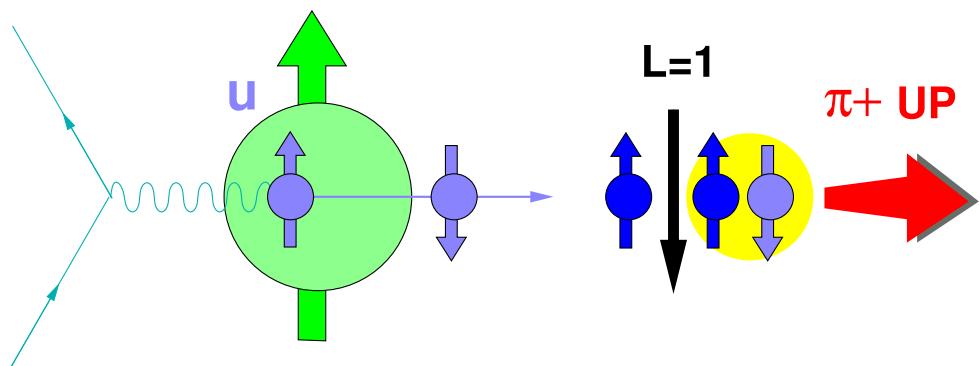


Understanding the Collins FF - String Model Interpretation (Artru)

transverse spin
of struck quark

(polarization component in lepton scattering
plane reversed by photoabsorption)

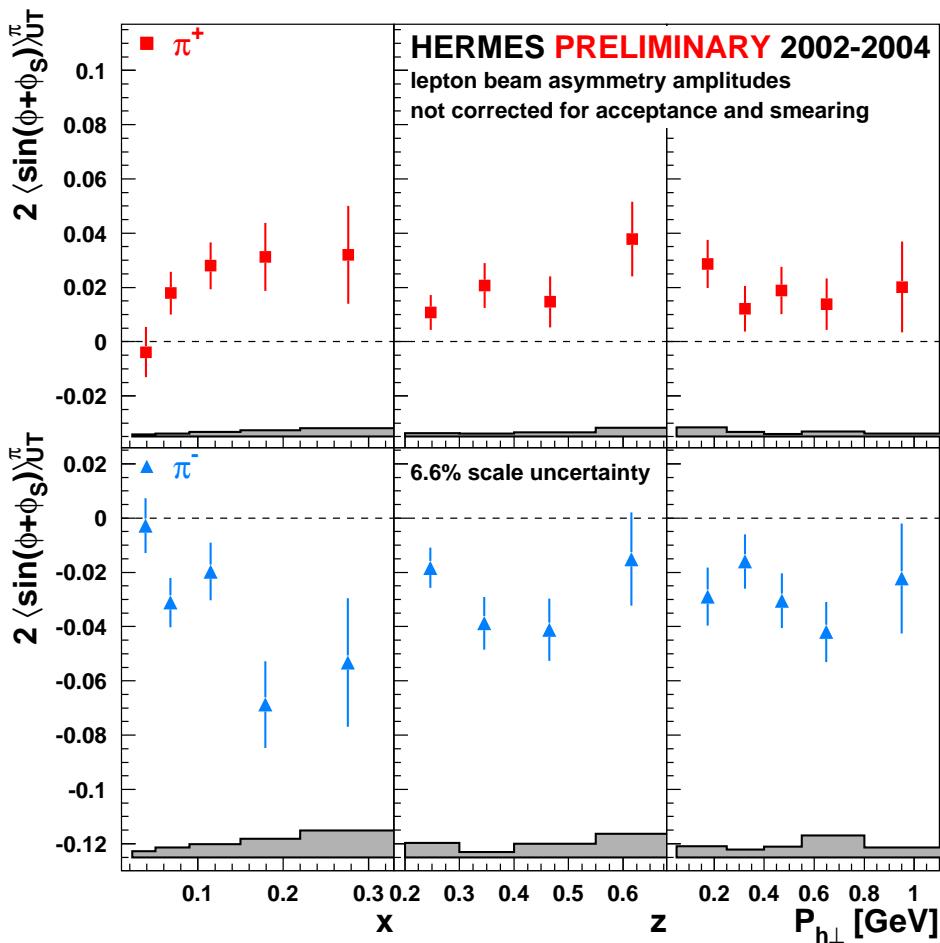




$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$

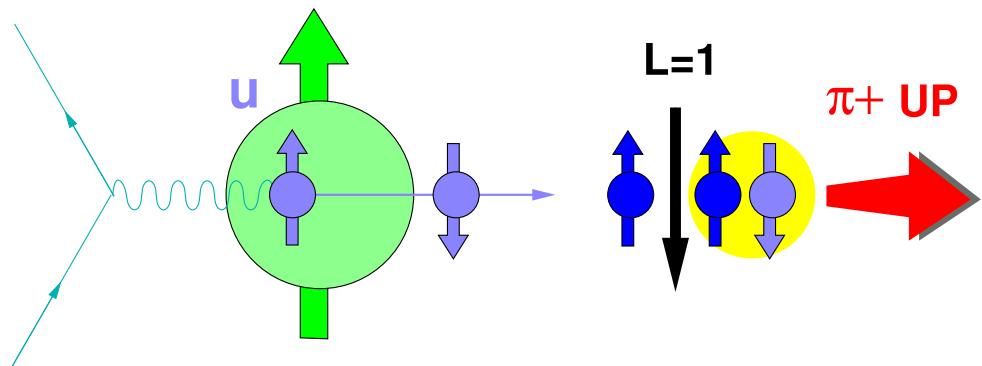
The Collins Effect

Artru Model vs. HERMES



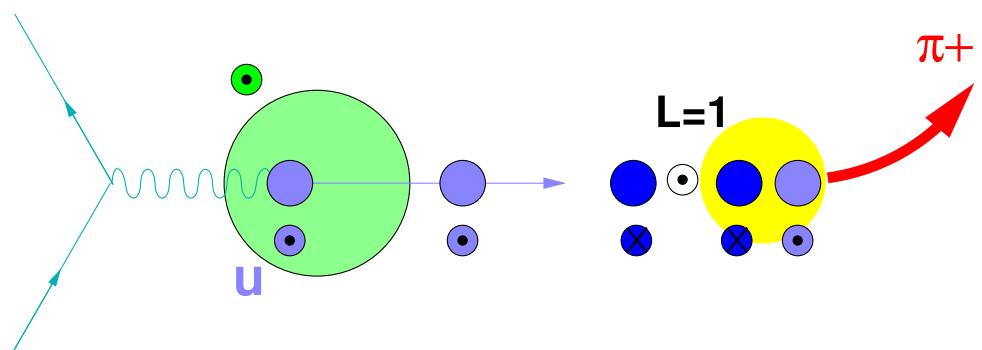
$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$



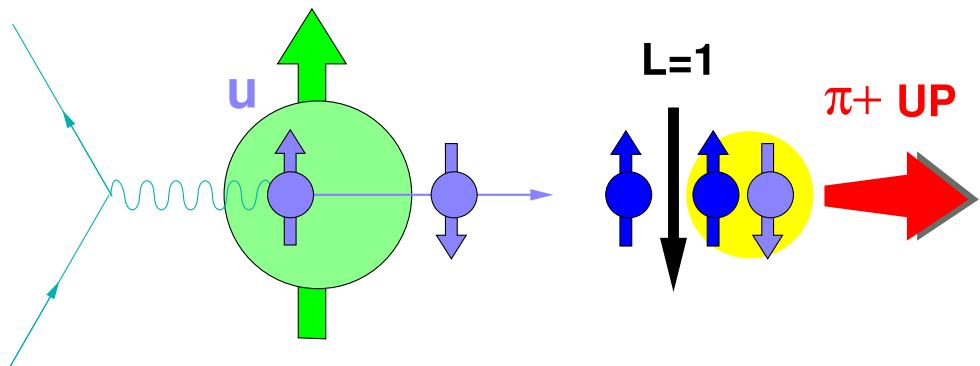


$$\left. \begin{array}{ll} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$

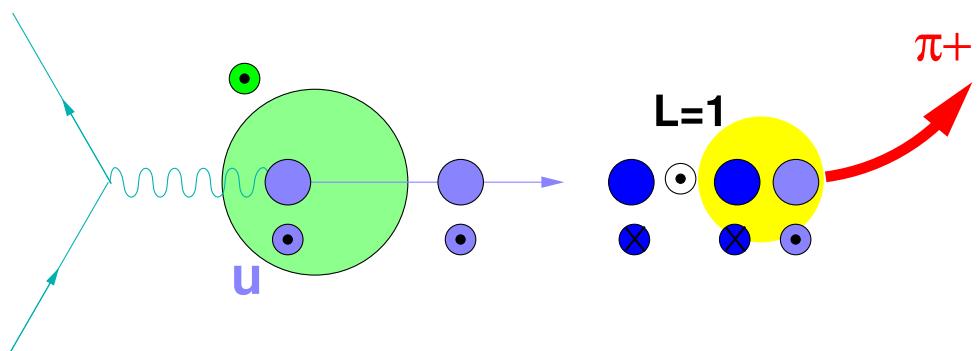
✓



$$\left. \begin{array}{ll} \phi_S = \pi/2 \\ \phi = 0 \end{array} \right\} \sin(\phi + \phi_S) > 0$$

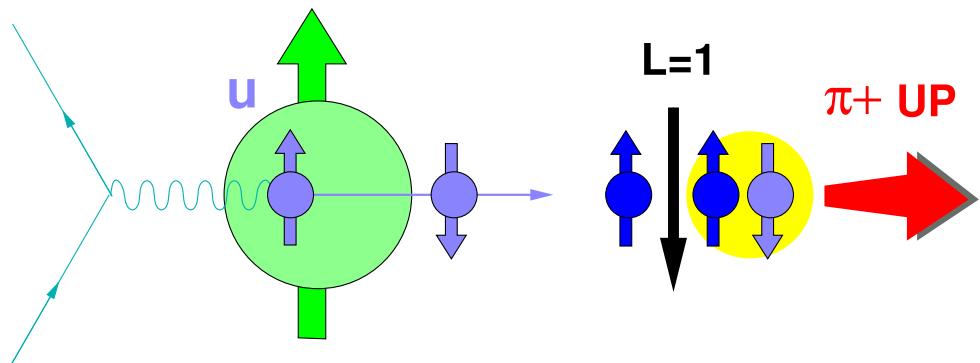


$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$

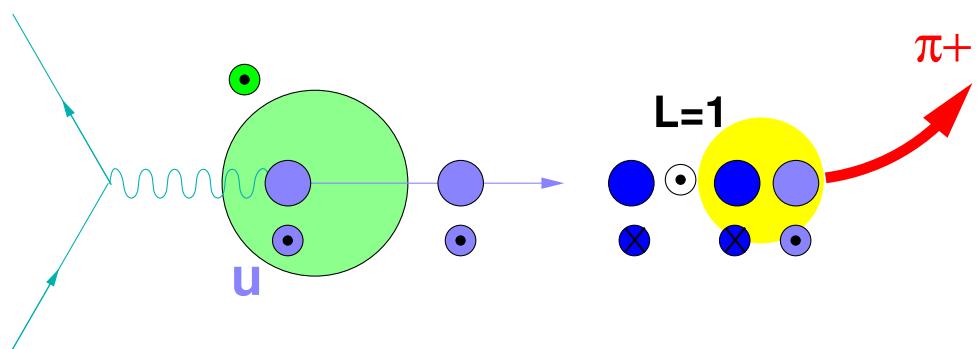


$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = 0 \end{array} \right\} \sin(\phi + \phi_S) > 0$$





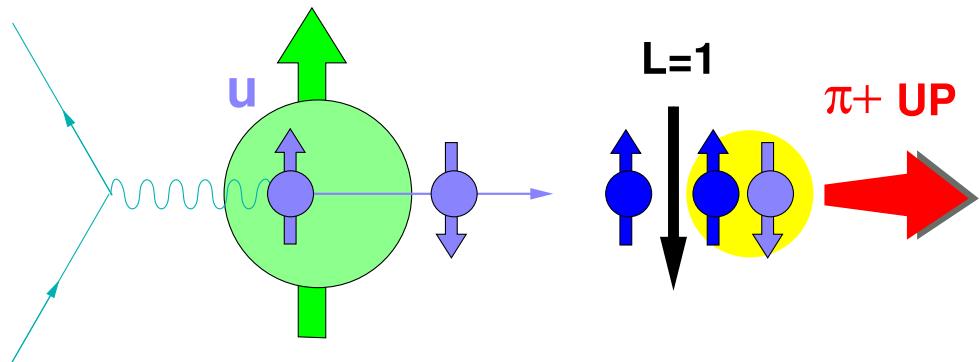
$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$



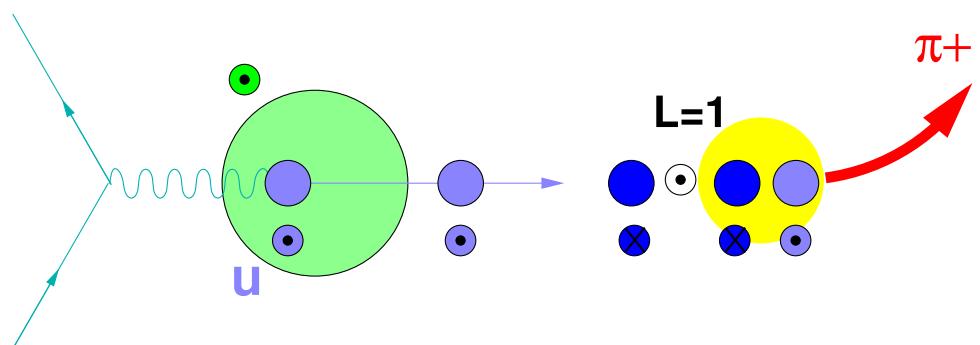
$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = 0 \end{array} \right\} \sin(\phi + \phi_S) > 0$$



Artru model and HERMES results in agreement!
(assuming u -quark transversity positive)



$$\left. \begin{array}{l} \phi_S = 0 \\ \phi = \pi/2 \end{array} \right\} \sin(\phi + \phi_S) > 0$$



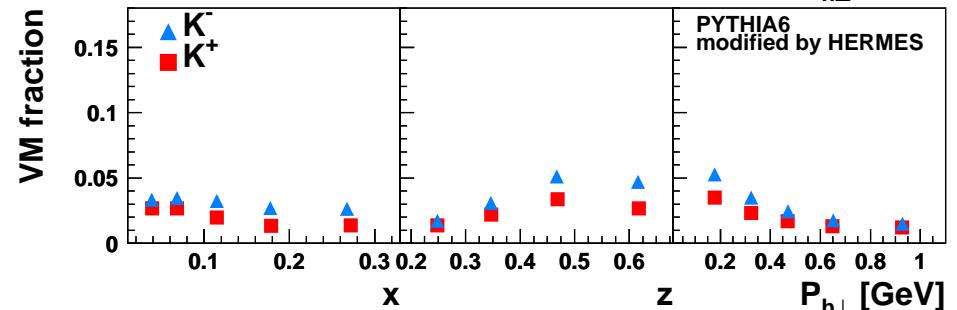
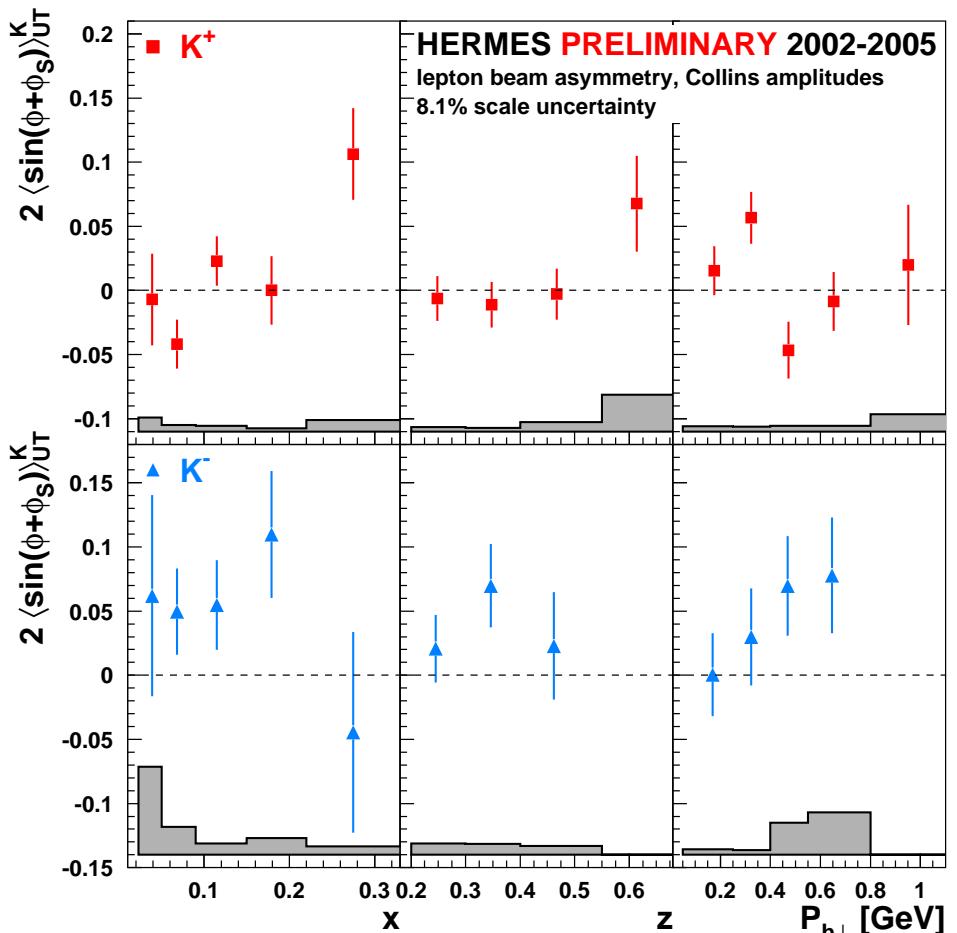
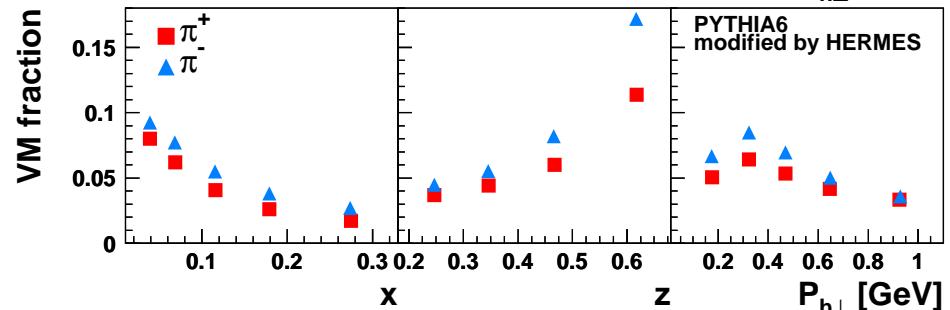
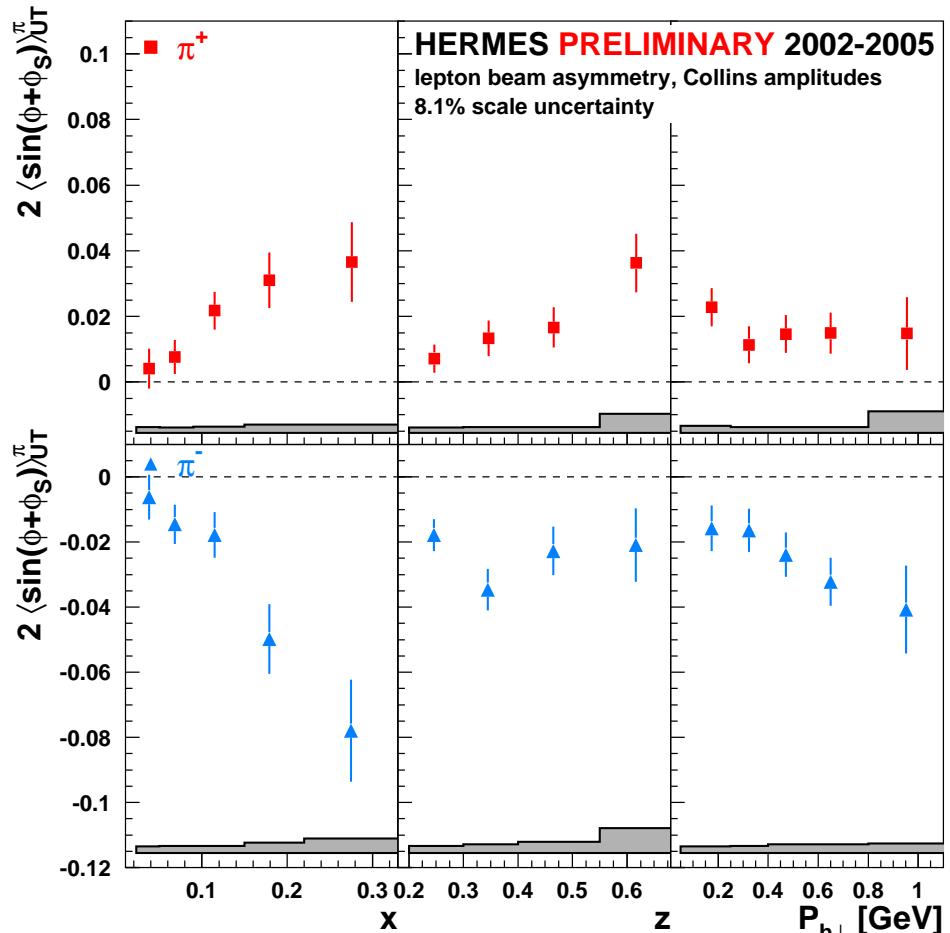
$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = 0 \end{array} \right\} \sin(\phi + \phi_S) > 0$$



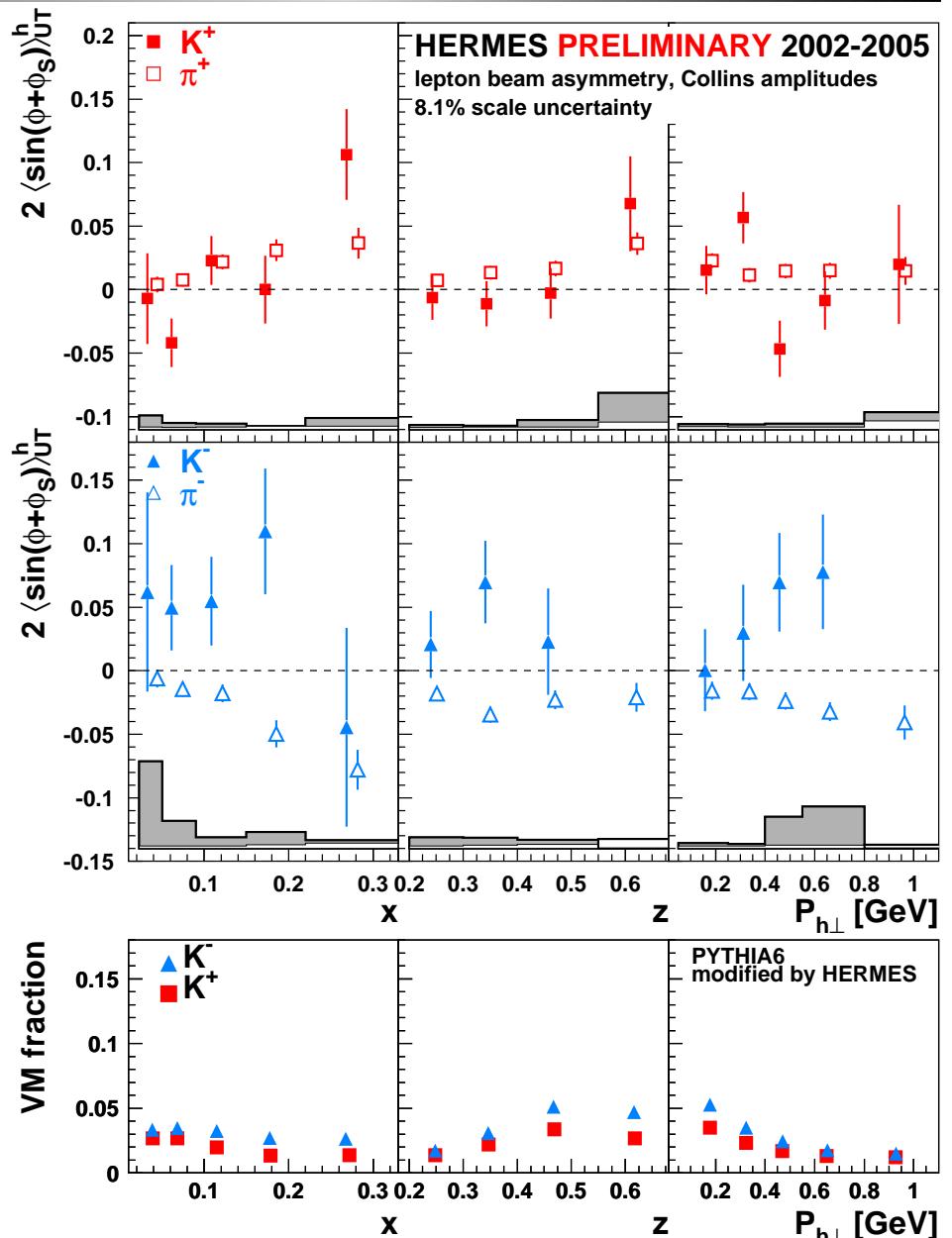
Artru model and HERMES results in agreement also for π^- !
 (e.g., assuming $h_1^u h_1^d < 0$ and using $H_1^{u \rightarrow \pi^-} H_1^{d \rightarrow \pi^-} < 0$)

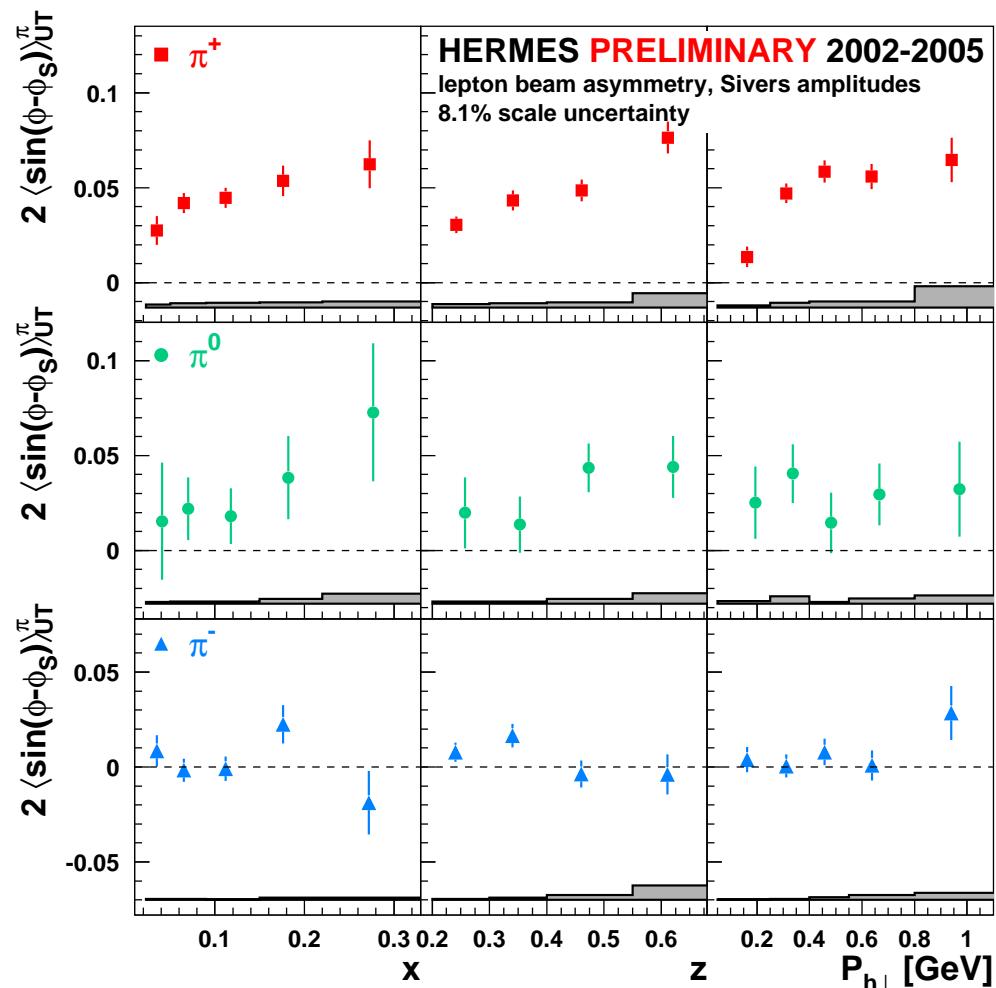
Collins Amplitudes

(2002-2005 data)



- none of the kaon amplitudes significantly non-zero
- however, K^+ amplitudes not different from π^+ amplitudes
- K^- amplitudes slightly positive, contrary to large negative π^- amplitudes
- K^- pure sea object
 \Rightarrow production dominated by u-quark scattering





- published[†] results confirmed with much higher statistical precision
- overall scale uncertainty of 8.1%
- π^+ : positive; π^- : consistent with zero
- ⇒ first evidence for non-zero Sivers fct.: $f_{1T}^{\perp, u} < 0$ (u -quark dominance)
- ⇒ non-zero orbital angular momentum
- Isospin symmetry for Sivers amplitudes fulfilled

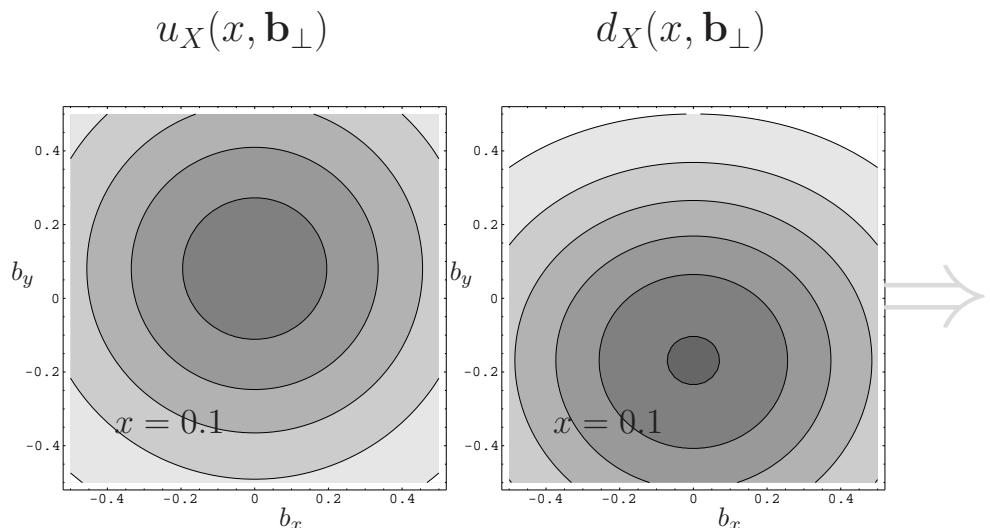
[†] [A. Airapetian et al, Phys. Rev. Lett. 94 (2005)
012002]

approach by M. Burkardt:

[hep-ph/0309269]

spatial distortion of q-distribution

(obtained using anom. magn. moments
& impact parameter dependent PDFs)



approach by M. Burkardt:

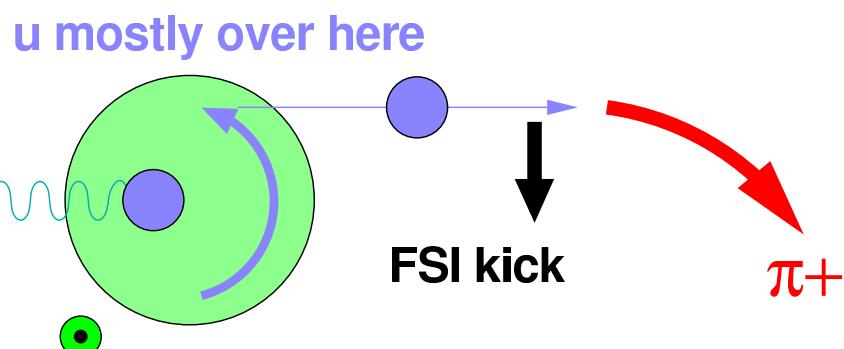
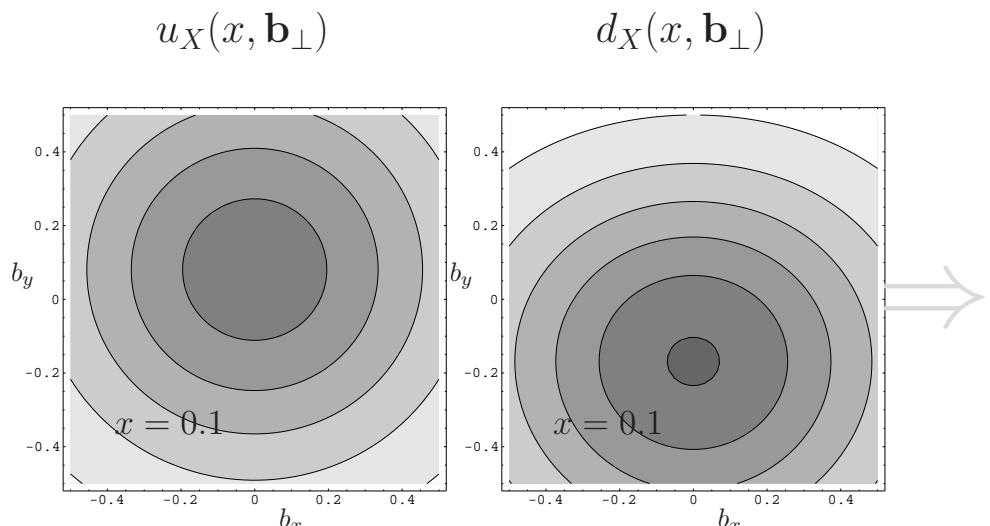
[hep-ph/0309269]

spatial distortion of q-distribution

(obtained using anom. magn. moments
& impact parameter dependent PDFs)

+ attractive QCD potential
(gluon exchange)

⇒ transverse asymmetries



$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = \pi \end{array} \right\} \sin(\phi - \phi_S) > 0$$

Chromodynamic Lensing

Understanding the Sivers Moments

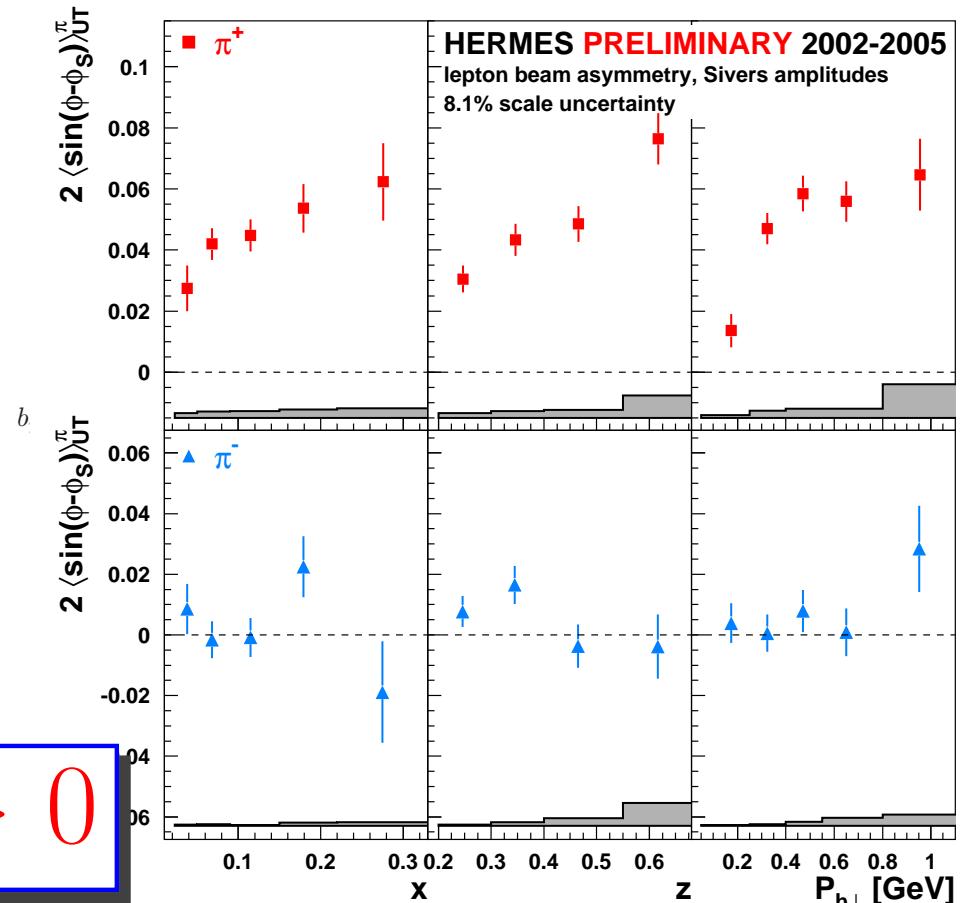
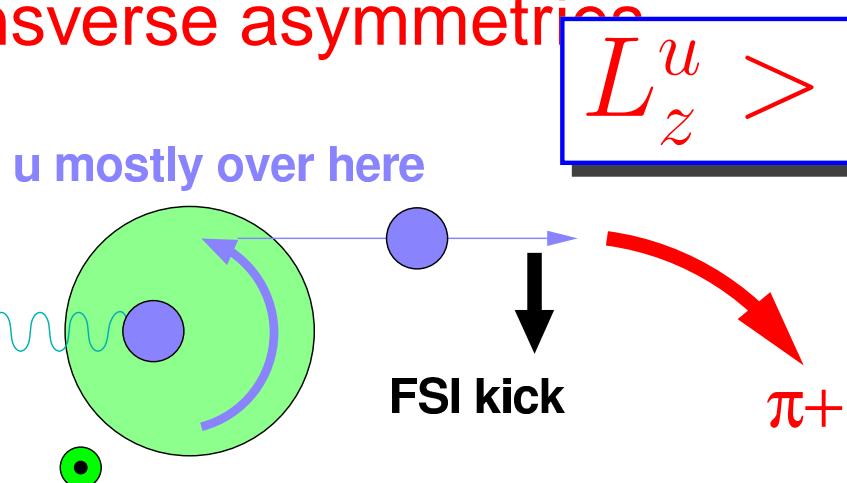
approach by M. Burkardt:

spatial distortion of q-distribution

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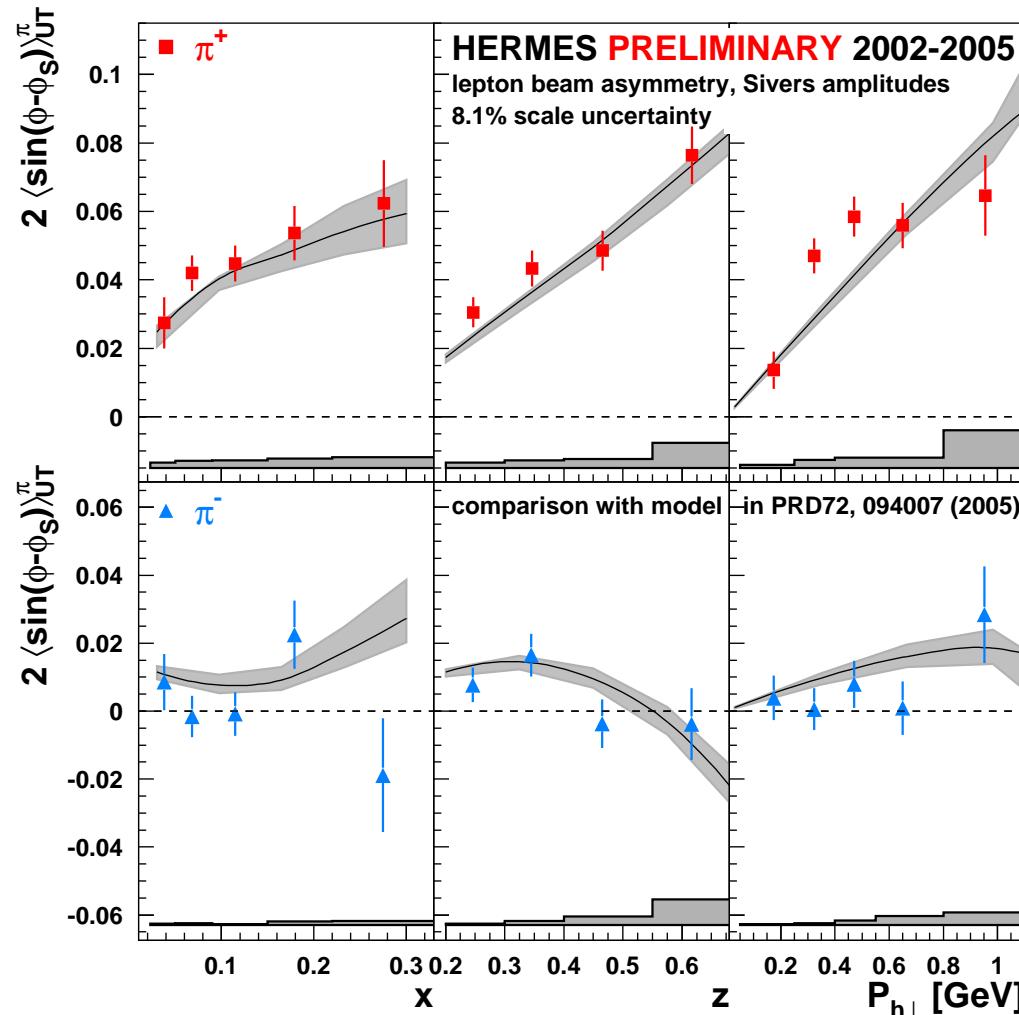
+ attractive QCD potential
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\Rightarrow transverse asymmetries



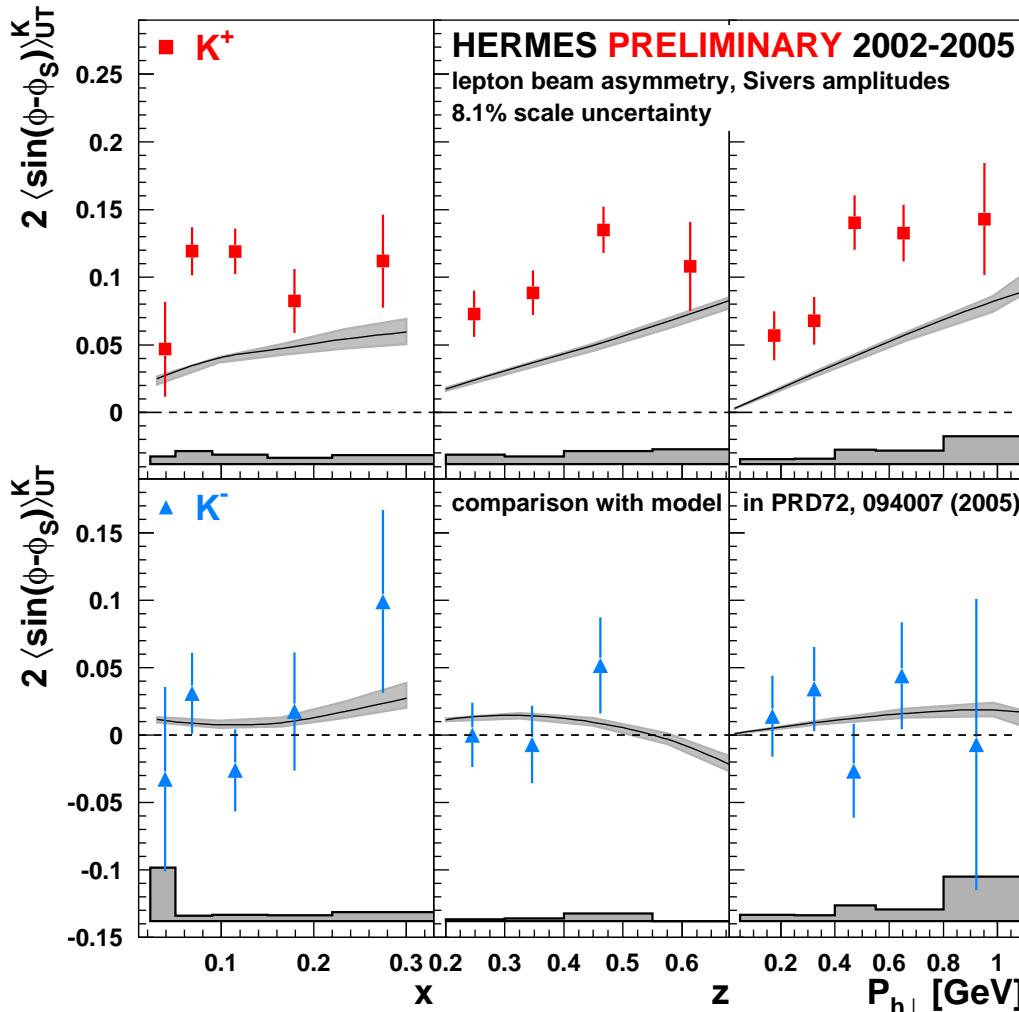
$$\left. \begin{array}{l} \phi_S = \pi/2 \\ \phi = \pi \end{array} \right\} \sin(\phi - \phi_S) > 0$$

Sivers Amplitudes 2002-2005



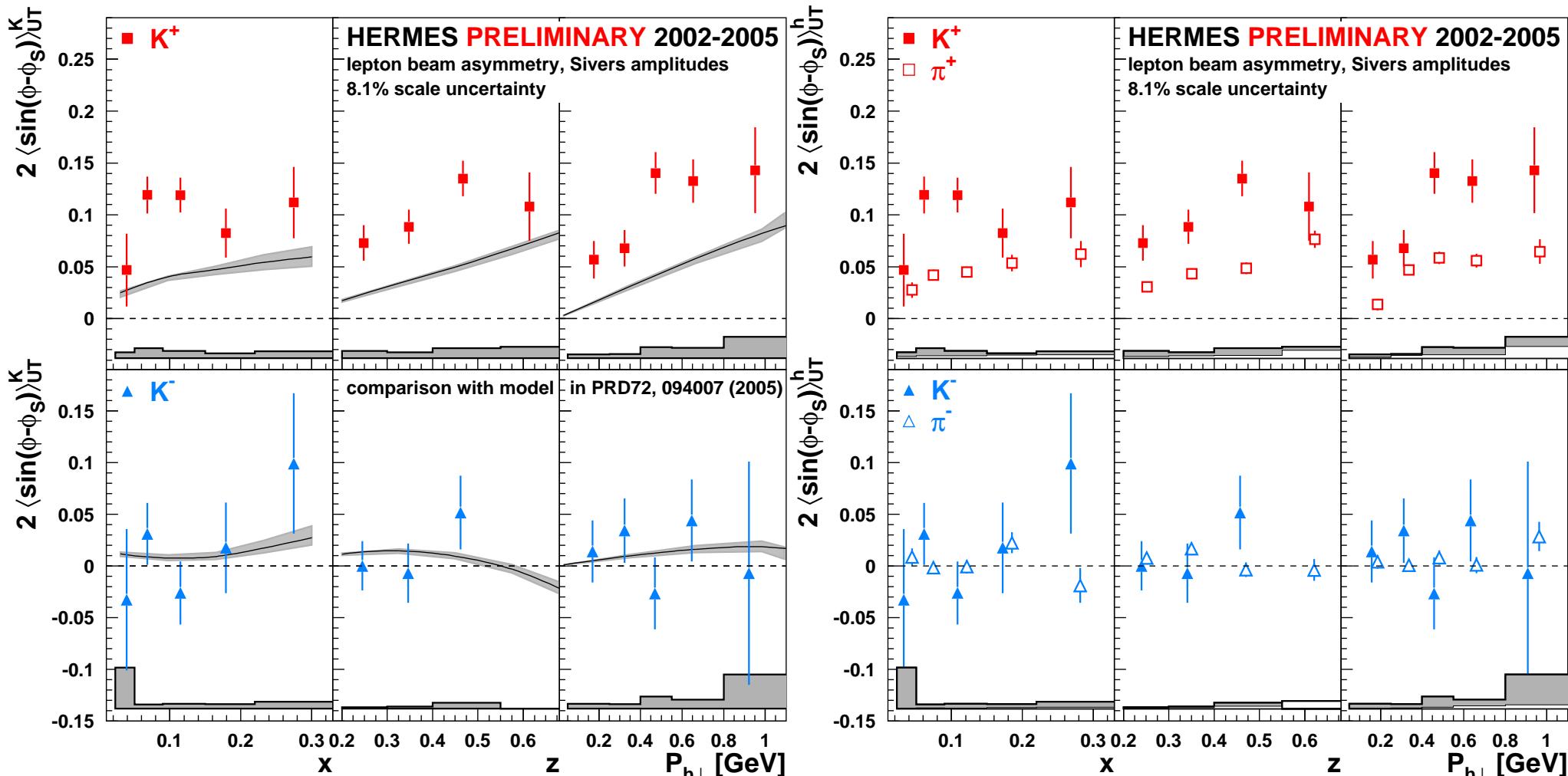
- comparison with model calculation by Anselmino et al., based on:
 - Gaussian Ansatz for Sivers fctn.
 - average transverse momenta from unpolarized $\cos \phi$ amplitudes
 - non-zero Sivers fctn. only for valence quarks
- excellent description of pion amplitudes from 2002-05 data

Sivers Amplitudes 2002-2005

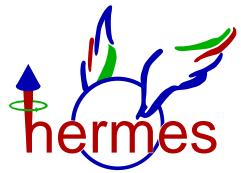


- comparison with model calculation by Anselmino et al., based on:
 - Gaussian Ansatz for Sivers fctn.
 - average transverse momenta from unpolarized $\cos \phi$ amplitudes
 - non-zero Sivers fctn. only for valence quarks
- excellent description of pion amplitudes from 2002-05 data
- fails to describe kaon amplitudes

Sivers Amplitudes 2002-2005

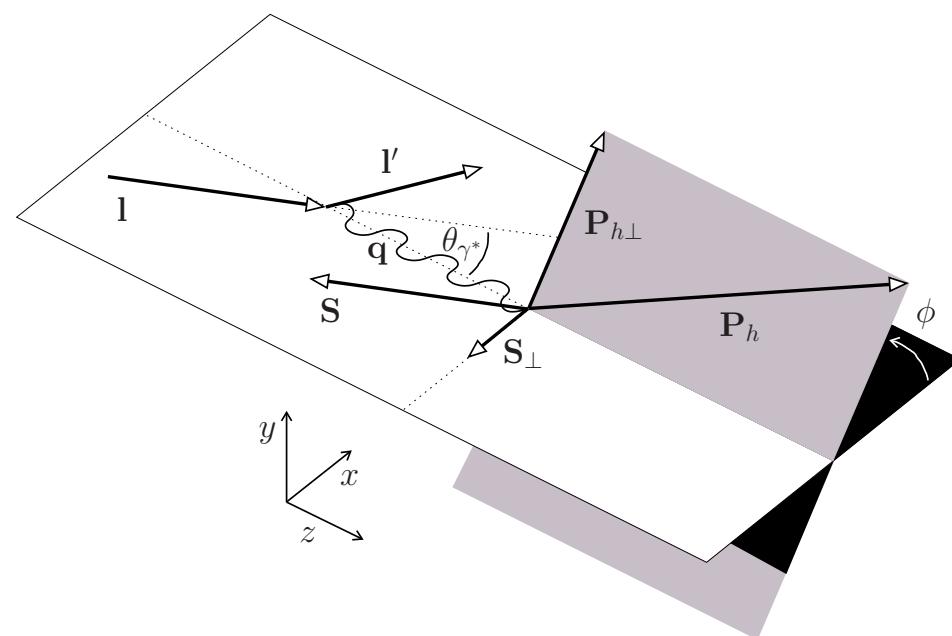


Non-trivial role of sea quarks!



"Longitudinal" SSAs

Mixing of Azimuthal Moments



Experiment: Target Polarization w.r.t. Beam Direction (\mathbf{l})!

Theory: Polarization along virtual photon direction (\mathbf{q})

⇒ mixing of “experimental” and “theory” asymmetries via:

[Diehl and Sapeta, Eur. Phys. J. C41 (2005)]

$$\begin{pmatrix} \langle \sin \phi \rangle_{UL}^{\perp} \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^{\perp} \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^{\perp} \end{pmatrix} = \begin{pmatrix} \cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\ \frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\ \frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*} \end{pmatrix} \begin{pmatrix} \langle \sin \phi \rangle_{UL}^q \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^q \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^q \end{pmatrix}$$

($\cos \theta_{\gamma^*} \simeq 1$, $\sin \theta_{\gamma^*}$ up to 15% at HERMES energies)

Mixing of Azimuthal Moments II

$$\begin{pmatrix} \langle \sin \phi \rangle_{UL}^{\text{I}} \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^{\text{I}} \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^{\text{I}} \end{pmatrix} = \begin{pmatrix} \cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\ \frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\ \frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*} \end{pmatrix} \begin{pmatrix} \langle \sin \phi \rangle_{UL}^{\text{q}} \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^{\text{q}} \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^{\text{q}} \end{pmatrix}$$

solve for photon-axis moments:

$$\langle \sin \phi \rangle_{UL}^{\text{q}} \simeq \langle \sin \phi \rangle_{UL}^{\text{I}} + \sin \theta_{\gamma^*} \left(\langle \sin(\phi + \phi_S) \rangle_{UT}^{\text{I}} + \langle \sin(\phi - \phi_S) \rangle_{UT}^{\text{I}} \right)$$

Mixing of Azimuthal Moments II

$$\begin{pmatrix} \langle \sin \phi \rangle_{UL}^{\text{I}} \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^{\text{I}} \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^{\text{I}} \end{pmatrix} = \begin{pmatrix} \cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\ \frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\ \frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*} \end{pmatrix} \begin{pmatrix} \langle \sin \phi \rangle_{UL}^{\text{q}} \\ \langle \sin(\phi - \phi_S) \rangle_{UT}^{\text{q}} \\ \langle \sin(\phi + \phi_S) \rangle_{UT}^{\text{q}} \end{pmatrix}$$

solve for photon-axis moments:

$$\langle \sin \phi \rangle_{UL}^{\text{q}} \simeq \langle \sin \phi \rangle_{UL}^{\text{I}} + \sin \theta_{\gamma^*} \left(\langle \sin(\phi + \phi_S) \rangle_{UT}^{\text{I}} + \langle \sin(\phi - \phi_S) \rangle_{UT}^{\text{I}} \right)$$

$$\begin{aligned} \langle \sin(\phi \pm \phi_S) \rangle_{UT}^{\text{q}} &\simeq \langle \sin(\phi \pm \phi_S) \rangle_{UT}^{\text{I}} \\ &\quad - \underbrace{\frac{1}{2} \sin \theta_{\gamma^*} \left(\langle \sin \phi \rangle_{UL}^{\text{I}} + \tan \theta_{\gamma^*} \langle \sin(\phi \mp \phi_S) \rangle_{UT}^{\text{I}} \right)}_{\substack{\text{max. 0.4% absolute} \\ \text{correction}}} \end{aligned}$$

max. 1% relative



What About Longitudinally Polarized Targets?

$$\langle \sin \phi \rangle_{UL}^q = \langle \sin \phi \rangle_{UL}^\perp + \sin \theta_{\gamma^*} \left(\langle \sin(\phi + \phi_S) \rangle_{UT}^\perp + \langle \sin(\phi - \phi_S) \rangle_{UT}^\perp \right)$$

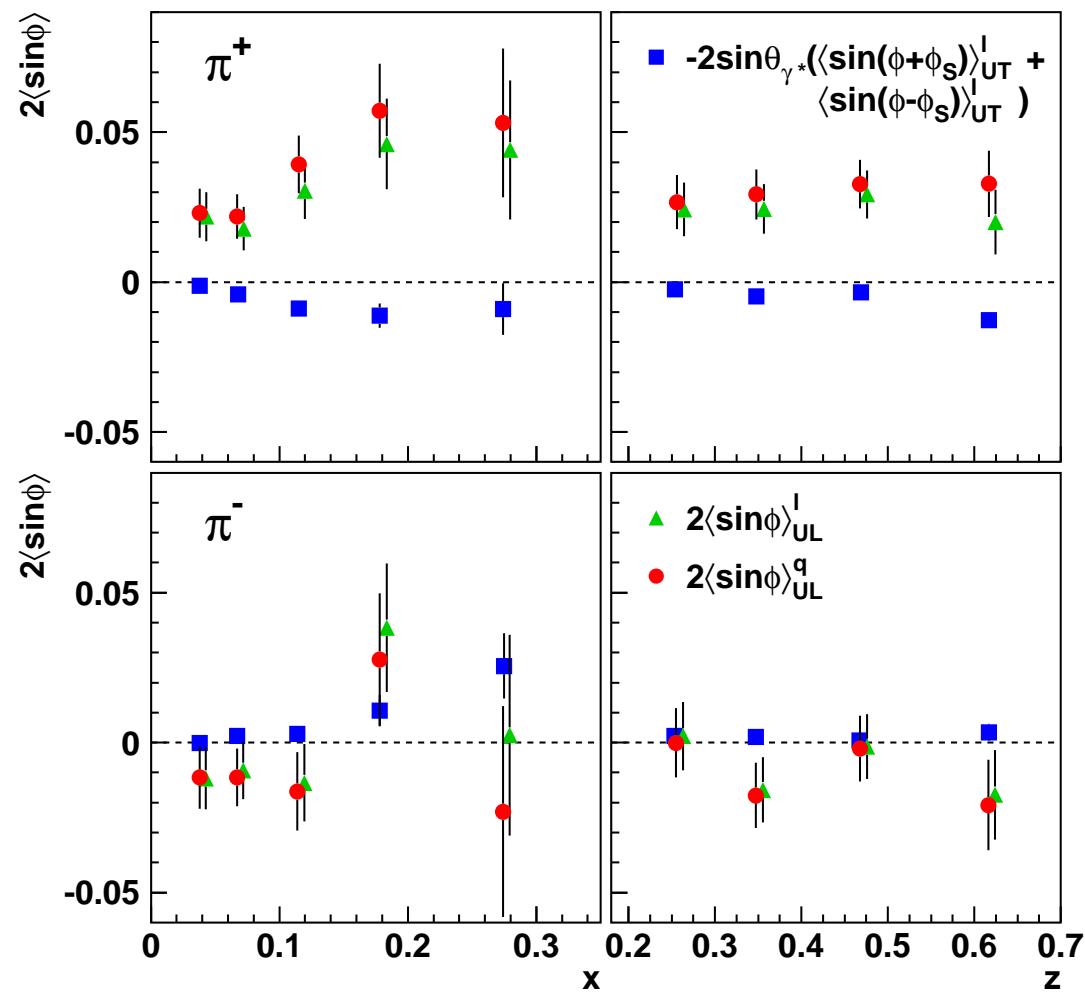
$$\begin{aligned} \langle \sin \phi \rangle_{UL}^q &\propto \frac{M}{Q} \mathcal{I} \left[\frac{\hat{P}_{h\perp} k_T}{M_h} \left(\frac{M_h}{z M} g_1 G^\perp + x h_L H_1^\perp \right) \right. \\ &\quad \left. + \frac{\hat{P}_{h\perp} p_T}{M} \left(\frac{M_h}{z M} h_{1L}^\perp \tilde{H} - x f_L^\perp D_1 \right) \right] \end{aligned}$$

Bacchetta et al., Phys. Lett. B 595 (2004) 309

⇒ they are all subleading-twist expressions!

- | | |
|--|--|
| $\langle \sin \phi \rangle_{UL}^\perp$ | ... Airapetian et al., Phys. Rev. Lett. 84 (2000) 4047 |
| $\langle \sin(\phi \pm \phi_S) \rangle_{UT}^\perp$ | ... Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002 |

$$\langle \sin \phi \rangle_{UL}^q = \langle \sin \phi \rangle_{UL}^I + \sin \theta_{\gamma^*} \left(\langle \sin(\phi + \phi_S) \rangle_{UT}^I + \langle \sin(\phi - \phi_S) \rangle_{UT}^I \right)$$



- twist-3 dominates measured asymmetries on longitudinally polarized targets!
- significantly positive for π^+
- consistent with zero for π^-
- twist-3 not necessarily small

Airapetian et al., Phys. Lett. B 622 (2005) 14

Conclusions

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- $\sin \phi$ amplitudes on long. polar. target dominated by twist-3

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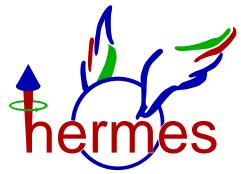
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Backup Slides



Extracting Quark Distributions

Purity Formalism

$$\begin{aligned} A_{UT}^{\sin(\phi - \phi_S), h}(x) &= \mathcal{C} \cdot \frac{\sum_q e_q^2 f_{1T}^{\perp(1), q}(x) \int dz D_1^{q, h}(z) \mathcal{A}(x, z)}{\sum_{q'} e_{q'}^2 f_1^{q'}(x) \int dz D_1^{q', h}(z) \mathcal{A}(x, z)} \\ &= \mathcal{C} \cdot \sum_q \frac{e_q^2 f_1^q(x) \mathcal{D}_1^{q, h}(x)}{\sum_{q'} e_{q'}^2 f_1^{q'}(x) \mathcal{D}_1^{q', h}(x)} \cdot \frac{f_{1T}^{\perp(1), q}}{f_1^q}(x) \\ &= \mathcal{C} \cdot \sum_q \mathcal{P}_q^h(x) \cdot \frac{f_{1T}^{\perp(1), q}}{f_1^q}(x) \end{aligned}$$

- **purities** are completely **unpolarized** objects → present Monte Carlo-tunes can be used
- probabilistic interpretation of purities possible
- “easy”: Sivers ← fragmentation function (D_1) known

$$\begin{aligned}
 A_{UT}^{\sin(\phi+\phi_S), h}(x) &= \mathcal{C} \cdot \frac{\sum_q e_q^2 \, h_1^q(x) \int dz \, H_1^{\perp(1), q, h}(z) \mathcal{A}(x, z)}{\sum_{q'} e_{q'}^2 \, f_1^{q'}(x) \int dz \, D_1^{q', h}(z) \mathcal{A}(x, z)} \\
 &= \mathcal{C} \cdot \sum_q \frac{e_q^2 \, f_1^q(x) \, \mathcal{H}_1^{\perp(1), q, h}(x)}{\sum_{q'} e_{q'}^2 \, f_1^{q'}(x) \, \mathcal{D}_1^{q', h}(x)} \cdot \frac{h_1^q}{f_1^q}(x) \\
 &= \mathcal{C} \cdot \sum_q \mathcal{P}_q^h(x) \cdot \frac{h_1^q}{f_1^q}(x)
 \end{aligned}$$

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- probabilistic interpretation of purities possible
- “easy”: Sivers ← fragmentation function (D_1) known
- Collins: these purities still **depend on parametrization** of Collins FF function