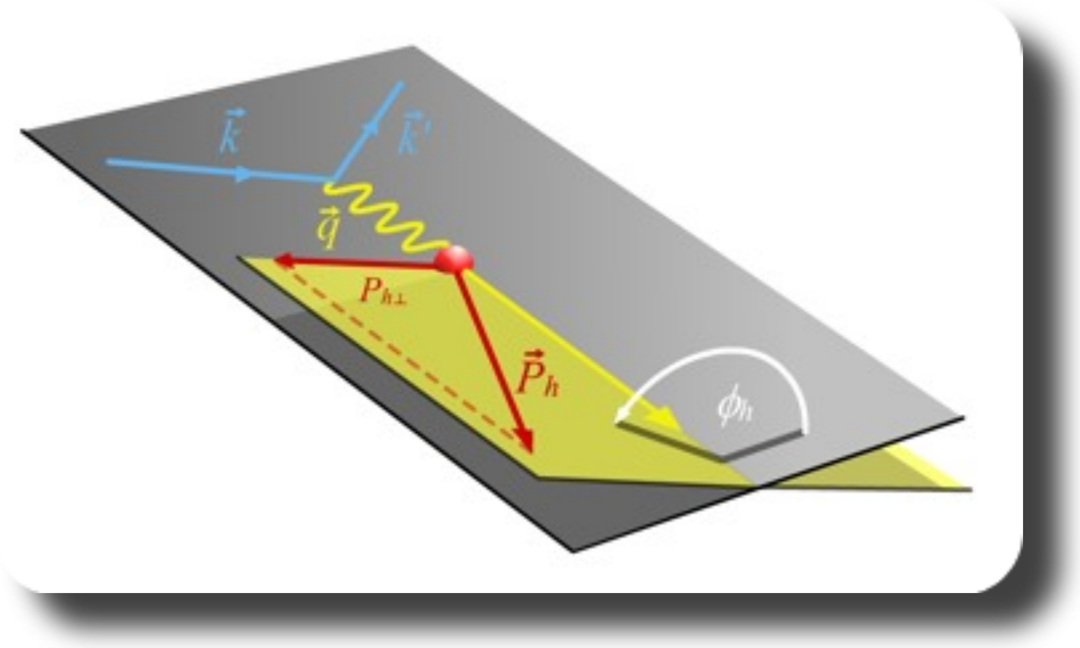
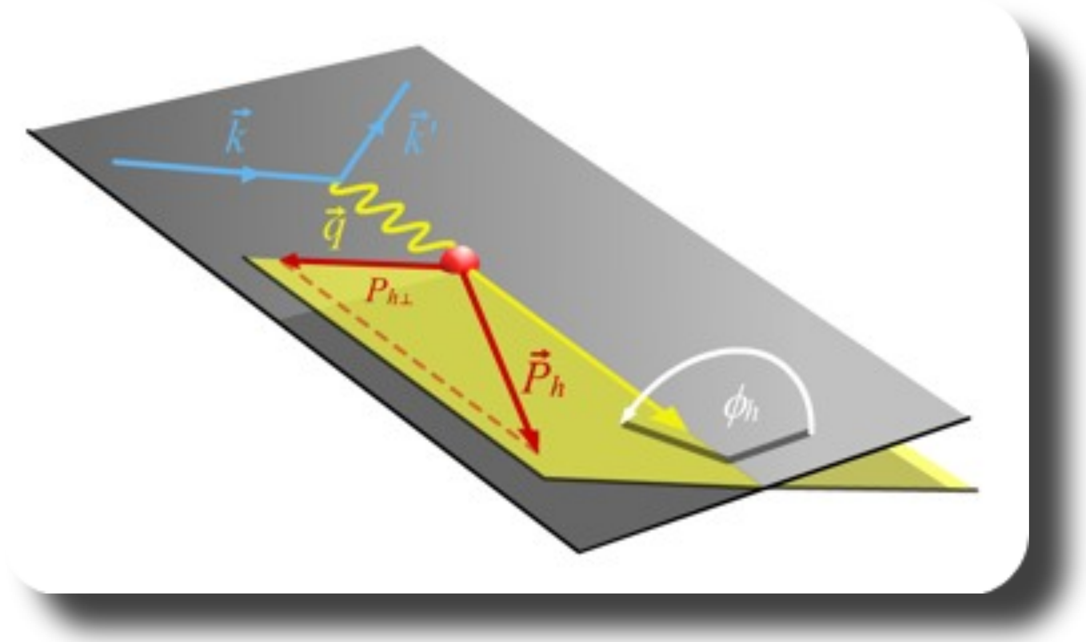
# Flavor dependent azimuthal cosine modulations in SIDIS unpolarized cross section

Francesca Giordano

November 17th, 2010, Urbana, Champaign, IL



# *Semi Inclusive Deep Inelastic Scattering*



## Collinear case

$$\frac{d^3\sigma}{dx\,dy\,dz} = \frac{\alpha^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\{A(y)F_{UU,T} + B(y)F_{UU,L}\}$$

| | |
|---|---|
| $Q^2$ | **Negative squared 4-momentum transfer to the target** |
| y | **Fractional energy of the virtual photon** |
| x | **Bjorken scaling variable** |
| z | **Fractional energy transfer to the produced hadron** |

**target polarization**

$$F_{X\overset{\Uparrow}{Y},Z} = F_{XY,Z}(x,y,z)$$

**beam polarization**

**virtual photon polarization**

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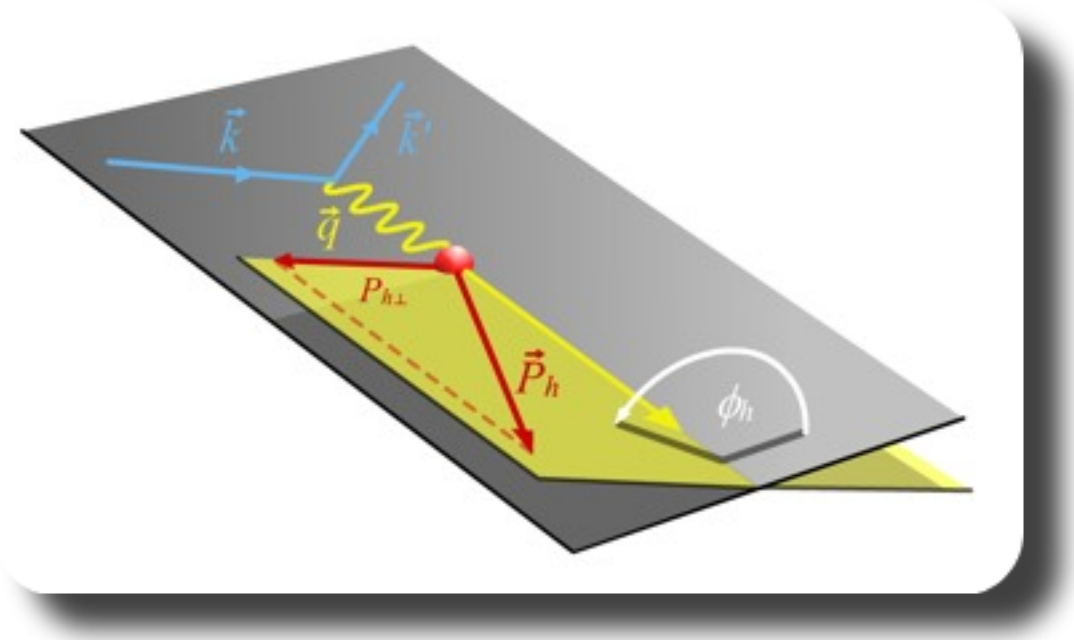

$$\frac{d^5\sigma}{dxdydzd\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\{A(y)F_{UU,T} + B(y)F_{UU,L}$$

$$+C(y)\cos\phi_h F_{UU}^{\cos\phi_h} + B(y)\cos 2\phi_h F_{UU}^{\cos 2\phi_h}\}$$

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$$F_{\underset{\underset{\text{beam polarization}}{\Downarrow}}{X}\overset{\overset{\text{target polarization}}{\Uparrow}}{Y},\underset{\underset{\text{virtual photon polarization}}{\Downarrow}}{Z}} = F_{XY,Z}(x, y, z, P_{h\perp})$$

# Semi Inclusive Deep Inelastic Scattering

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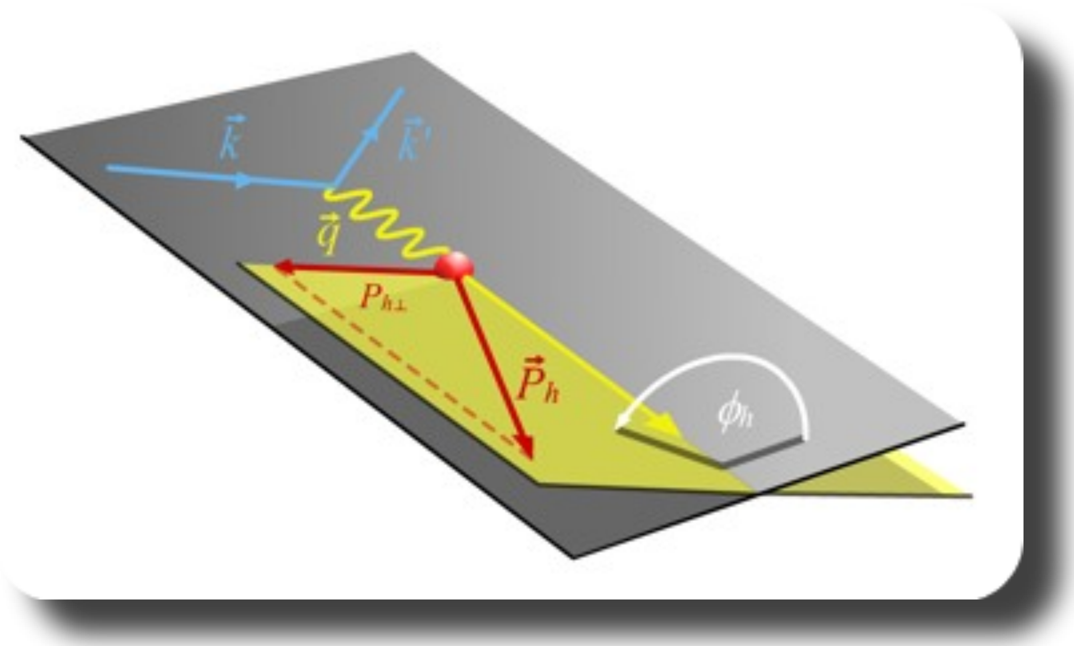

$$\langle \cos n\phi_h \rangle = \frac{\int \cos n\phi_h \frac{d^5\sigma}{dx dy dz d\phi_h dP_{h\perp}^2} d\phi_h}{\int \frac{d^5\sigma}{dx dy dz d\phi_h dP_{h\perp}^2} d\phi_h}$$

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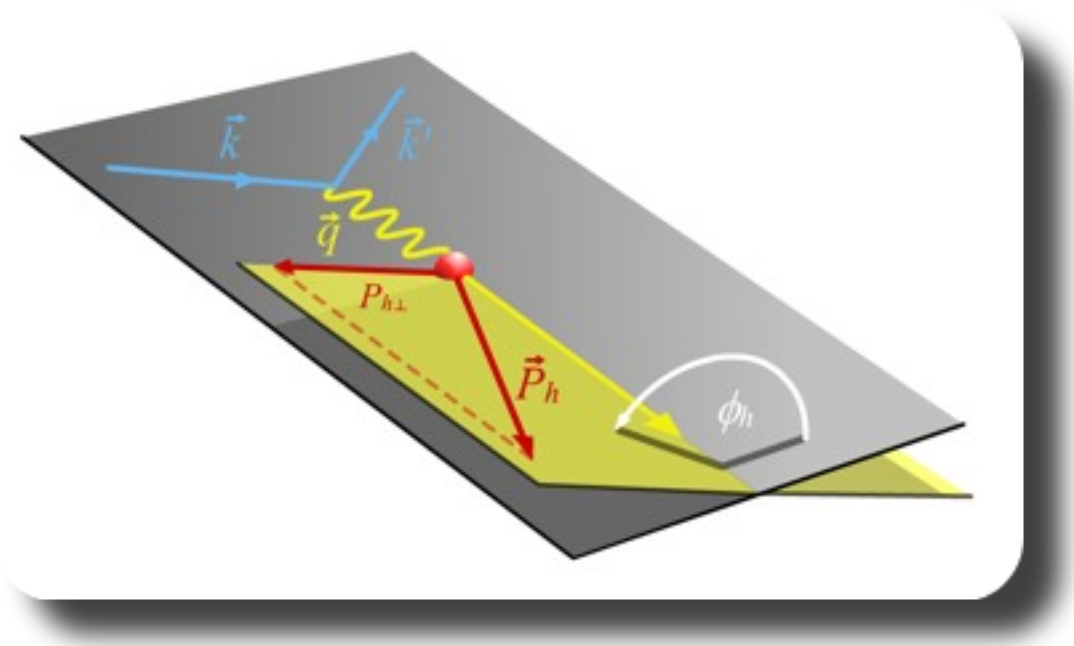

$$\langle \cos n\phi_h \rangle = \frac{\int \cos n\phi_h \frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP^2_{h\perp}} d\phi_h}{\int \frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP^2_{h\perp}} d\phi_h}$$

$$\boxed{\langle \cos \phi_h \rangle \approx \frac{C(y)}{2A(y)} \frac{F_{UU}^{\cos\phi_h}}{F_{UU,T}}}$$

$$\boxed{\langle \cos 2\phi_h \rangle \approx \frac{B(y)}{2A(y)} \frac{F_{UU}^{\cos 2\phi_h}}{F_{UU,T}}}$$

$$\frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP^2_{h\perp}} = \frac{\alpha^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\{A(y)F_{UU,T} + B(y)F_{UU,L}$$
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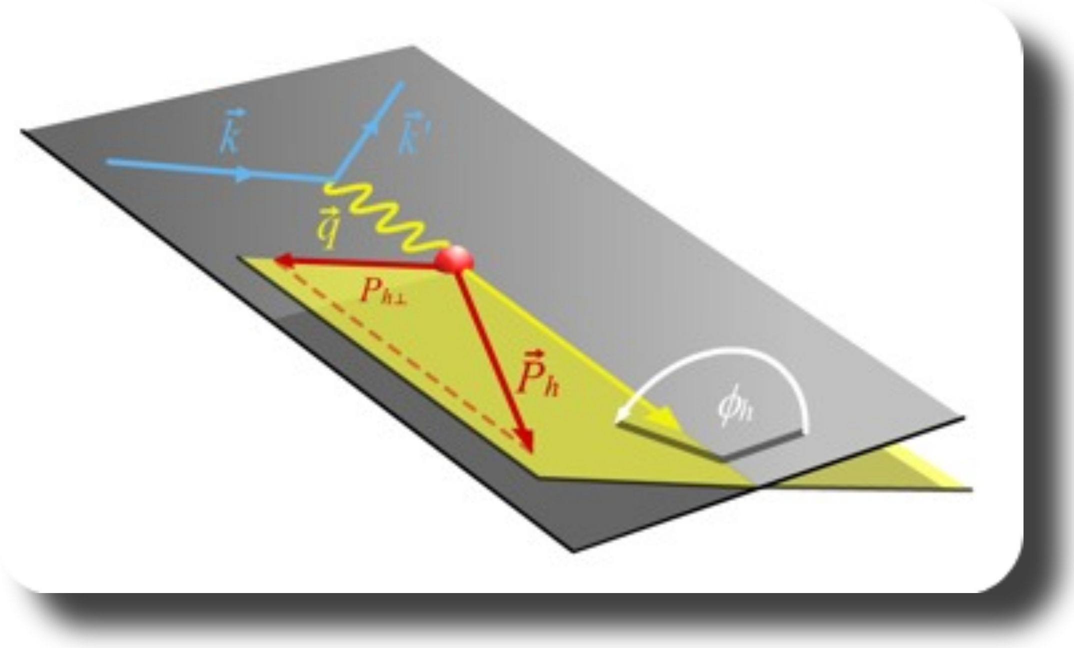
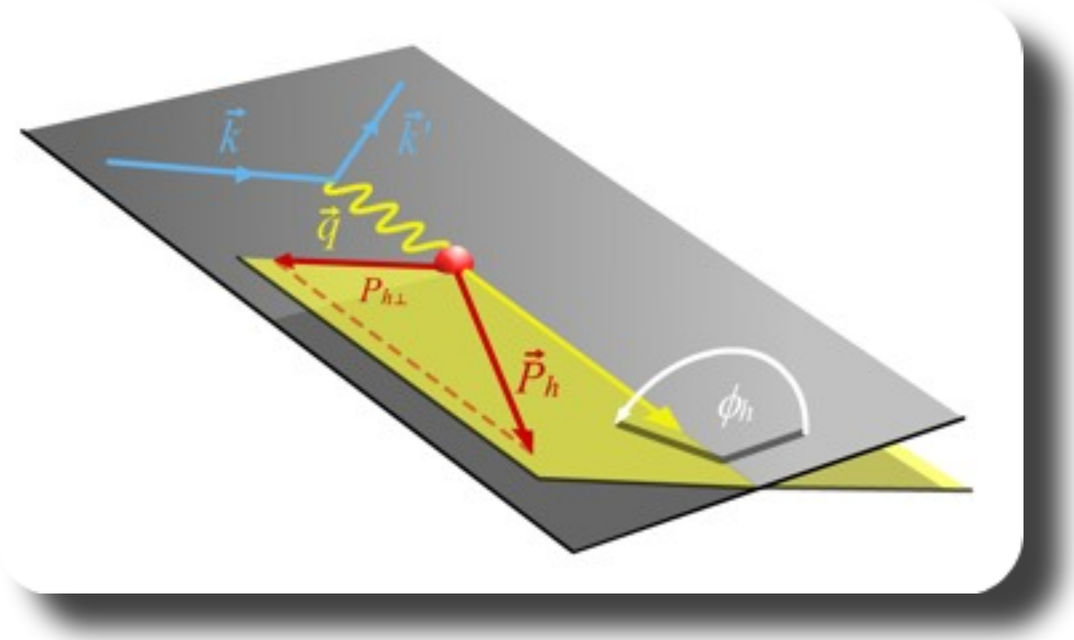
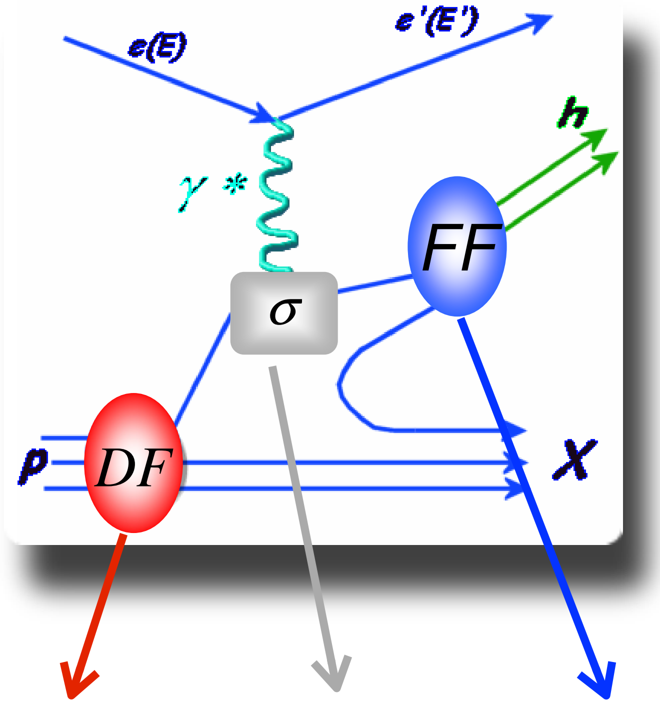


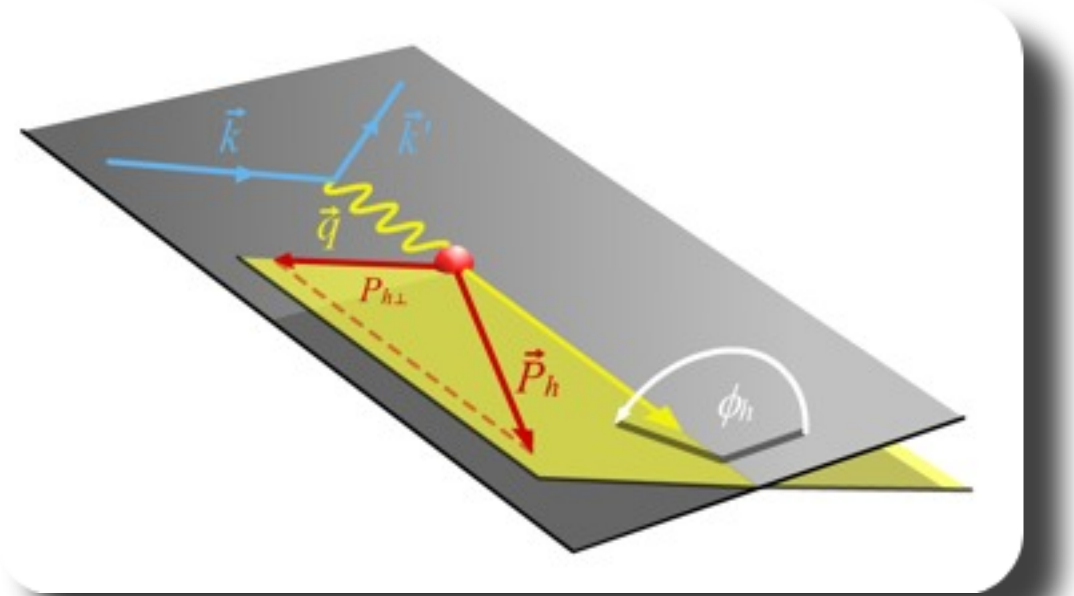
# *Transverse Momentum Dependent Functions (TMDs)*



Factorization Theorem



$$\sigma^{ep\to ehX} = \sum_q DF \times \sigma^{eq\to eq} \times FF$$



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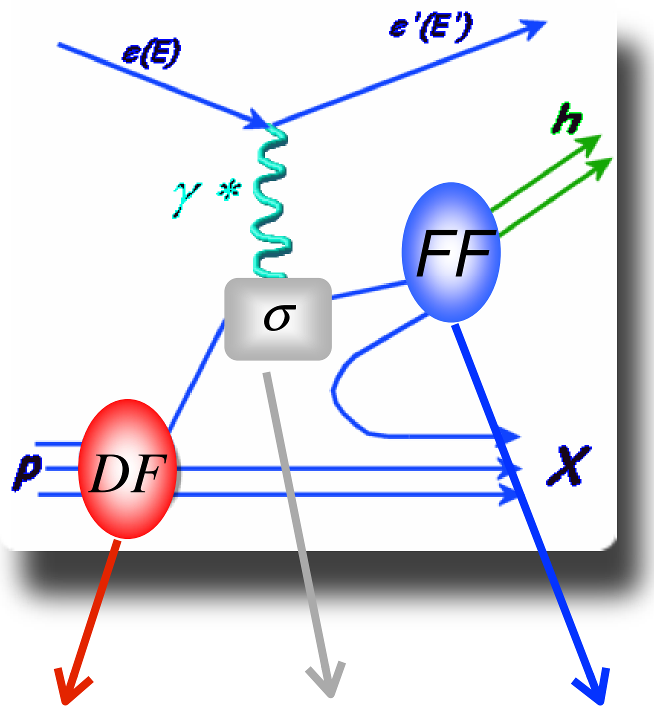

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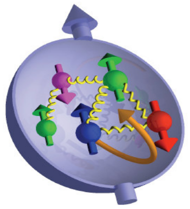
## distribution functions

|  | | quark | | |
|---|---|---|---|---|
| | | **U** | **L** | **T** |
| **nucleon** | **U** | $f_1$ | | $h_1^\perp$ |
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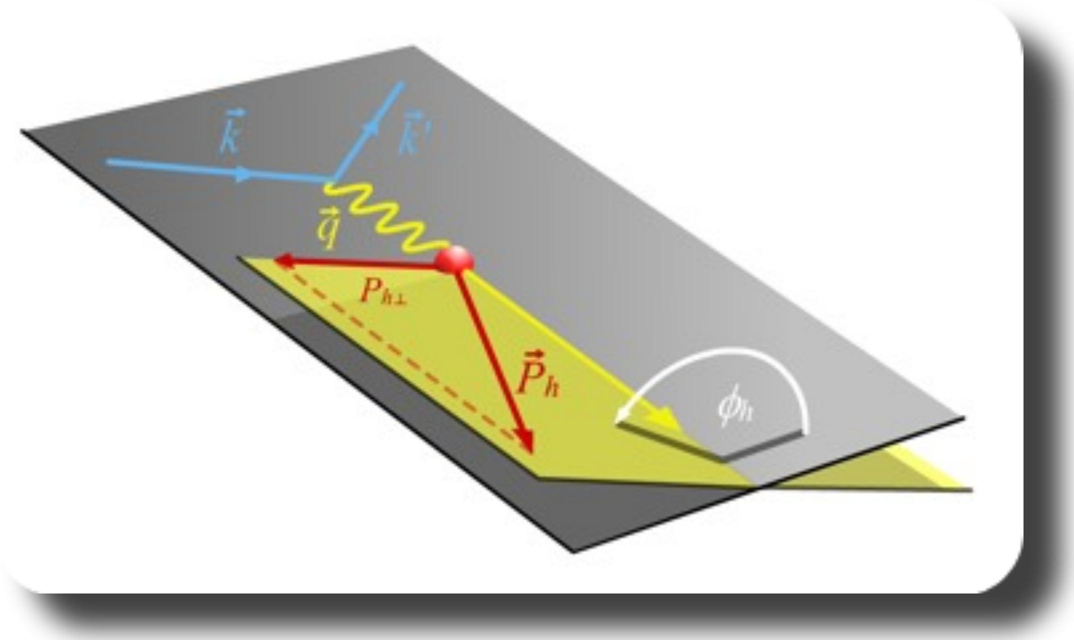

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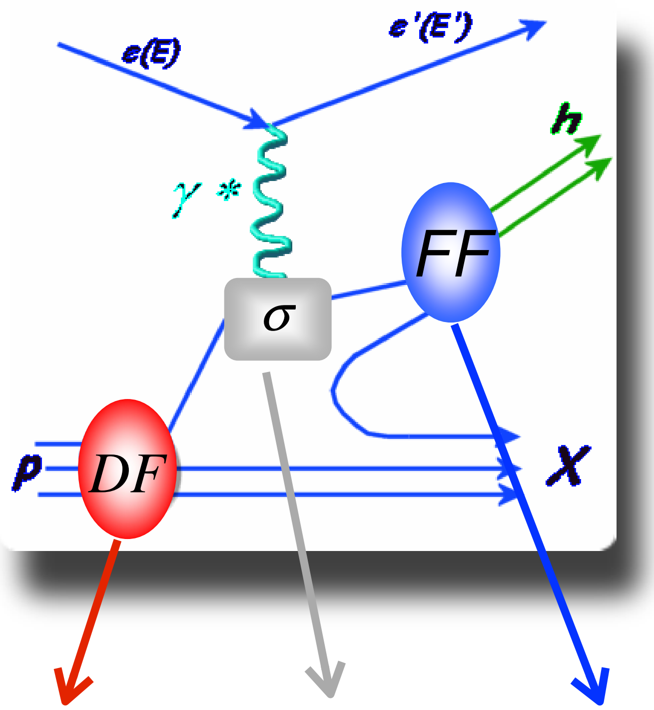

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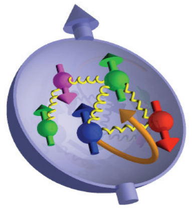
|  | | quark | | |
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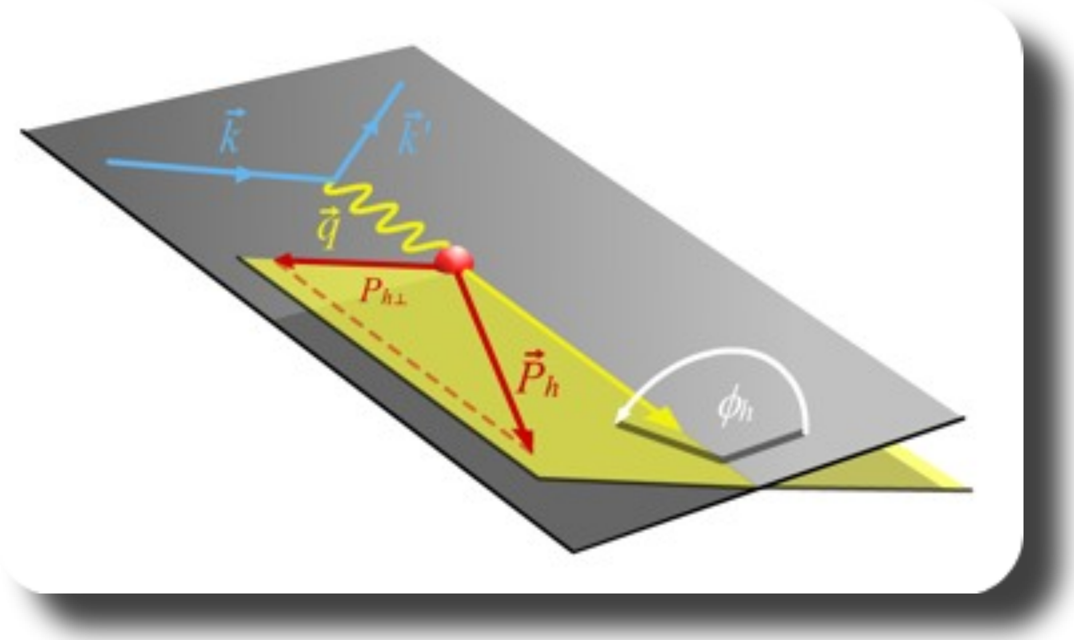
fragmentation functions

| | | **quark** | | |
|---|---|---|---|---|
| | | **U** | **L** | **T** |
| **had** | **U** | $D_1$ | | $H_1^\perp$ |



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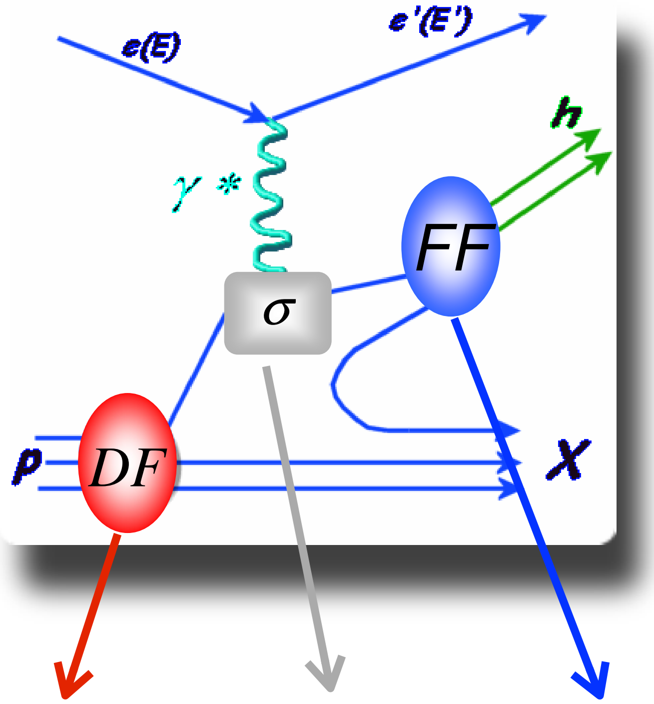

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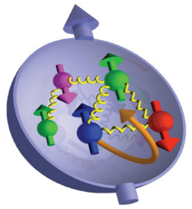
|  | | quark | | |
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fragmentation functions

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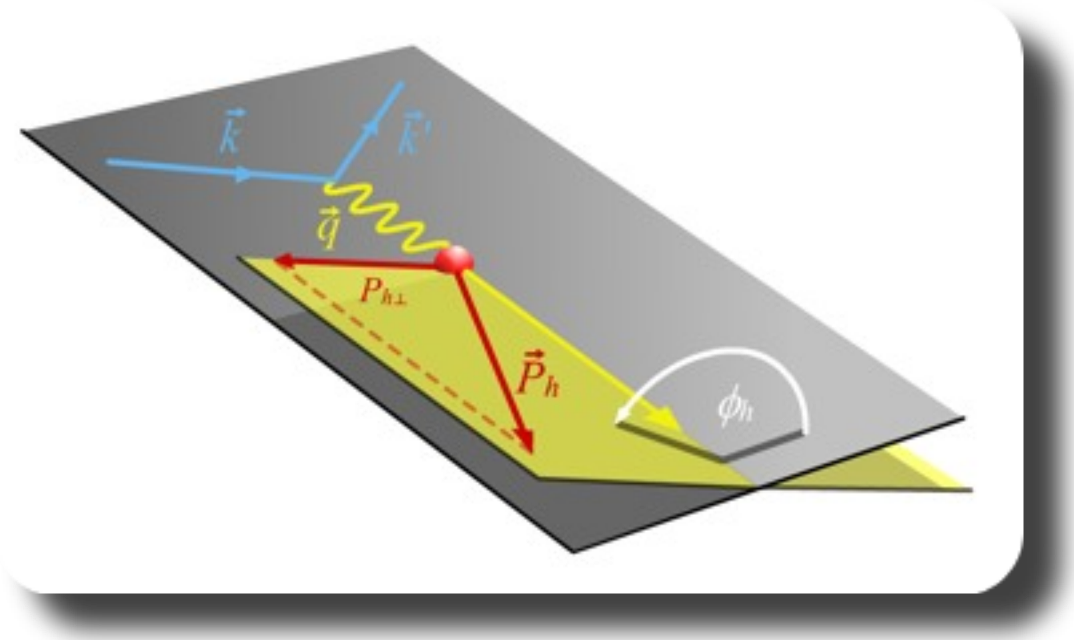

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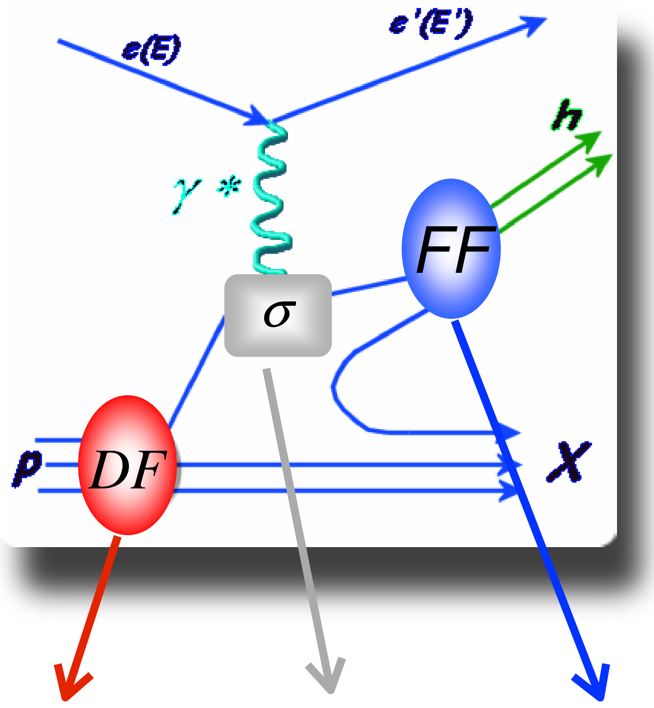

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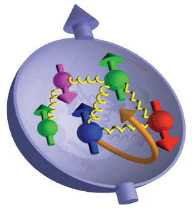
|  | | quark | | |
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Boer-Mulders DF

fragmentation functions

| | | quark | | |
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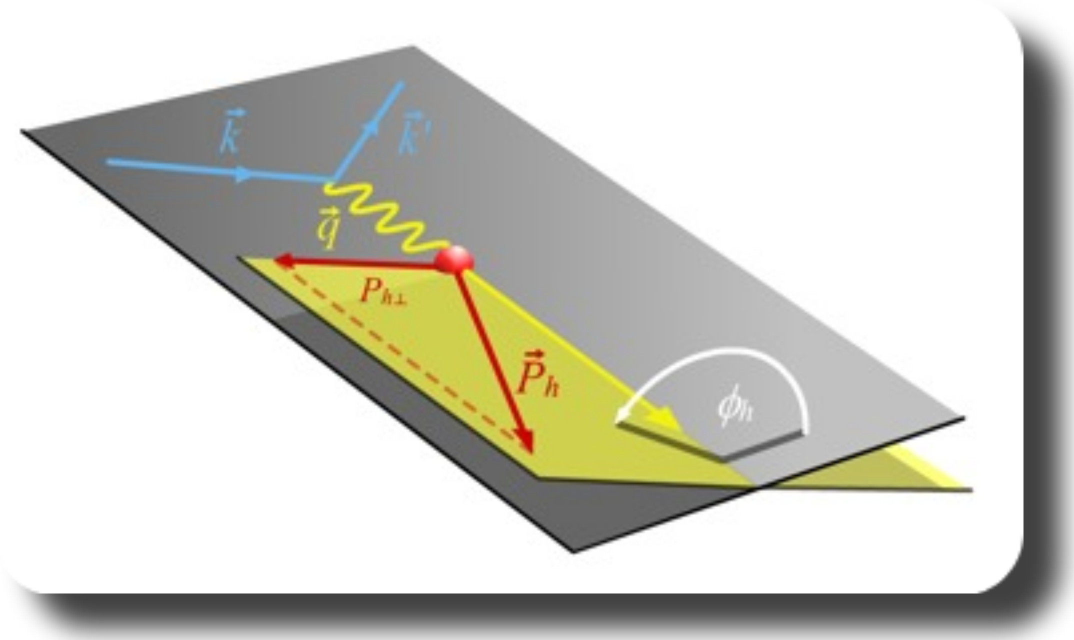

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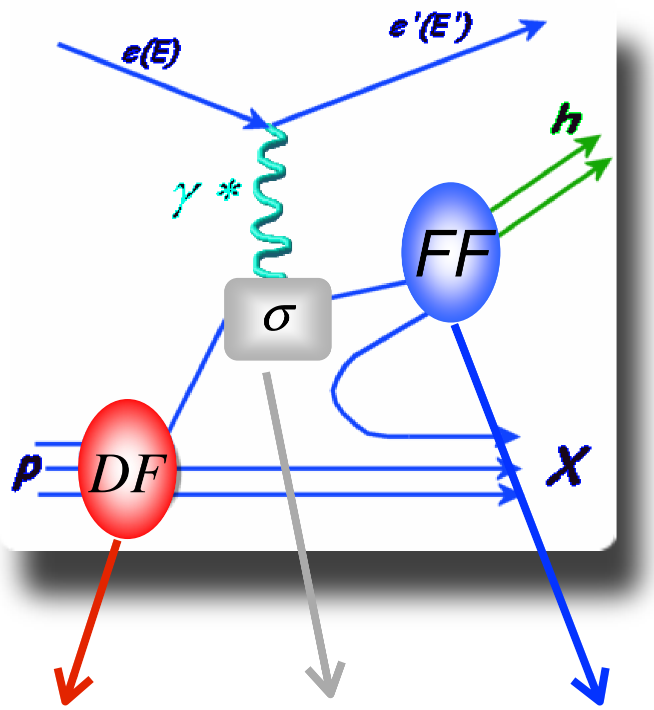

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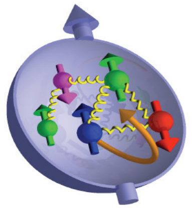
|  | | quark | | |
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Boer-Mulders DF

Collins FF

fragmentation functions

| | | quark | | |
|---|---|---|---|---|
| | | **U** | **L** | **T** |
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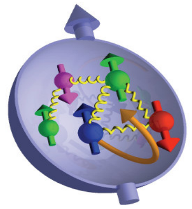
**Cahn effect** $\propto C[f_1 D_1]$

distribution functions

|  | | quark | | |
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fragmentation functions

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|---|---|---|---|---|
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## *Transverse **Momentum** **Dependent** Functions (TMDs)*

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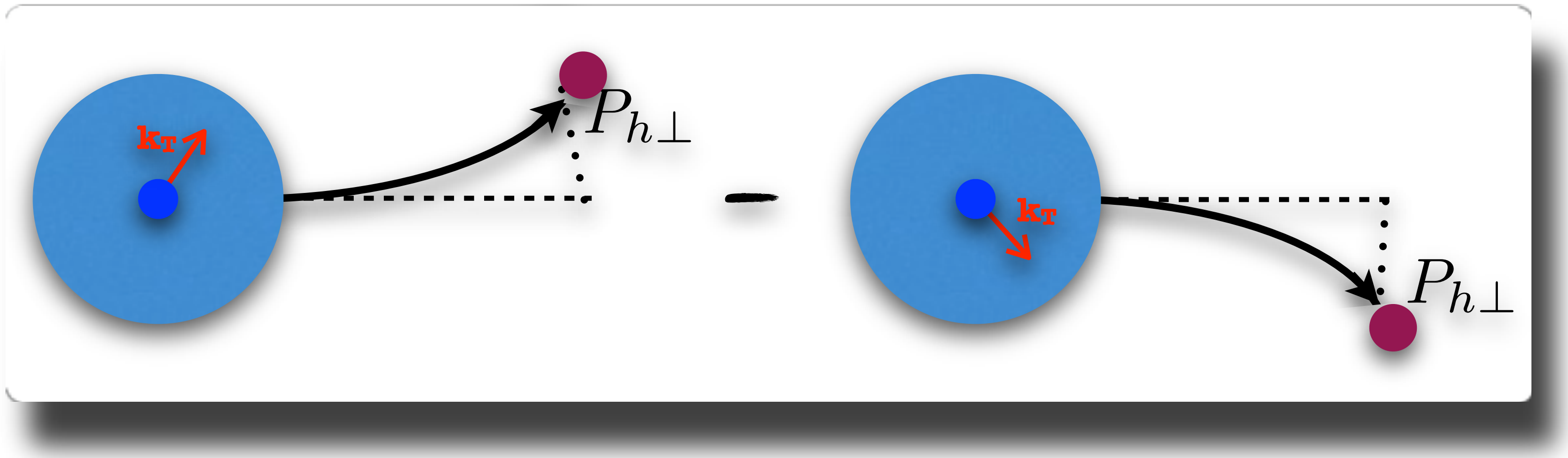
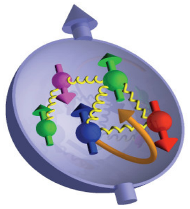


distribution functions

|  | | **quark** | | |
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fragmentation functions

| | | **quark** | | |
|---|---|---|---|---|
| | | **U** | **L** | **T** |
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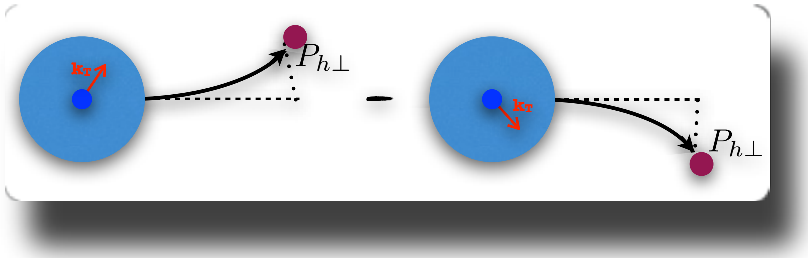
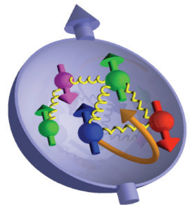
## *Transverse Momentum Dependent Functions (TMDs)*

**Cahn effect** $\propto C[f_1 D_1]$

✱ kinematic effect (1978)



distribution functions

|  | | **quark** | | |
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fragmentation functions

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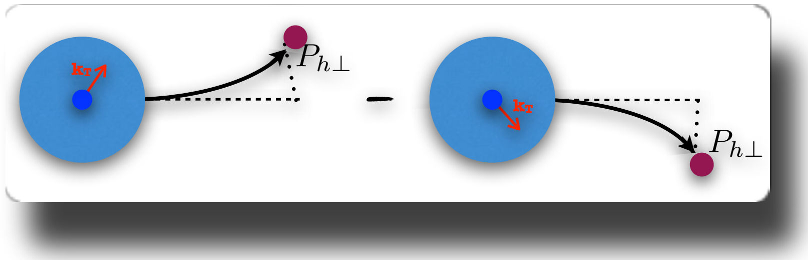
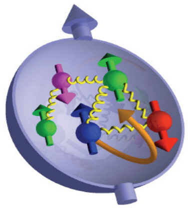


* kinematic effect (1978)

$$\langle \kappa_T^q \rangle \rightarrow \cos\phi_h^q \rightarrow \cos\phi_h^H$$

distribution functions

|  | | quark | | |
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fragmentation functions

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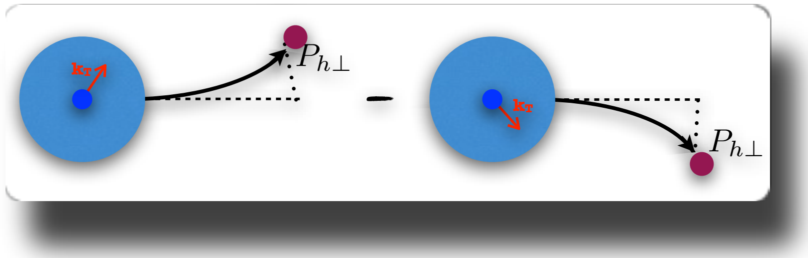

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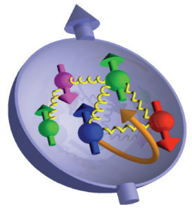
|  | | **U** | **L** | **T** |
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fragmentation functions

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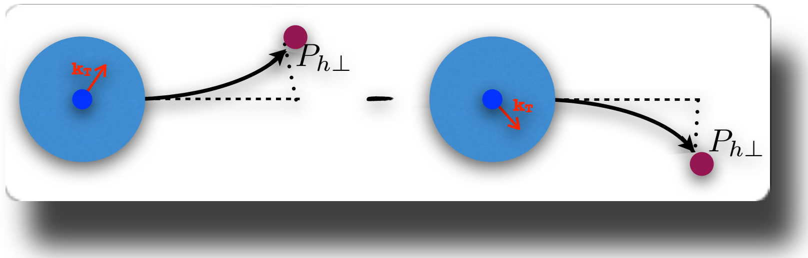

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- ✱ access to quark transverse momenta

**Boer-Mulders effect** $\propto C[-h_1^\perp H_1^\perp]$

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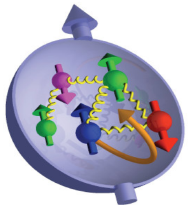
|  | | quark | | |
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fragmentation functions

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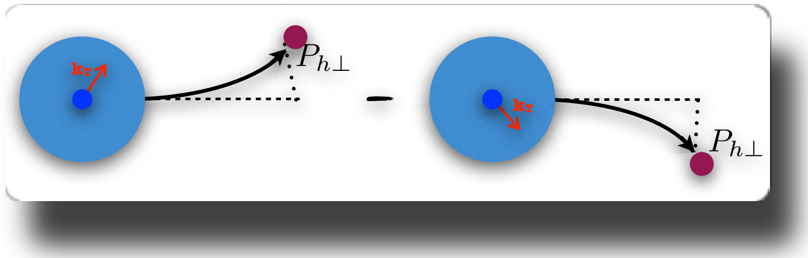

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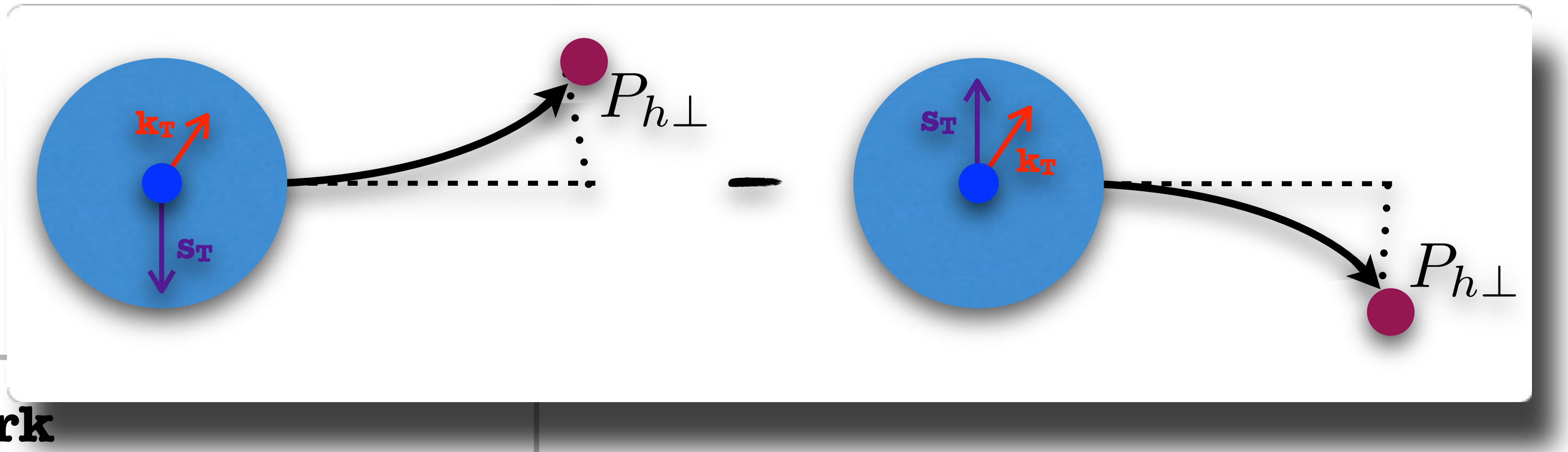

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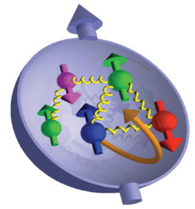
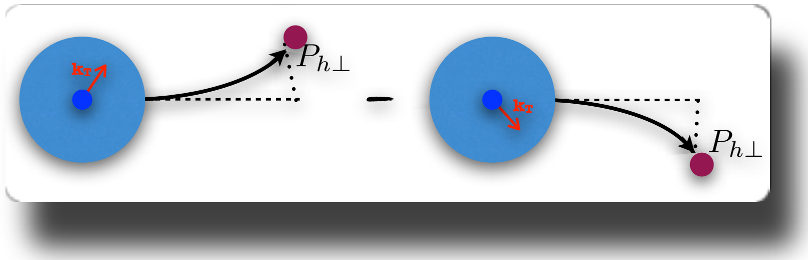
|  | | quark | | |
|---|---|---|---|---|
| | | **U** | **L** | **T** |
| **nucleon** | **U** | $f_1$ | | $h_1^\perp$ |
| | **L** | | $g_1$ | $h_{1L}^\perp$ |
| | **T** | $f_{1T}^\perp$ | $g_{1T}^\perp$ | $h_1$ $h_{1T}^\perp$ |

fragmentation functions

| | | **quark** | | |
|---|---|---|---|---|
| | | **U** | **L** | **T** |
| **had** | **U** | $D_1$ | | $H_1^\perp$ |

# *Transverse Momentum Dependent Functions (TMDs)*

**Cahn effect** $\propto C[f_1 D_1]$



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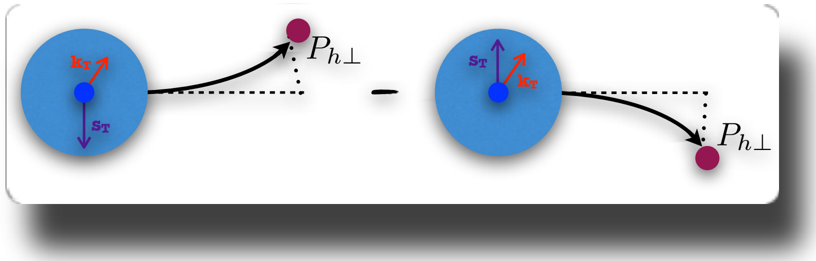

- * chiral odd functions

$$\underbrace{\underset{\text{chiral odd}}{h_1^\perp}\quad \underset{\text{chiral odd}}{H_1^\perp}}_{\text{chiral even!}}$$

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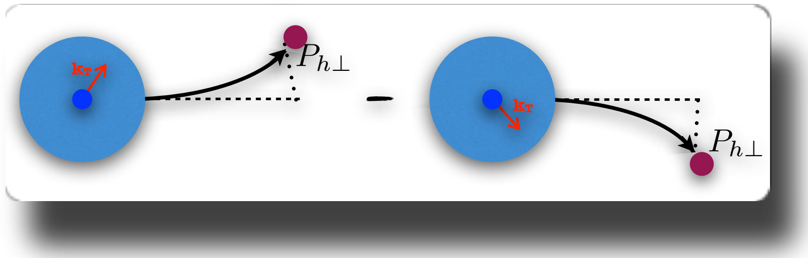

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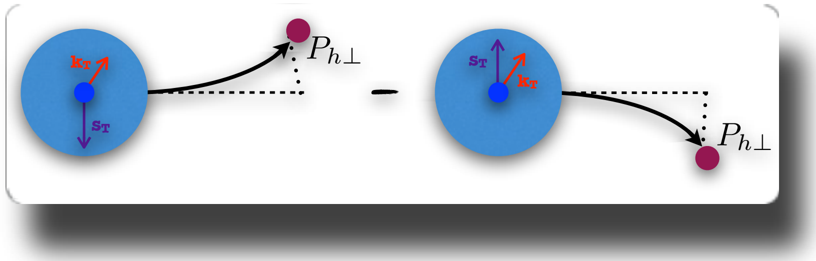

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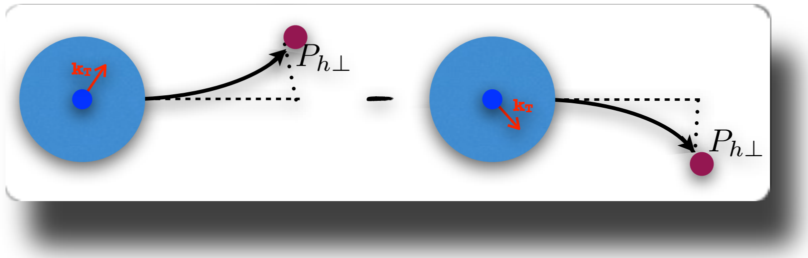

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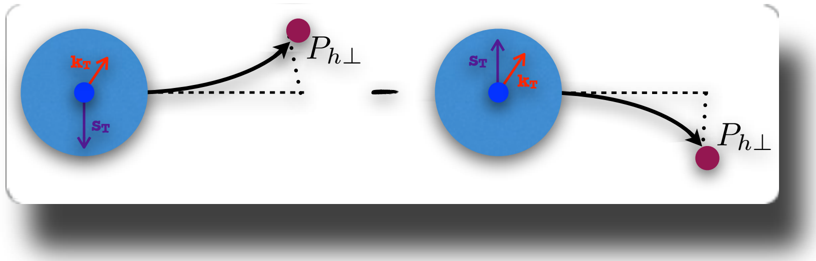

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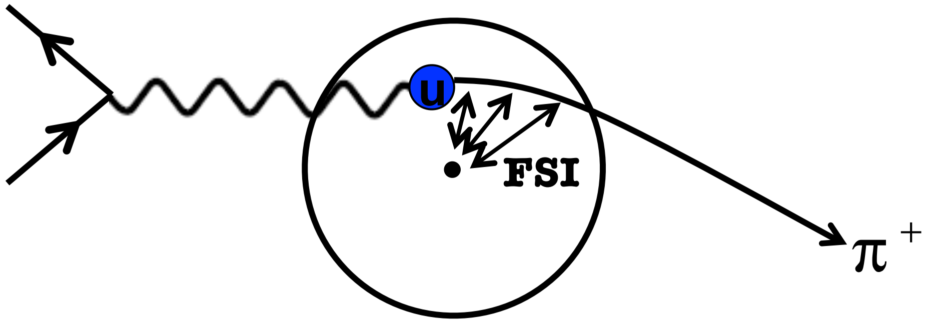

Final State Interactions

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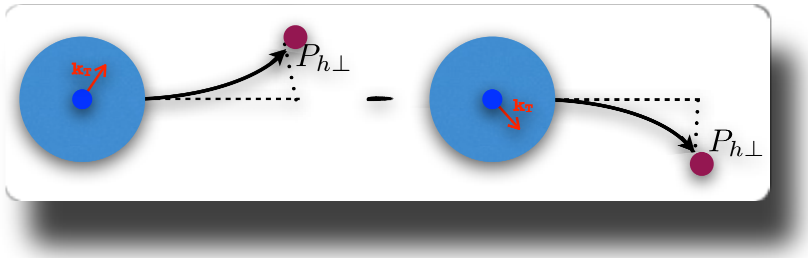

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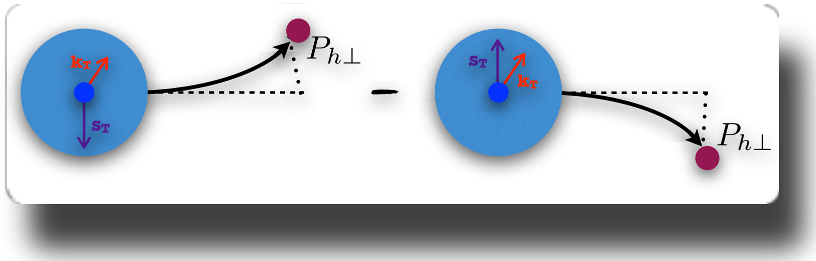

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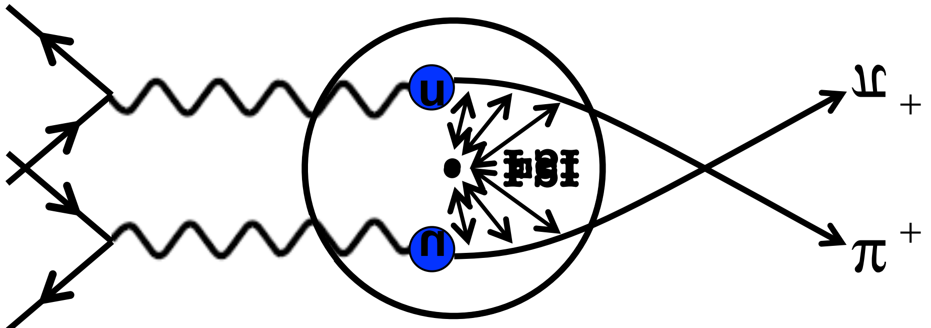

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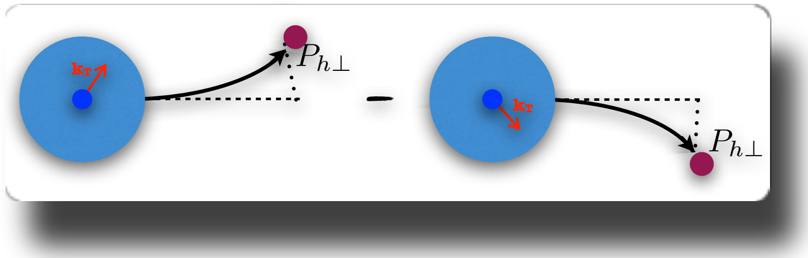

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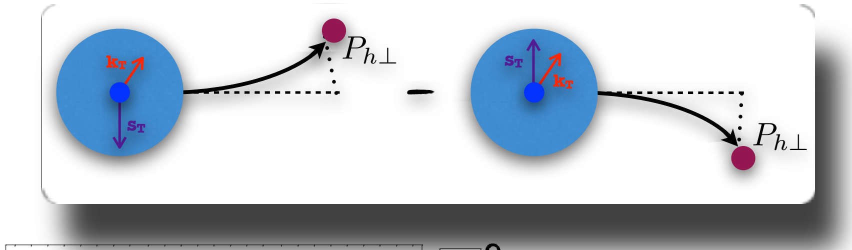

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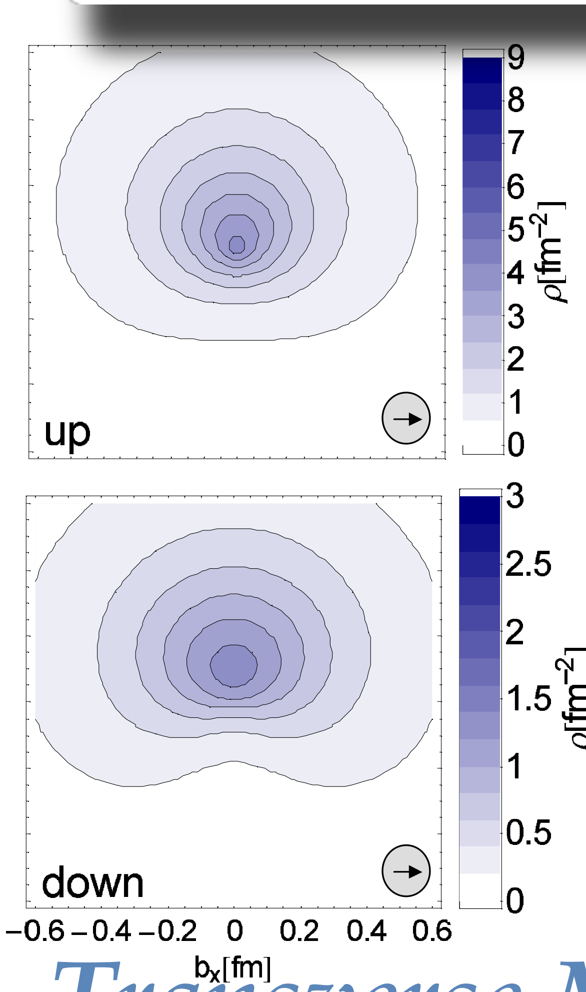

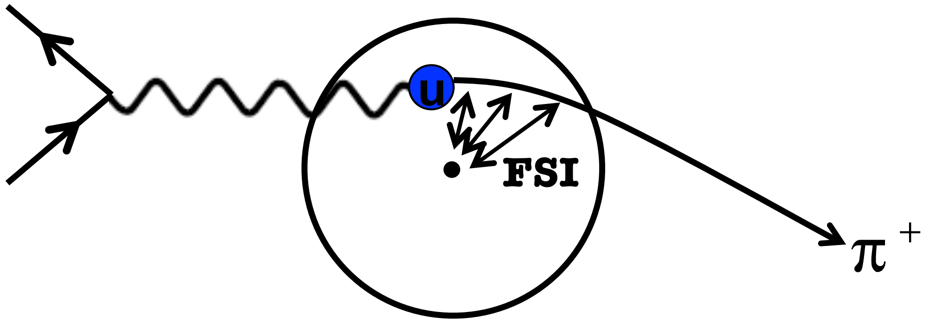

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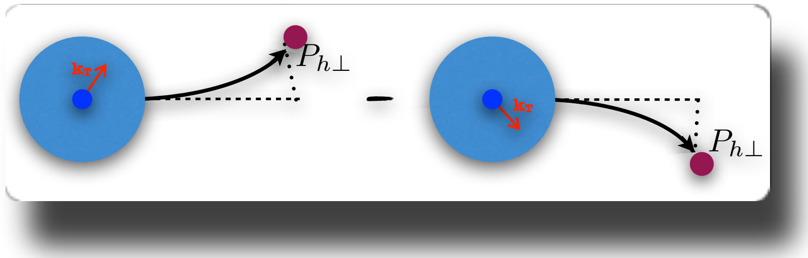

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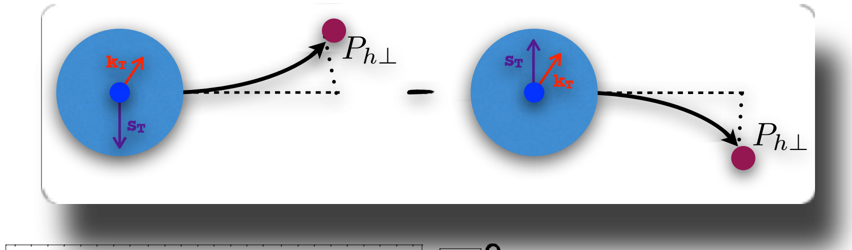

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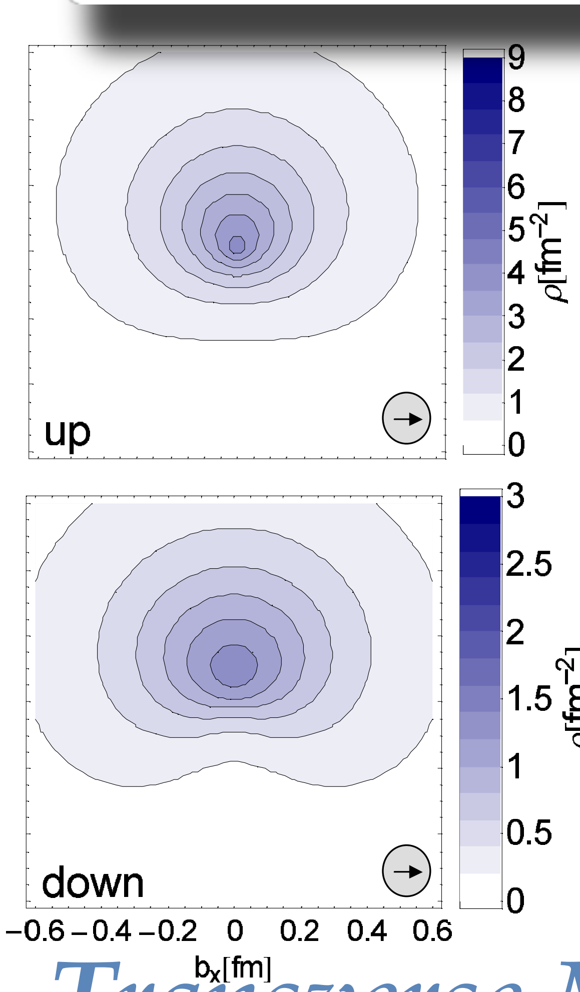

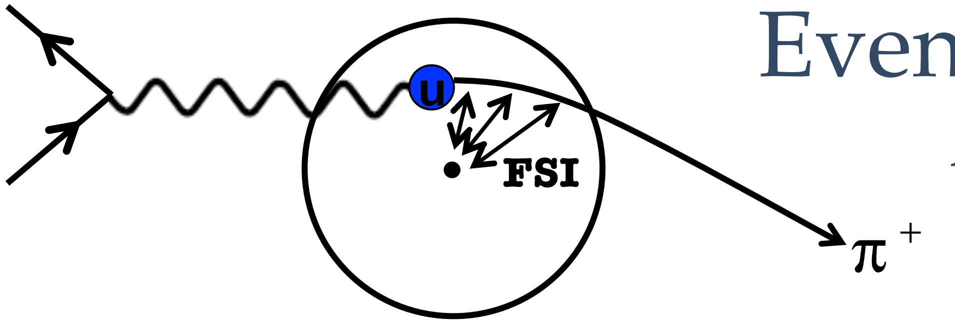

Final State Interactions

Even in unpolarized reactions the spin plays a role!

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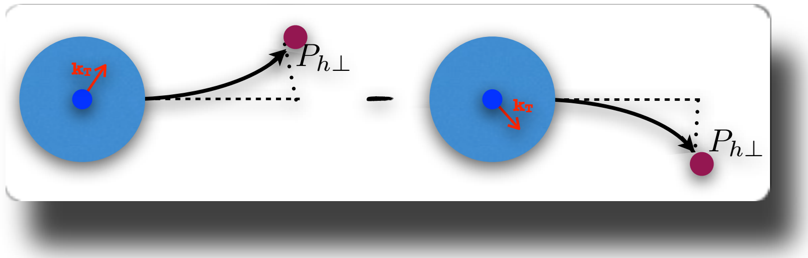

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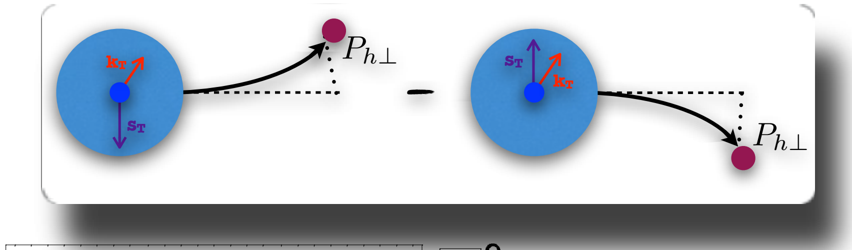

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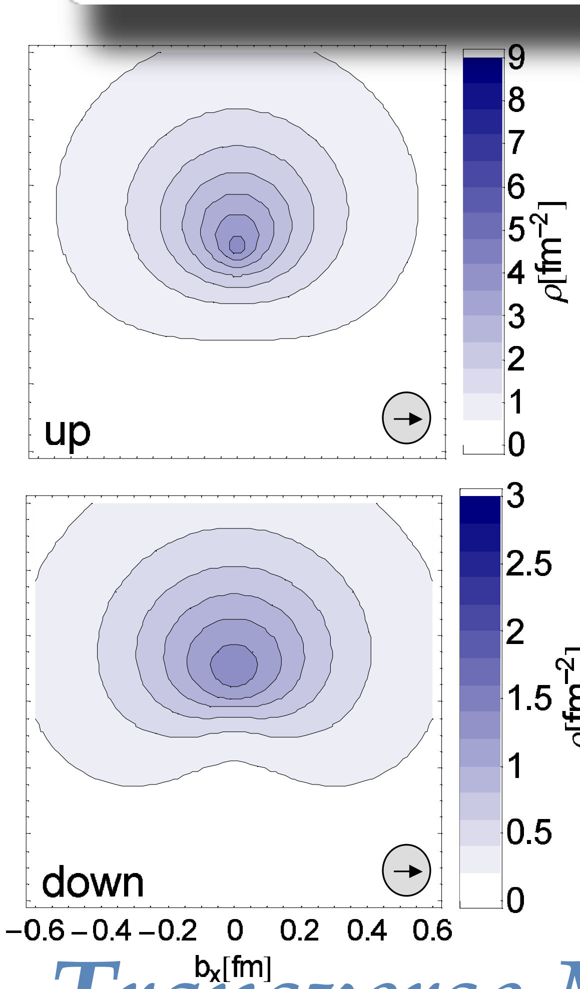

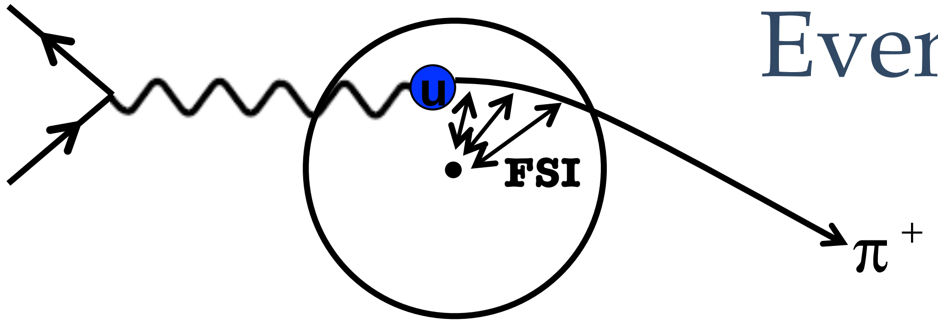

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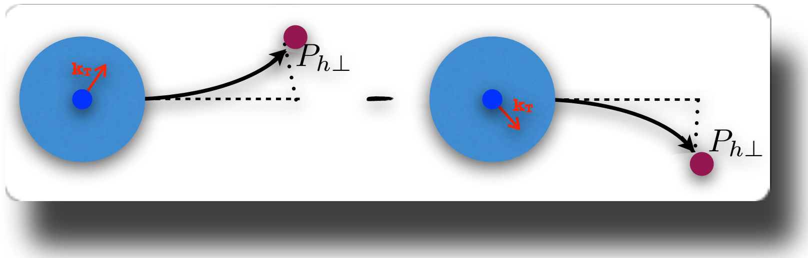

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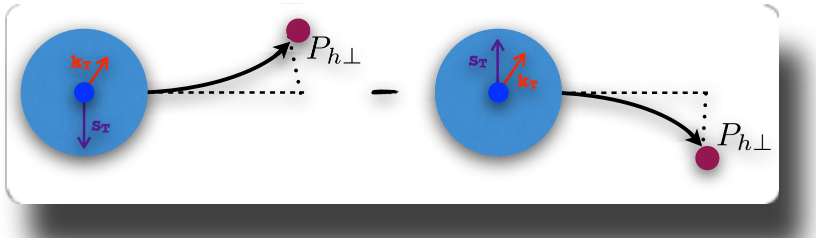

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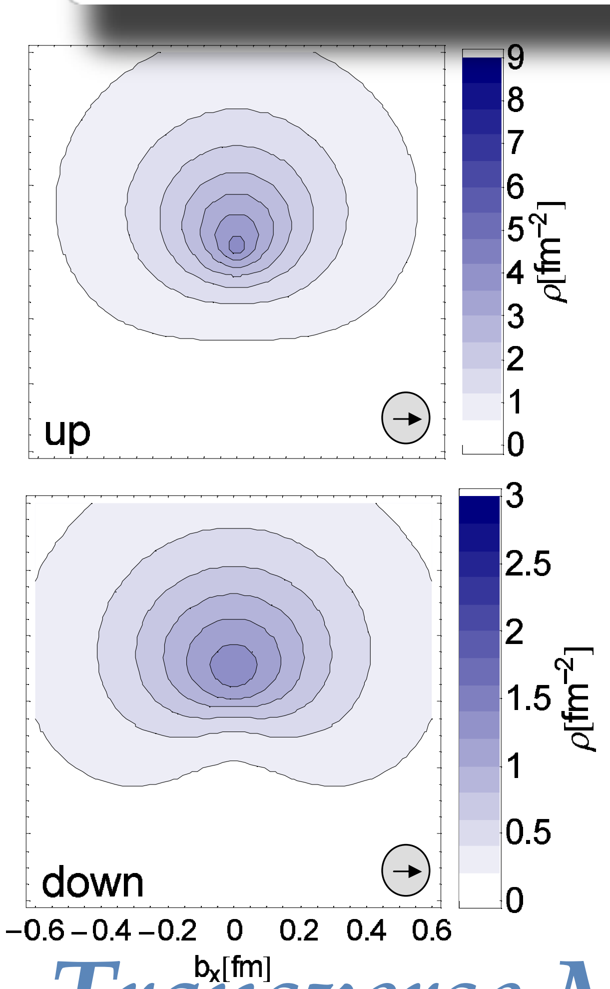

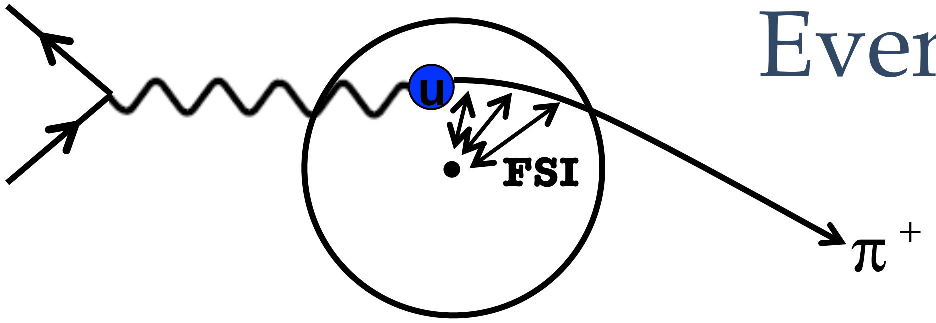

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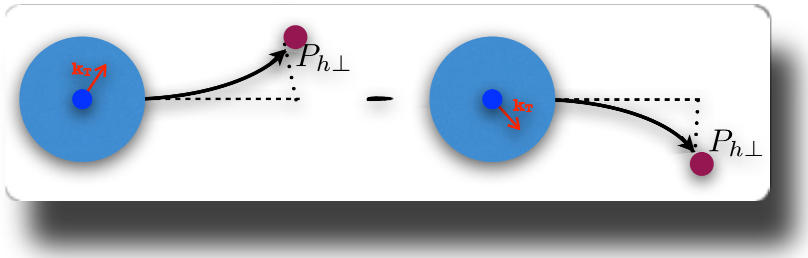

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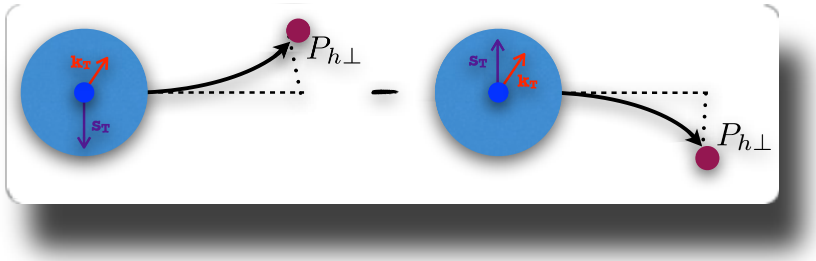

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implicit sum over quark flavors

# *Leading Twist Terms*

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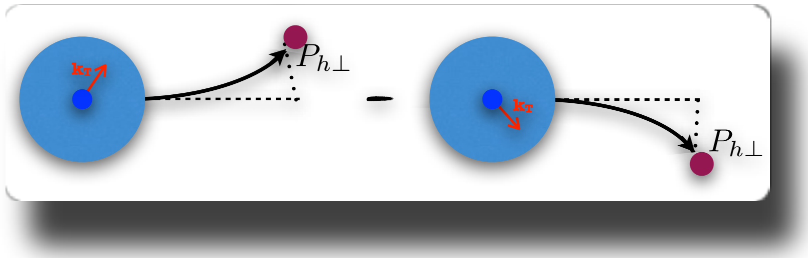

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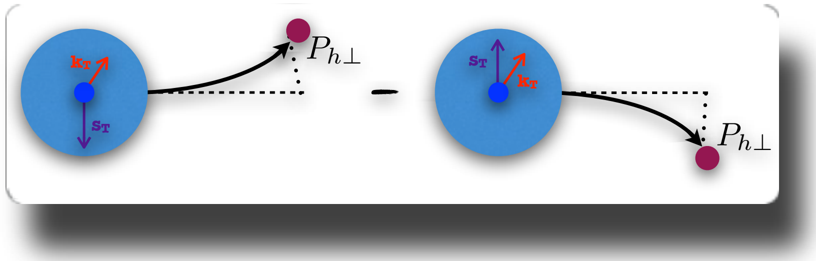

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# Leading And Next-to-Leading Twist Terms

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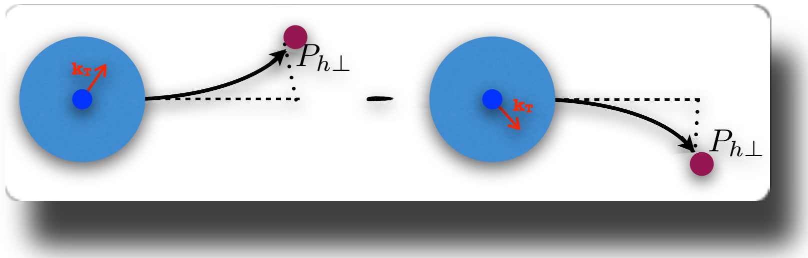

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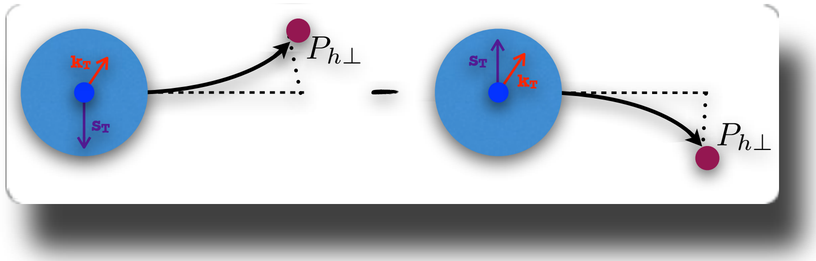

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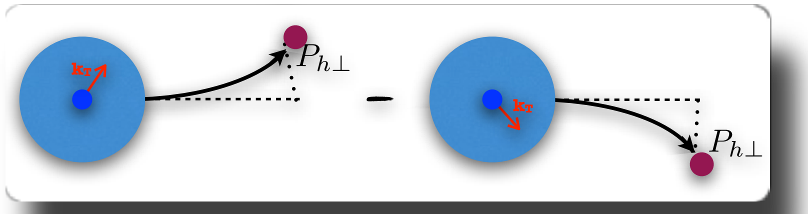

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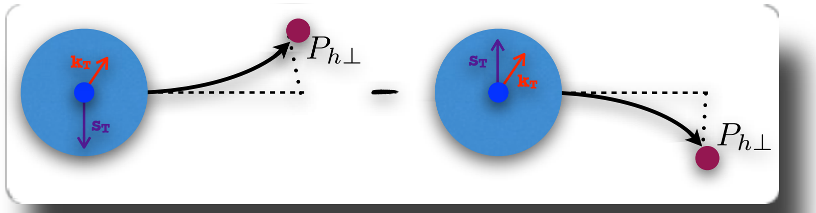

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implicit sum over quark flavors

interaction dependent terms neglected

Francesca Giordano

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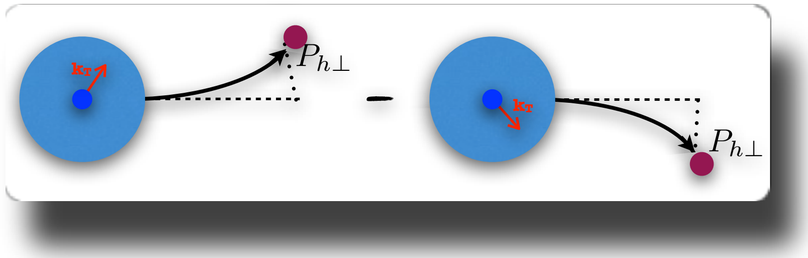

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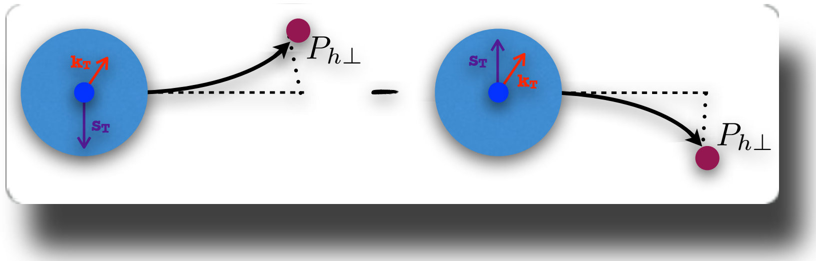

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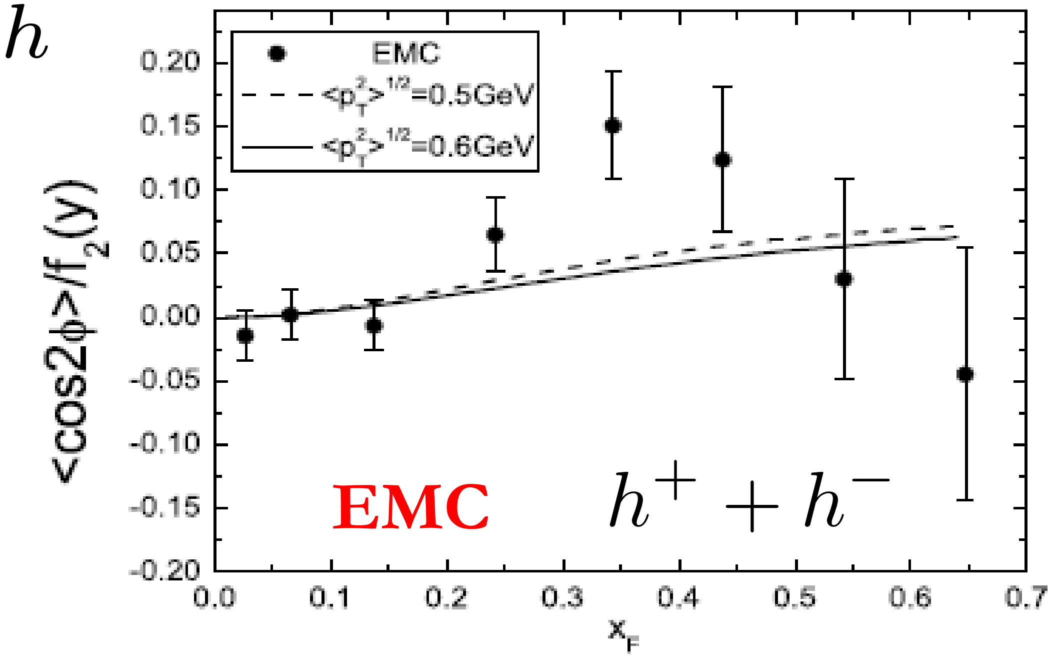
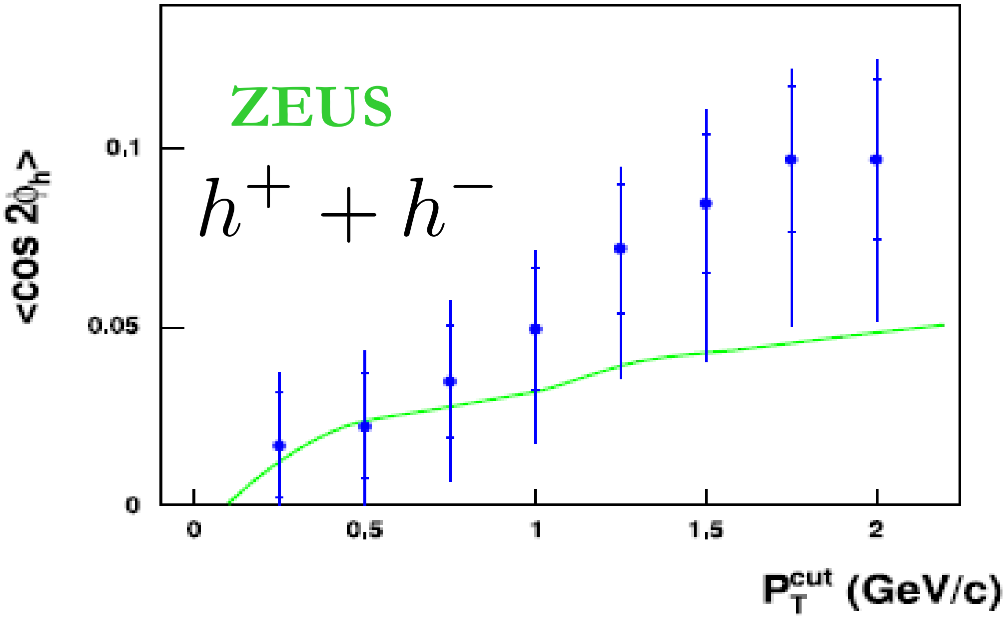
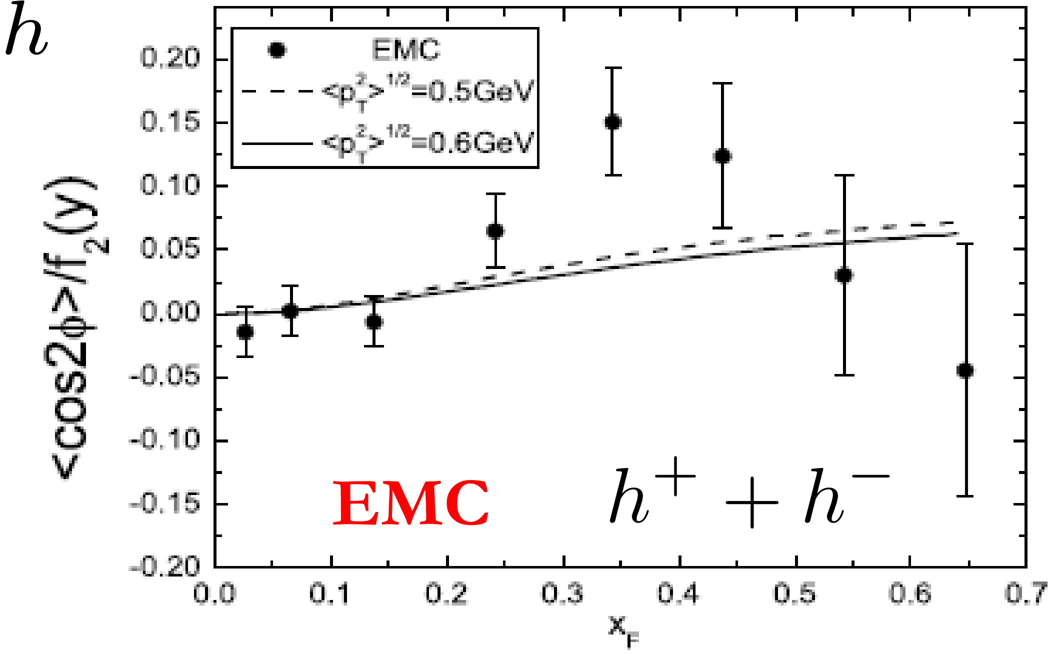
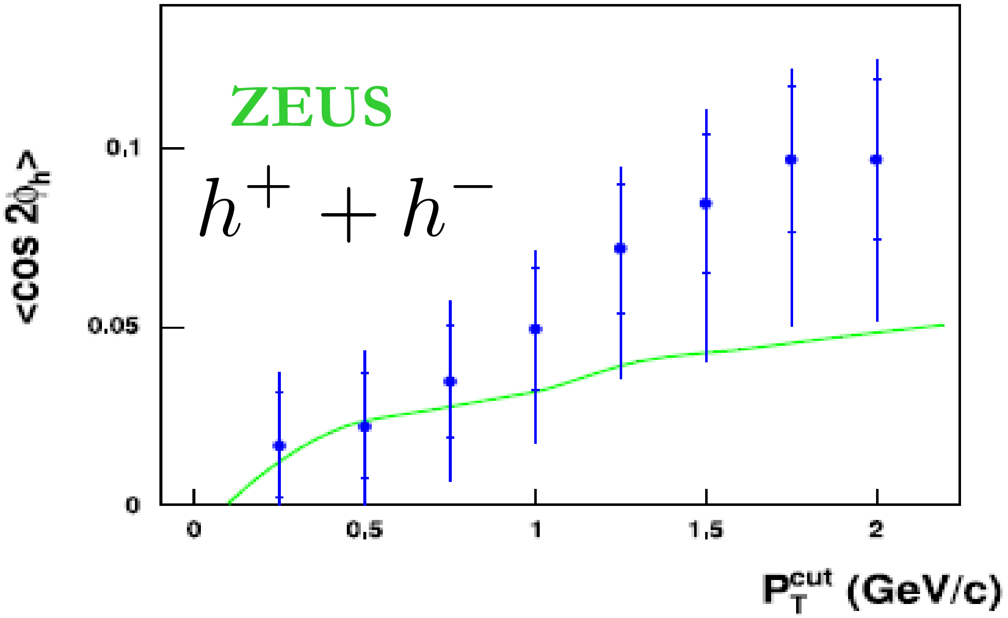




# Existing Measurements

$\cos 2\phi_h$





No distinction between hadron type or charge > any possible flavor dependence is averaged out!

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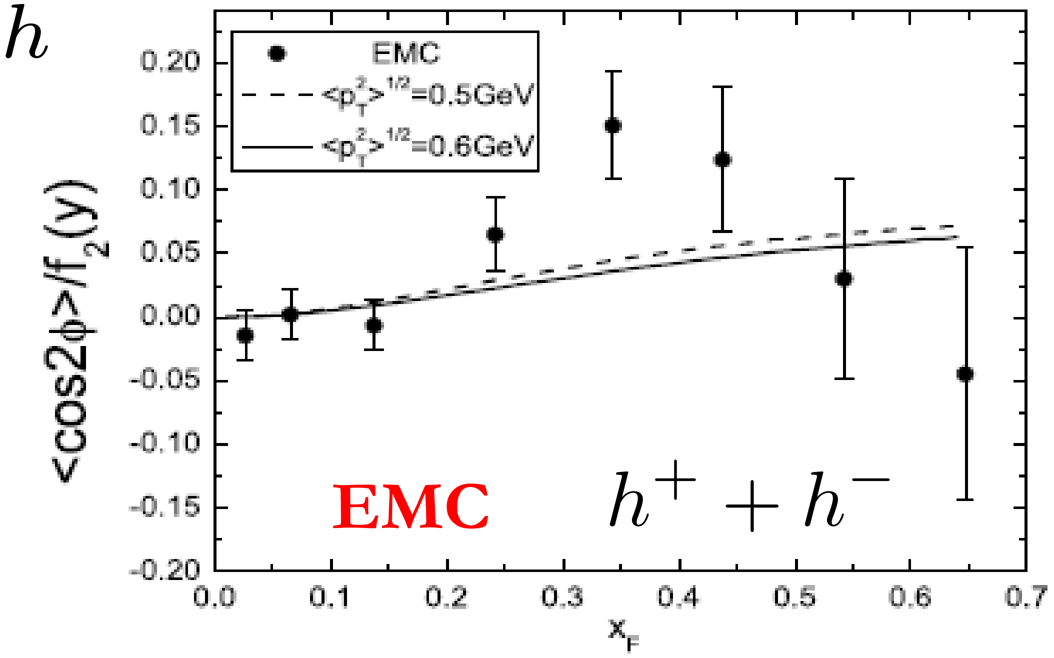

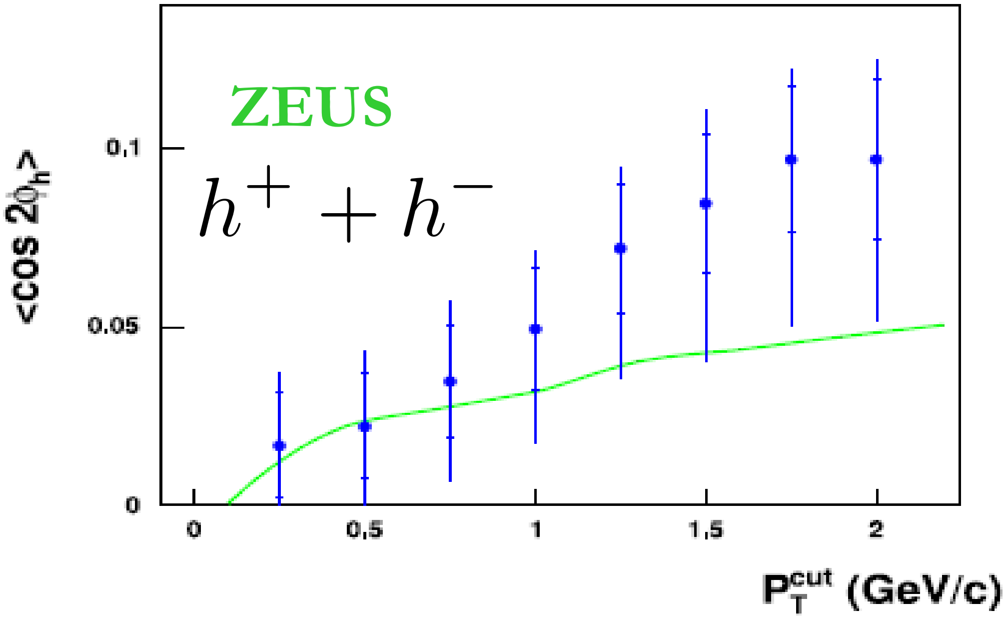

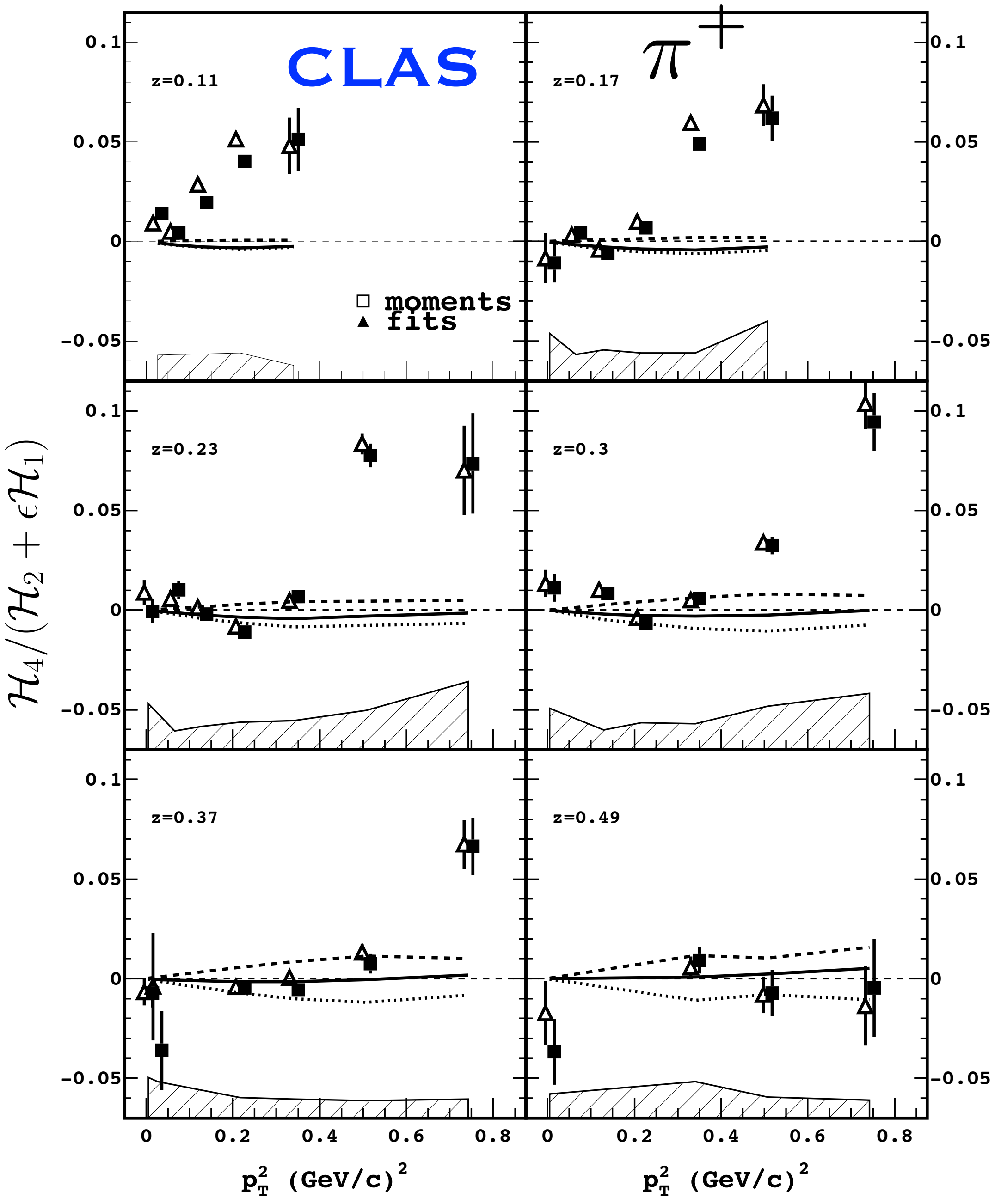

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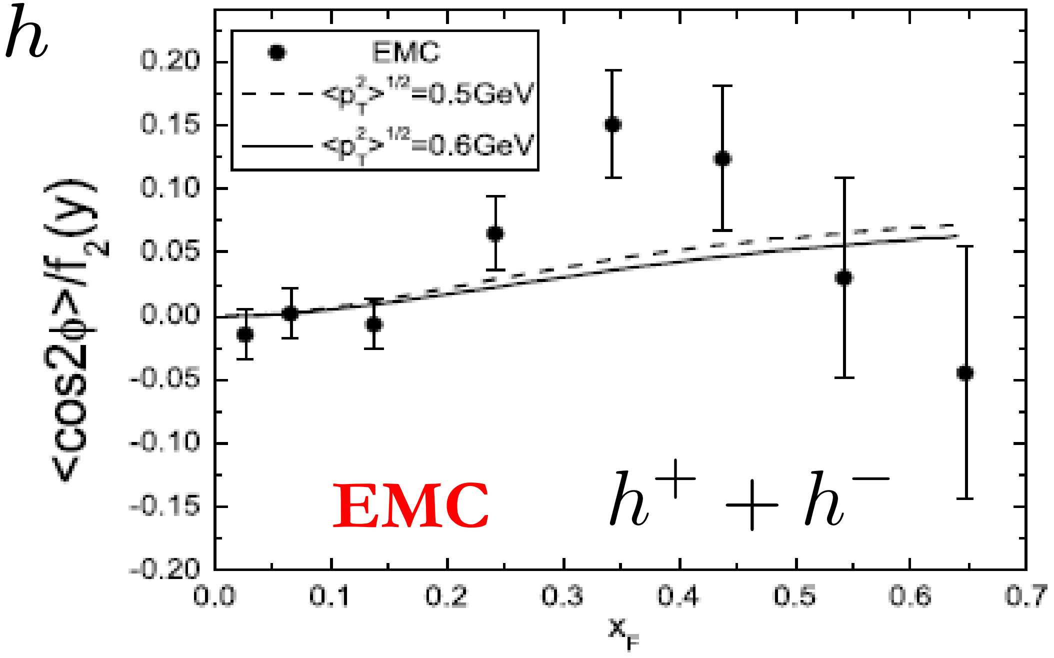

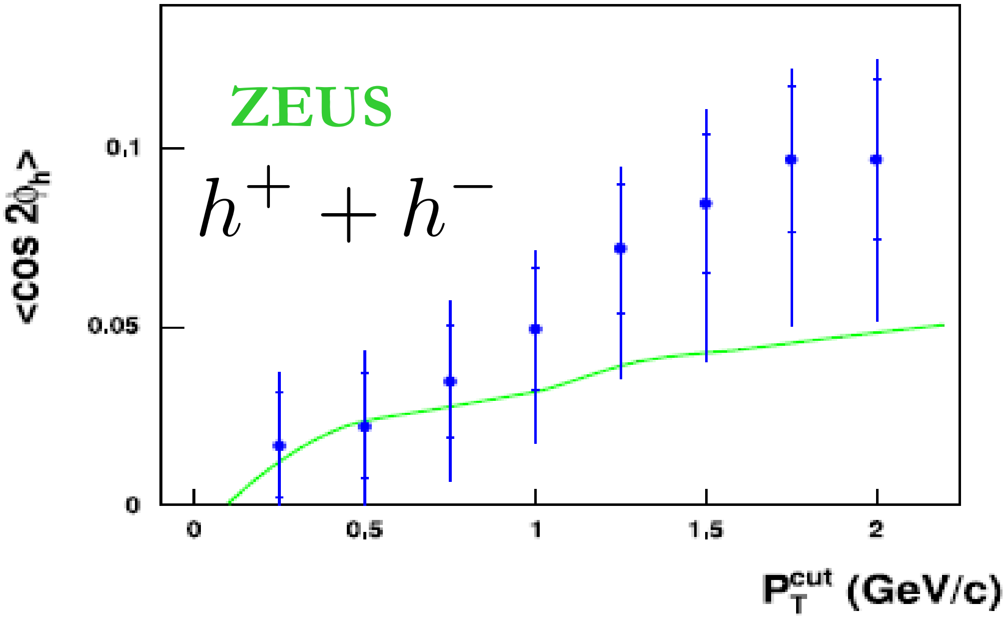

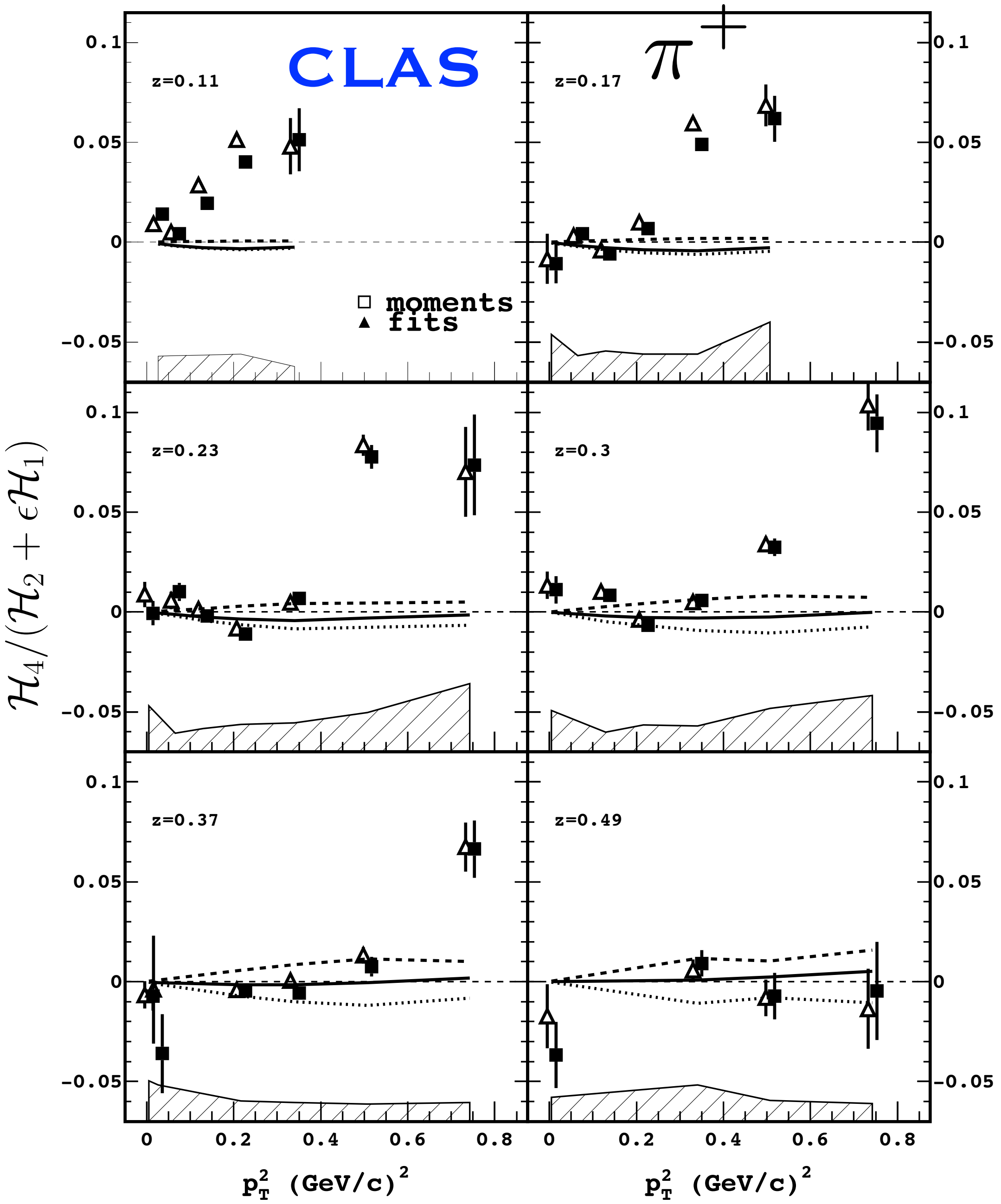

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**COMPASS**

**SEE NEXT SLIDES...**

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$\cos\phi_h$

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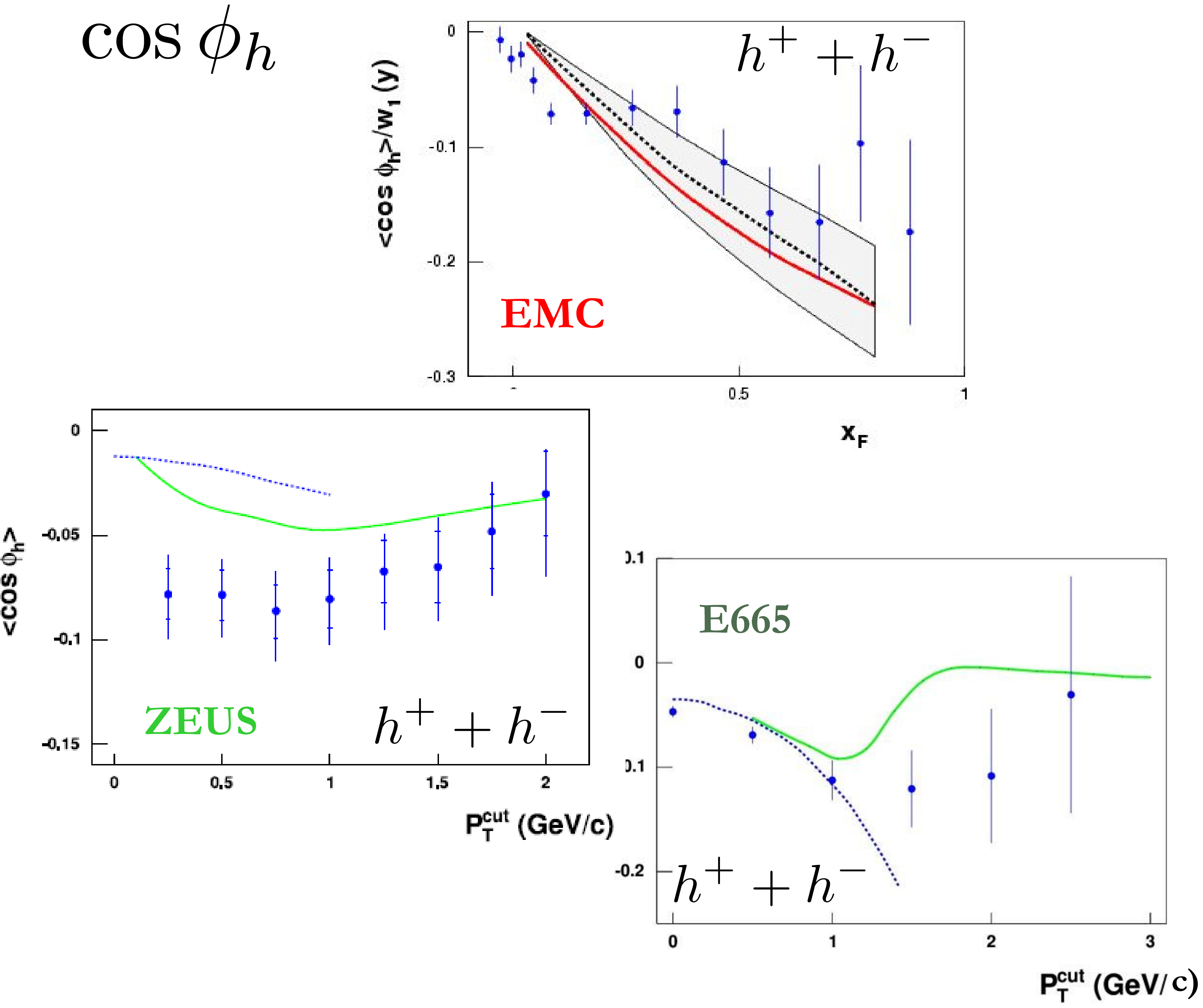

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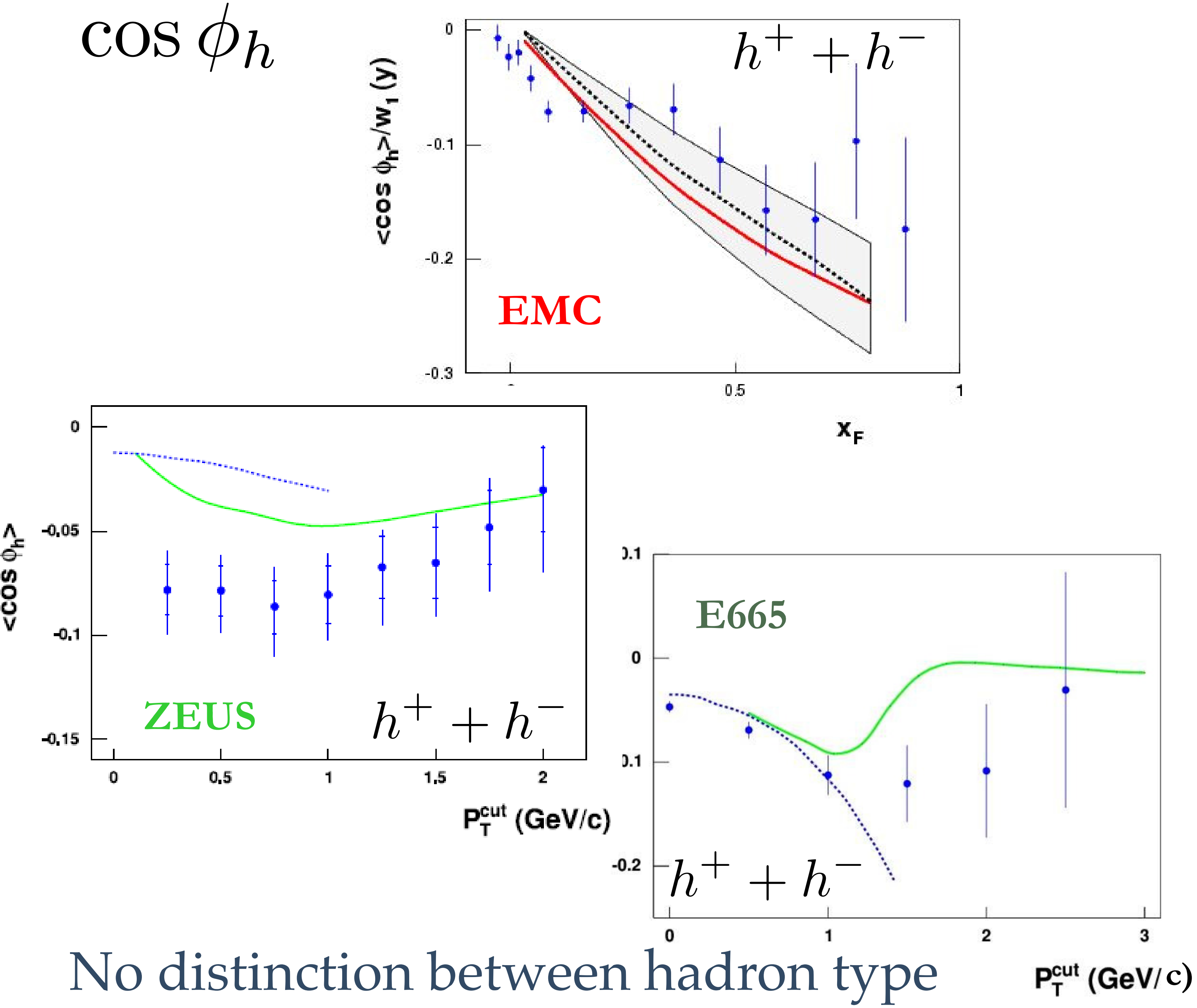
Francesca Giordano

# Existing Measurements

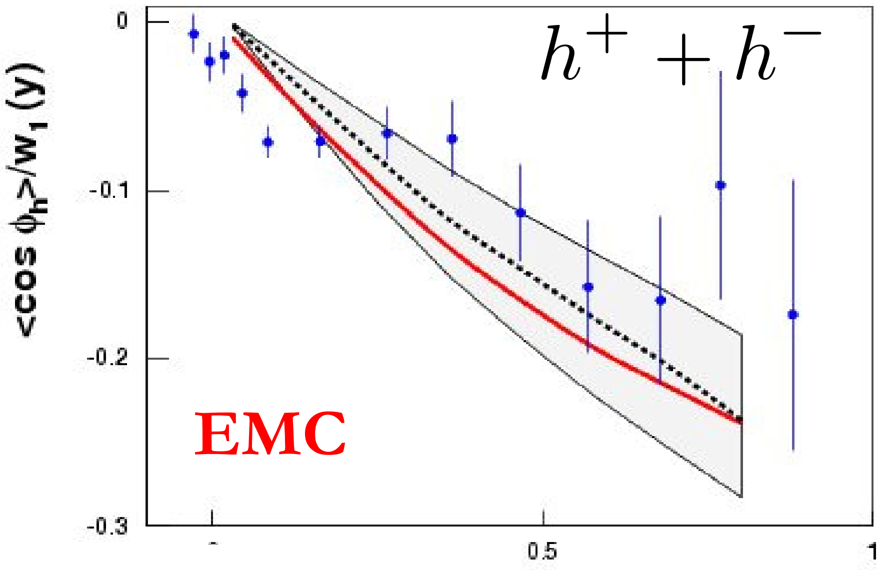
$\cos\phi_h$

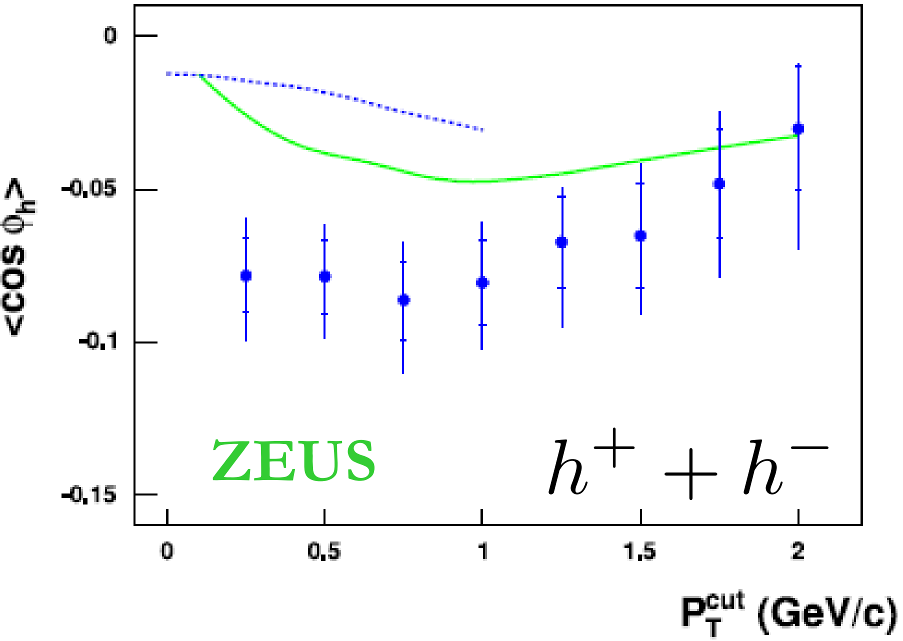


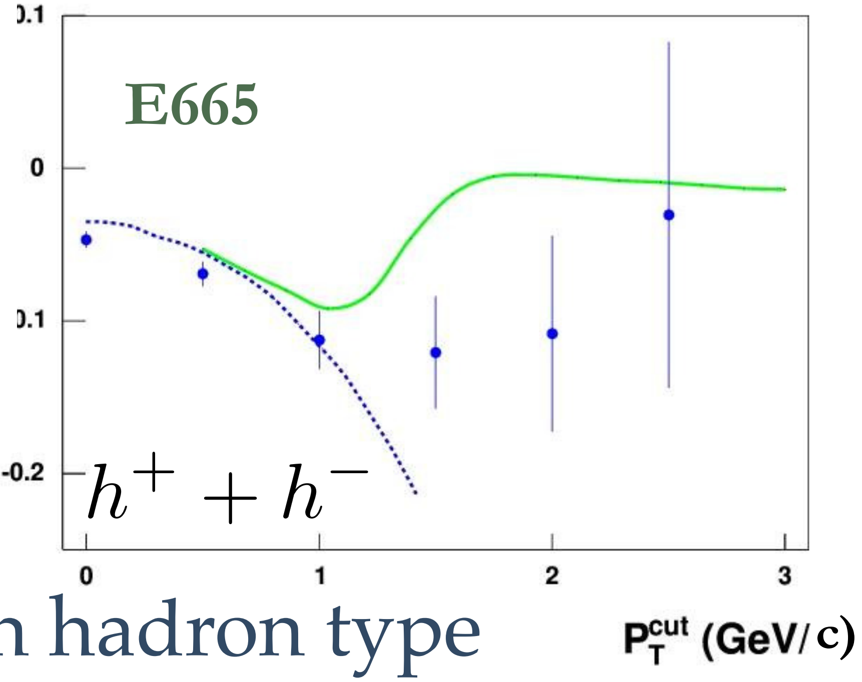
No distinction between hadron type or charge > any possible flavor dependence is averaged out!

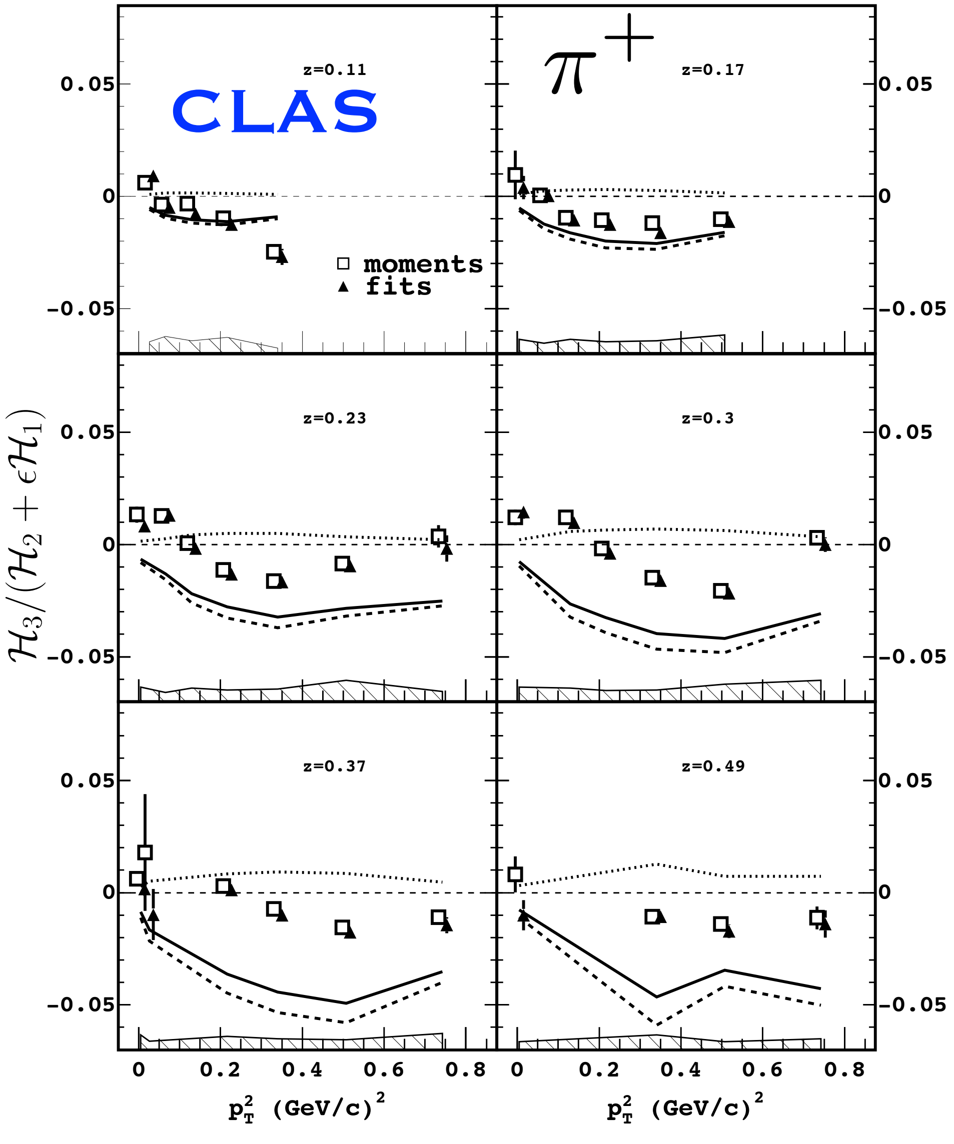
Francesca Giordano

$\cos\phi_h$









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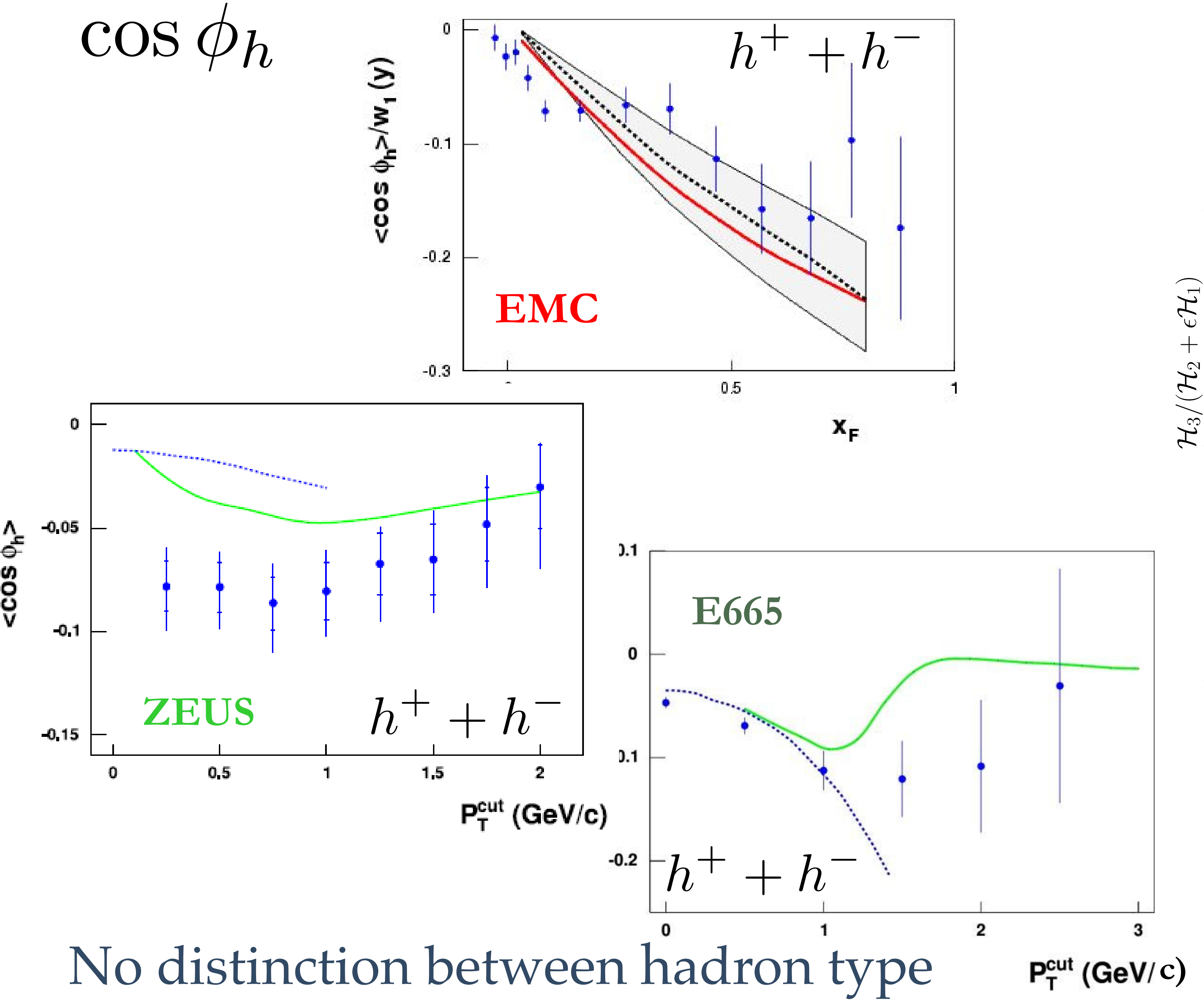
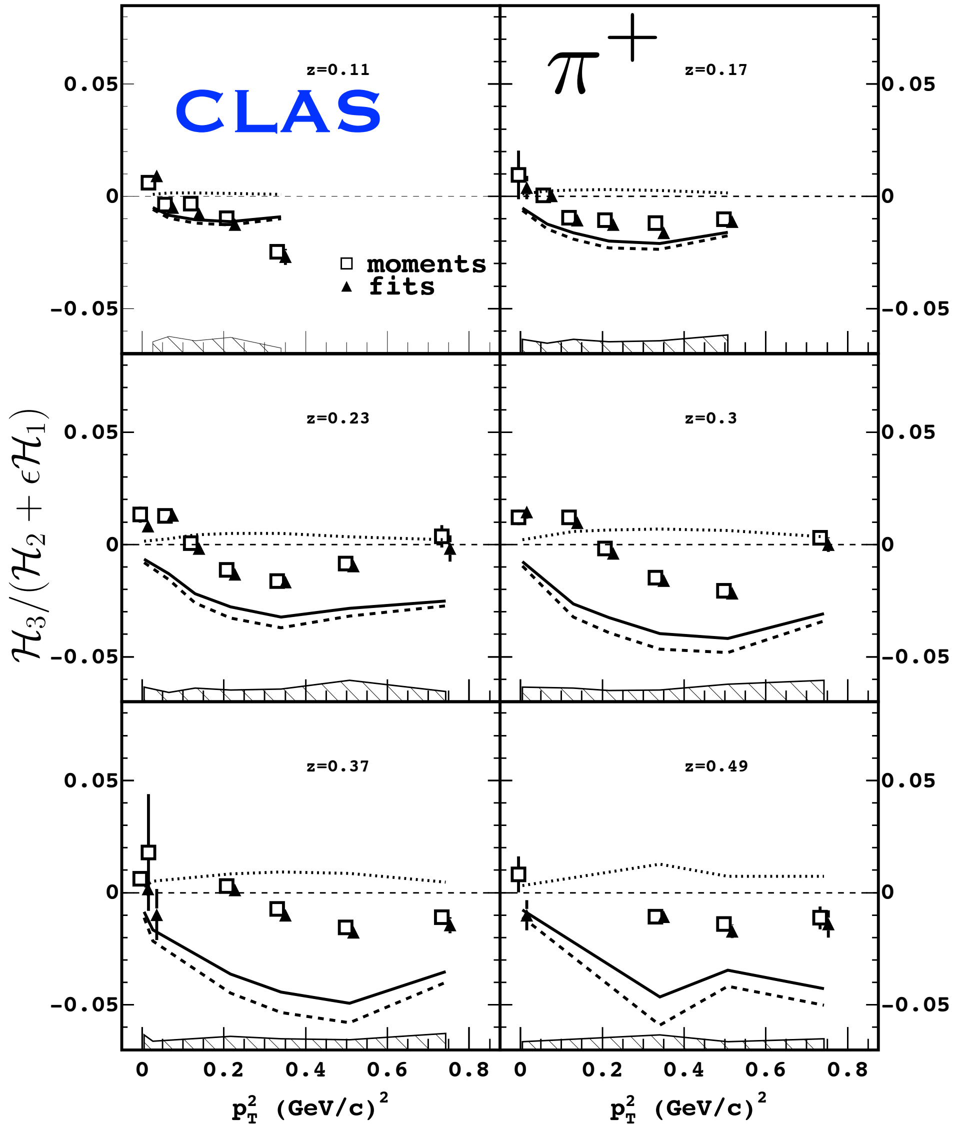
# Existing Measurements

Francesca Giordano

## $\cos\phi_h$





No distinction between hadron type or charge > any possible flavor dependence is averaged out!

**COMPASS**

**SEE NEXT SLIDES...**

# Existing Measurements

$$
\begin{aligned}
d\sigma = {} & d\sigma^{0}_{UU} + \cos 2\phi \; d\sigma^{1}_{UU} + \frac{1}{Q}\cos\phi \; d\sigma^{2}_{UU} + \lambda_{e}\frac{1}{Q}\sin\phi \; d\sigma^{3}_{LU} \\
& + S_{L}\left\{ \sin 2\phi \; d\sigma^{4}_{UL} + \frac{1}{Q}\sin\phi \; d\sigma^{5}_{UL} + \lambda_{e}\left[ d\sigma^{6}_{LL} + \frac{1}{Q}\cos\phi \; d\sigma^{7}_{LL} \right]\right]\right\} \\
& + S_{T}\Big\{ \sin(\phi-\phi_{S}) \; d\sigma^{8}_{UT} + \sin(\phi+\phi_{S}) \; d\sigma^{9}_{UT} + \sin(3\phi-\phi_{S}) \; d\sigma^{10}_{UT} \\
& + \frac{1}{Q}\sin(2\phi-\phi_{S}) \; d\sigma^{11}_{UT} + \frac{1}{Q}\sin\phi_{S} d\sigma^{12}_{UT} \\
& + \lambda_{e}\left[ \cos(\phi-\phi_{S}) \; d\sigma^{13}_{LT} + \frac{1}{Q}\cos\phi_{S} d\sigma^{14}_{LT} + \frac{1}{Q}\cos(2\phi-\phi_{S}) d\sigma^{15}_{LT} \right]\Big]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
d\sigma = {} & d\sigma_{UU}^{0} + \cos 2\phi \, d\sigma_{UU}^{1} + \frac{1}{Q}\cos\phi \, d\sigma_{UU}^{2} + \lambda_e \frac{1}{Q}\sin\phi \, d\sigma_{LU}^{3} \\
& + S_L \left\{ \sin 2\phi \, d\sigma_{UL}^{4} + \frac{1}{Q}\sin\phi \, d\sigma_{UL}^{5} + \lambda_e \left[ d\sigma_{LL}^{6} + \frac{1}{Q}\cos\phi \, d\sigma_{LL}^{7} \right] \right\} \\
& + S_T \left\{ \sin(\phi-\phi_S) \; d\sigma_{UT}^{8} + \sin(\phi+\phi_S) \; d\sigma_{UT}^{9} + \sin(3\phi-\phi_S) \; d\sigma_{UT}^{10} \right. \\
& \quad + \frac{1}{Q}\sin(2\phi-\phi_S) \; d\sigma_{UT}^{11} + \frac{1}{Q}\sin\phi_S \, d\sigma_{UT}^{12} \\
& \quad \left. + \lambda_e \left[ \cos(\phi-\phi_S) \; d\sigma_{LT}^{13} + \frac{1}{Q}\cos\phi_S \, d\sigma_{LT}^{14} + \frac{1}{Q}\cos(2\phi-\phi_S) \, d\sigma_{LT}^{15} \right] \right\}
\end{aligned}
$$

# Modulation Extractions: Why Is It So Difficult?

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$$+ \boxed{S_T} \left\{ \boxed{\sin(\phi-\phi_S)\ d\sigma^{8}_{UT}} + \sin(\phi+\phi_S)\ d\sigma^{9}_{UT} + \sin(3\phi-\phi_S)\ d\sigma^{10}_{UT} \right.$$

$$+ \frac{1}{Q}\sin(2\phi-\phi_S)\ d\sigma^{11}_{UT} + \frac{1}{Q}\sin\phi_S\, d\sigma^{12}_{UT}$$

$$\left. + \lambda_e \left[ \cos(\phi - \phi_S)\ d\sigma^{13}_{LT} + \frac{1}{Q}\cos\phi_S\, d\sigma^{14}_{LT} + \frac{1}{Q}\cos(2\phi - \phi_S) d\sigma^{15}_{LT} \right] \Big] \right\}$$

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$$+ \lambda_e \left[ \cos(\phi - \phi_S) \; d\sigma^{13}_{LT} + \frac{1}{Q}\cos\phi_S d\sigma^{14}_{LT} + \frac{1}{Q}\cos(2\phi - \phi_S) d\sigma^{15}_{LT} \right] \Big\}$$

$$A_{UT} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

# *Modulation Extractions: Why Is It So Difficult?*

$$d\sigma = d\sigma^0_{UU} + \cos 2\phi\, d\sigma^1_{UU} + \frac{1}{Q}\cos\phi\, d\sigma^2_{UU} + \lambda_e \frac{1}{Q}\sin\phi\, d\sigma^3_{LU}$$

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$$+ \frac{1}{Q}\sin(2\phi-\phi_S)\ d\sigma^{11}_{UT} + \frac{1}{Q}\sin\phi_S\, d\sigma^{12}_{UT}$$

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$$A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{\boxed{d\sigma^\uparrow + d\sigma^\downarrow}} \longrightarrow \frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP^2_{h\perp}} = \frac{\alpha^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\{A(y)F_{UU,T} + B(y)F_{UU,L}$$

$$+ C(y)\cos\phi_h F^{\cos\phi_h}_{UU} + B(y)\cos 2\phi_h F^{\cos 2\phi_h}_{UU}\}$$

# Modulation Extractions: Why Is It So Difficult?

Francesca Giordano

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& + \frac{1}{Q}\sin(2\phi-\phi_S)\ d\sigma^{11}_{UT} + \frac{1}{Q}\sin\phi_S d\sigma^{12}_{UT} \\
& \left. + \lambda_e\left[\cos(\phi-\phi_S)\ d\sigma^{13}_{LT} + \frac{1}{Q}\cos\phi_S d\sigma^{14}_{LT} + \frac{1}{Q}\cos(2\phi-\phi_S) d\sigma^{15}_{LT}\right]\right\}
\end{aligned}
$$

$$
A_{UT} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{\boxed{d\sigma^{\uparrow} + d\sigma^{\downarrow}}} \longrightarrow \frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP^2_{h\perp}} = \frac{\alpha^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\left\{A(y)F_{UU,T} + B(y)F_{UU,L} + C(y)\cos\phi_h F_{UU}^{\cos\phi_h} + B(y)\cos 2\phi_h F_{UU}^{\cos 2\phi_h}\right\}
$$

cancels large part of acceptance effects

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& + \frac{1}{Q}\sin(2\phi - \phi_S) \; d\sigma^{11}_{UT} + \frac{1}{Q}\sin\phi_S d\sigma^{12}_{UT} \\
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\end{aligned}
$$

For modulations of the unpolarized cross section it is not possible to define an asymmetry!

$$
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$$

$$
= \frac{\alpha^2}{xyQ^2}\left(1 + \frac{\gamma^2}{2x}\right)\{A(y) F_{UU,T} + B(y) F_{UU,L} + C(y)\cos\phi_h F_{UU}^{\cos\phi_h} + B(y)\cos 2\phi_h F_{UU}^{\cos 2\phi_h}\}
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$$= \frac{\alpha^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\{A(y)F_{UU,T} + B(y)F_{UU,L}$$

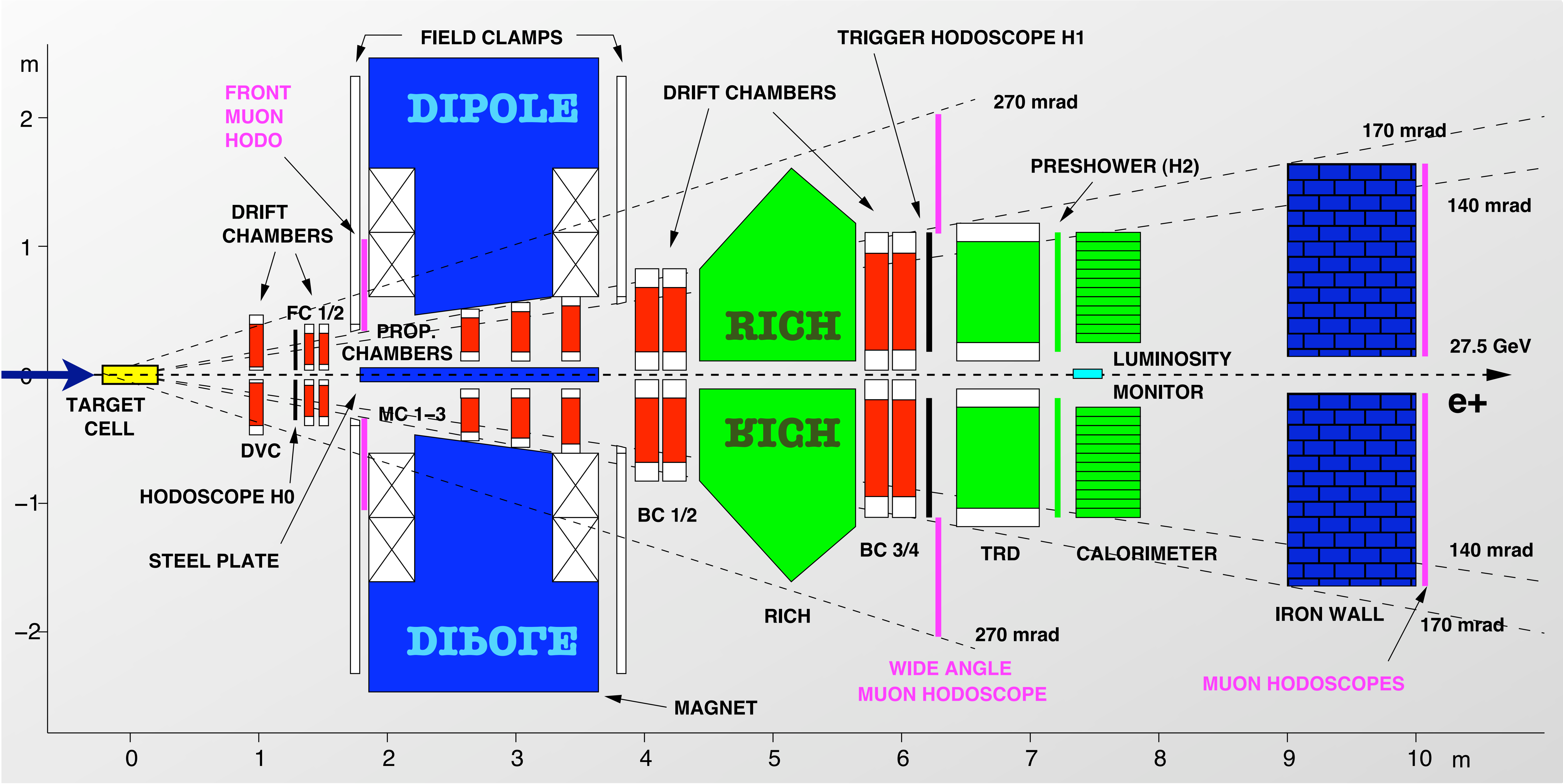
$$+C(y)\cos\phi_h F_{UU}^{\cos\phi_h} + B(y)\cos 2\phi_h F_{UU}^{\cos 2\phi_h}\}$$

cancels large part of acceptance effects

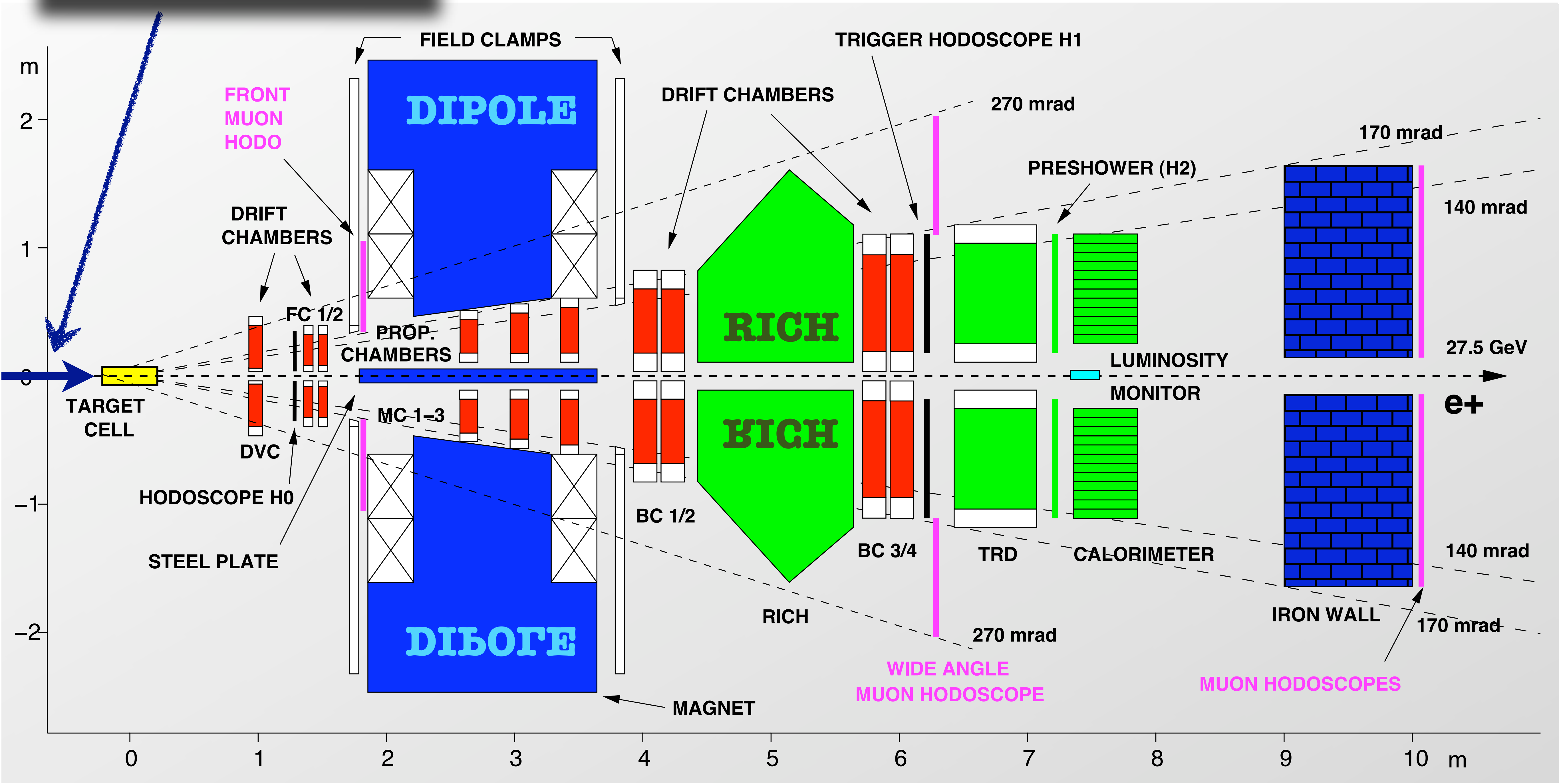
contribution from unpol. cross section cosine modulation

# *Modulation Extractions: Why Is It So Difficult?*


HERA
hermes
PETRA

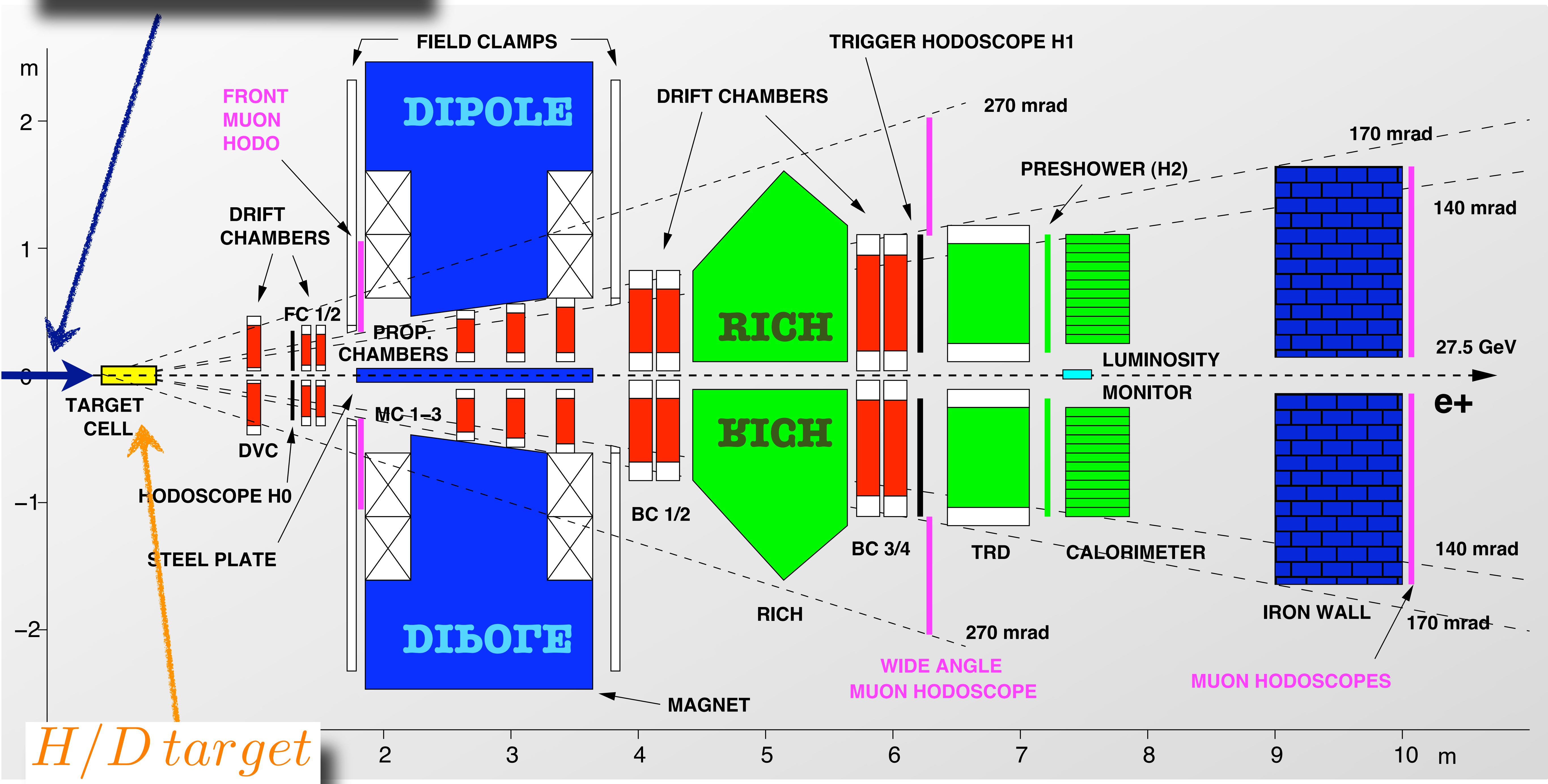

FIELD CLAMPS
TRIGGER HODOSCOPE H1
m
2
1
0
−1
−2
FRONT
MUON
HODO
DIPOLE
DRIFT CHAMBERS
270 mrad
170 mrad
PRESHOWER (H2)
140 mrad
DRIFT
CHAMBERS
FC 1/2
PROP.
CHAMBERS
RICH
27.5 GeV
LUMINOSITY
MONITOR
e+
TARGET
CELL
MC 1–3
DVC
RICH
HODOSCOPE H0
BC 1/2
STEEL PLATE
BC 3/4
TRD
CALORIMETER
140 mrad
RICH
IRON WALL
170 mrad
DIPOLE
270 mrad
WIDE ANGLE
MUON HODOSCOPE
MUON HODOSCOPES
MAGNET
0
1
2
3
4
5
6
7
8
9
10
m

$27.6\,GeV\ e^{\pm}\ beam$

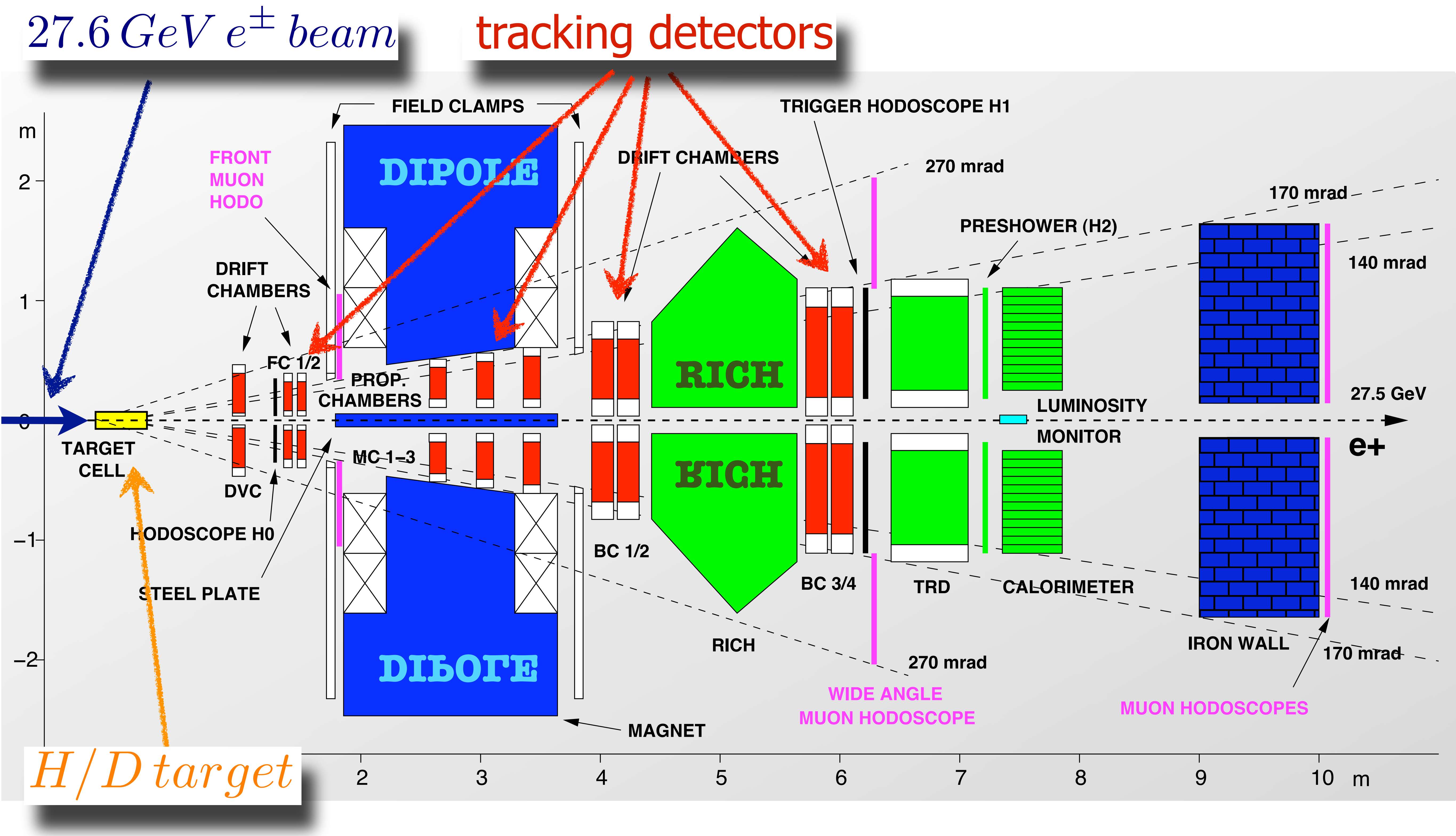


Francesca Giordano

$27.6\, GeV\ e^{\pm}\ beam$

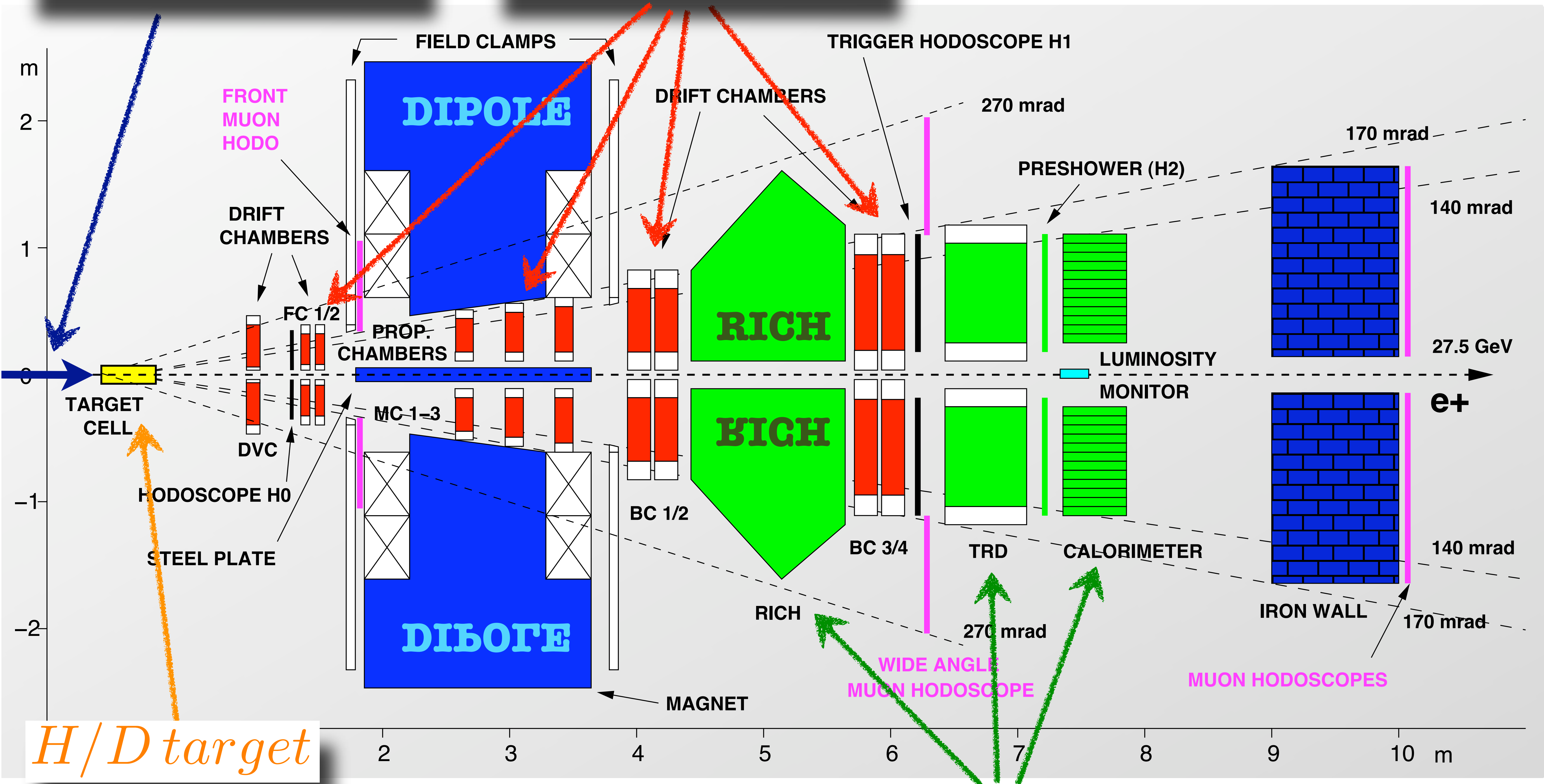


*H/D target*



$27.6\,GeV\ e^{\pm}\ beam$

tracking detectors

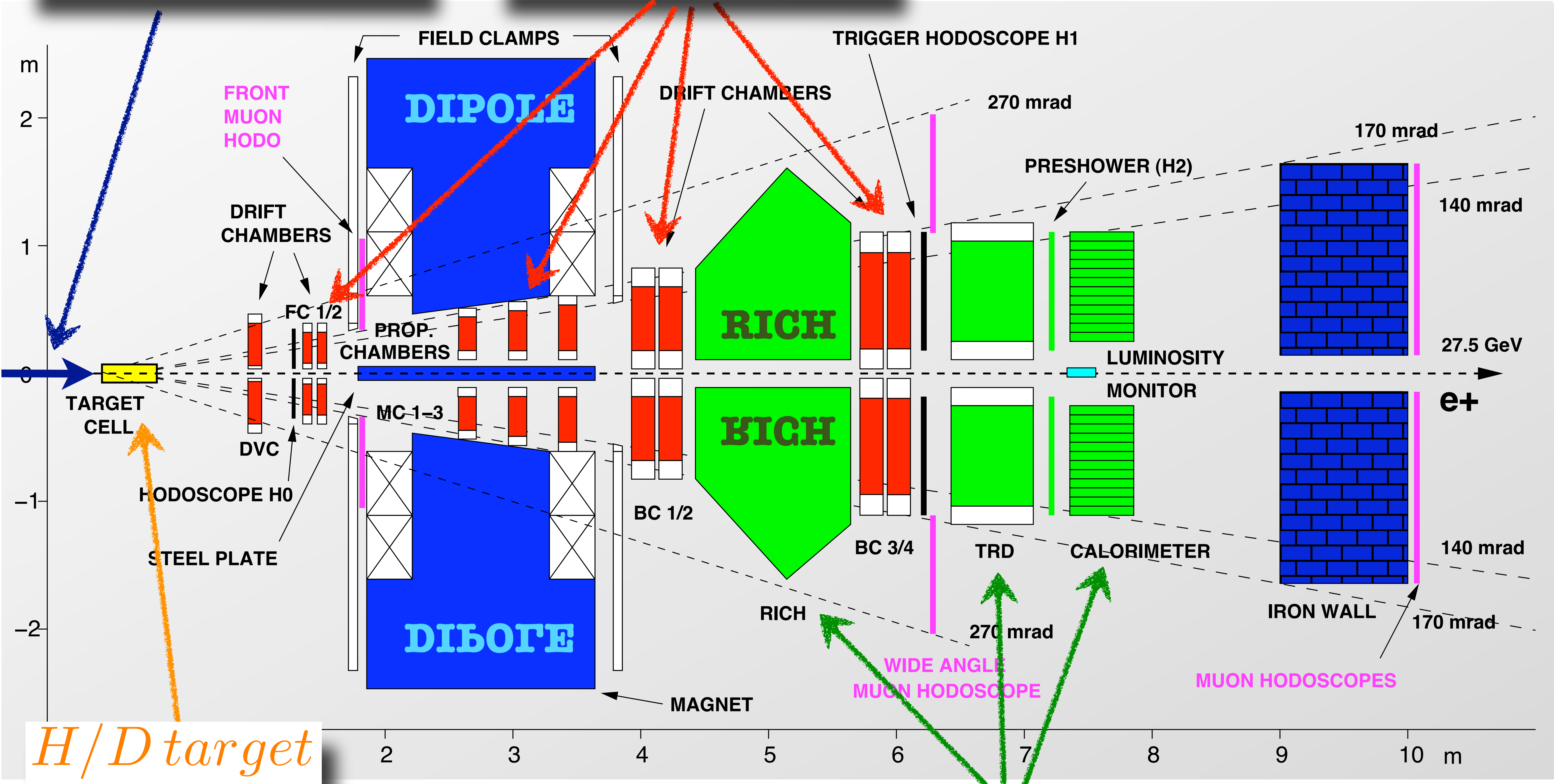


$H/D\,target$

particle identification: lepton/hadron separation

$27.6\, GeV\ e^{\pm}\ beam$

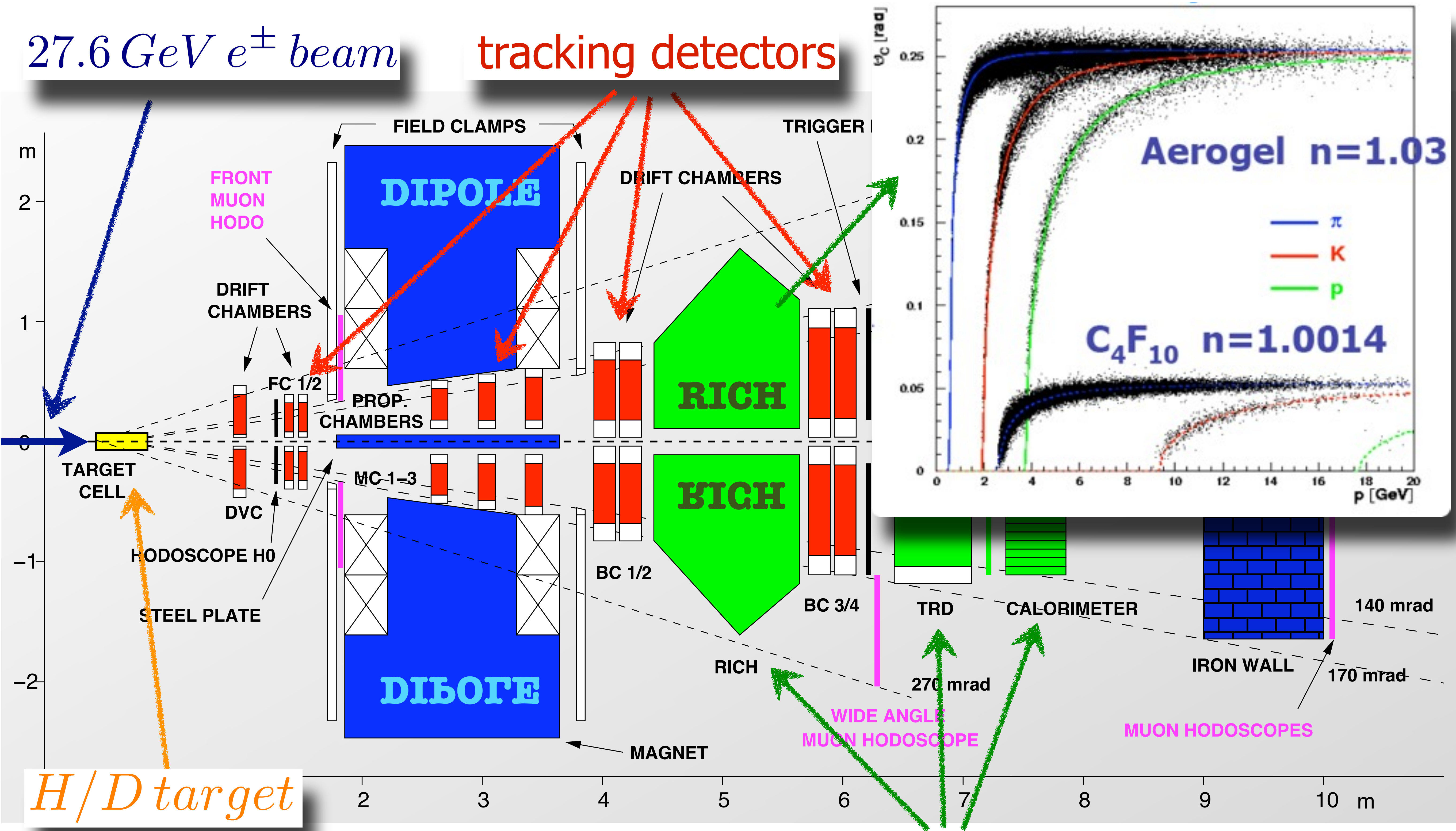
tracking detectors



*H/D target*

particle identification: lepton/hadron separation

RICH: discrimination between charged $\pi$ / K / p



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particle identification: lepton/hadron separation

RICH: discrimination between charged π / K / p

$$w = (x, y, z, P_{h\perp})$$

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$$\sigma_w^0[1 + 2\langle\cos\phi_h\rangle_w + 2\langle\cos 2\phi_h\rangle_w]$$

# *Acceptance Correction*

$$w = (x, y, z, P_{h\perp})$$

$$\sigma_w^0 [1 + 2\langle \cos\phi_h \rangle_w + 2\langle \cos 2\phi_h \rangle_w]$$

# *Acceptance Correction*

Francesca Giordano

$$w = (x, y, z, P_{h\perp})$$

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# *Acceptance Correction*

$$w = (x, y, z, P_{h\perp})$$

$$n = \int L\sigma_w^0[1 + 2\langle\cos\phi_h\rangle_w + 2\langle\cos 2\phi_h\rangle_w] \qquad dw$$

# *Acceptance Correction*

$$w = (x, y, z, P_{h\perp})$$

$$n = \int L\sigma_w^0 [1 + 2\langle \cos\phi_h \rangle_w + 2\langle \cos 2\phi_h \rangle_w] \epsilon_{w,\phi_h}^{acc} \epsilon_{w,\phi_h}^{rad} dw$$

# *Acceptance Correction*

$$w = (x, y, z, P_{h\perp})$$

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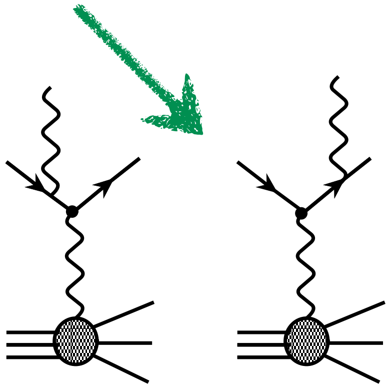

## Acceptance Correction

Francesca Giordano

$$w = (x, y, z, P_{h\perp})$$

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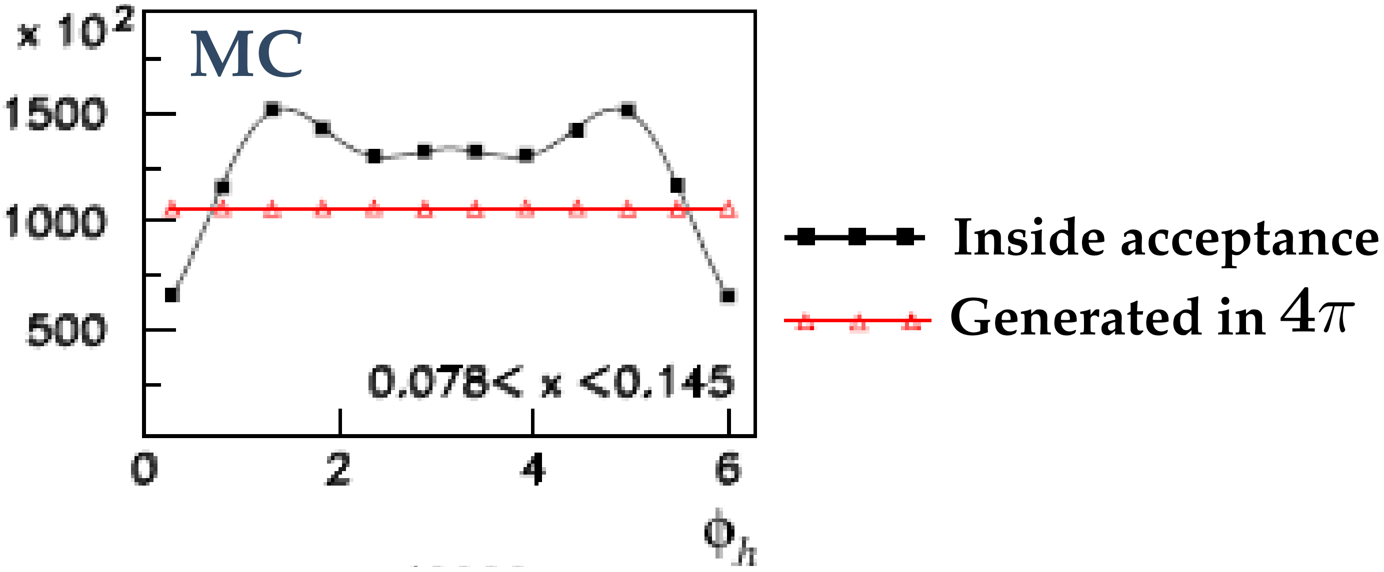

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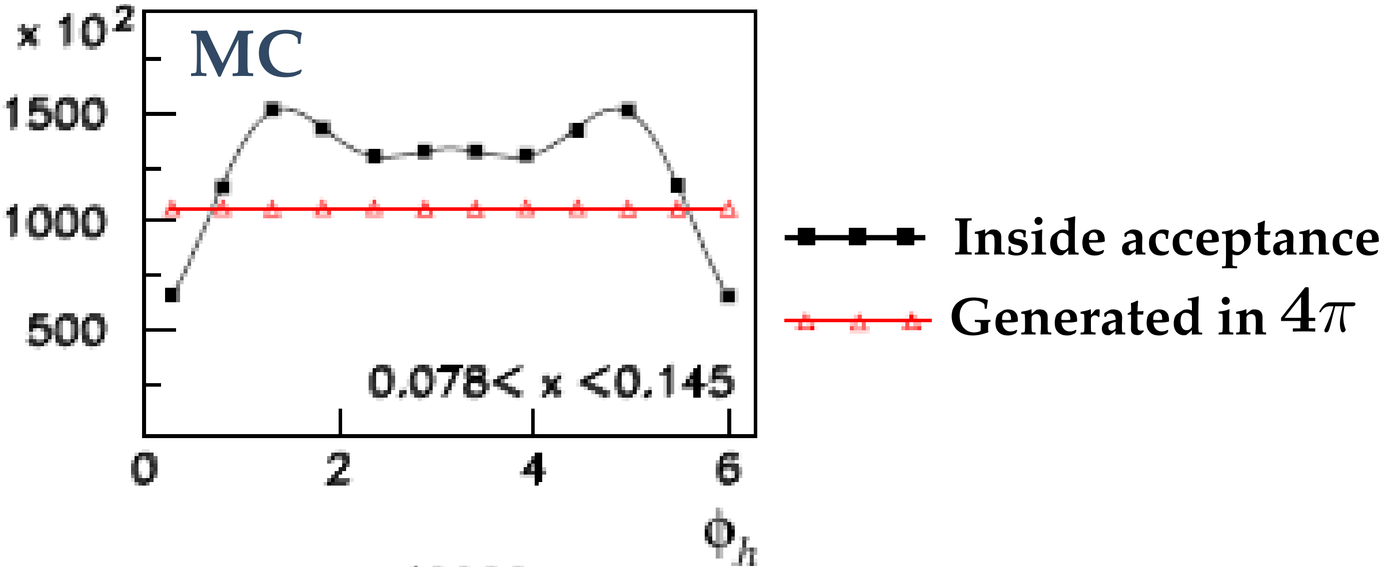
*MC simulation of spectrometers to correct for acceptance/QED radiation*



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$$w = (x, y, z, P_{h\perp})$$

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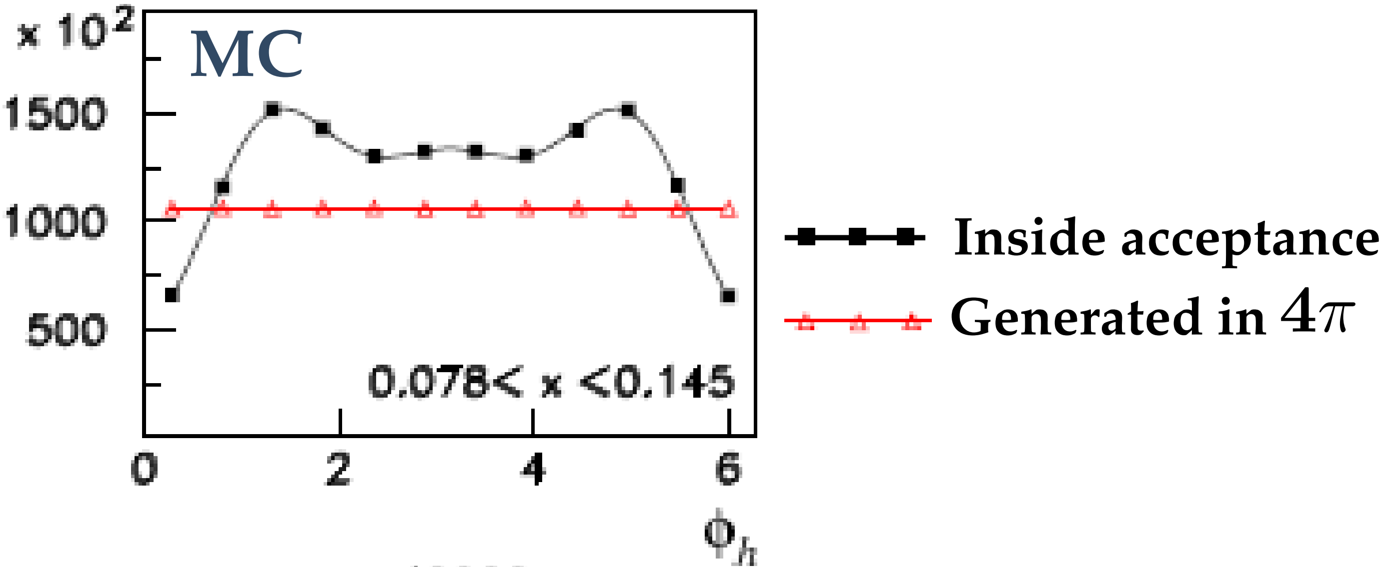

⇨

$$n_A = \int \sigma_w^0|_{mc}\epsilon_{w,\phi_h}^{acc}\,\epsilon_{w,\phi_h}^{rad}\,dw$$

# *Acceptance Correction*

$$w = (x, y, z, P_{h\perp})$$

$$n = \int L\sigma^0_w[1 + 2\langle\cos\phi_h\rangle_w + 2\langle\cos 2\phi_h\rangle_w]\epsilon^{acc}_{w,\phi_h}\epsilon^{rad}_{w,\phi_h}dw$$

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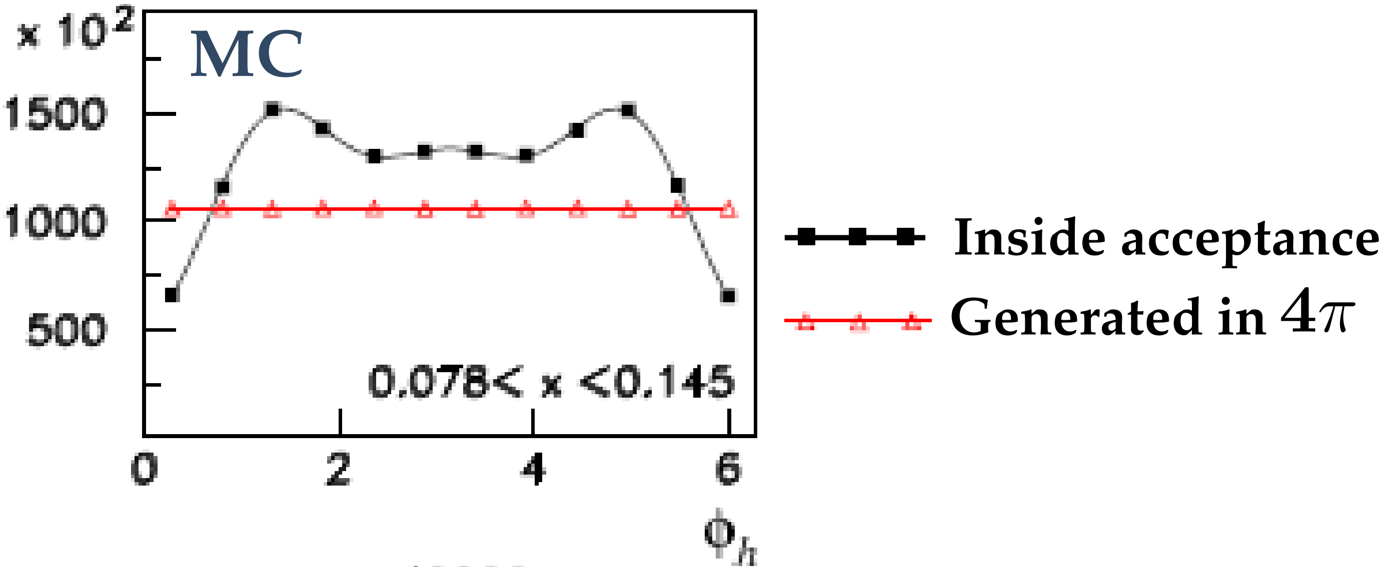


$$n_A = \int \sigma^0_w|_{mc}\epsilon^{acc}_{w,\phi_h}\epsilon^{rad}_{w,\phi_h}dw$$

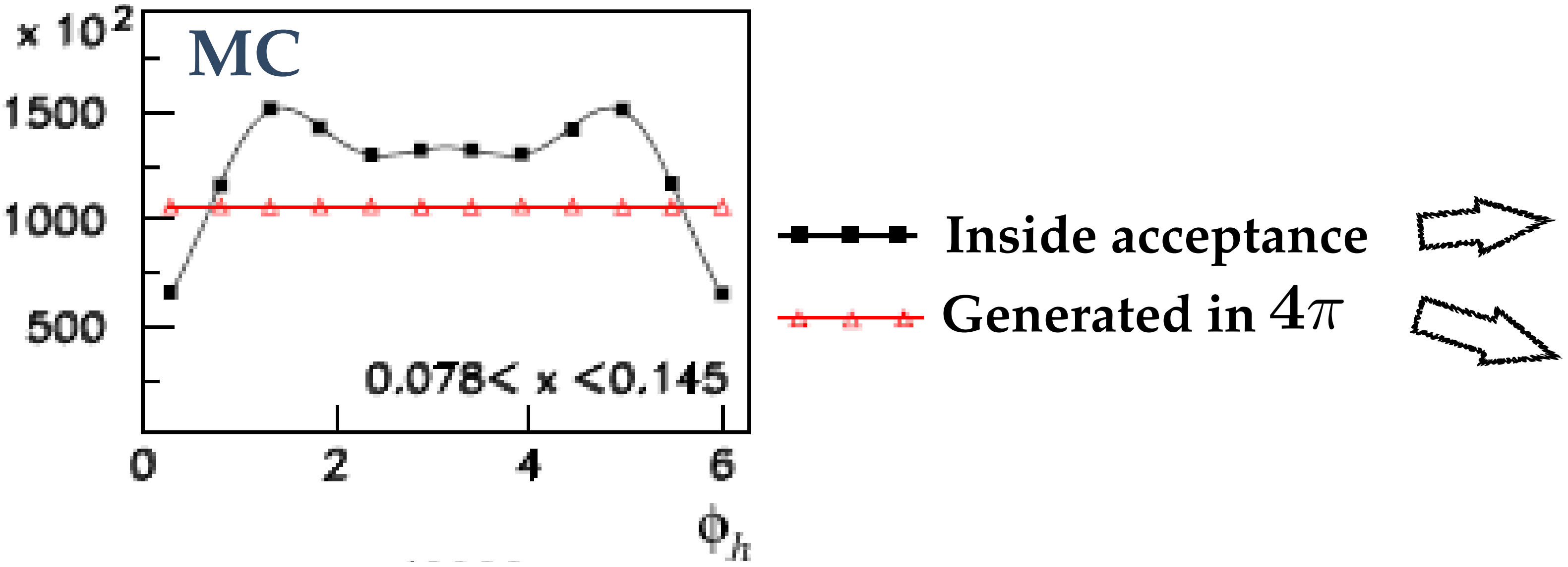
$$n_{4\pi} = \int \sigma^0_w|_{mc}dw$$

# Acceptance Correction

$$w = (x, y, z, P_{h\perp})$$

$$n = \int L\sigma^0_w[1 + 2\langle\cos\phi_h\rangle_w + 2\langle\cos 2\phi_h\rangle_w]\epsilon^{acc}_{w,\phi_h}\epsilon^{rad}_{w,\phi_h}dw$$

*MC simulation of spectrometers to correct for acceptance/QED radiation*



$$\frac{n_A = \int \sigma^0_w|_{mc}\epsilon^{acc}_{w,\phi_h}\epsilon^{rad}_{w,\phi_h}dw}{n_{4\pi} = \int \sigma^0_w|_{mc}dw}$$

$$w = (x, y, z, P_{h\perp})$$

$$n = \int L\sigma_w^0 [1 + 2\langle\cos\phi_h\rangle_w + 2\langle\cos 2\phi_h\rangle_w]\epsilon^{acc}_{w,\phi_h}\epsilon^{rad}_{w,\phi_h}dw$$

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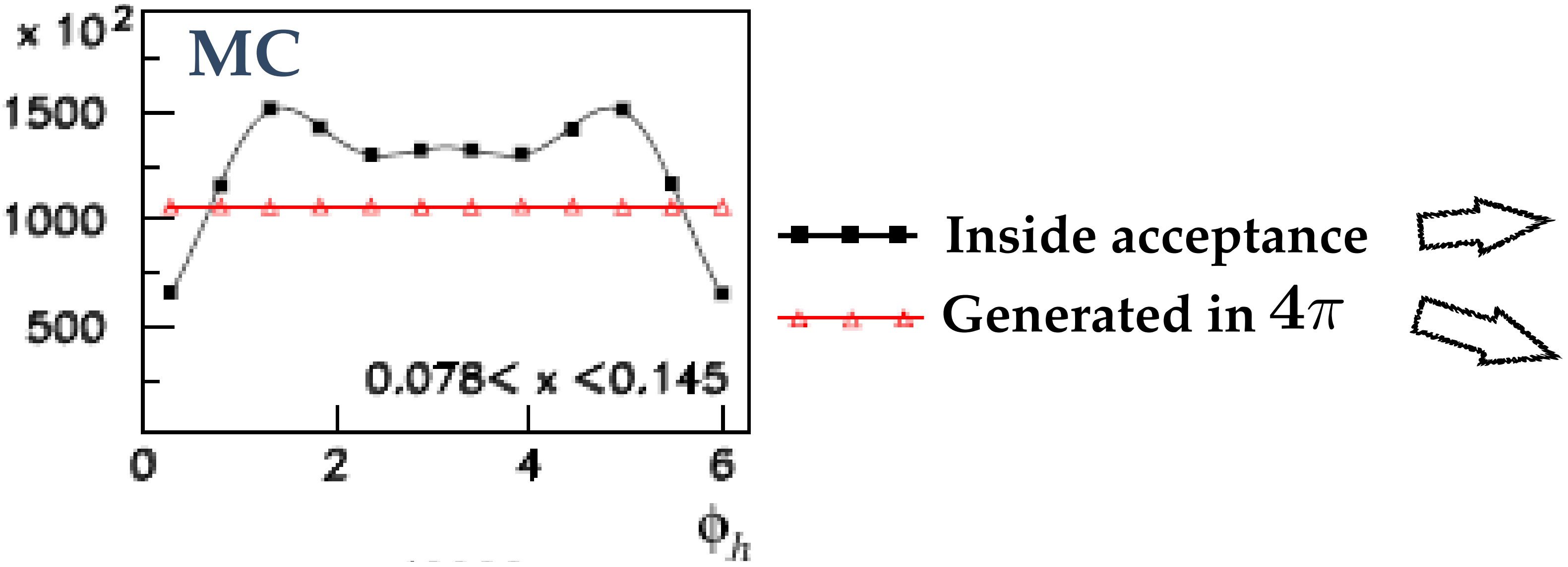

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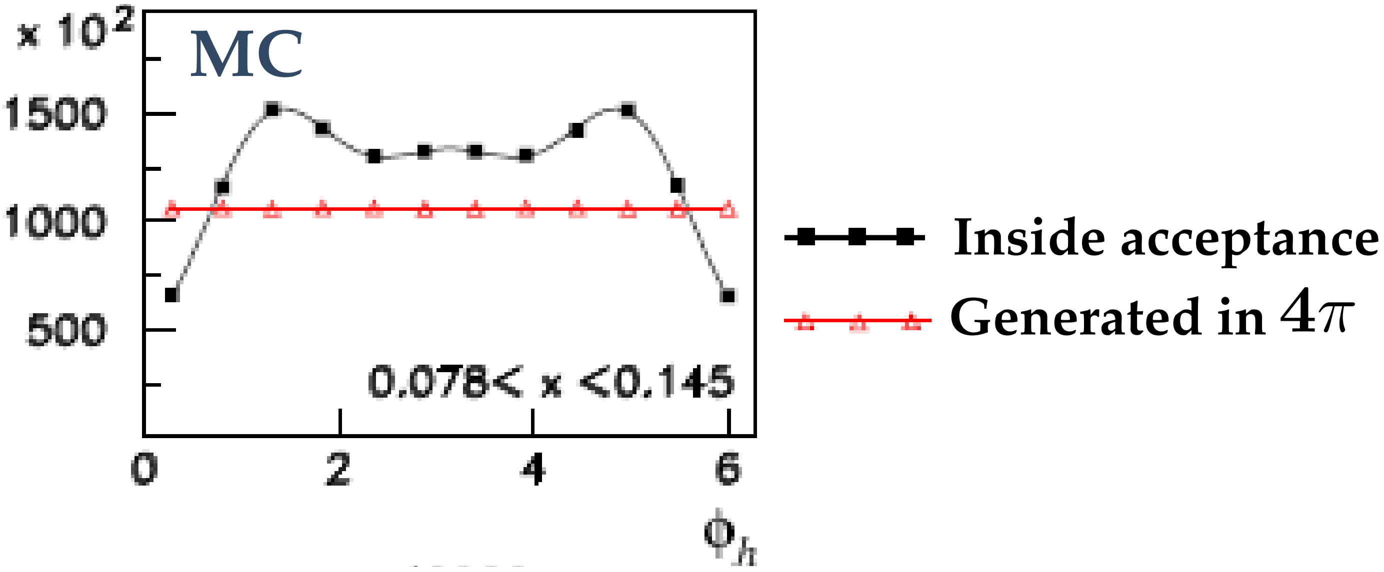

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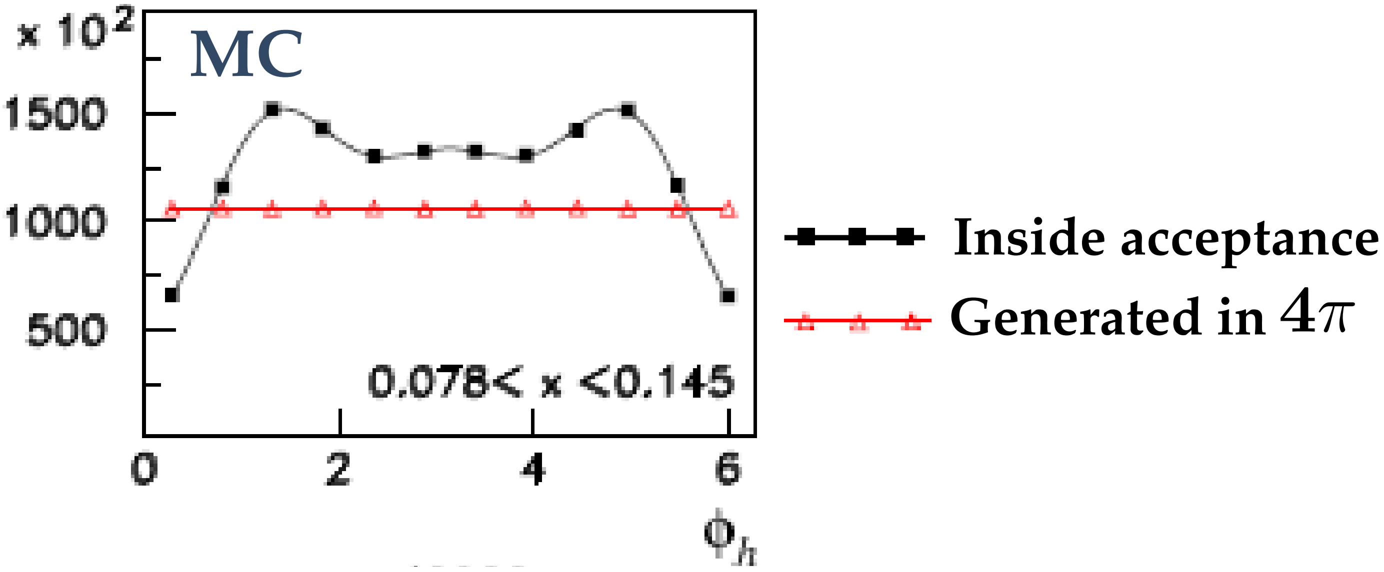

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**Not allowed!**

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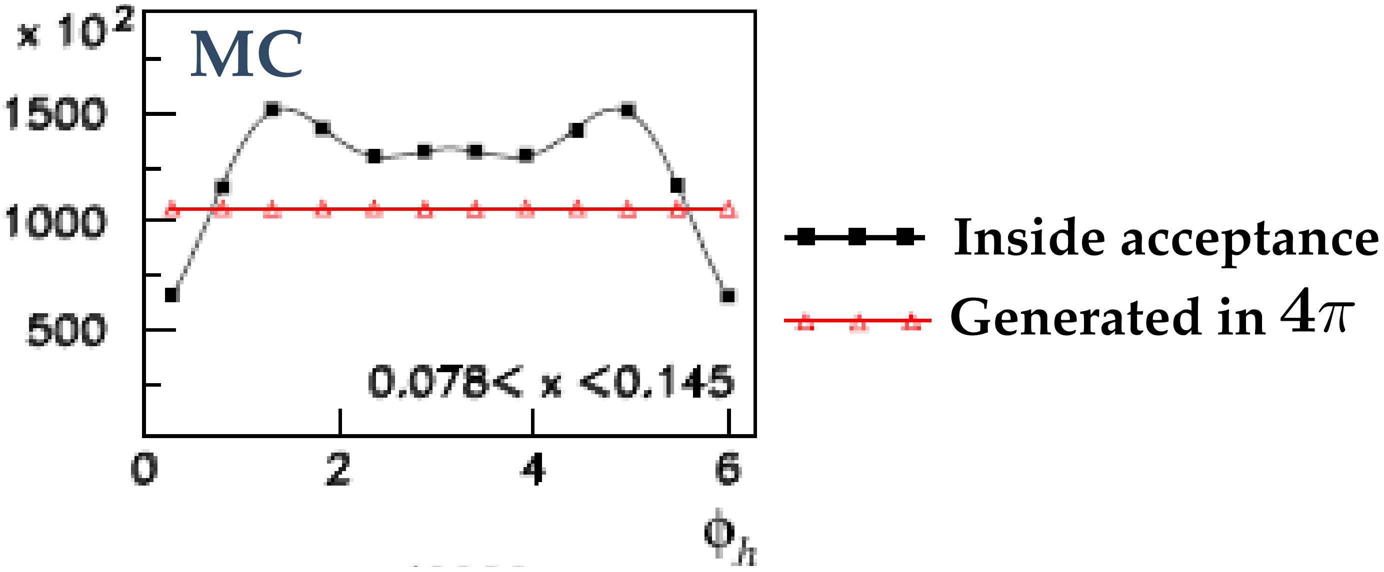

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**only if fully differential ratio (4D binning)**

**and only in the limit of infinitely small bins**

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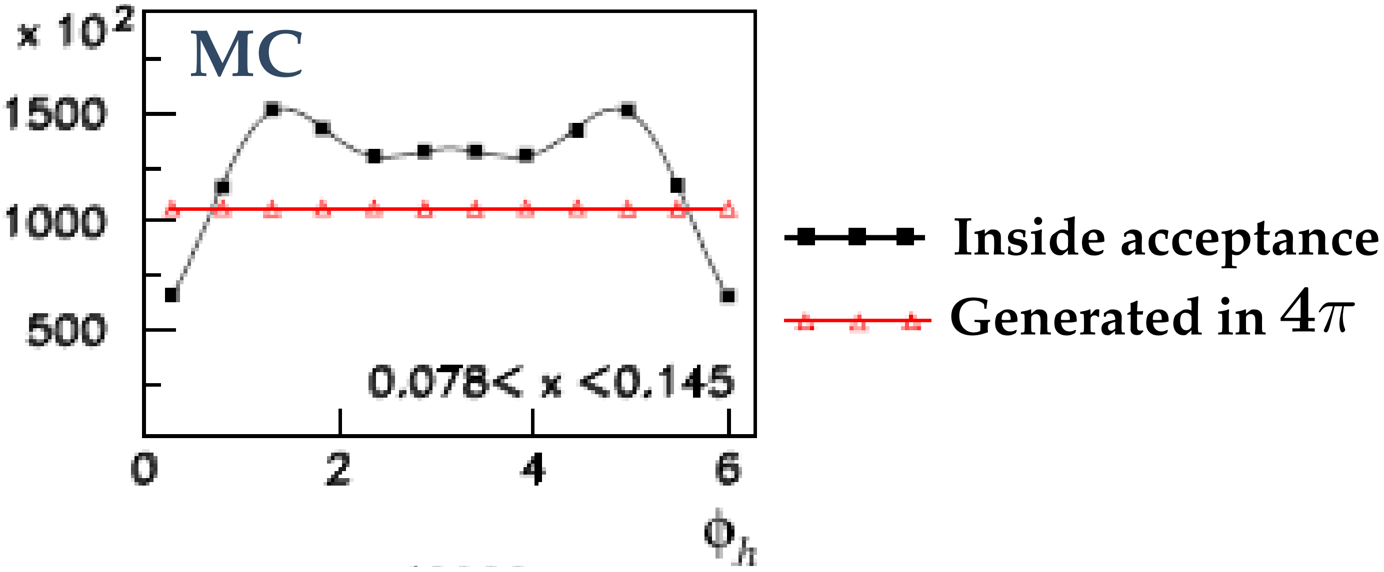

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**Model independent correction**

# *Acceptance Correction*

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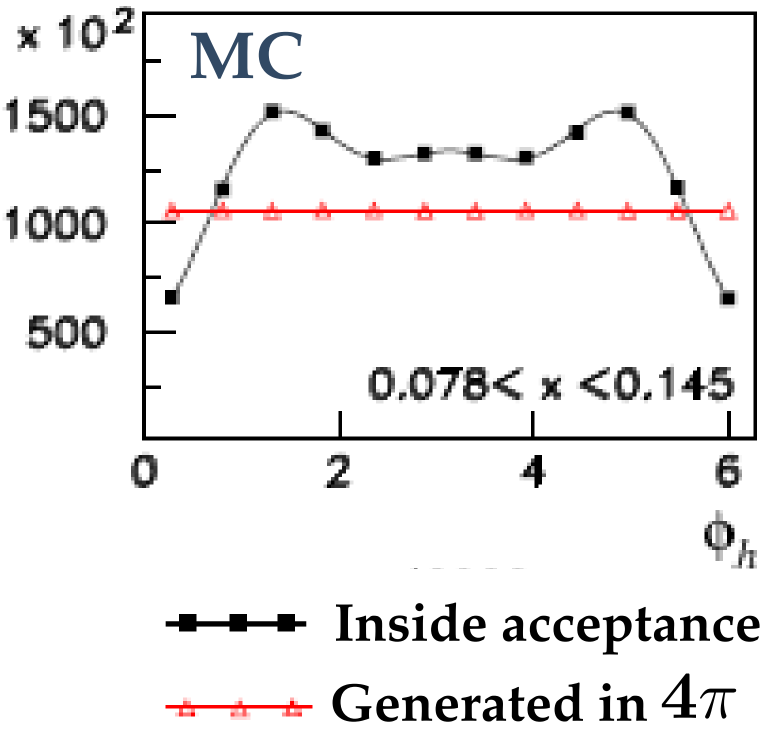

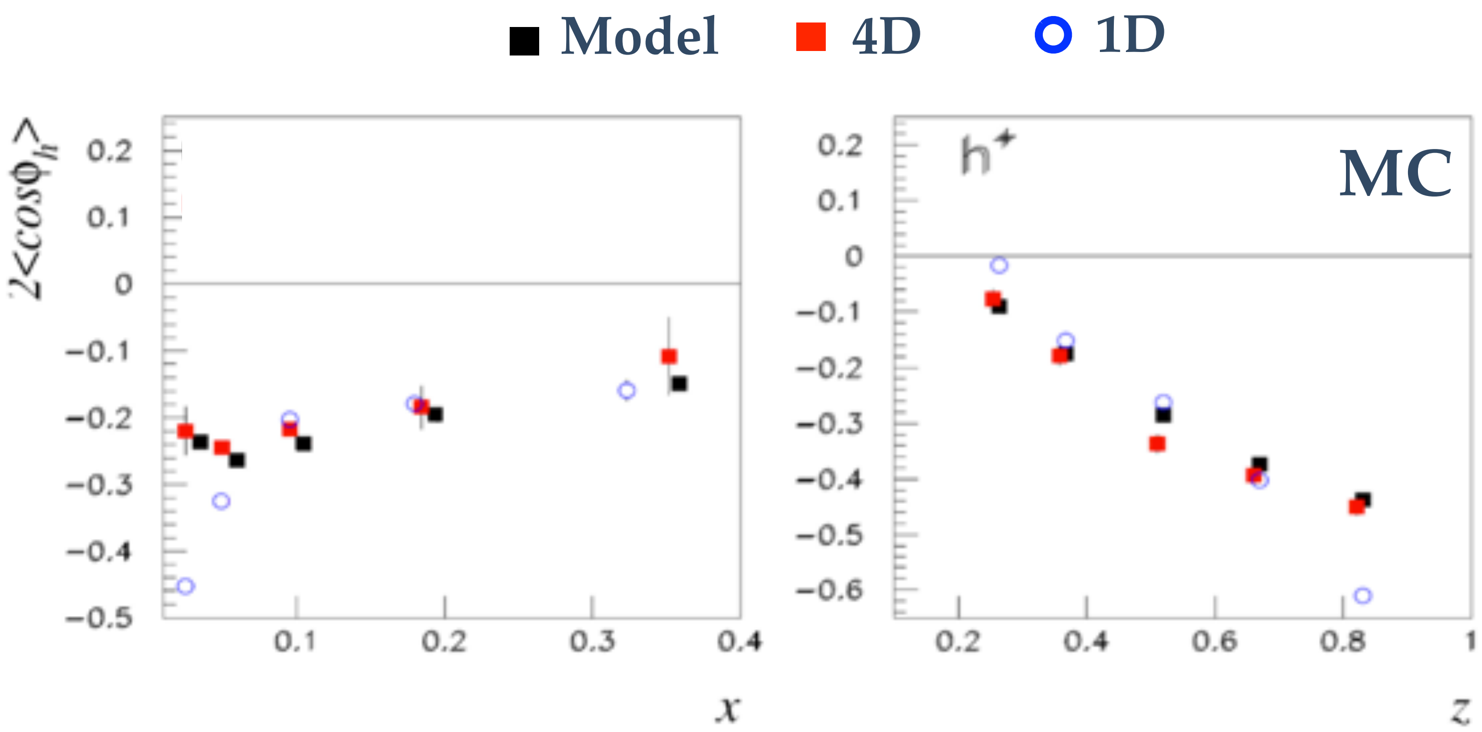

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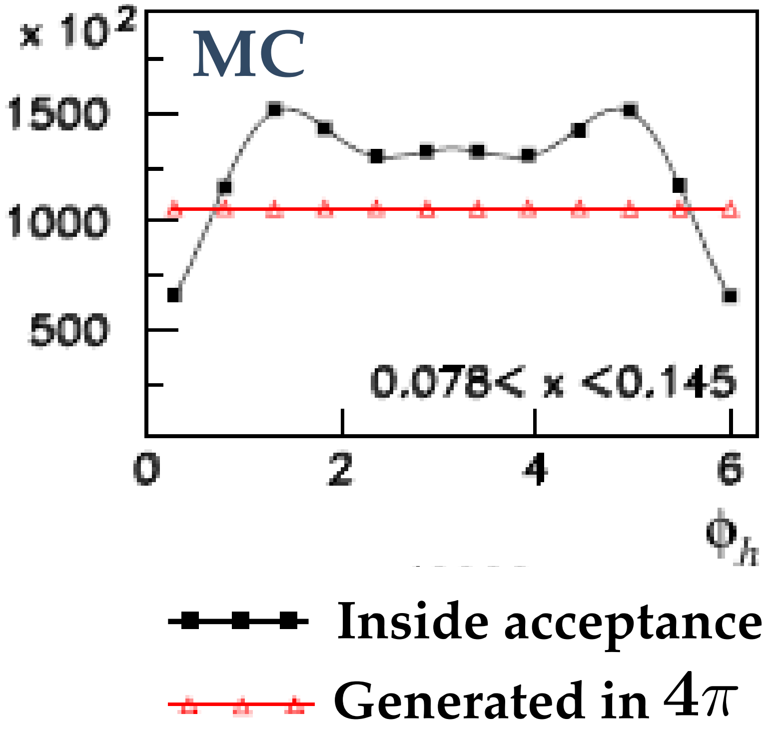
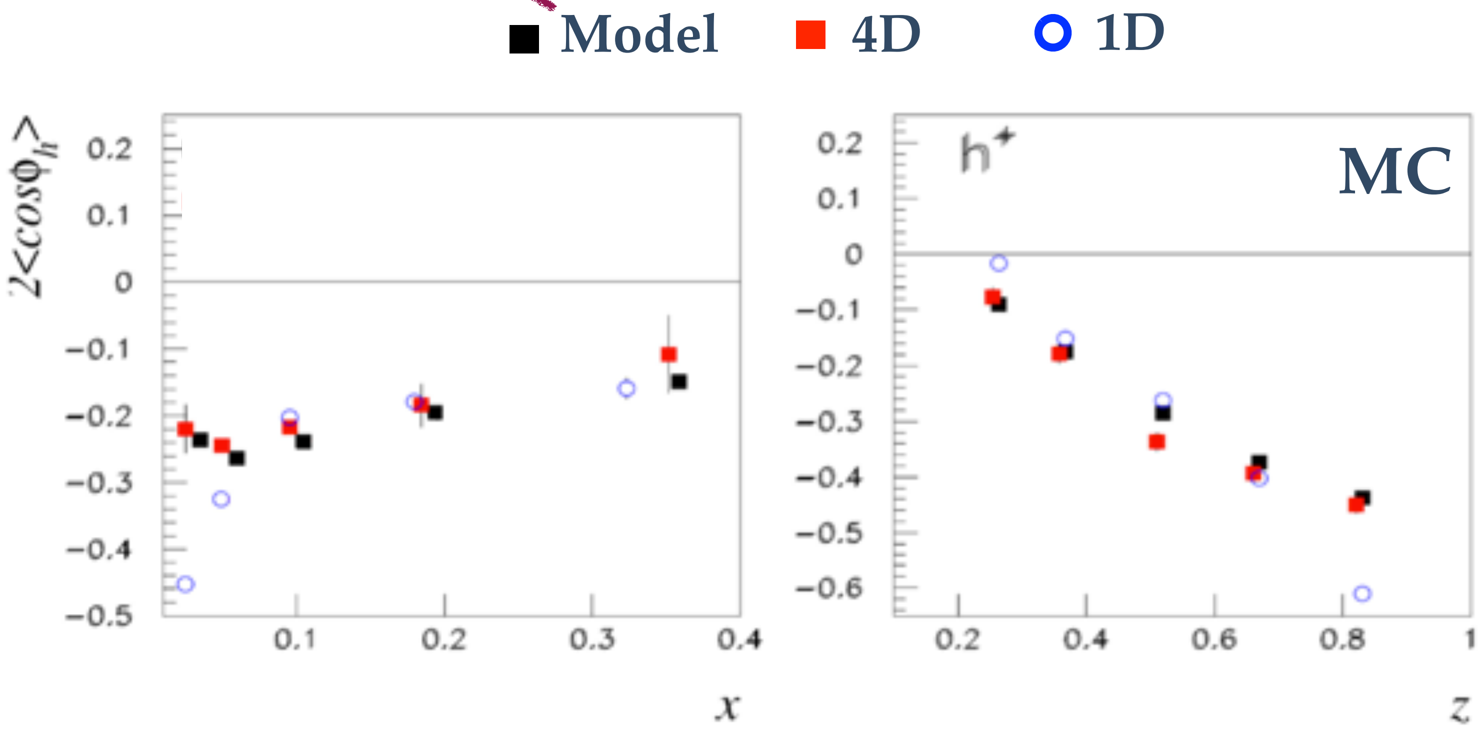


$$\sigma^0_w|_{mc}M(\cos\phi_h, \cos 2\phi_h)$$



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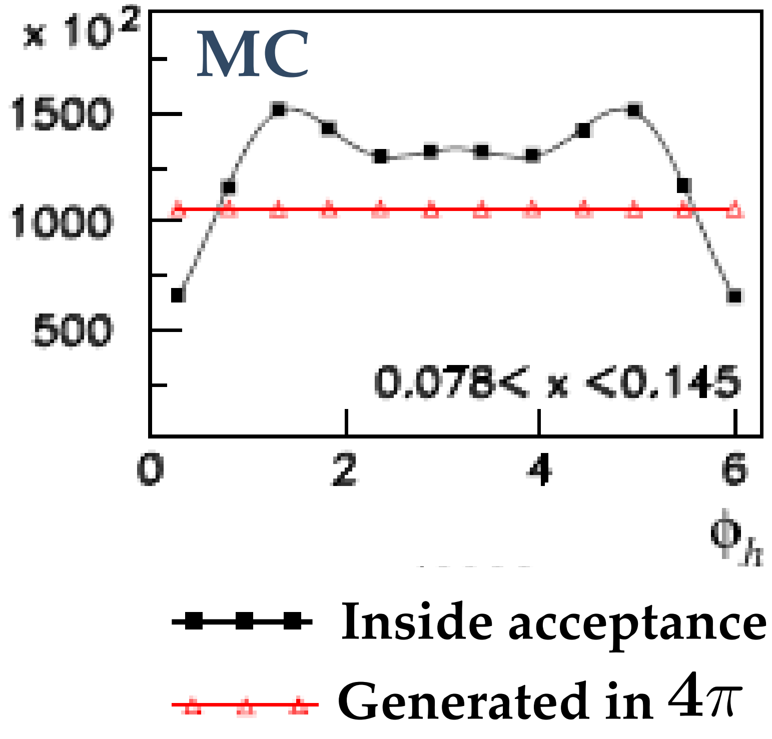

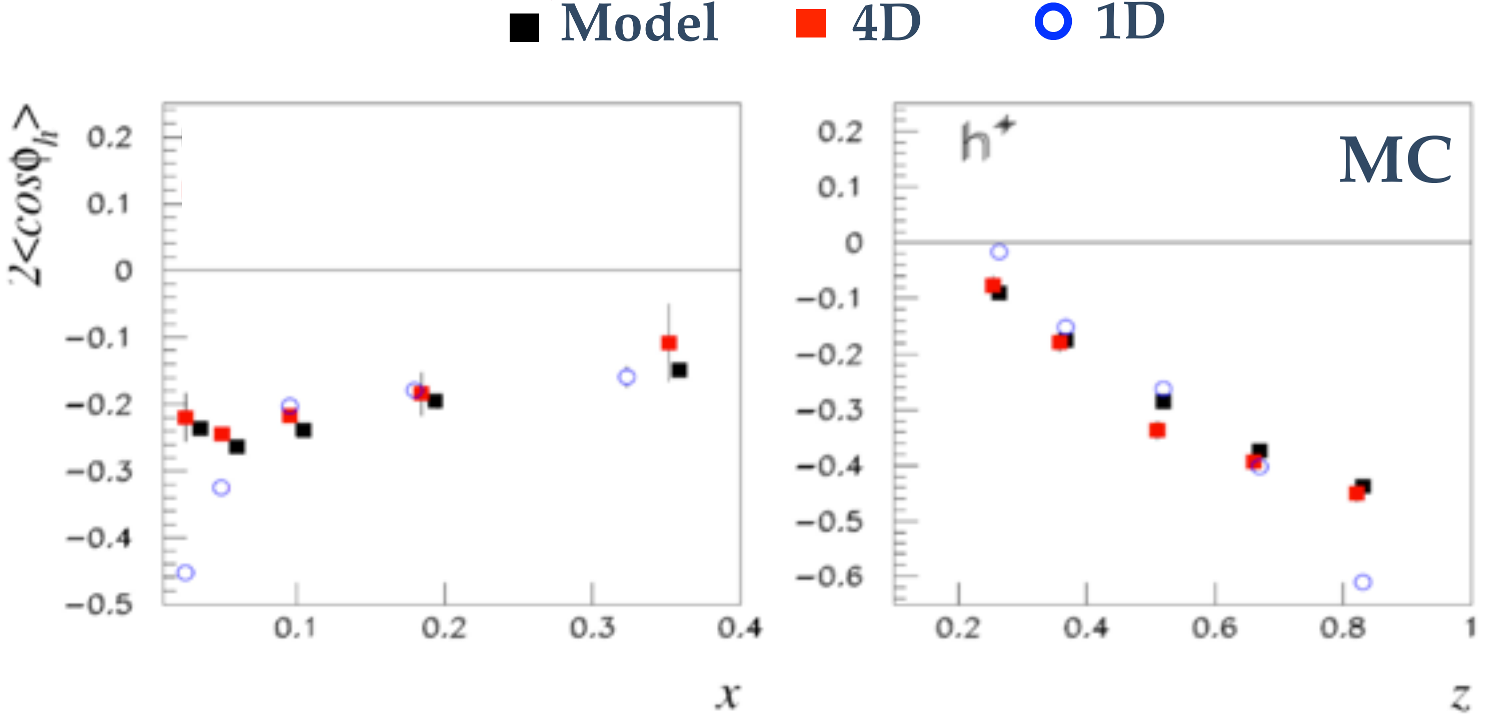

# *Acceptance Correction*

Francesca Giordano

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*4-Dimension analysis*

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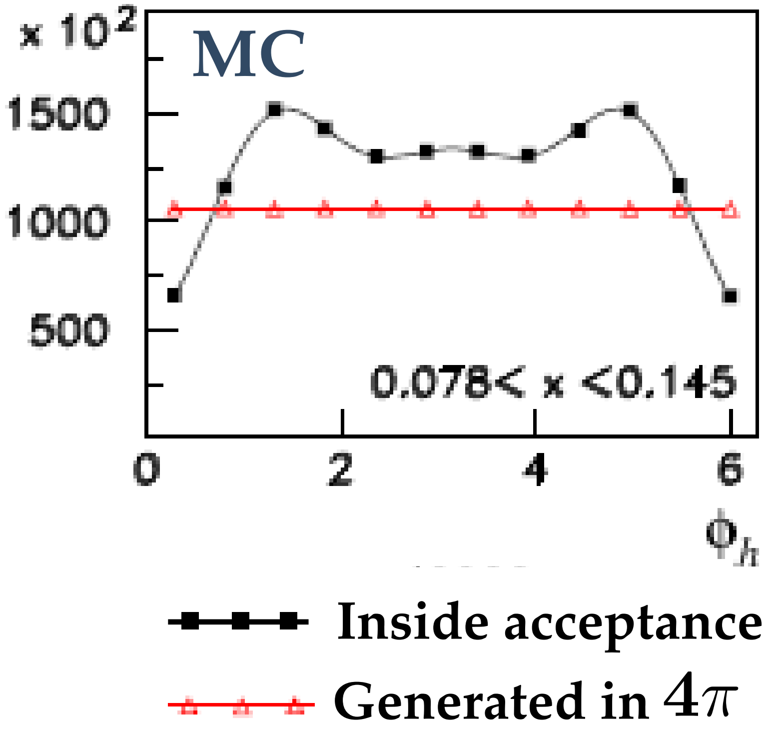

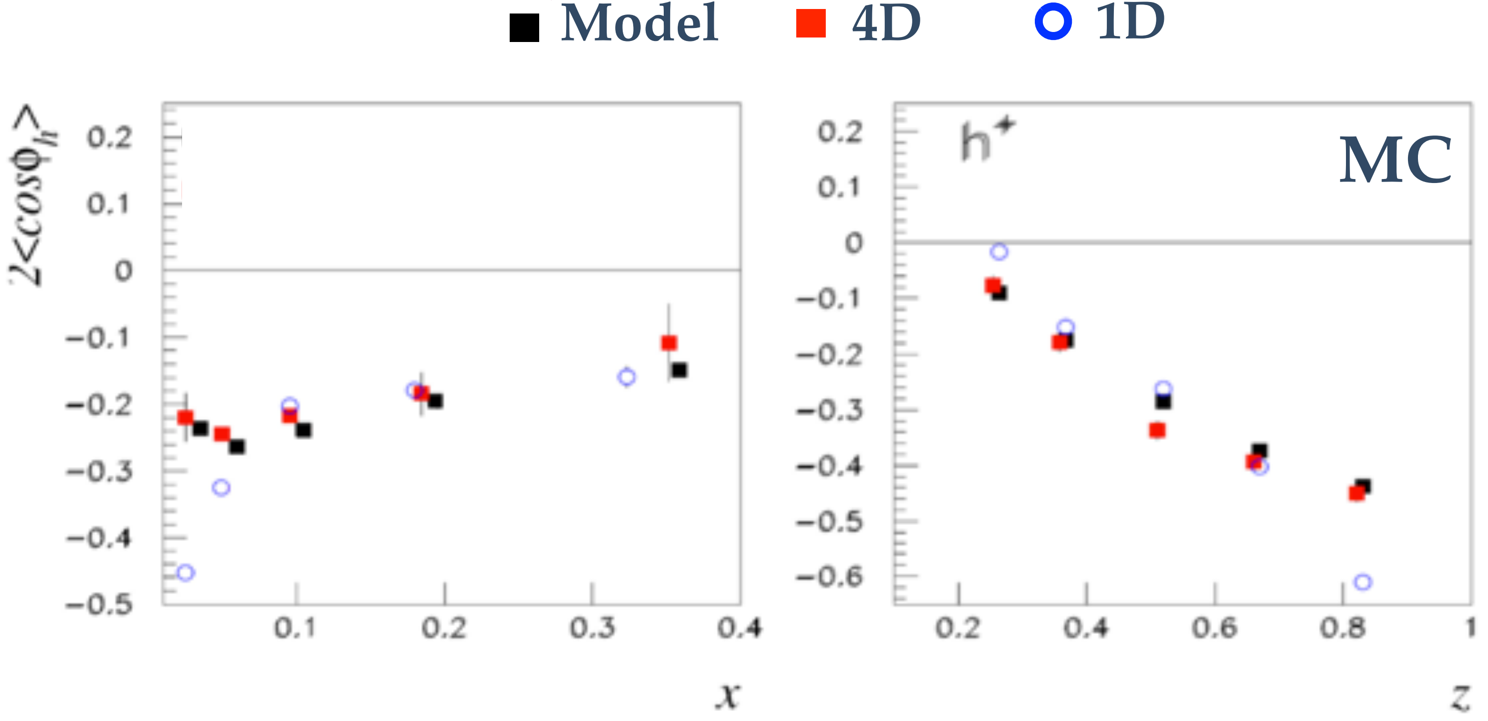

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## 4-dimensional $(w)$ unfolding

| Binning<br>900 kinematic bins x 12 $\phi_h$ -bins | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Variable | Bin limits | | | | | | | # |
| x | 0.023 | 0.042 | 0.078 | 0.145 | 0.27 | 0.6 | | 5 |
| y | 0.2 | 0.3 | 0.45 | 0.6 | 0.7 | 0.85 | | 5 |
| z | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.75 | 1 | 6 |
| $P_{h\perp}$ | 0.05 | 0.2 | 0.35 | 0.5 | 0.7 | 1 | 1.3 | 6 |

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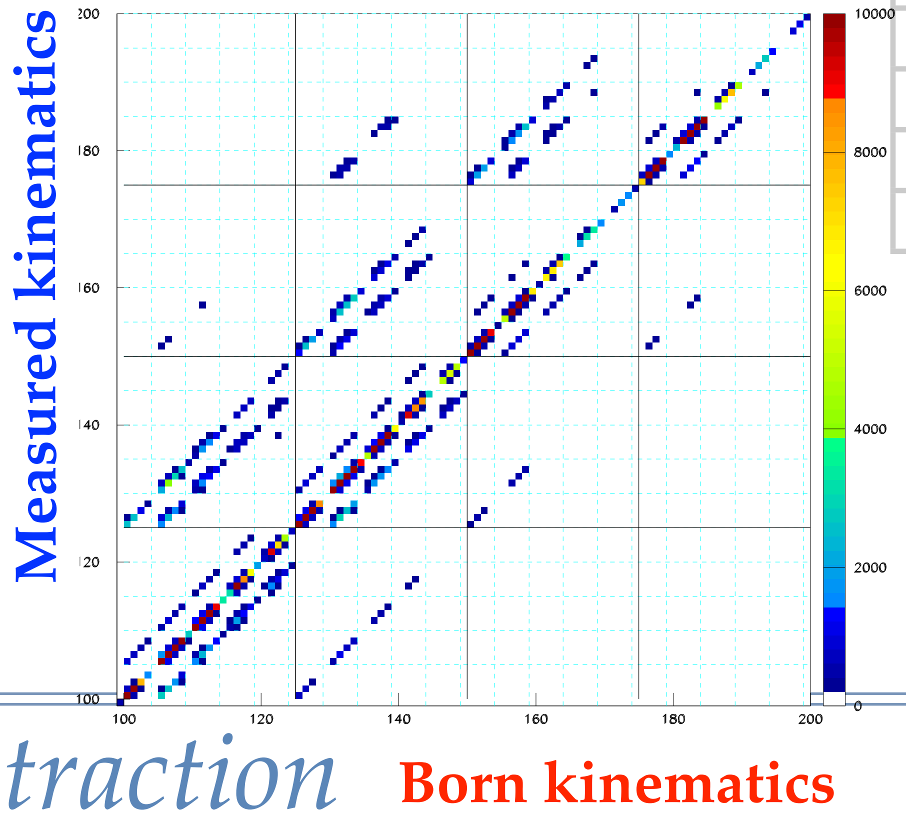

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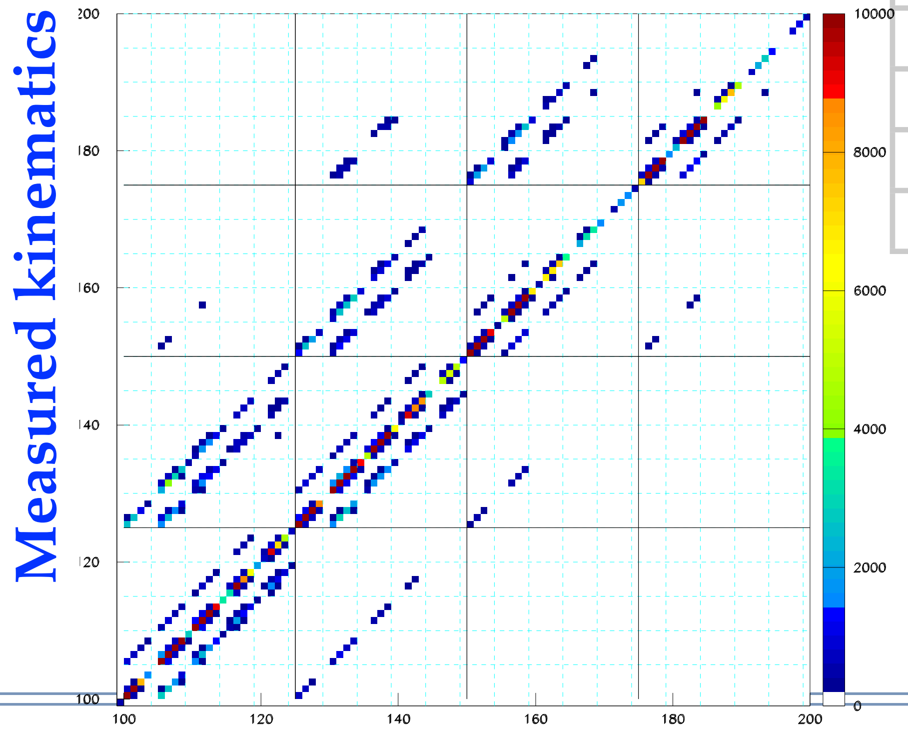

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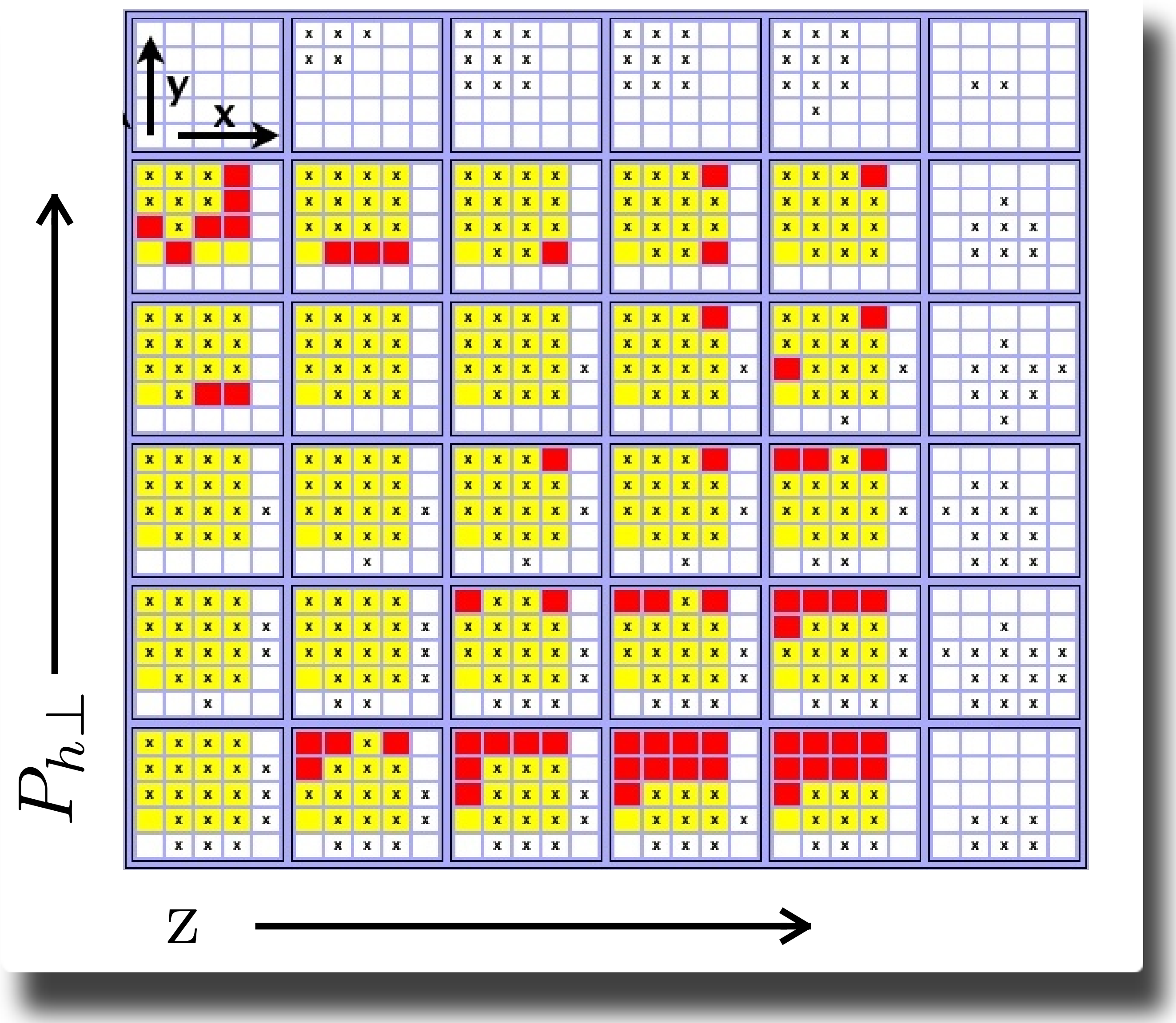

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# *Projection Versus The Single Variable*

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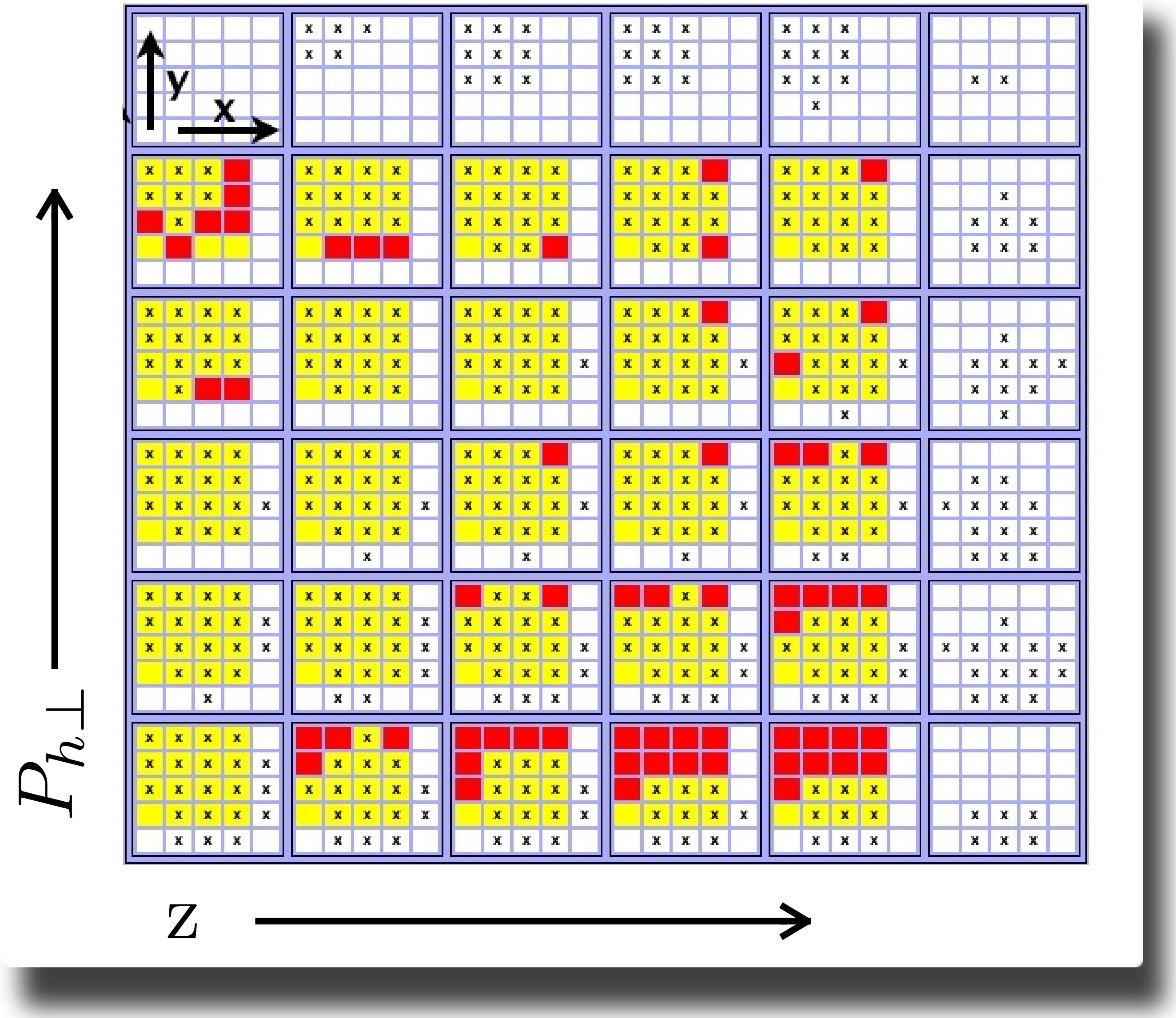

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*Systematic Checks*

Francesca Giordano

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1. $\sigma_w^0|_{mc}$ **Pythia**
2. $\sigma_w^0|_{mc} M(\cos\phi_h, \cos 2\phi_h)$ **Pythia + azimuthal modulations**

$$
\begin{aligned}
P = & \Big[\big(A_1 + A_2 x + A_3 y + A_4 z + A_5 P_{h\perp} + A_6 x^2 + A_7 y^2 + A_8 z^2 + A_9 P_{h\perp}^2 + \\
& A_{10} x\, y + A_{11} x\, z + A_{12} x\, P_{h\perp} + A_{13} y\, z + A_{14} y\, P_{h\perp} + A_{15} z\, P_{h\perp} + \\
& + A_{16} x^3 + A_{17} y^3 + A_{18} z^3 + A_{19} P_{h\perp}^3\big)\Big] cos\phi_h + \\
& \Big[\big(A_{20} + A_{21} x + A_{22} y + A_{23} z + A_{24} P_{h\perp} + A_{25} x^2 + A_{26} y^2 + A_{27} z^2 + A_{28} P_{h\perp}^2 + \\
& A_{29} x\, y + A_{30} x\, z + A_{31} x\, P_{h\perp} + A_{32} y\, z + A_{33} y\, P_{h\perp} + A_{34} z\, P_{h\perp} + \\
& + A_{35} x^3 + A_{36} y^3 + A_{37} z^3 + A_{38} P_{h\perp}^3\big)\Big] cos2\phi_h
\end{aligned}
$$

# *4D Parametrization*

$$
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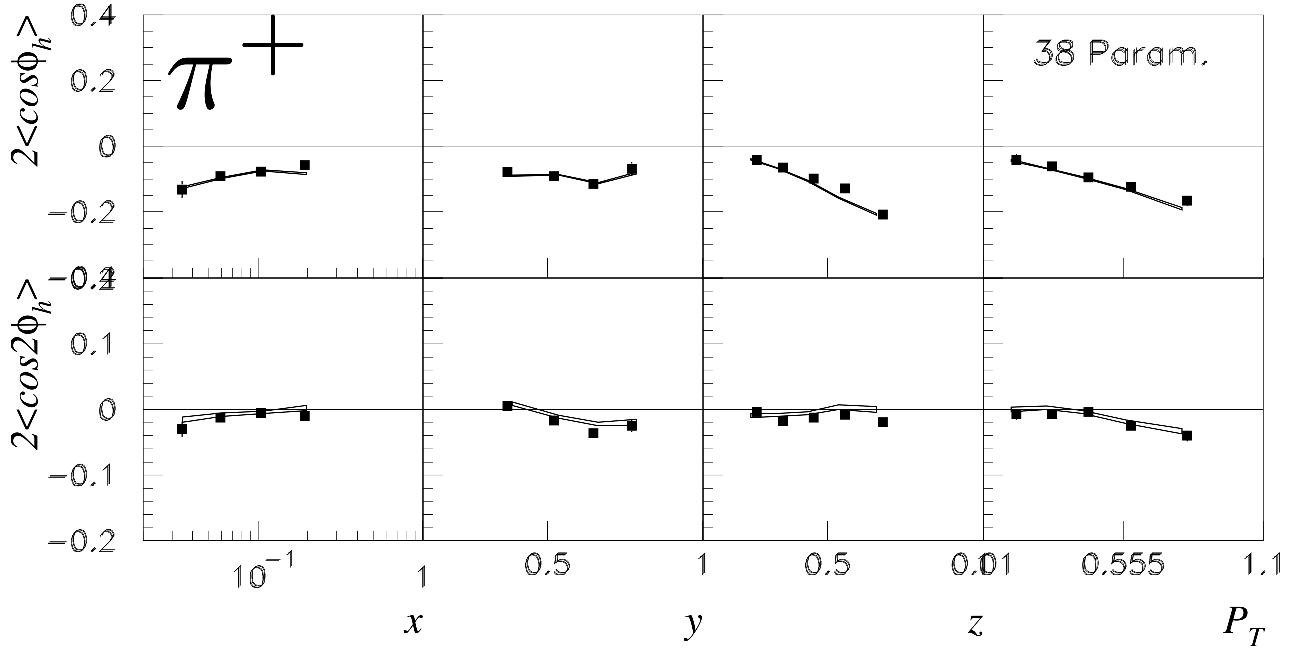
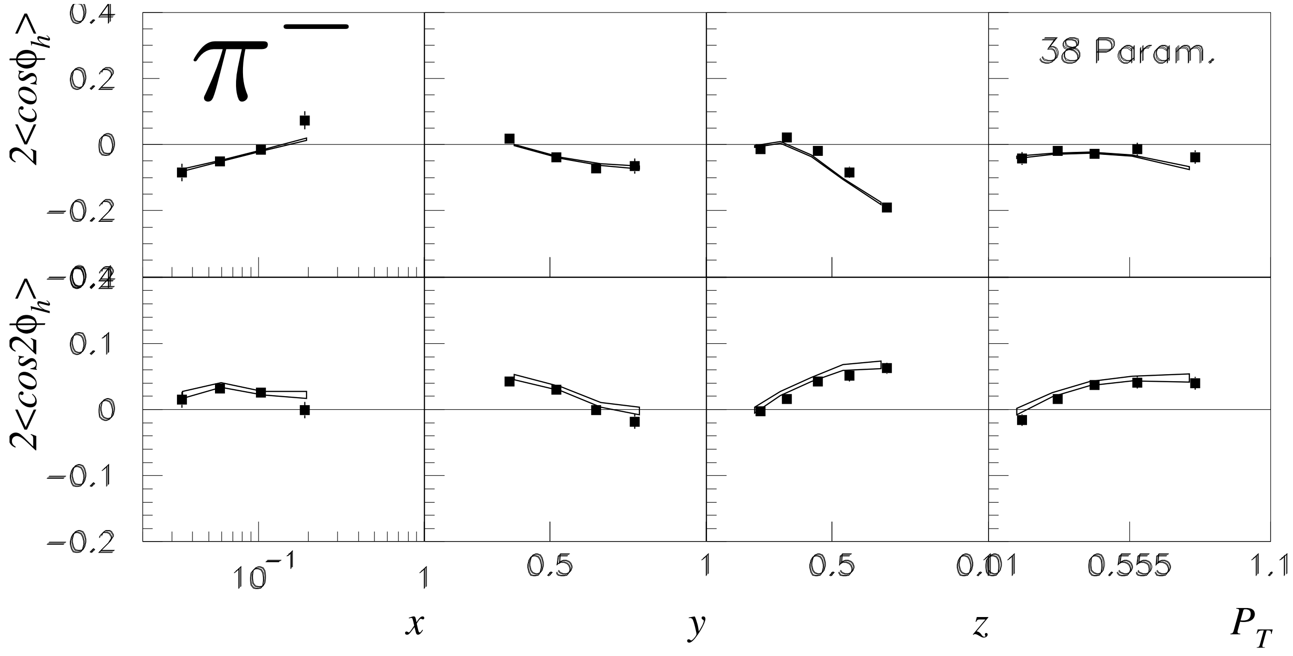
$$





# 4D Parametrization

$$P = \Big[\big(A_1 + A_2 x + A_3 y + A_4 z + A_5 P_{h\perp} + A_6 x^2 + A_7 y^2 + A_8 z^2 + A_9 P_{h\perp}^2 +$$

$$A_{10} x\, y + A_{11} x\, z + A_{12} x\, P_{h\perp} + A_{13} y\, z + A_{14} y\, P_{h\perp} + A_{15} z\, P_{h\perp} +$$

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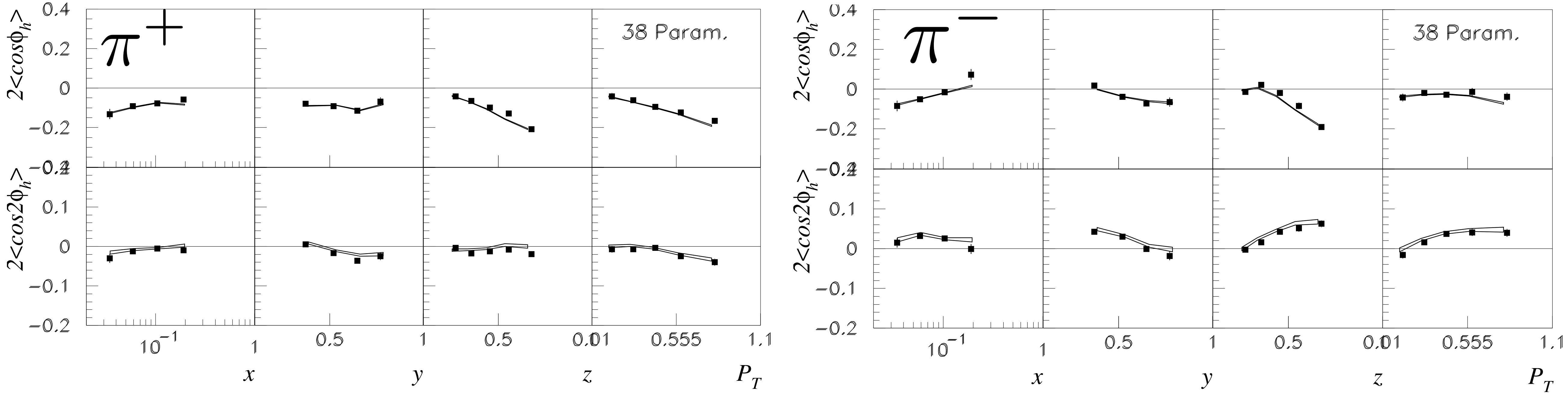
$$A_{29} x\, y + A_{30} x\, z + A_{31} x\, P_{h\perp} + A_{32} y\, z + A_{33} y\, P_{h\perp} + A_{34} z\, P_{h\perp} +$$

$$+ A_{35} x^3 + A_{36} y^3 + A_{37} z^3 + A_{38} P_{h\perp}^3\big)\Big] cos2\phi_h$$

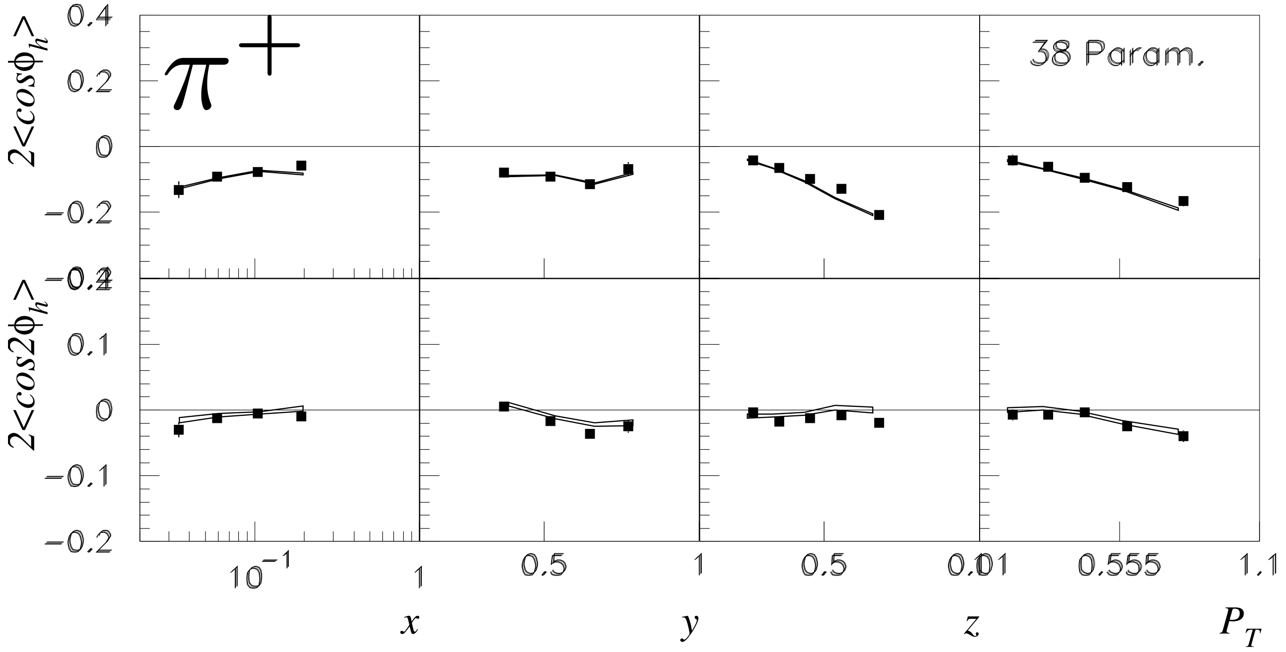


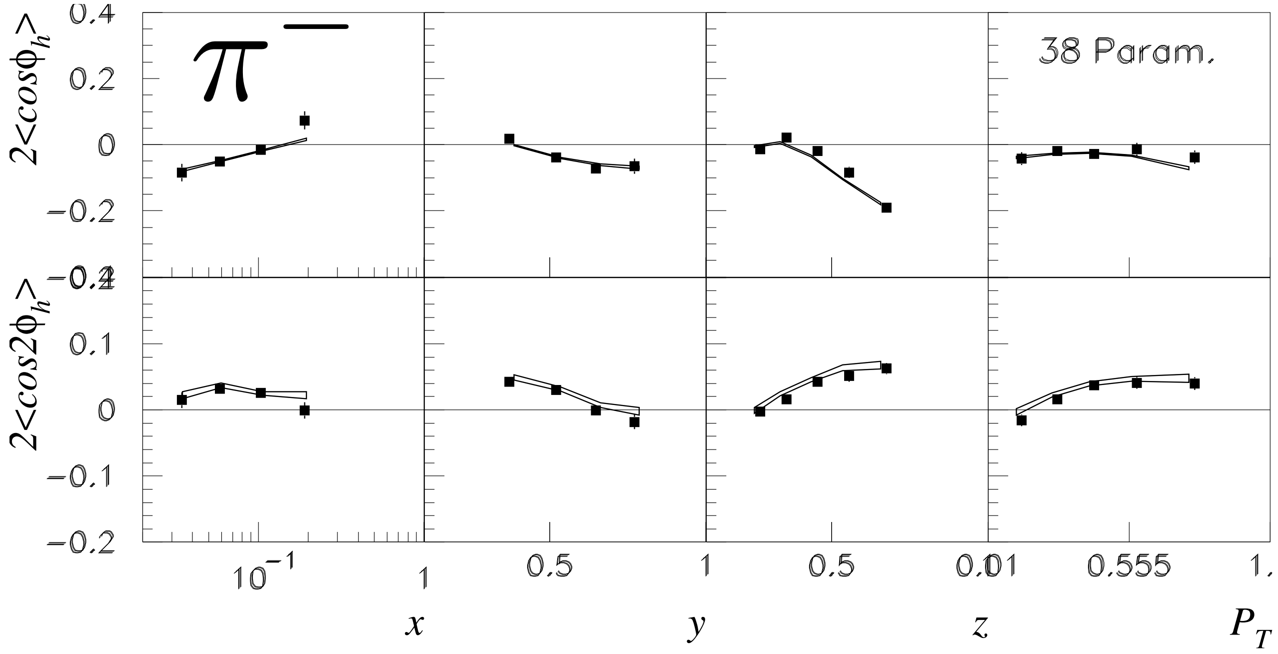
Used to generate a MC production whose cross section resemble the cosine modulation extracted from data

# 4D Parametrization

Francesca Giordano

$$
\begin{aligned}
P = \Big[ & \big(A_1 + A_2 x + A_3 y + A_4 z + A_5 P_{h\perp} + A_6 x^2 + A_7 y^2 + A_8 z^2 + A_9 P_{h\perp}^2 + \\
& A_{10} x\, y + A_{11} x\, z + A_{12} x\, P_{h\perp} + A_{13} y\, z + A_{14} y\, P_{h\perp} + A_{15} z\, P_{h\perp} + \\
& + A_{16} x^3 + A_{17} y^3 + A_{18} z^3 + A_{19} P_{h\perp}^3\big)\Big] cos\phi_h + \\
& \Big[\big(A_{20} + A_{21} x + A_{22} y + A_{23} z + A_{24} P_{h\perp} + A_{25} x^2 + A_{26} y^2 + A_{27} z^2 + A_{28} P_{h\perp}^2 + \\
& A_{29} x\, y + A_{30} x\, z + A_{31} x\, P_{h\perp} + A_{32} y\, z + A_{33} y\, P_{h\perp} + A_{34} z\, P_{h\perp} + \\
& + A_{35} x^3 + A_{36} y^3 + A_{37} z^3 + A_{38} P_{h\perp}^3\big)\Big] cos2\phi_h
\end{aligned}
$$





Used to generate a MC production whose cross section resemble the cosine modulation extracted from data

Used to correct or evaluate sensitivity of other **HERMES** analysis on cosine modulations

# *4D Parametrization*

## ☑ Model dependence of unfolding procedure:

$$n_{born} = S^{-1}[n - B_0]$$

**Model independent only if fully differential ratio (4D binning)**

**only in the limit of infinitely small bins**

**are the bins small enough?**

## ☑ Model dependence of unfolding procedure:

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Different cross section models used to build:

# ☑ Model dependence of unfolding procedure:

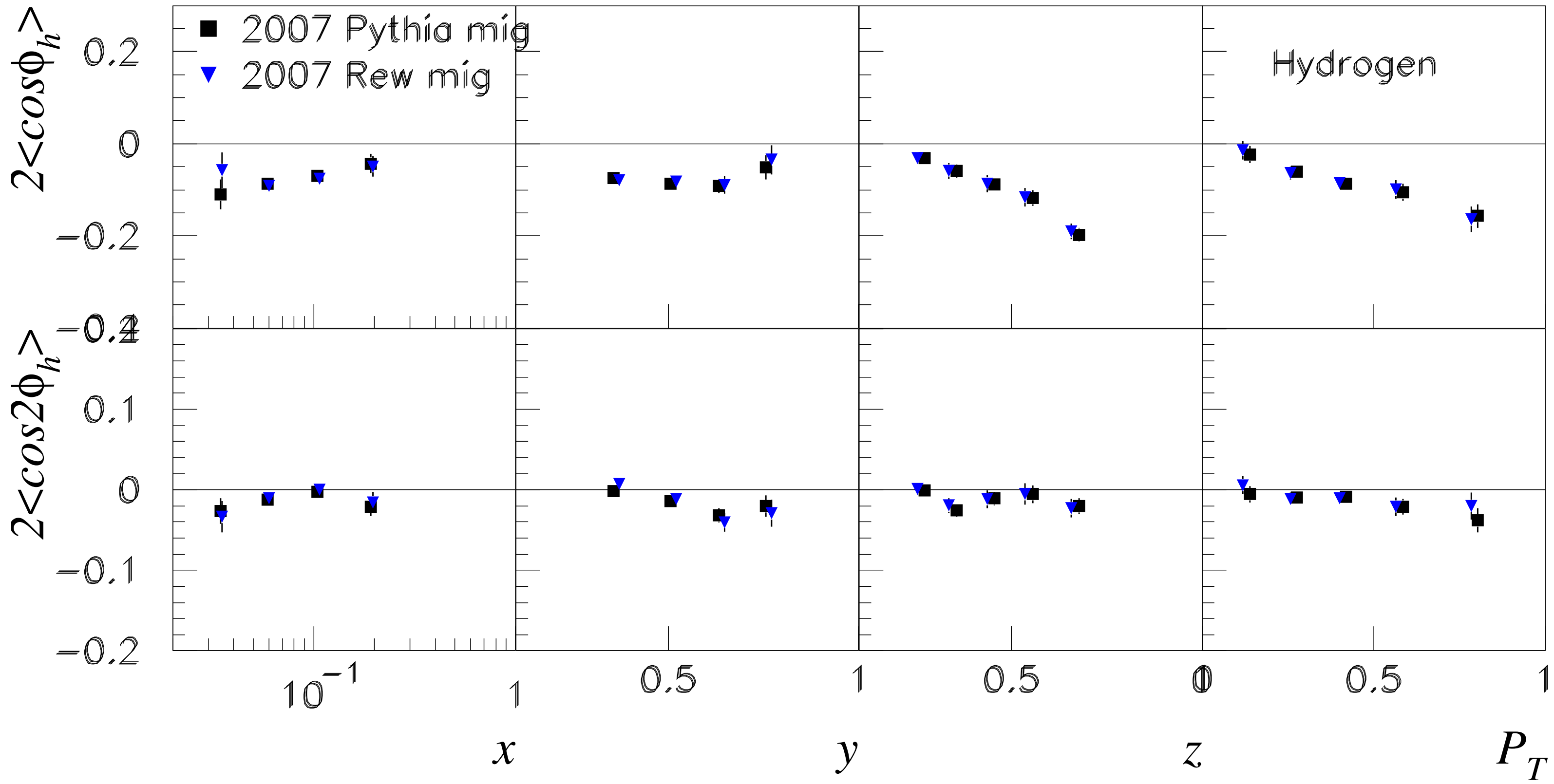
$$n_{born} = S^{-1}[n - B_0]$$

**Model independent only if fully differential ratio (4D binning)**

**only in the limit of infinitely small bins**

**are the bins small enough?**

Different cross section models used to build: the smearing matrix:



■ Subsample corrected with flat MC

▼ Subsample corrected with MC that resemble data modulations

## *Systematic Checks*

Francesca Giordano

# ☑ Model dependence of unfolding procedure:

Model independent only if fully differential ratio (4D binning)

only in the limit of infinitely small bins

are the bins small enough?

$$n_{born} = S^{-1}[n - B_0]$$

Different cross section models used to build:

*Systematic Checks*

Francesca Giordano

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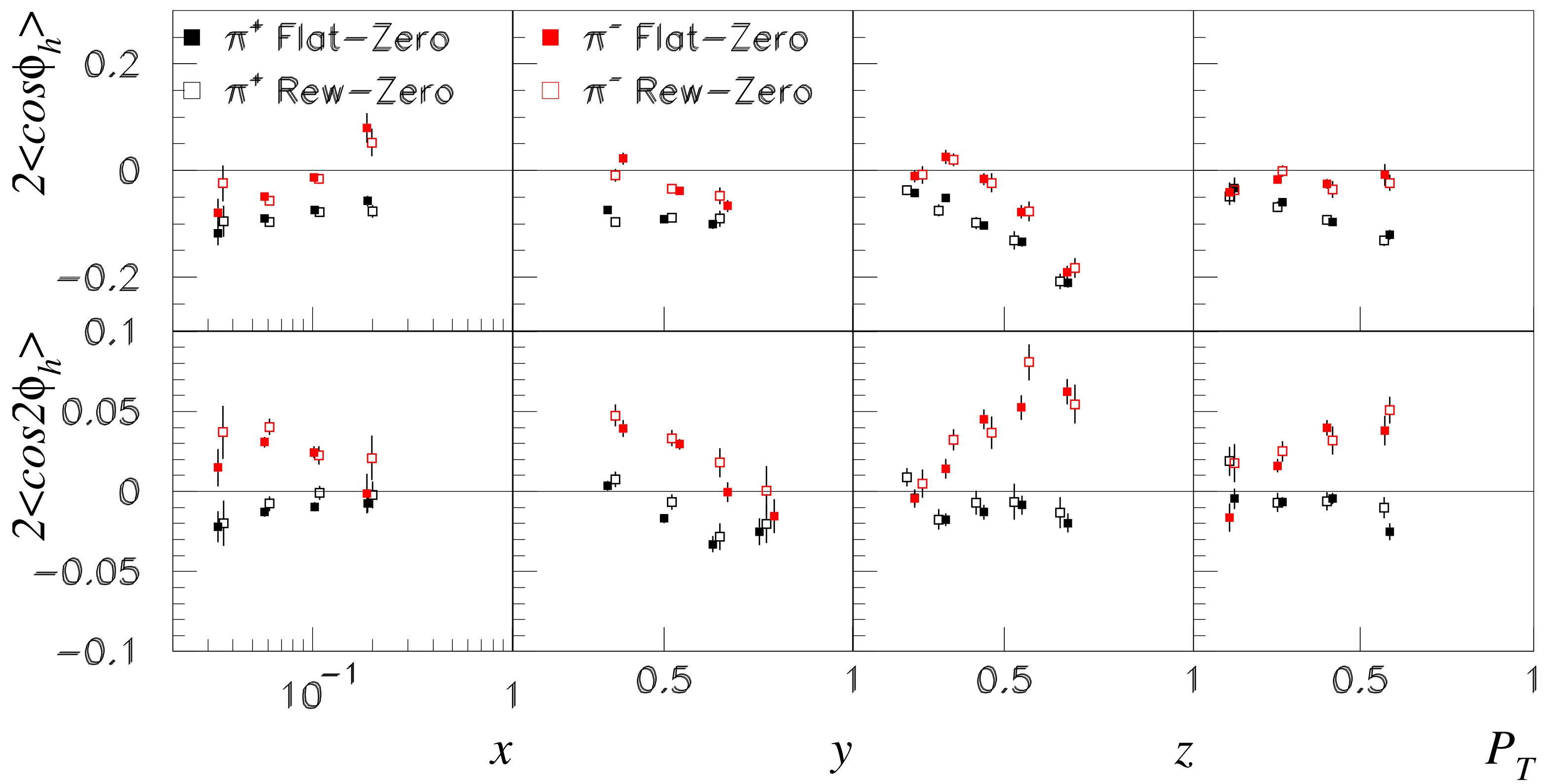
$$n_{born} = S^{-1}[n - B_0]$$

Model independent only if fully differential ratio (4D binning)

only in the limit of infinitely small bins

are the bins small enough?

Different cross section models used to build: the background vector



$\pi^+$

■ Subsample corrected with flat background

□ Subsample corrected with background that resemble data modulations

$\pi^-$

🟥 Subsample corrected with flat background

□ Subsample corrected with background that resemble data modulations

☑ Model dependence of unfolding procedure:
different cross section models

☑ RICH efficiency& contamination

☑ Binning effect

☑ Unfolding method

☑ Time stability

# *Systematic Checks*

☑ Model dependence of unfolding procedure: different cross section models

☑ RICH efficiency& contamination

☑ Binning effect

☑ Unfolding method

☑ Time stability

☐ Dipole Magnet misalignement on different charges tested

☐ Beam/Detector misalignment effects tested

☐ Effects due to target transverse/longitudinal magnets

☐ Residual modulations related to acceptance

☐ Effects at the edge of spectrometer

☐ ....

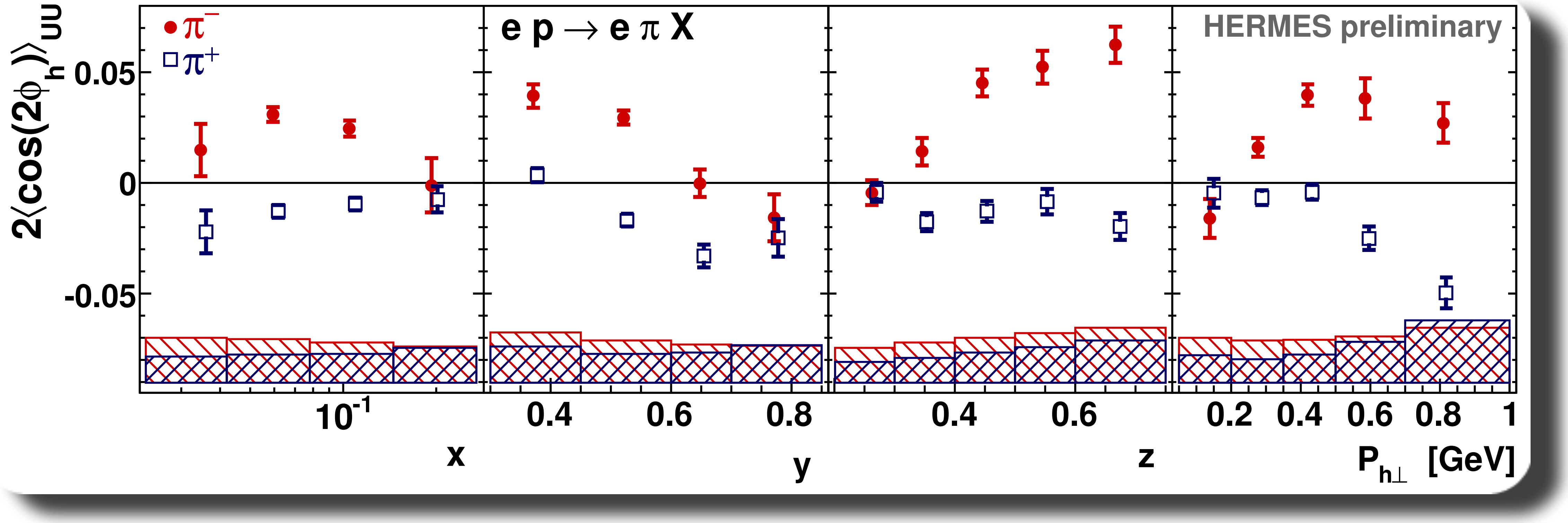
# *Systematic Checks*

$\cos 2\phi_h$

pions

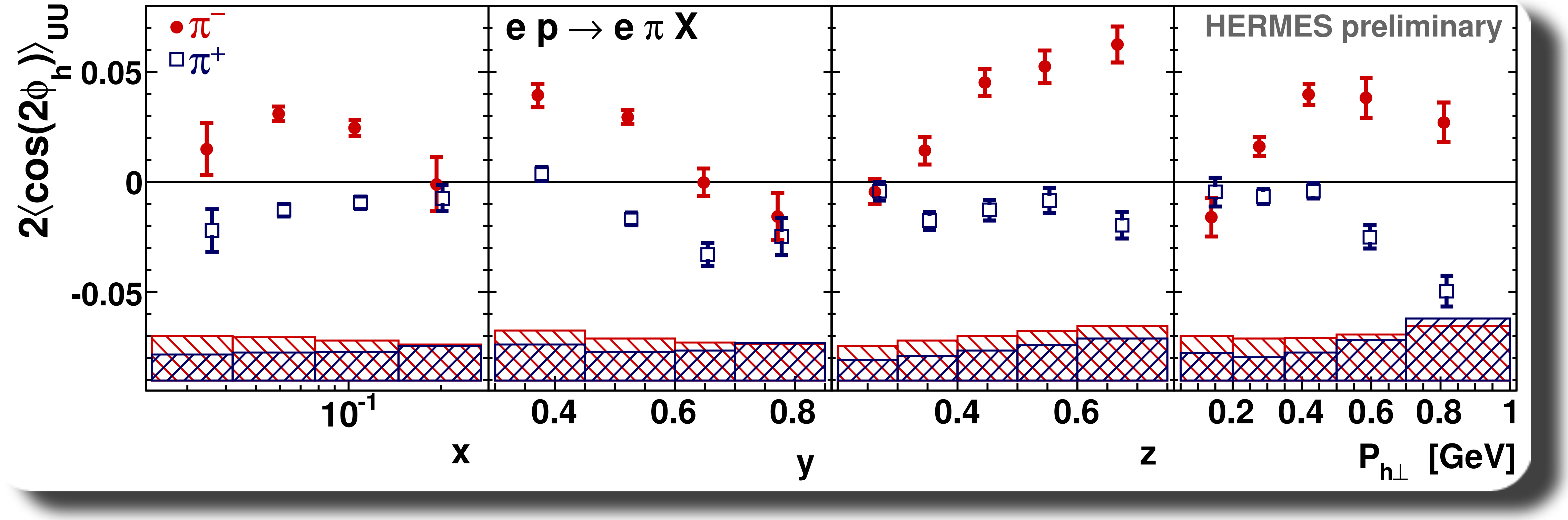
$\cos 2\phi_h$

pions



*Pions, Hydrogen*

# Pions, Hydrogen

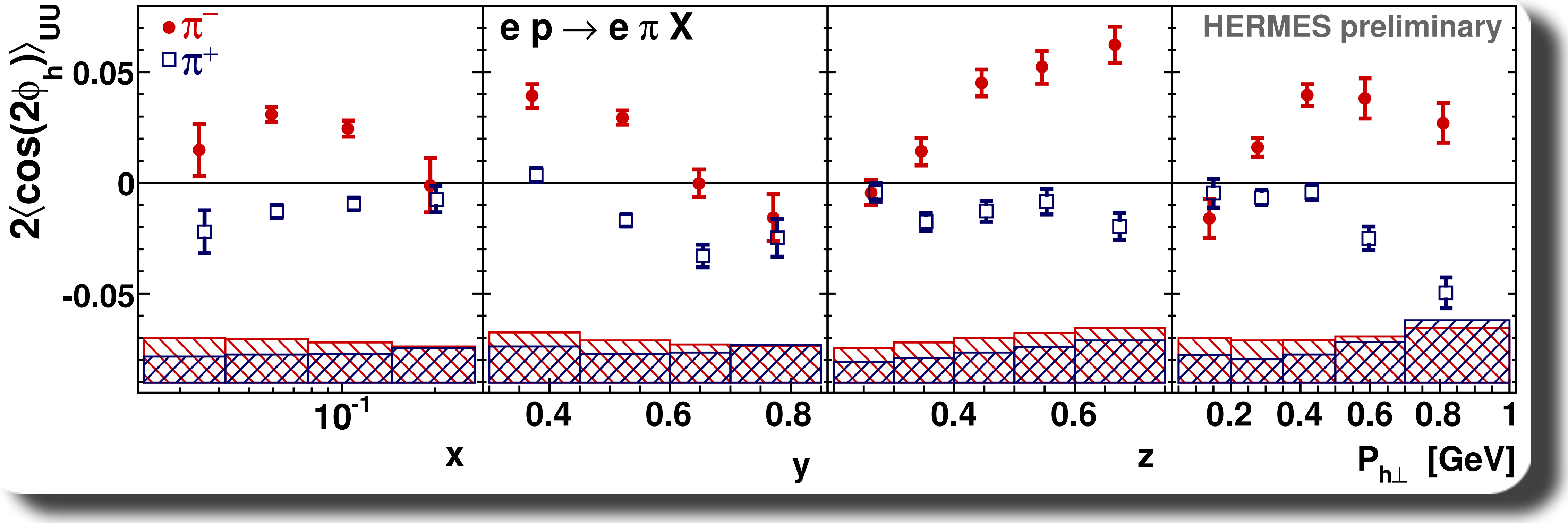


$\cos 2\phi_h$

pions

$$\propto C[\overset{\text{Boer-Mulders}}{-h_1^{\perp}H_1^{\perp}} + \frac{\kappa_T^2}{Q^2}\underset{\text{Cahn}}{f_1 D_1} + ....]$$

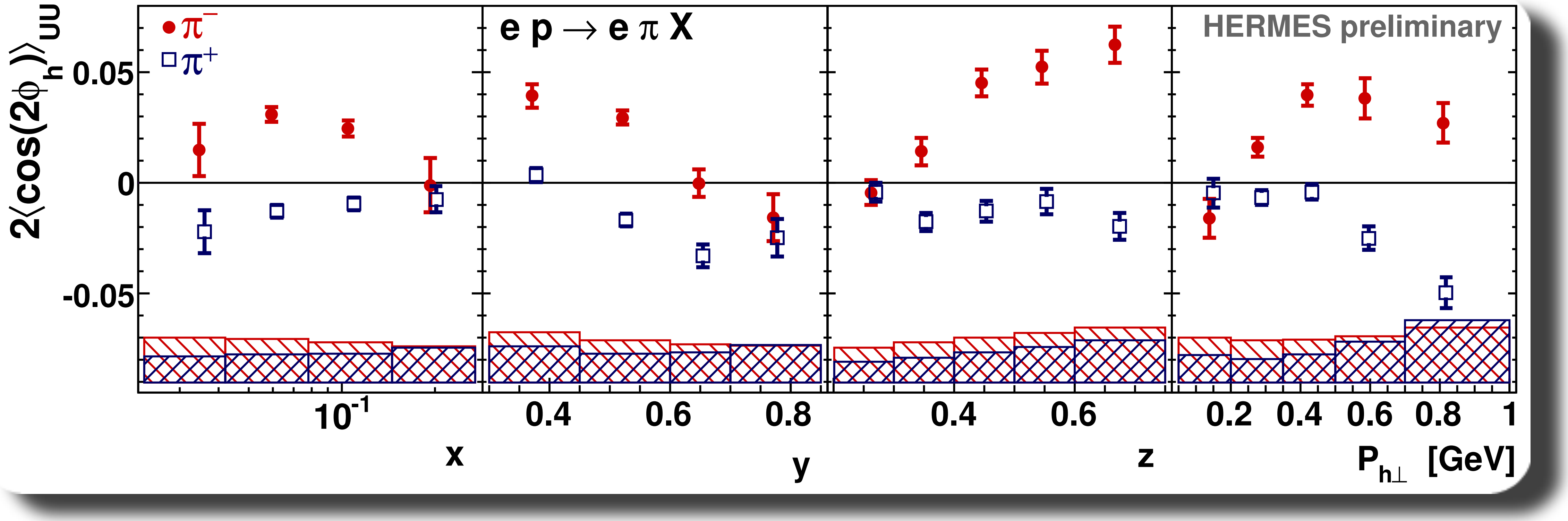
## $\cos 2\phi_h$
## pions



$$\propto C[\overset{\text{Boer-Mulders}}{-h_1^\perp H_1^\perp} + \frac{\kappa_T^2}{Q^2}\underset{\text{Cahn}}{f_1 D_1} + \ldots.]$$

Cahn expected flavor blind

## *Pions, Hydrogen*



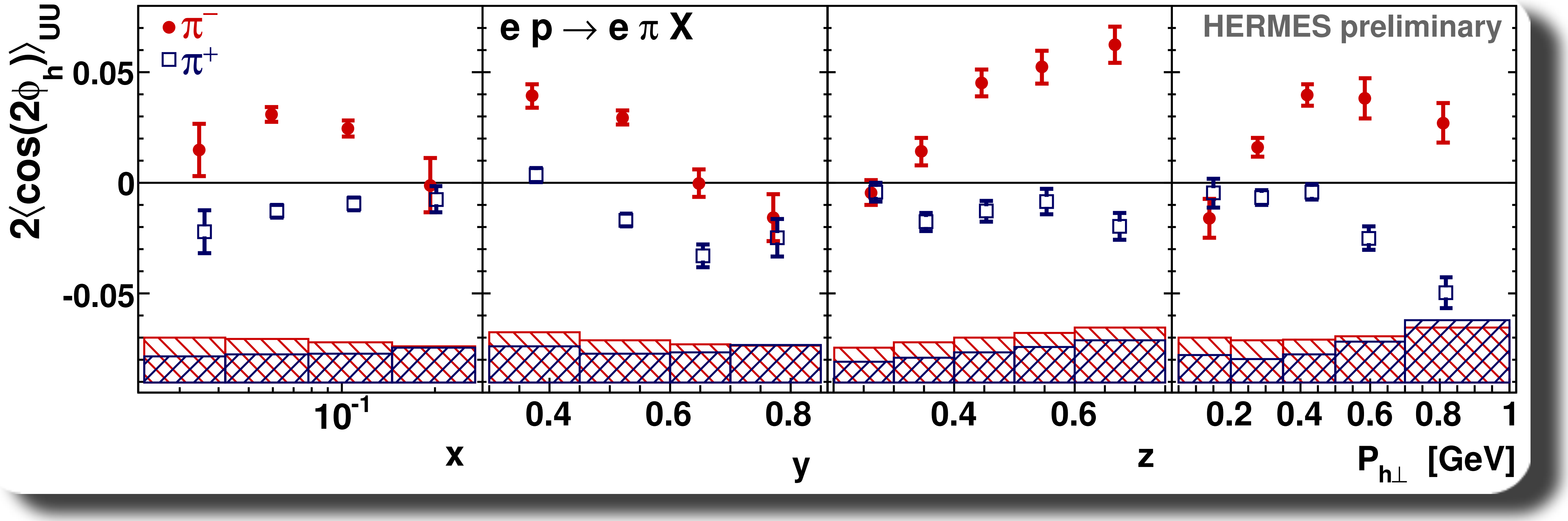
# $\cos 2\phi_h$ pions

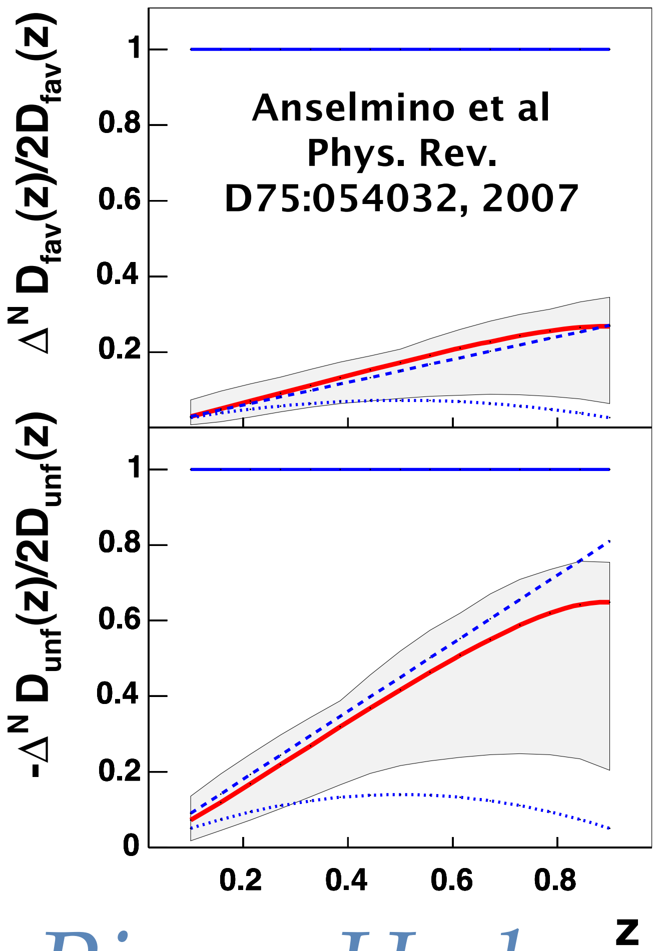
$$\propto C[\overset{\text{Boer-Mulders}}{-h_1^\perp H_1^\perp} + \underset{\text{Cahn}}{\frac{\kappa_T^2}{Q^2} f_1 D_1} + ....]$$

Cahn expected flavor blind

different $\pi^+/\pi^-$amplitudes
⇨Boer-Mulders effect

# $\cos 2\phi_h$ pions





Anselmino et al
Phys. Rev.
D75:054032, 2007

$$\propto C[-\overset{\text{Boer-Mulders}}{h_1^\perp H_1^\perp} + \frac{\kappa_T^2}{Q^2}\underset{\text{Cahn}}{f_1 D_1} + ....]$$

$$H_1^{\perp,u\rightarrow\pi^-} \approx -H_1^{\perp,u\rightarrow\pi^+}$$

Cahn expected flavor blind

different $\pi^+/\pi^-$amplitudes
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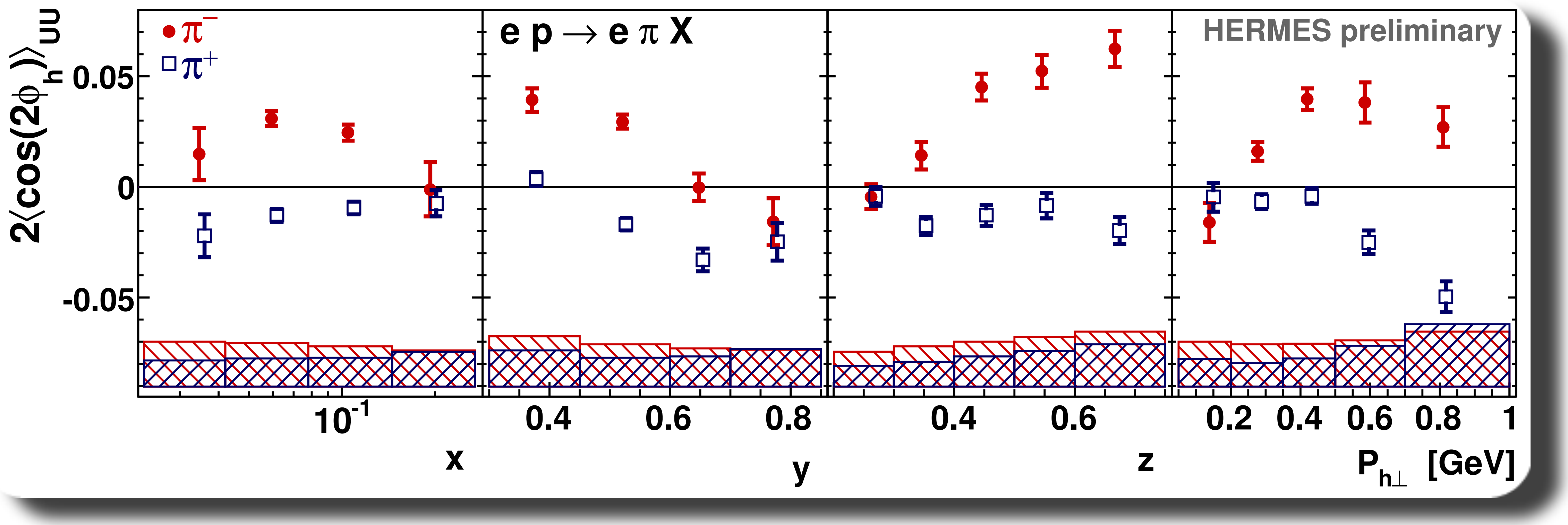
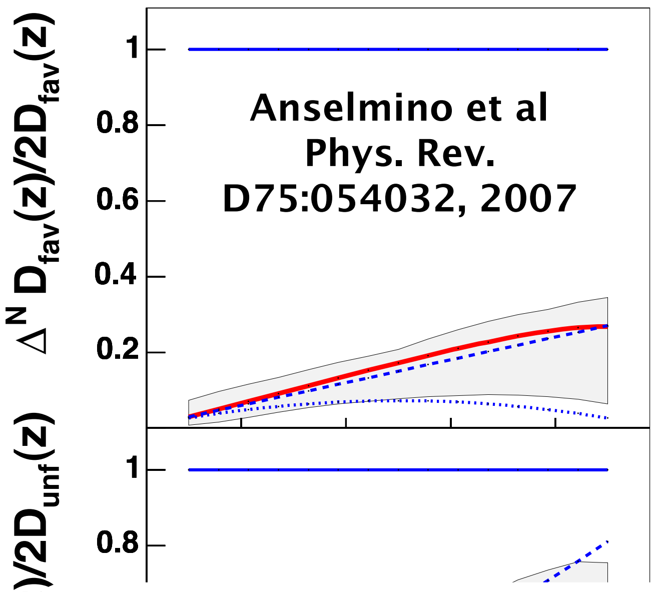
## *Pions, Hydrogen*



# $\cos 2\phi_h$
# pions

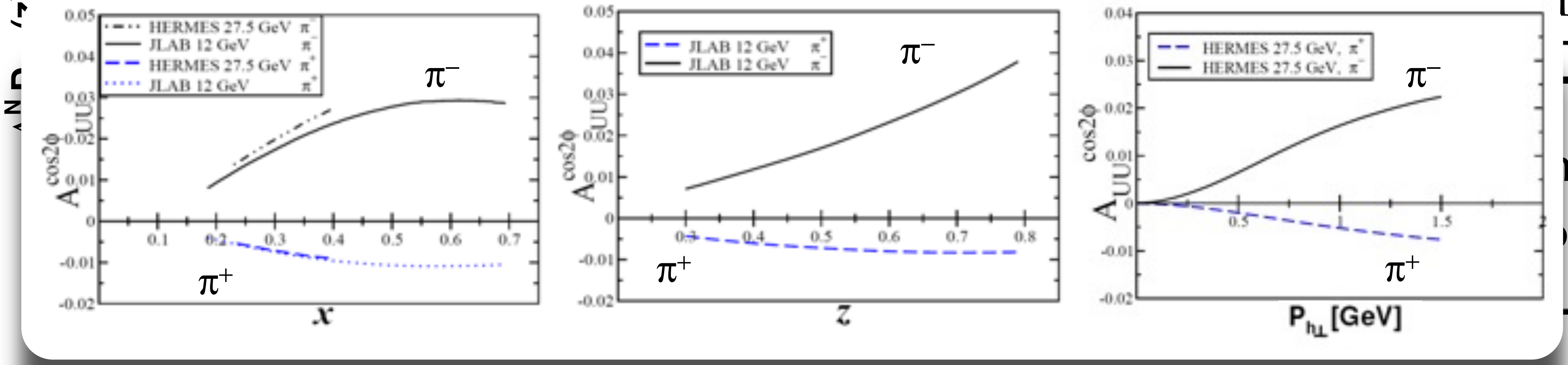


Anselmino et al
Phys. Rev. D75:054032, 2007

$$\propto C[\overset{\text{Boer-Mulders}}{-h_1^\perp H_1^\perp} + \frac{\kappa_T^2}{Q^2}\underset{\text{Cahn}}{f_1 D_1} + \ldots]$$

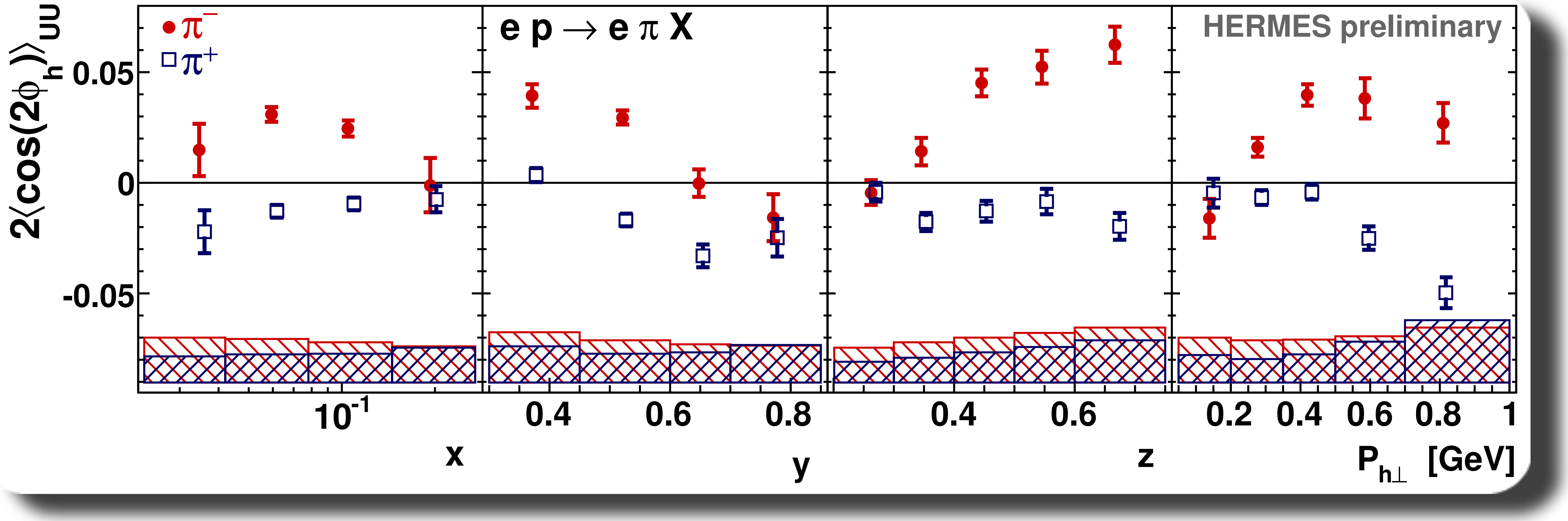
**Gamberg, Goldstein**
**Phys. Rev. D77:094016, 2008**

$$H_1^{\perp,u\to\pi^-} \approx -H_1^{\perp,u\to\pi^+}$$



C n expected flavor blind

t $\pi^+/\pi^-$ amplitudes
er-Mulders effect



$\cos 2\phi_h$
pions

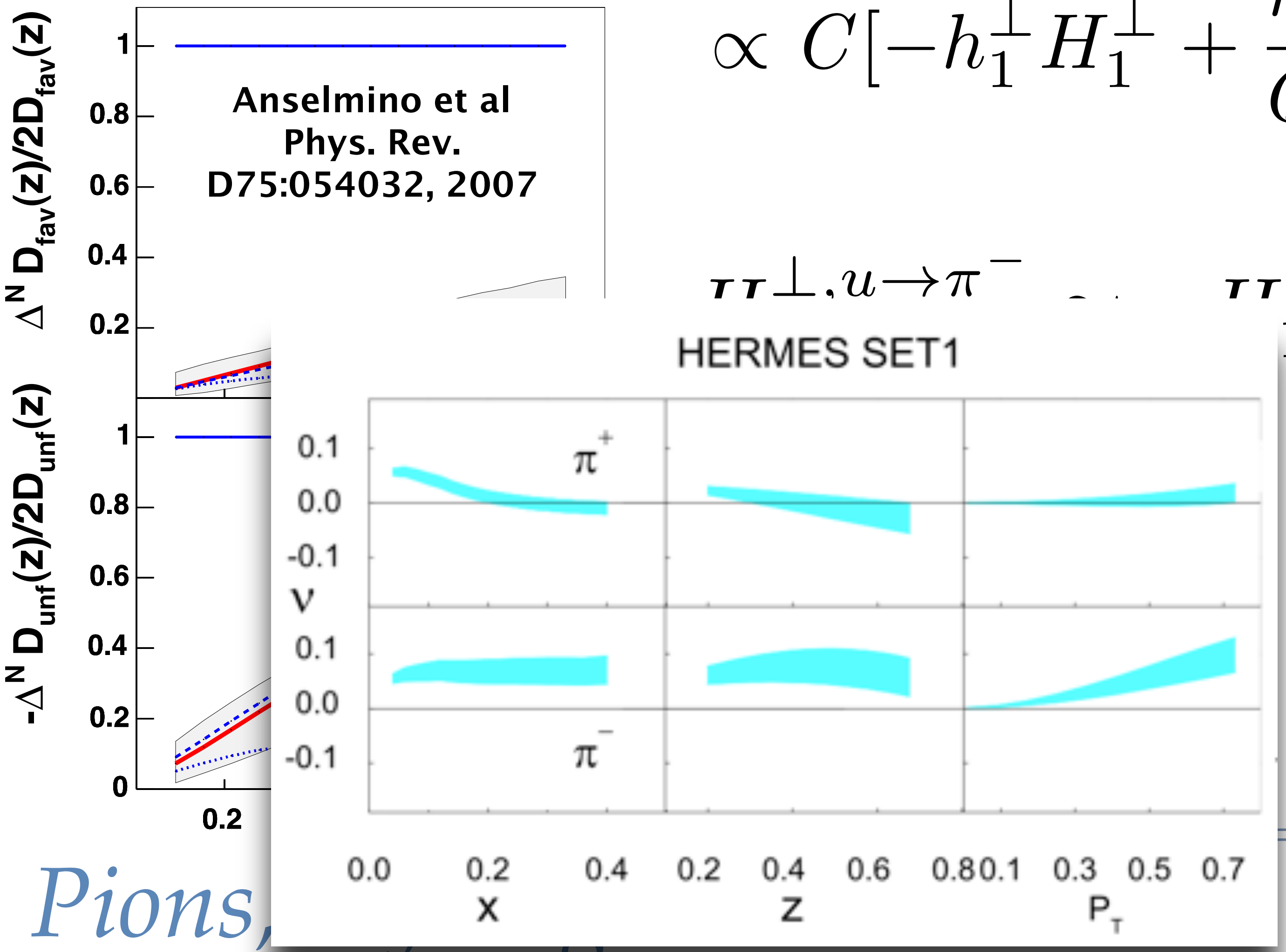
Boer-Mulders

$$\propto C[-h_1^{\perp}H_1^{\perp} + \frac{\kappa_T^2}{Q^2} f_1 D_1 + \ldots]$$

Cahn

**Gamberg, Goldstein**
**Phys. Rev. D77:094016, 2008**

**Zhang et al**
**Phys. Rev. D78:034035, 2008**



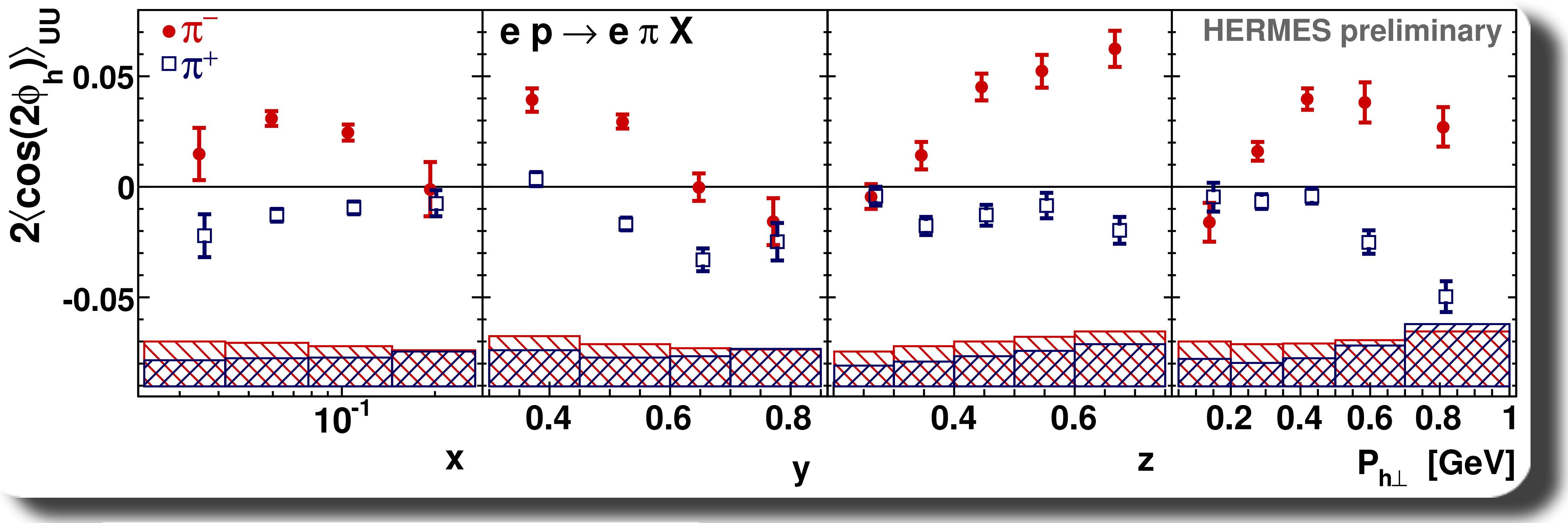
**Anselmino et al**
**Phys. Rev.**
**D75:054032, 2007**

$$H^{\perp,u\to\pi^-} \quad H_1^{\perp,u\to\pi^+}$$

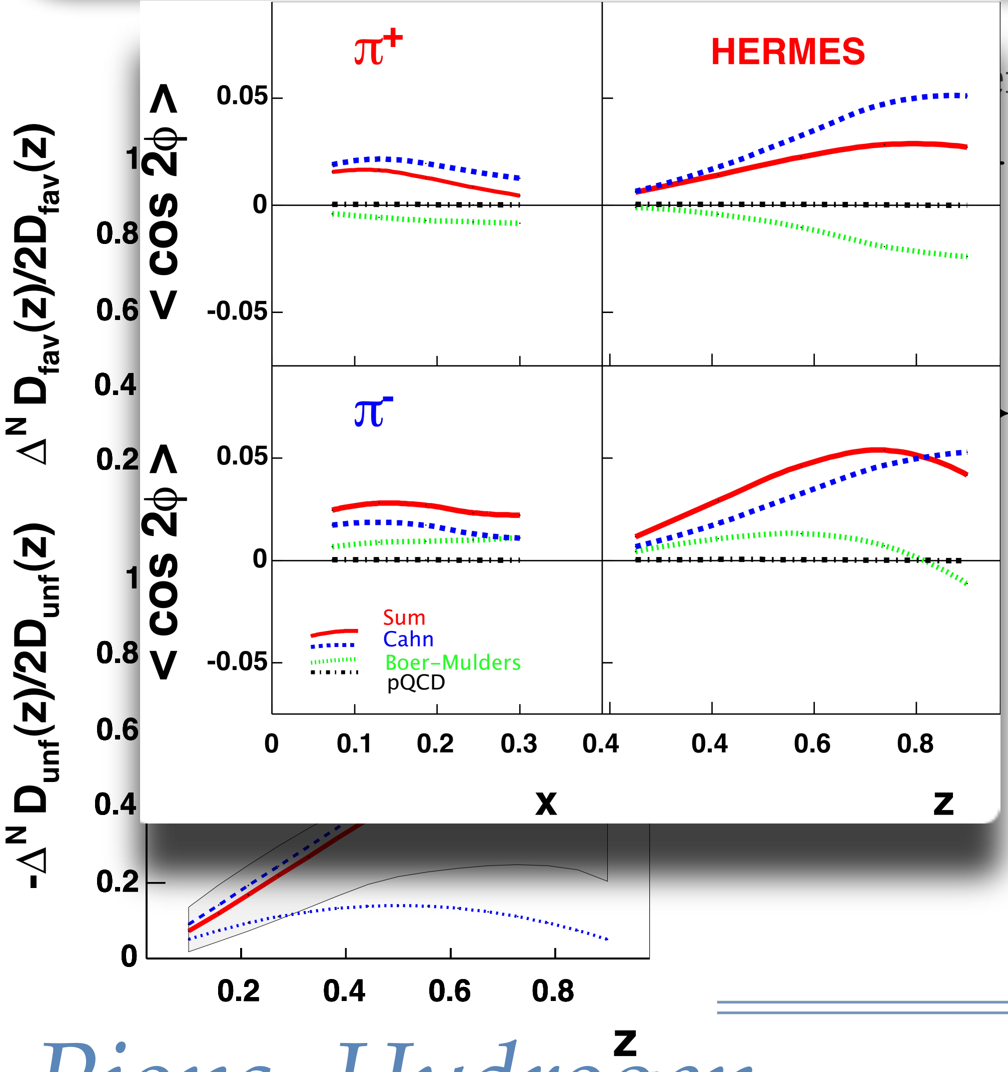
Cahn expected flavor blind

different $\pi^+/\pi^-$ amplitudes
⇨Boer-Mulders effect

*Pions,*



# $\cos 2\phi_h$ pions



r-Mulders

$$h_1^\perp H_1^\perp + \frac{\kappa_T^2}{Q^2} f_1 D_1 + \ldots]$$

Cahn

$$\pi^- \approx -H_1^{\perp,u\rightarrow\pi^+}$$

Gamberg, Goldstein
Phys. Rev. D77:094016, 2008

Zhang et al
Phys. Rev. D78:034035, 2008

Barone et al
Phys. Rev. D78:045022, 2008

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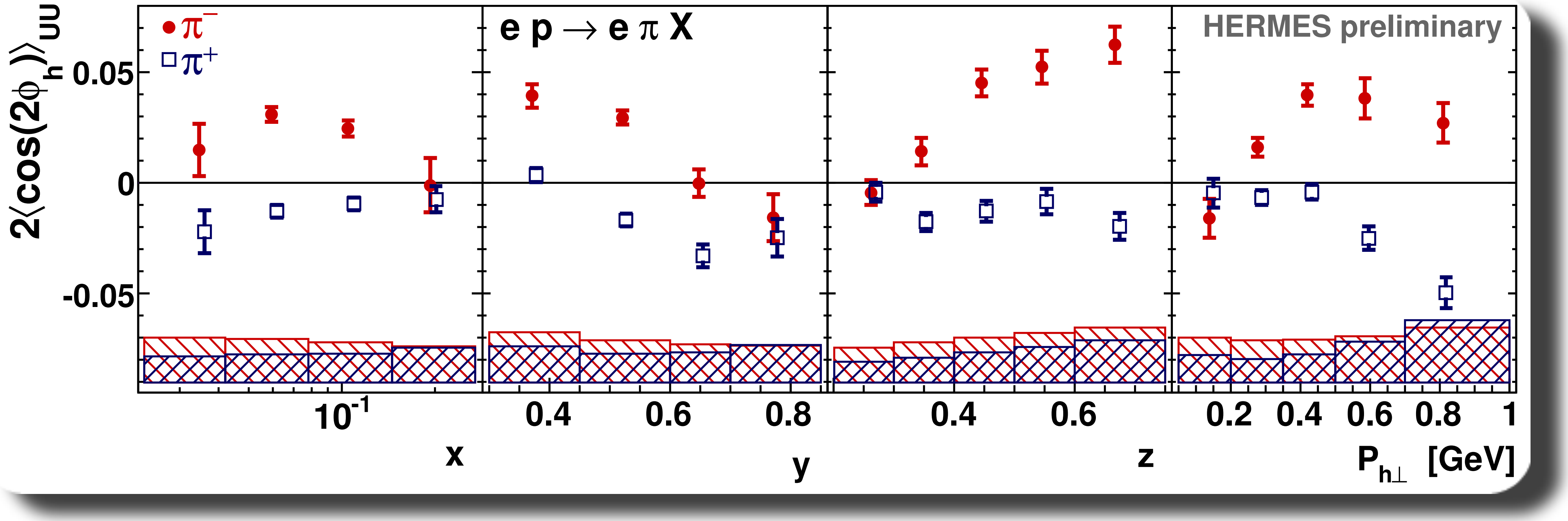
*Pions, Hydrogen*



## $\cos 2\phi_h$ pions

$$\propto C[\underbrace{-h_1^\perp H_1^\perp}^{\text{Boer-Mulders}} + \underbrace{\frac{\kappa_T^2}{Q^2} f_1 D_1}_{\text{Cahn}} + \ldots]$$

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**Barone, Melis, Prokudin**
**Phys. Rev. D81:114026, 2010**

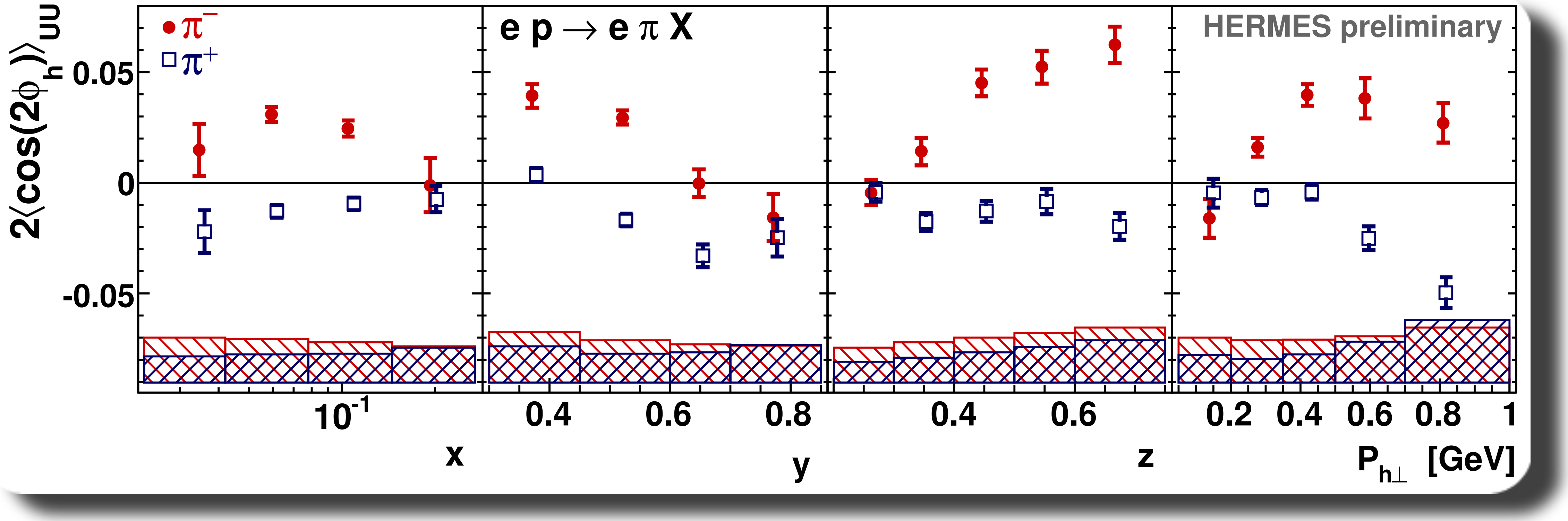
Cahn expected flavor blind

different $\pi^+/\pi^-$amplitudes
⇨Boer-Mulders effect

*Pions, Hydrogen*

Francesca Giordano

# $\cos 2\phi_h$
# pions



Boer-Mulders

$$\propto C[-h_1^\perp H_1^\perp + \frac{k_T^2}{Q^2}\boxed{f_1}D_1 + \dots]$$

Cahn

**Gamberg, Goldstein**
**Phys. Rev. D77:094016, 2008**

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**Barone, Melis, Prokudin**
**Phys. Rev. D81:114026, 2010**

Gaussian ansatz

$$f_1^q(x, k_T^2) = \boxed{f_1^q(x)}\frac{e^{-k_T^2/\langle k_T^2\rangle}}{\pi\langle k_T^2\rangle}$$

GRV98 fit

Cahn expected flavor blind

different $\pi^+/\pi^-$ amplitudes
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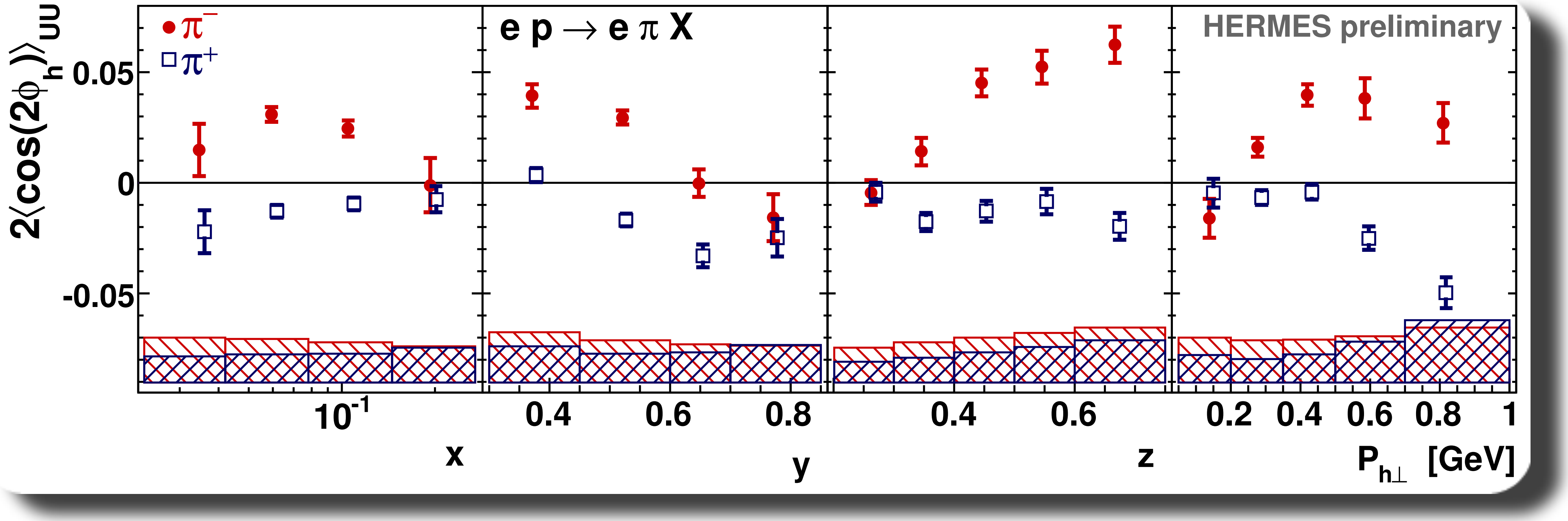

$\cos 2\phi_h$

pions

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$$\propto C[-h_1^\perp H_1^\perp + \frac{\kappa_T^2}{Q^2} \boxed{f_1} \boxed{D_1} + \ldots]$$

Cahn

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GRV98 fit

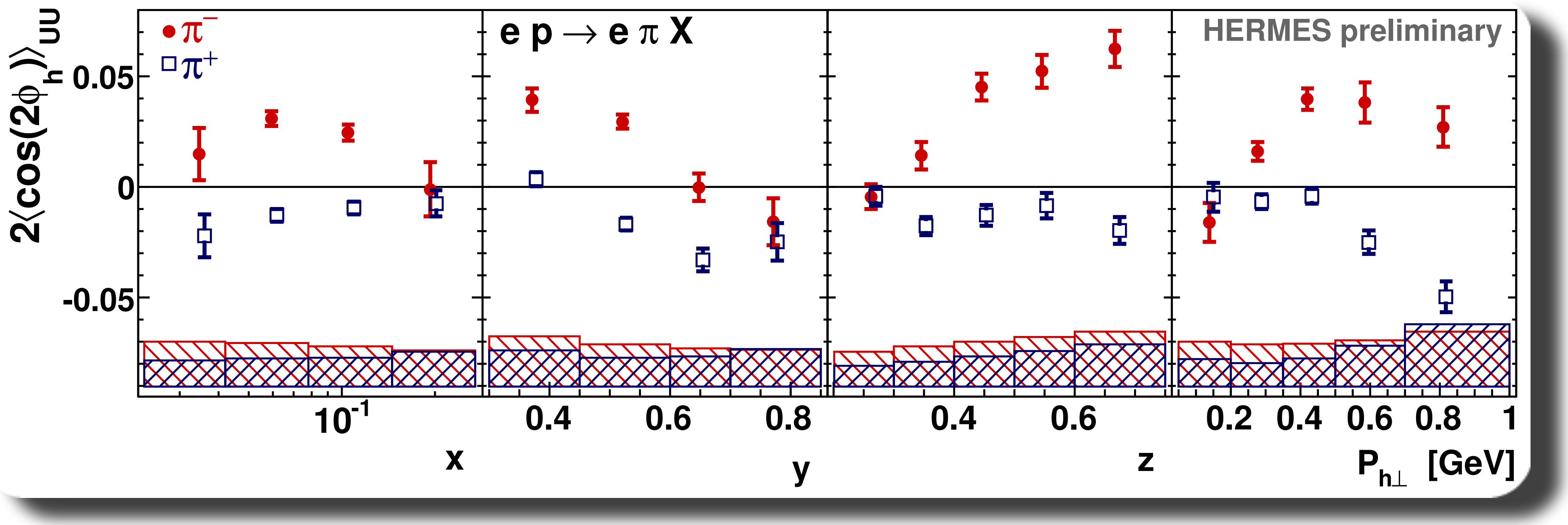
DSS fit

**Gaussian ansatz**

$$D_1(z, p_T^2) = \boxed{D_1(z)} \frac{e^{-p_T^2/\langle p_T^2 \rangle}}{\pi \langle p_T^2 \rangle}$$

*Pions, Hydrogen*

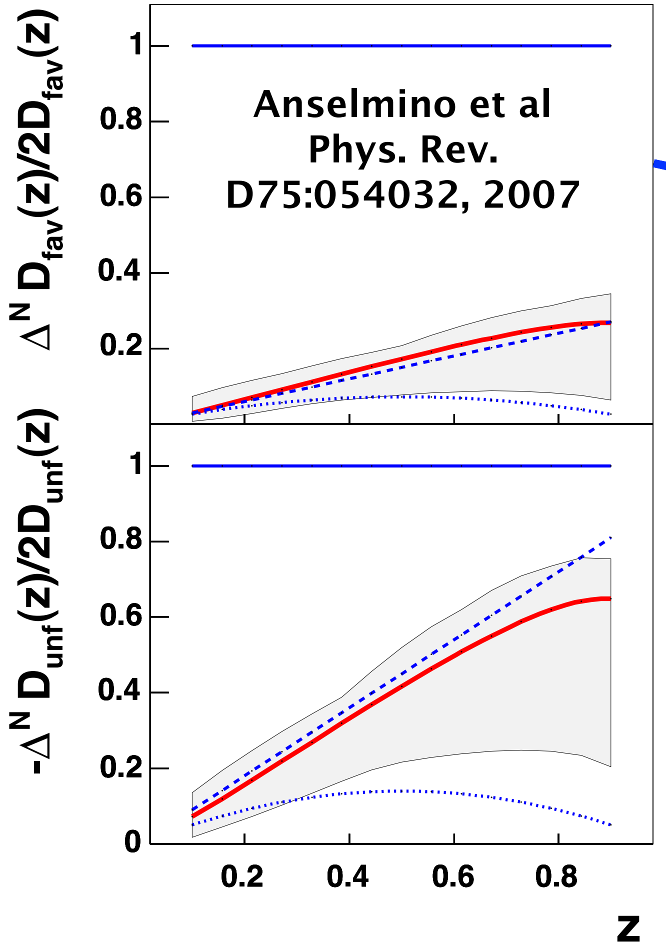
# $\cos 2\phi_h$ pions



Boer-Mulders

$$\propto C[-h_1^\perp H_1^\perp + \frac{\kappa_T^2}{Q^2} f_1 D_1 + \ldots]$$

Cahn



$$\boxed{H_1^{\perp q}(z, p_T^2) = \rho_q^C(z)\,\eta^C(p_T)\,D_1(z, p_T^2)}$$

Gamberg, Goldstein
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GRV98 fit

DSS fit

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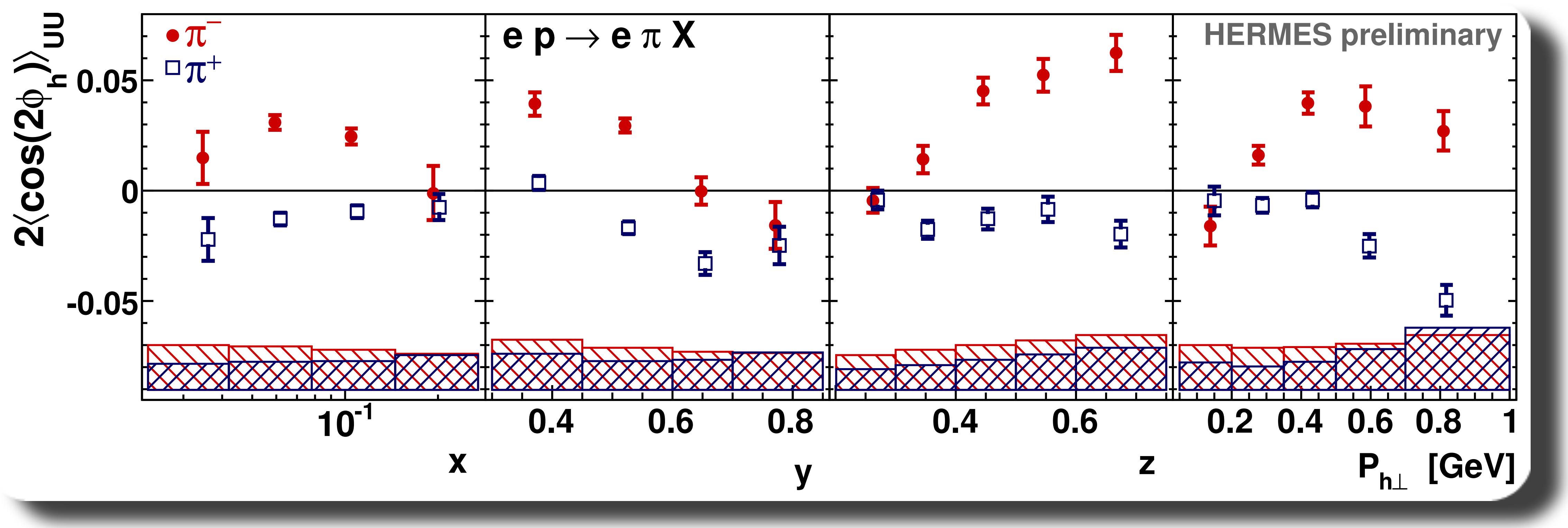
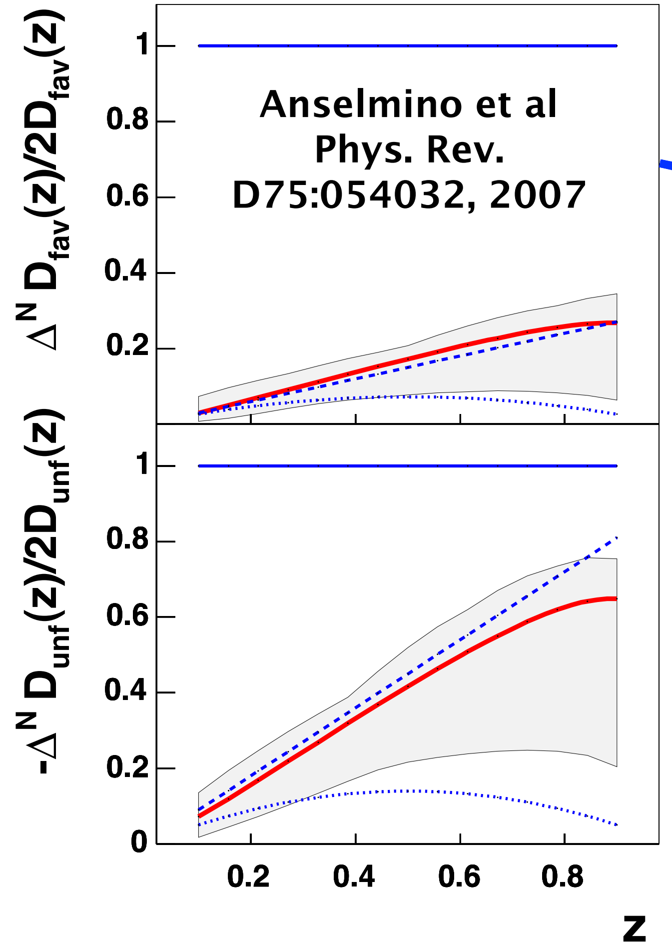
## *Pions, Hydrogen*

# $\cos 2\phi_h$
# pions





Boer-Mulders

$$\propto C[-h_1^\perp H_1^\perp + \frac{\kappa_T^2}{Q^2} f_1 D_1 + \ldots]$$

Cahn

$$H_1^{\perp q}(z, p_T^2) = \rho_q^C(z)\,\eta^C(p_T)\,D_1(z, p_T^2)$$

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Phys. Rev. D81:114026, 2010

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GRV98 fit

DSS fit

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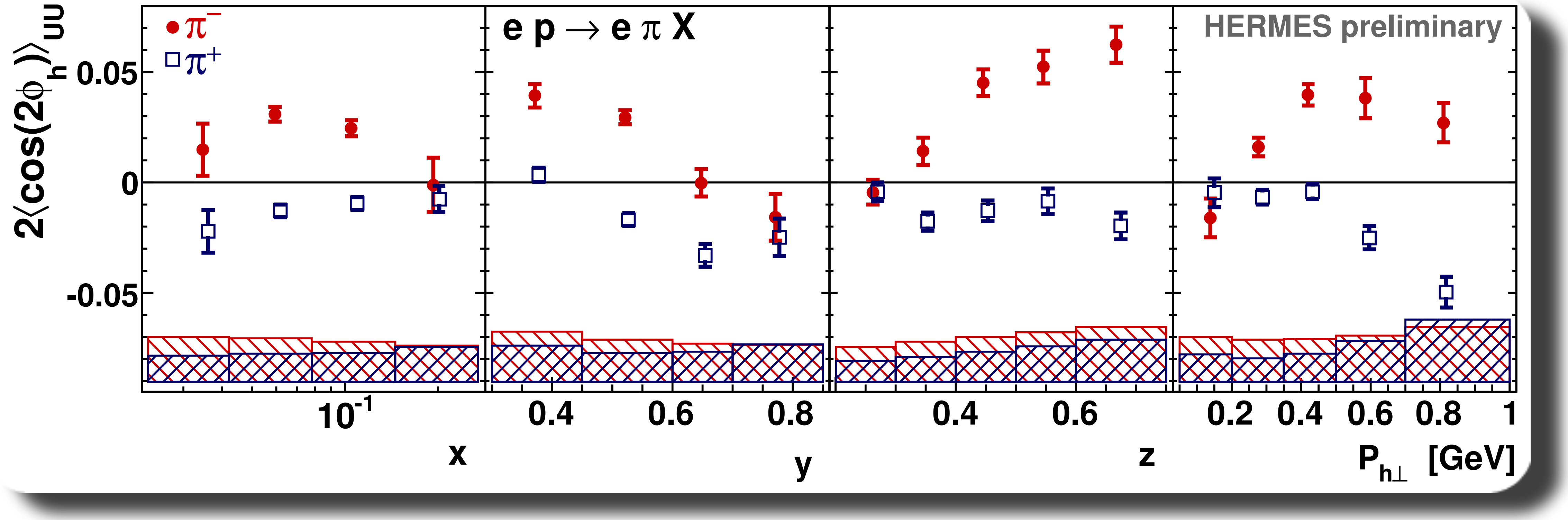
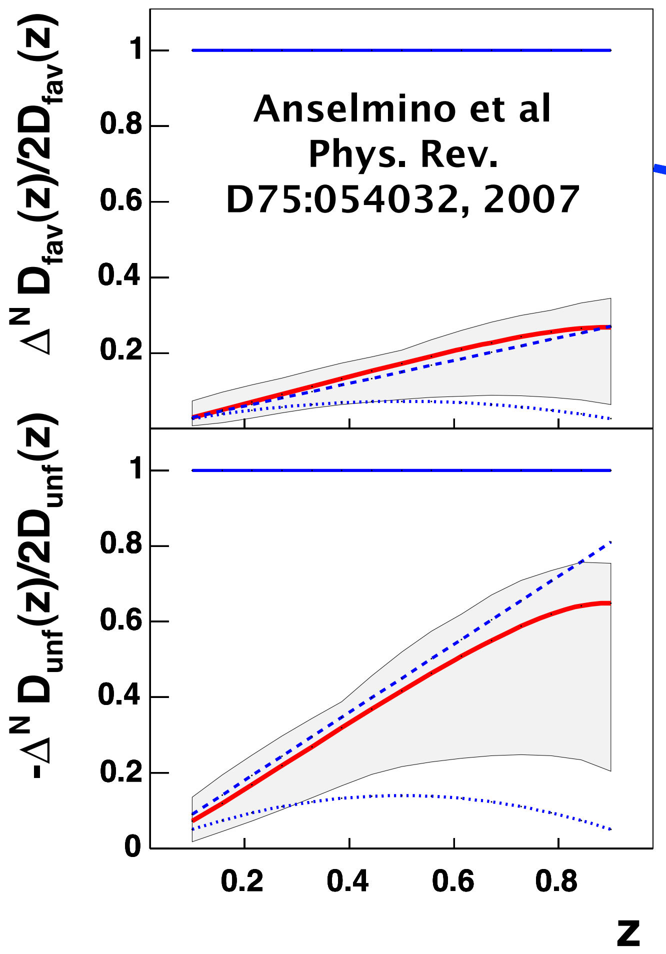
# $\cos 2\phi_h$

# pions





**Anselmino et al**
**Phys. Rev.**
**D75:054032, 2007**

**Boer-Mulders**

$$\propto C[-h_1^{\perp} H_1^{\perp} + \frac{\kappa_T^2}{Q^2} f_1 D_1 + \ldots]$$

**Cahn**

$$\boxed{H_1^{\perp q}(z, p_T^2) = \rho_q^C(z)\,\eta^C(p_T)\,D_1(z, p_T^2)}$$

$$h_1^{\perp q}(x, k_T^2) = \lambda_q\,\rho_q(x)\,\eta(k_T)\,f_1^q(x, \mathbf{k}_T^2)$$

**Gamberg, Goldstein**
**Phys. Rev. D77:094016, 2008**

**Zhang et al**
**Phys. Rev. D78:034035, 2008**

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GRV98 fit

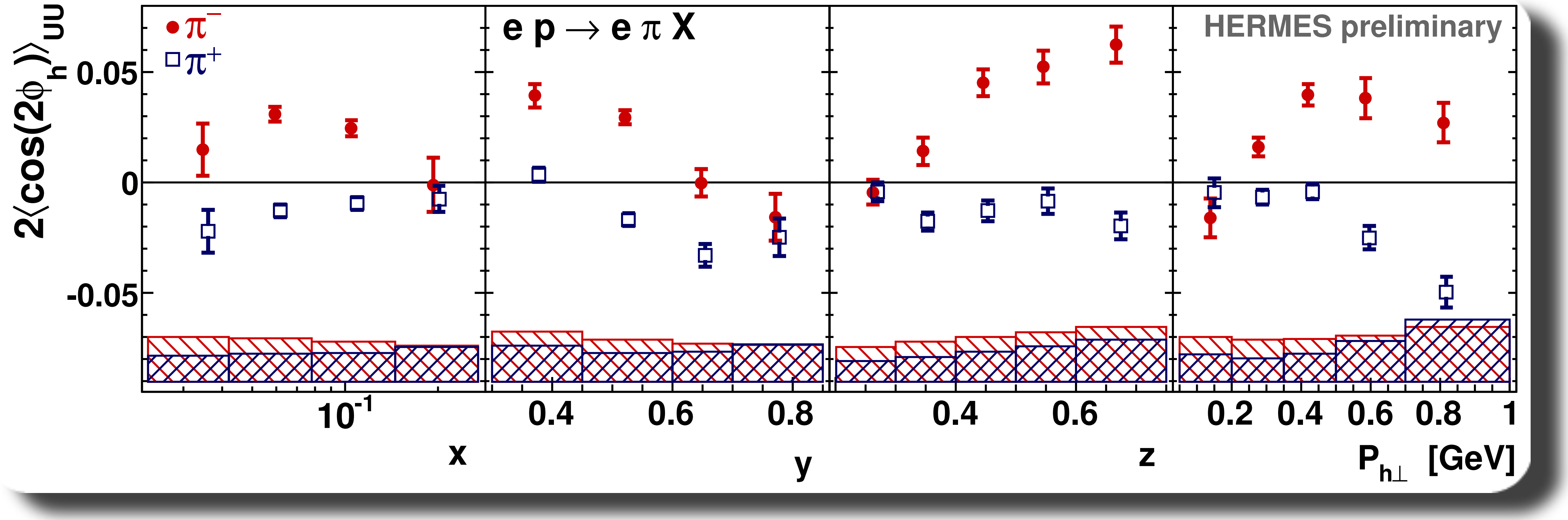
DSS fit

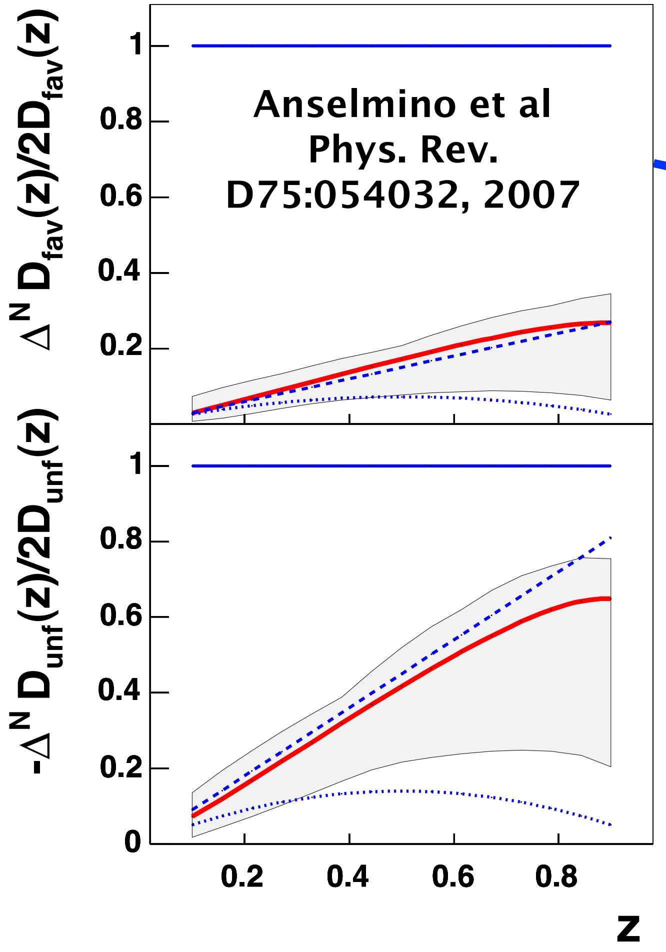
**Gaussian ansatz**

$$D_1(z, p_T^2) = \boxed{D_1(z)}\,\frac{e^{-p_T^2/\langle p_T^2\rangle}}{\pi\langle p_T^2\rangle}$$

# $\cos 2\phi_h$

# pions





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Boer-Mulders

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Cahn

$$\boxed{H_1^{\perp q}(z, p_T^2) = \rho_q^C(z)\, \eta^C(p_T)\, D_1(z, p_T^2)}$$

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$$f_1^q(x, k_T^2) = \boxed{f_1^q(x)}\, \frac{e^{-k_T^2/\langle k_T^2\rangle}}{\pi\langle k_T^2\rangle}$$

GRV98 fit

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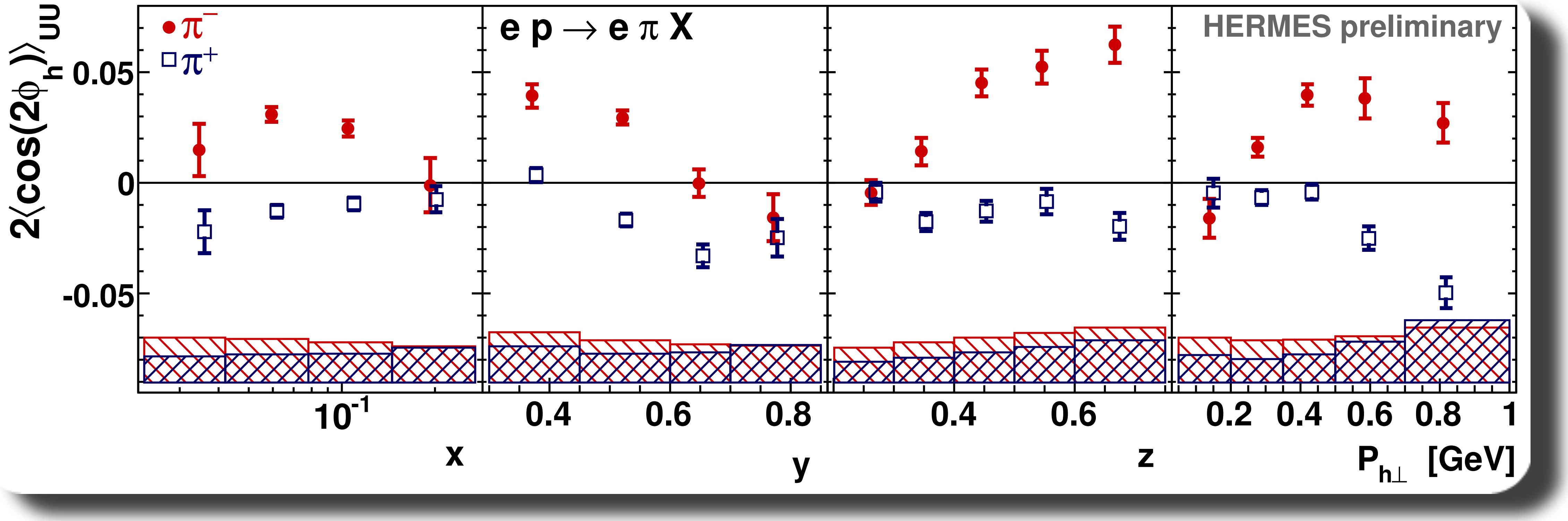
**Phys. Rev. D81:114026, 2010**

*Pions, Hydrogen*

# $\cos 2\phi_h$ pions



$$\propto C[\overset{\text{Boer-Mulders}}{-h_1^\perp H_1^\perp} + \frac{\kappa_T^2}{Q^2}\underset{\text{Cahn}}{f_1 D_1} + \ldots]$$

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**Barone, Melis, Prokudin**
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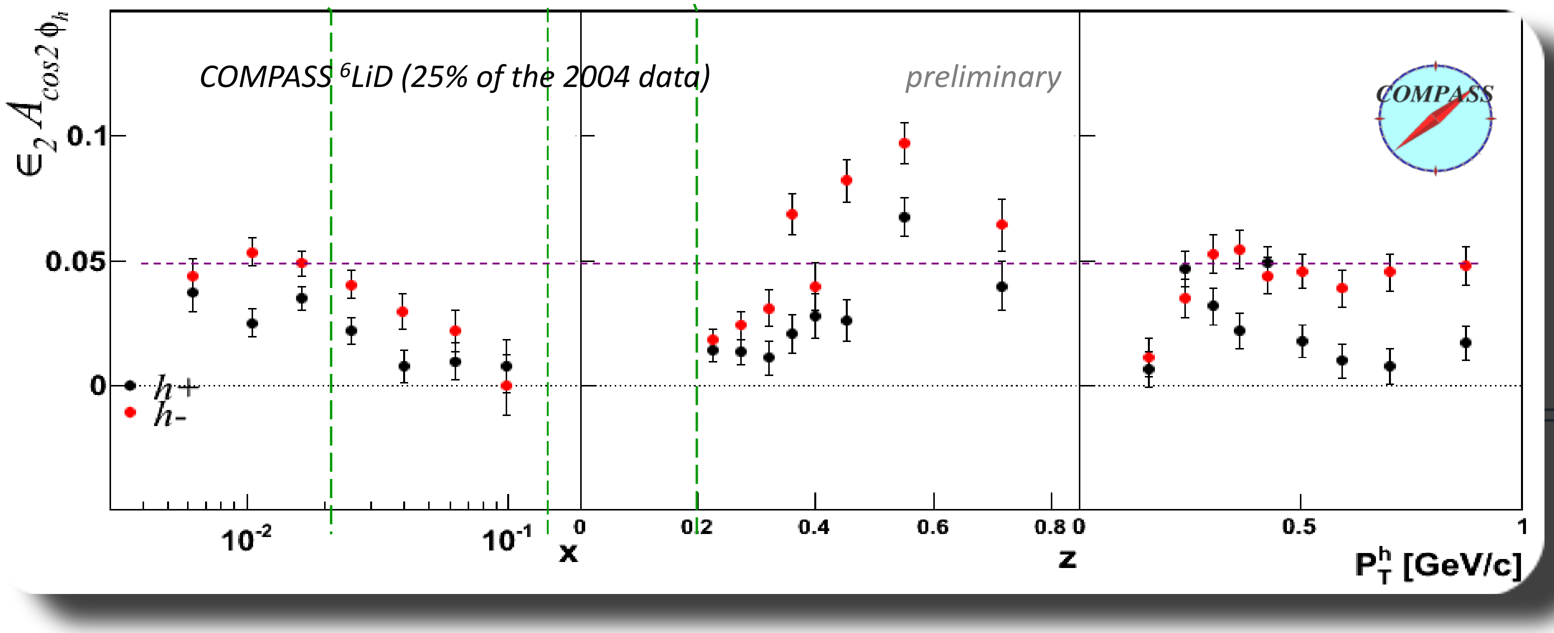
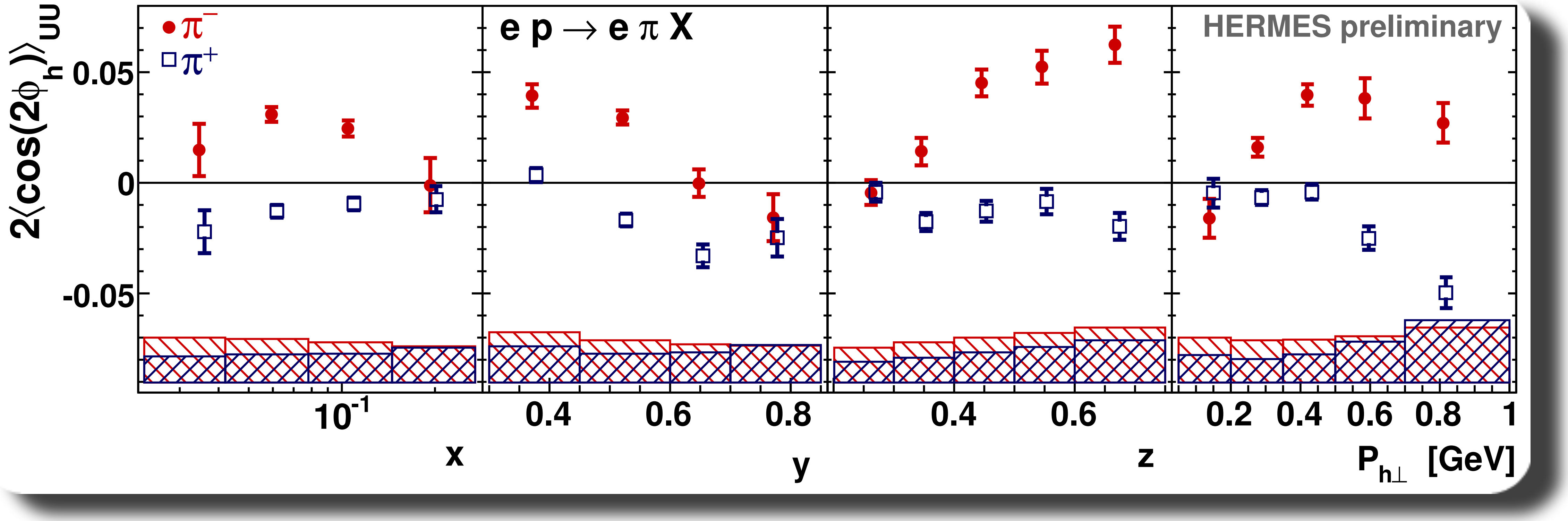


*Pions, Hydrogen*



$\cos 2\phi_h$

pions

$$\propto C[\overset{\text{Boer-Mulders}}{-h_1^\perp H_1^\perp} + \frac{\kappa_T^2}{Q^2}\underset{\text{Cahn}}{f_1 D_1} + \dots]$$

dominates

**Gamberg, Goldstein**
**Phys. Rev. D77:094016, 2008**

**Zhang et al**
**Phys. Rev. D78:034035, 2008**

**Barone et al**
**Phys. Rev. D78:045022, 2008**

**Barone, Melis, Prokudin**
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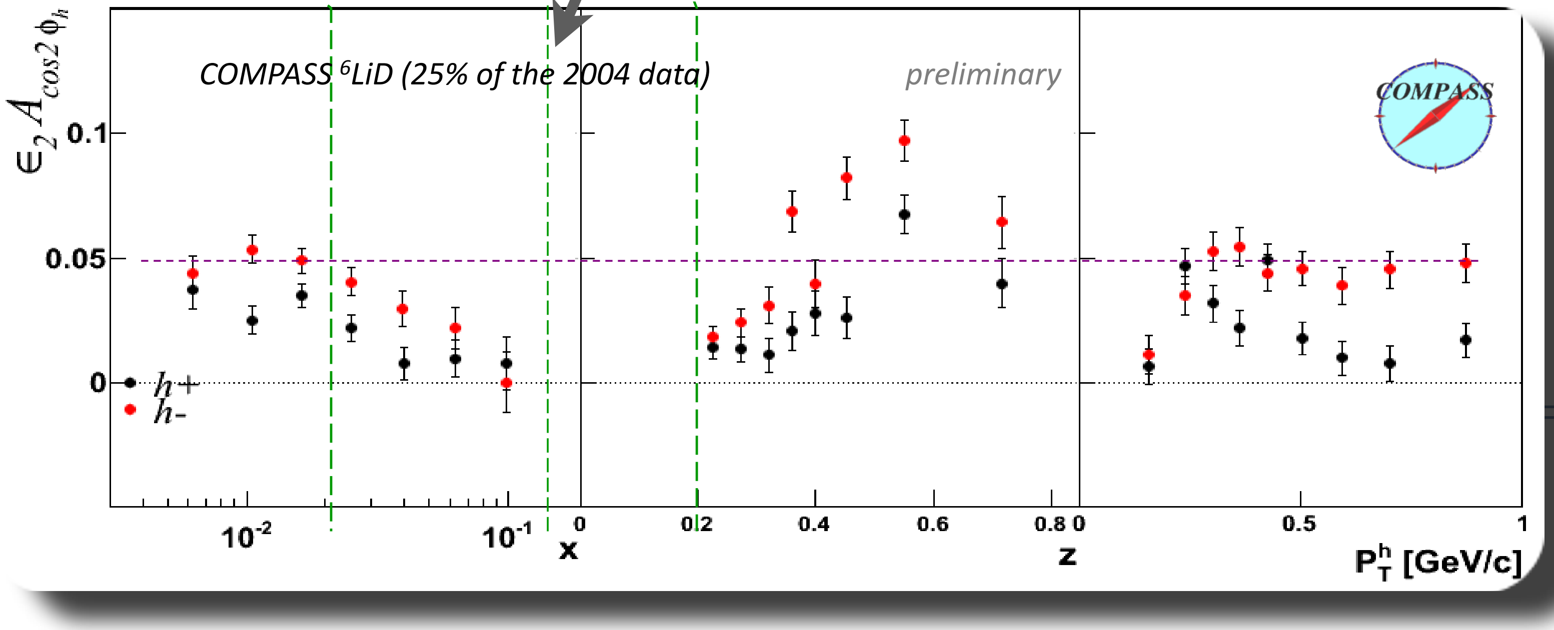
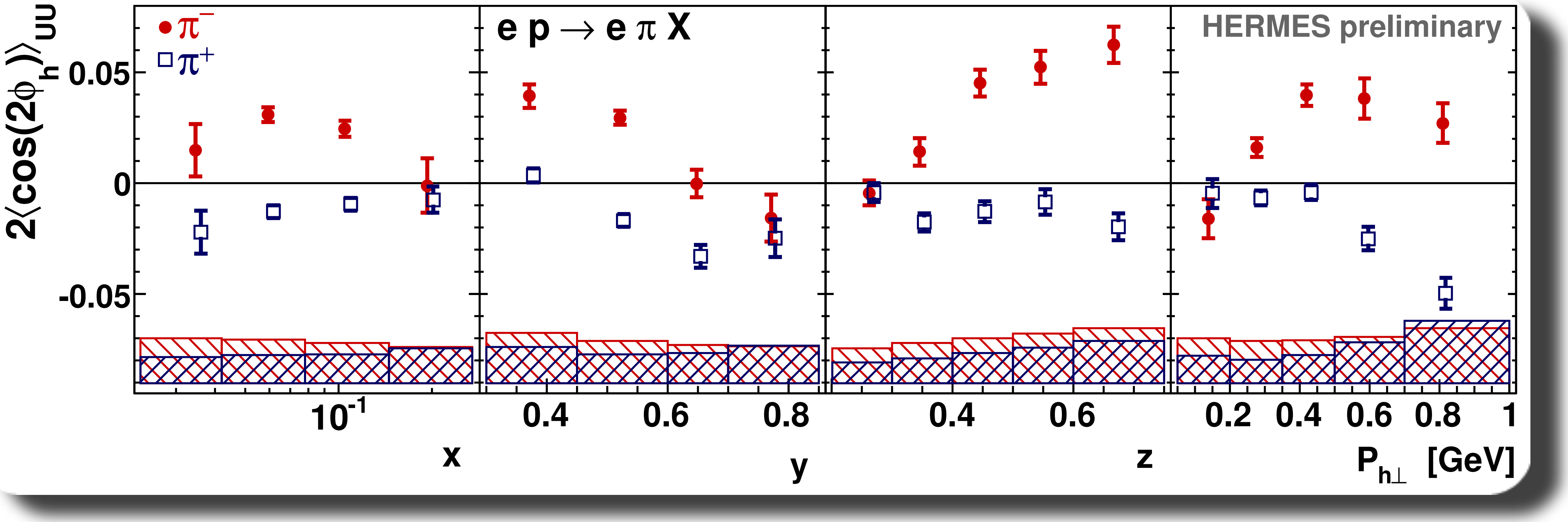


# Pions, Hydrogen

Francesca Giordano

# $\cos 2\phi_h$ pions



**Boer-Mulders**

$$\propto C[-h_1^\perp H_1^\perp + \frac{\kappa_T^2}{Q^2} f_1 D_1 + \ldots]$$

**Cahn** dominates

hermes $\langle \kappa_T \rangle$ < COMPASS $\langle \kappa_T \rangle$

**Gamberg, Goldstein**
**Phys. Rev. D77:094016, 2008**

**Zhang et al**
**Phys. Rev. D78:034035, 2008**

**Barone et al**
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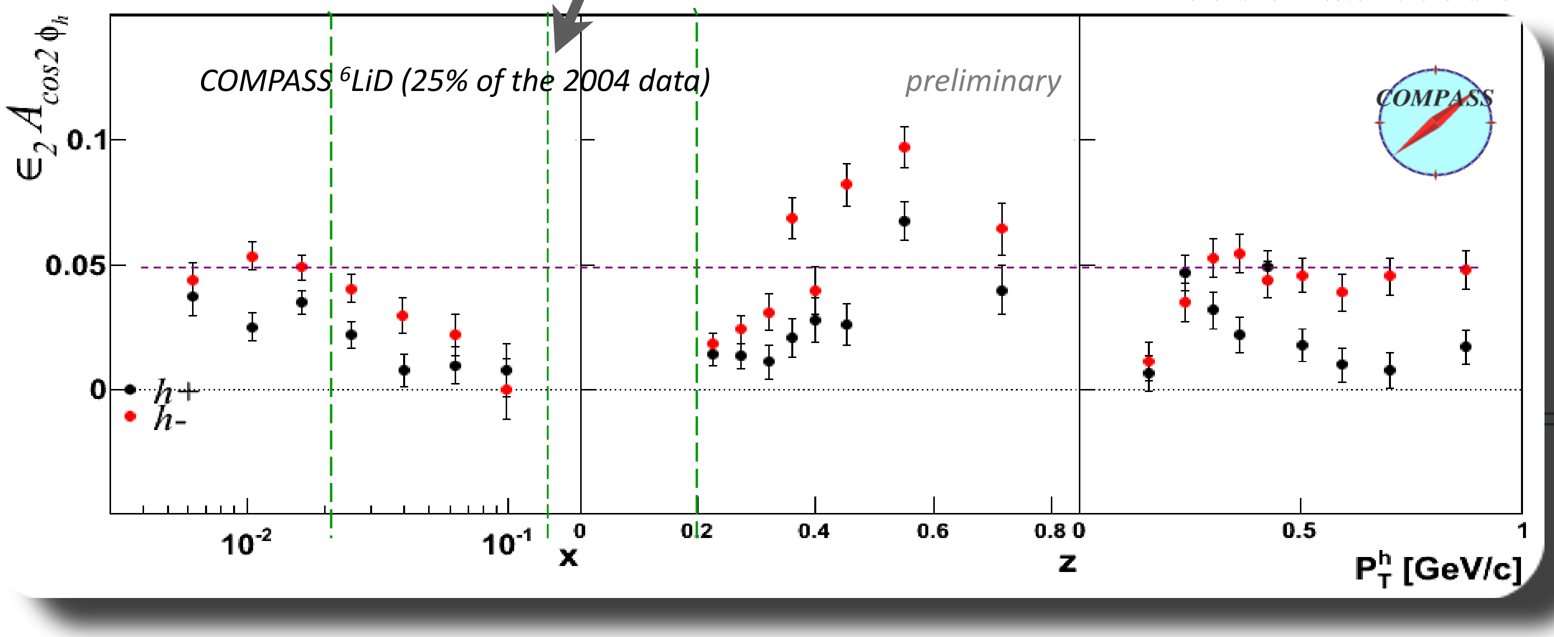
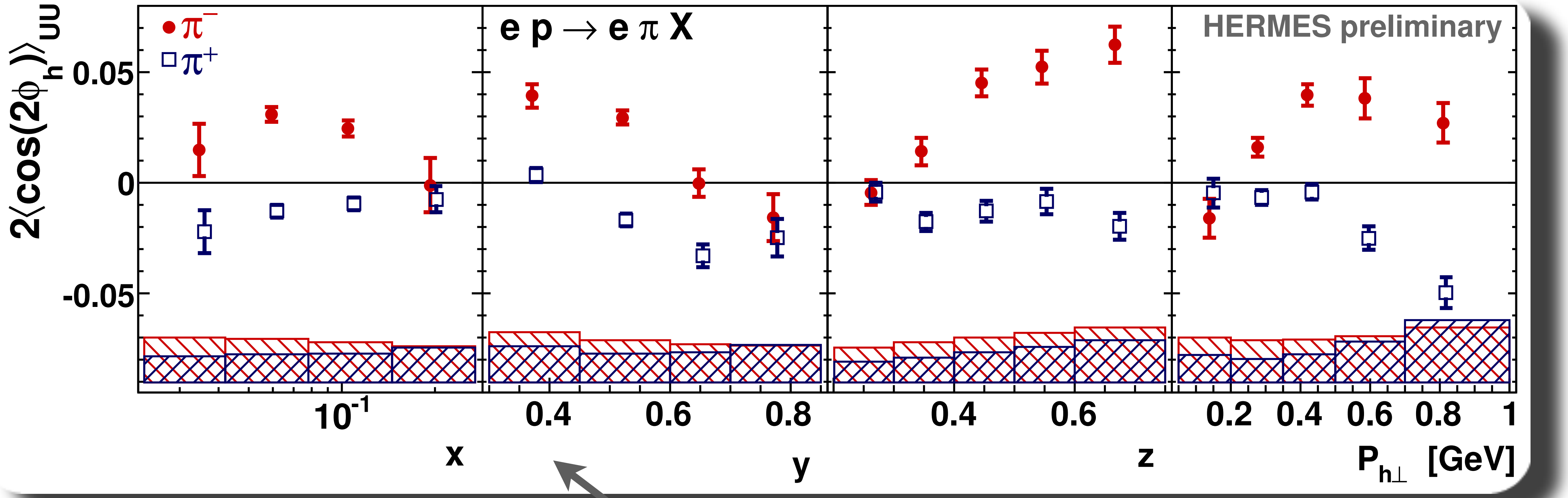


# $\cos 2\phi_h$ pions



dominates
**Boer-Mulders**

$$\propto C[-h_1^\perp H_1^\perp + \frac{\kappa_T^2}{Q^2} f_1 D_1 + \dots]$$

**Cahn**
dominates

**Gamberg, Goldstein**
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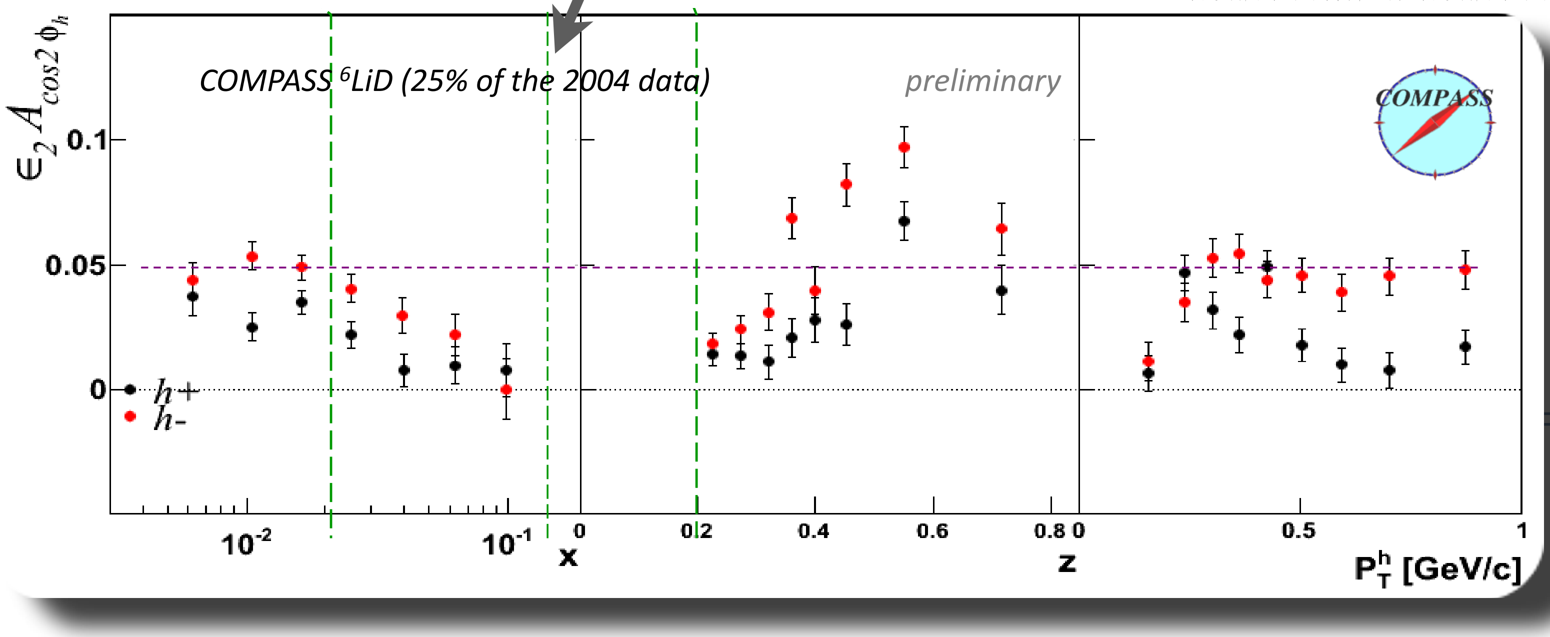
**Barone, Melis, Prokudin**
**Phys. Rev. D81:114026, 2010**

$$\langle\kappa_T\rangle < \langle\kappa_T\rangle$$



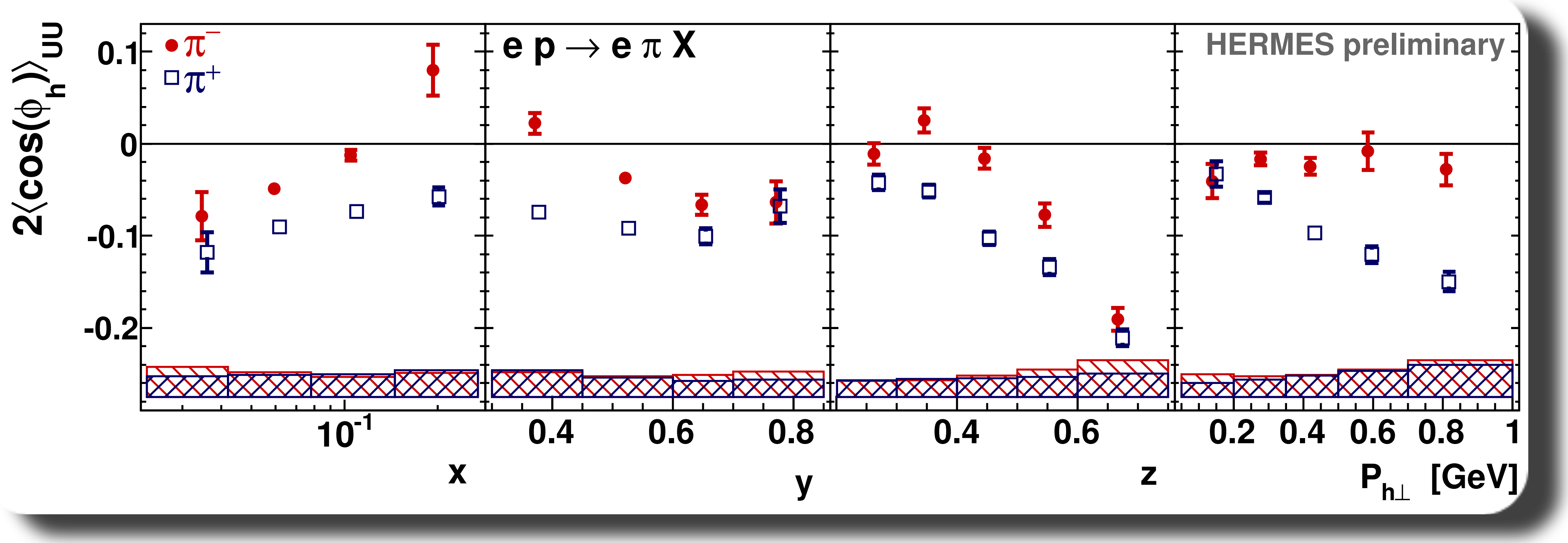
## *Pions, Hydrogen*

Francesca Giordano

$\cos \phi_h$

pions

*Pions, Hydrogen*

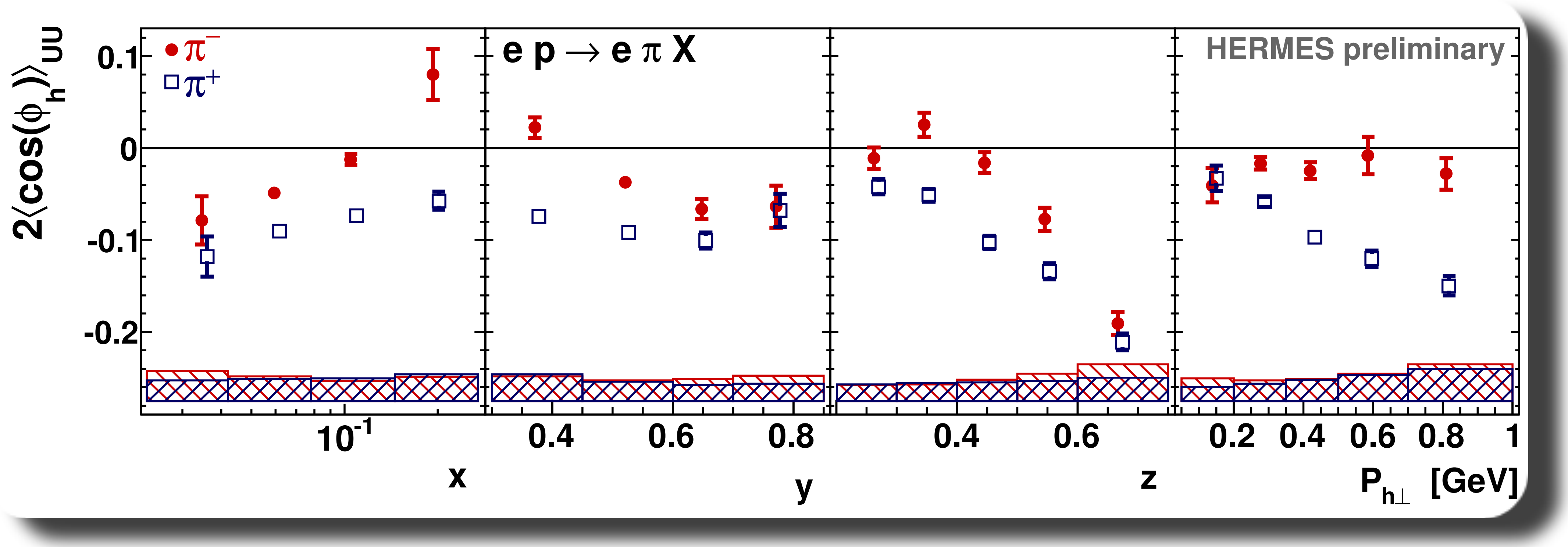


$\cos\phi_h$

pions

## *Pions, Hydrogen*

Francesca Giordano



$\cos\phi_h$

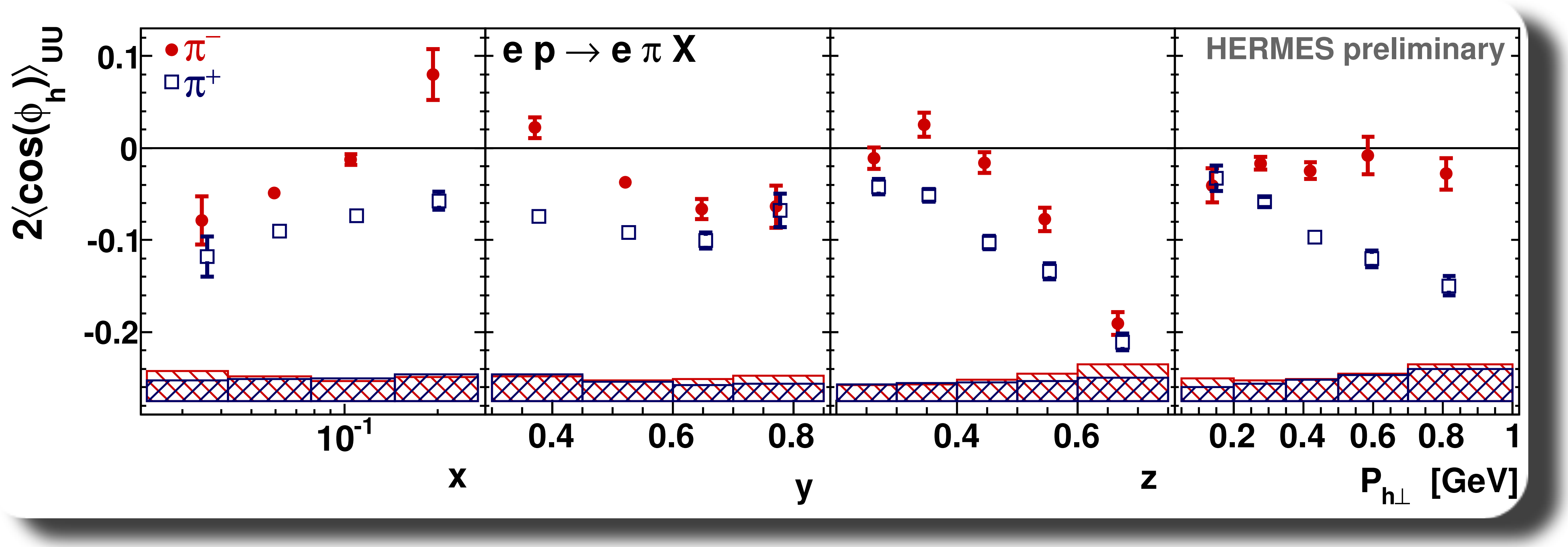
pions

$$\propto \frac{2M}{Q} C[\underbrace{-h_1^\perp H_1^\perp}_{}\overset{\text{Boer-Mulders}}{} - \underbrace{f_1 D_1}_{\text{Cahn}} + ....]$$

# *Pions, Hydrogen*

Francesca Giordano

# $\cos\phi_h$ pions



Boer-Mulders

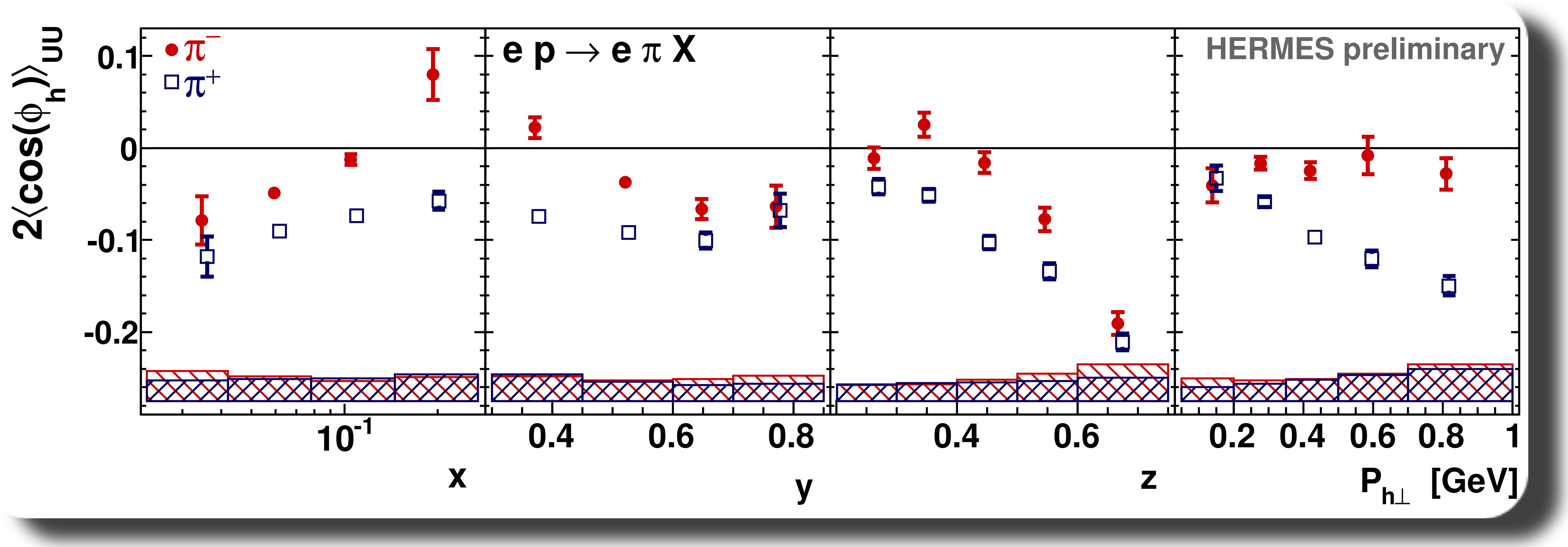
$$\propto \frac{2M}{Q} C[-h_1^\perp H_1^\perp - f_1 D_1 + ....]$$

Cahn

Cahn expected flavor blind

different $\pi^+/\pi^-$amplitudes

⇨Boer-Mulders effect



$\cos\phi_h$

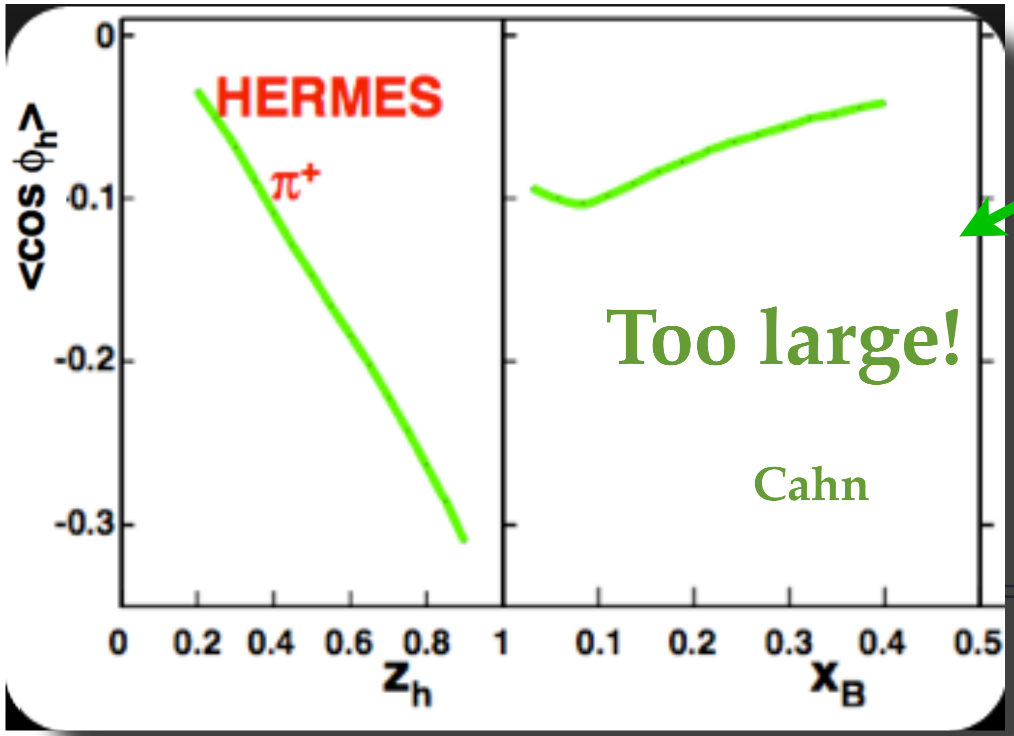
pions

Boer-Mulders

$$\propto \frac{2M}{Q} C[-h_1^{\perp} H_1^{\perp} - f_1 D_1 + ....]$$

Cahn

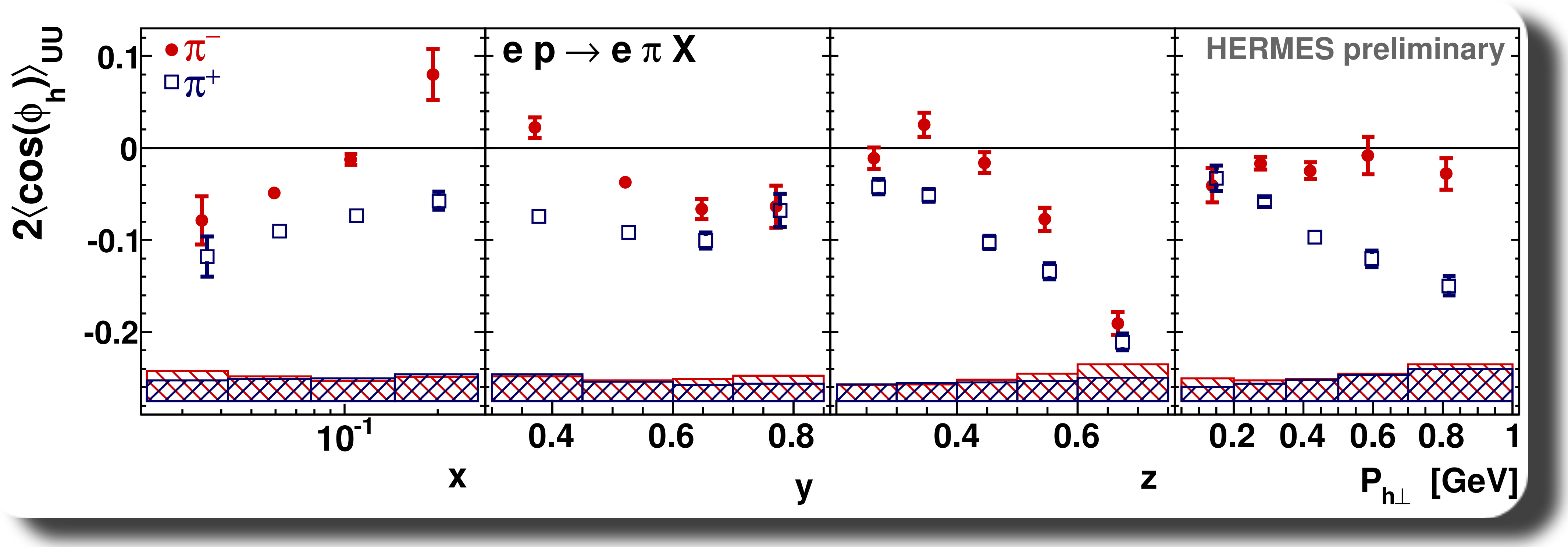
M. Anselmino et al.,
Phys. Rev. D71:074006, 2005
Eur. Phys. J. A31:373, 2007



Too large!

Cahn expected flavor blind

different $\pi^+/\pi^-$amplitudes
⇨Boer-Mulders effect



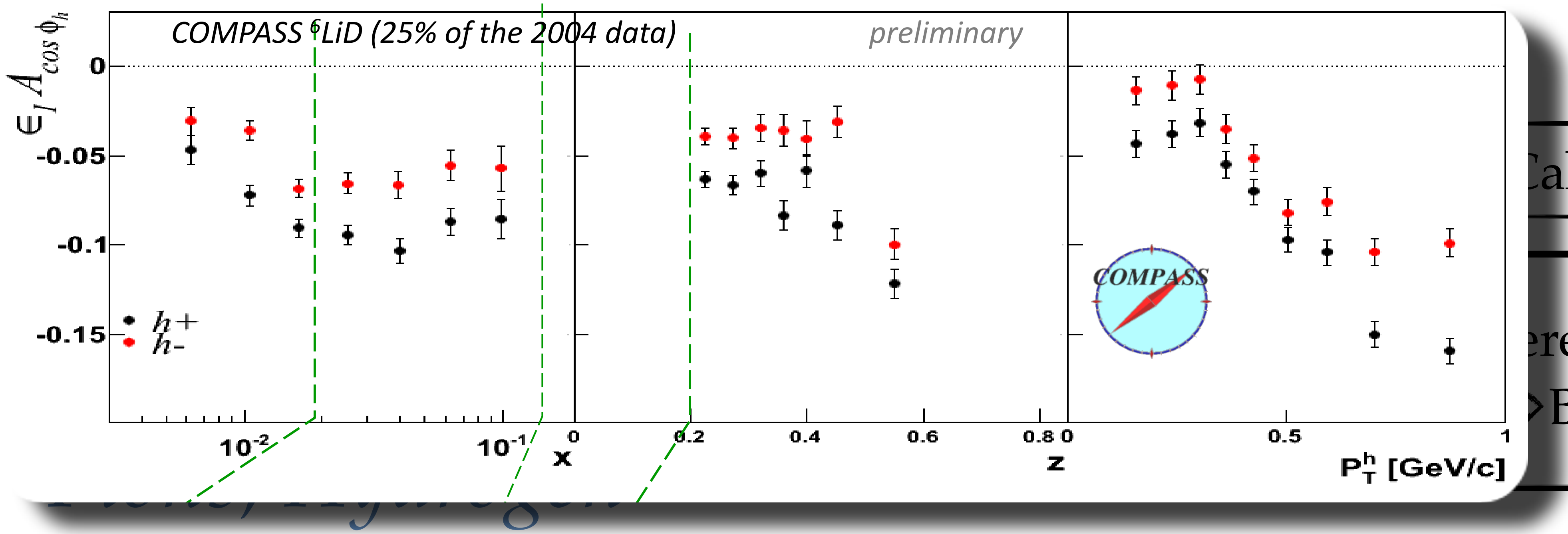
# $\cos\phi_h$

## pions

Boer-Mulders

$$\propto \frac{2M}{Q} C[-h_1^\perp H_1^\perp - f_1 D_1 + ....]$$

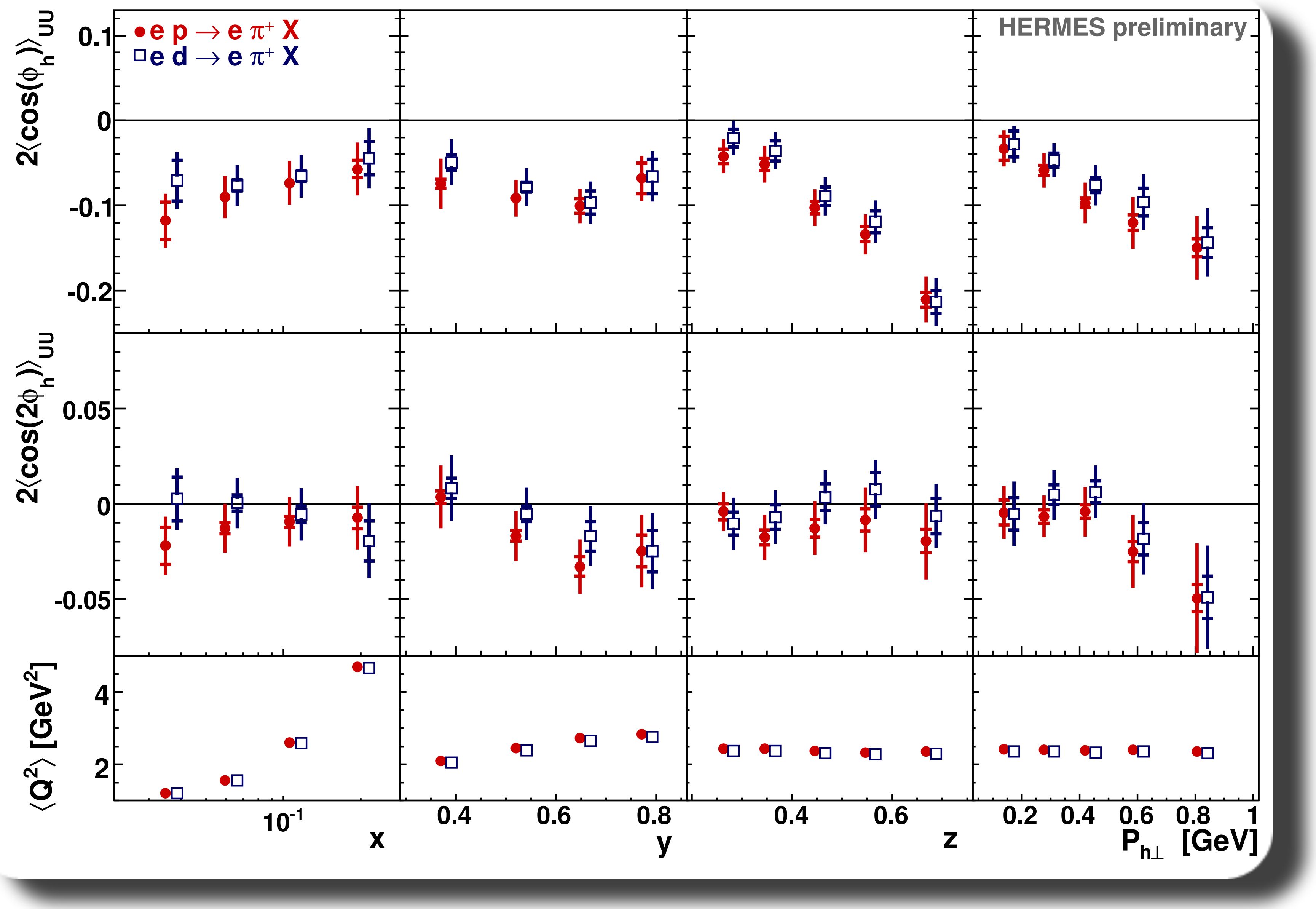
Cahn



Cahn expected flavor blind

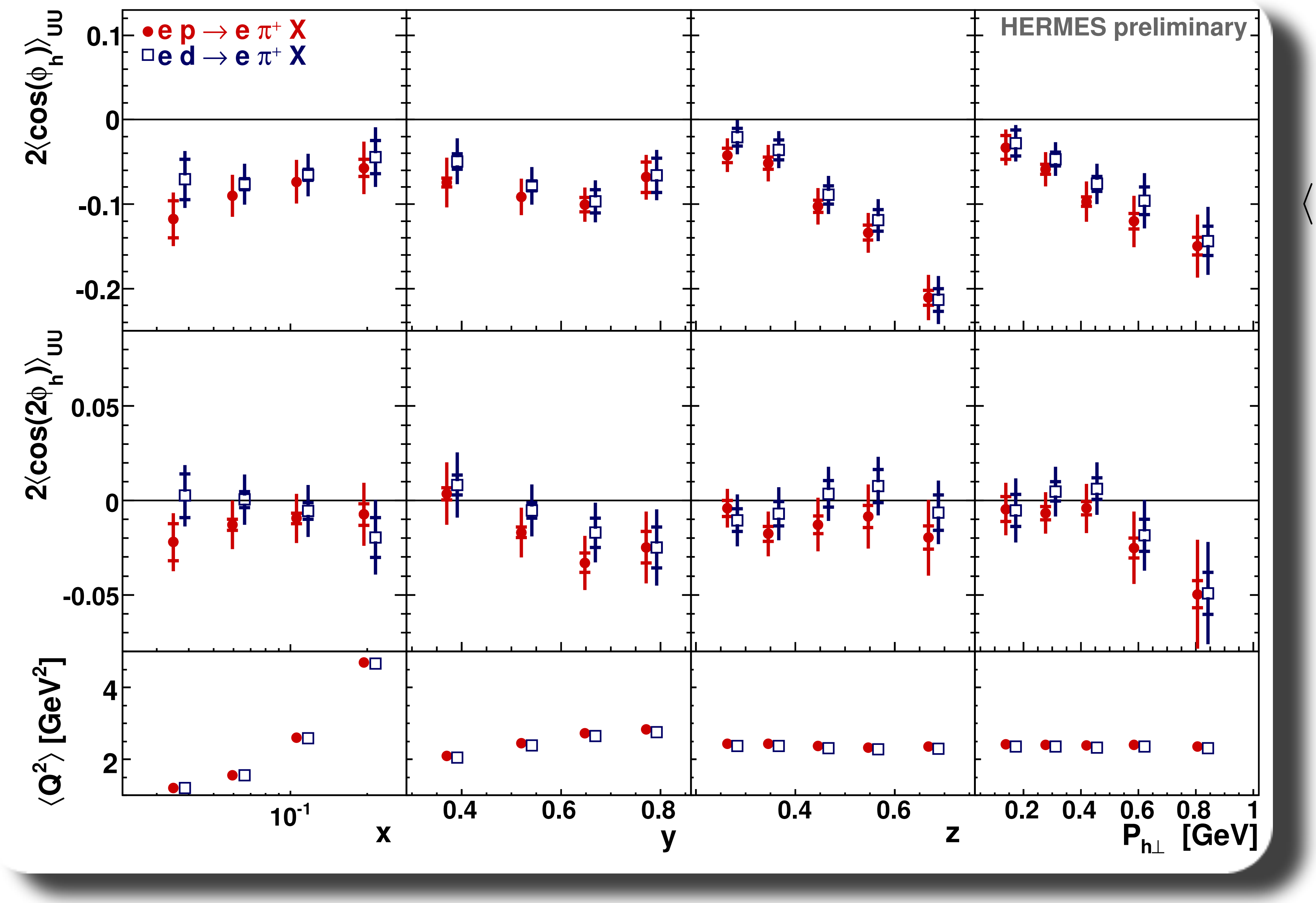
erent $\pi^+/\pi^-$ amplitudes
>Boer-Mulders effect

$\pi^+$



# *Pions, Hydrogen vs. Deuterium*

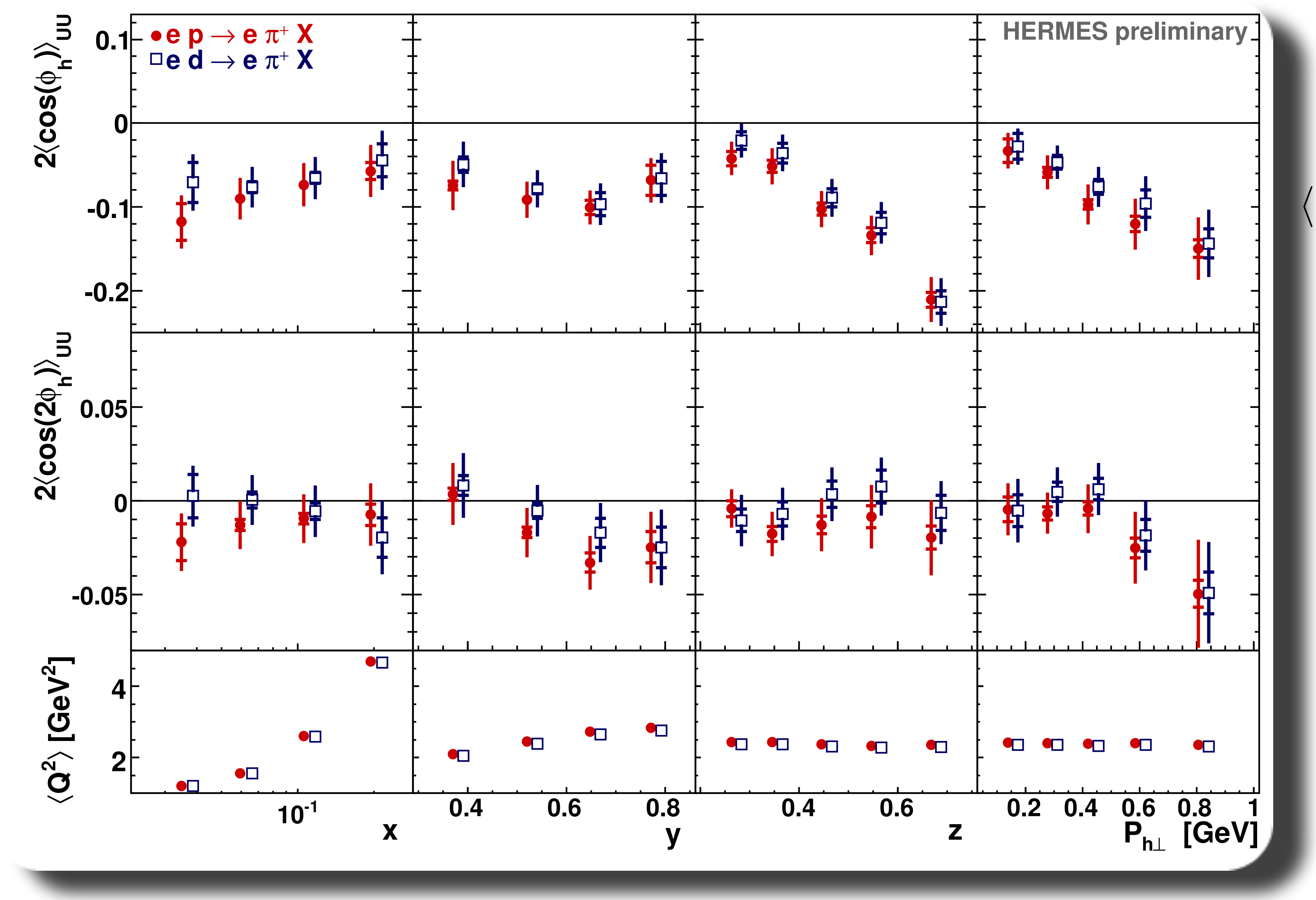
Francesca Giordano



$\pi^+$

$$\langle \cos 2\phi_h \rangle^h_{target} = \frac{\sum_q e_q h^{\perp}_{1,q}(x) H^{\perp}_{1,q\to h}(z)}{\sum_q e_q f_{1,q}(x) D_{1,q}(z)}$$

# *Pions, Hydrogen vs. Deuterium*



$$\pi^+$$

$$\langle \cos 2\phi_h \rangle^h_{target} = \frac{\sum_q e_q h^{\perp}_{1,q}(x) H^{\perp}_{1,q\rightarrow h}(z)}{\sum_q e_q f_{1,q}(x) D_{1,q}(z)}$$

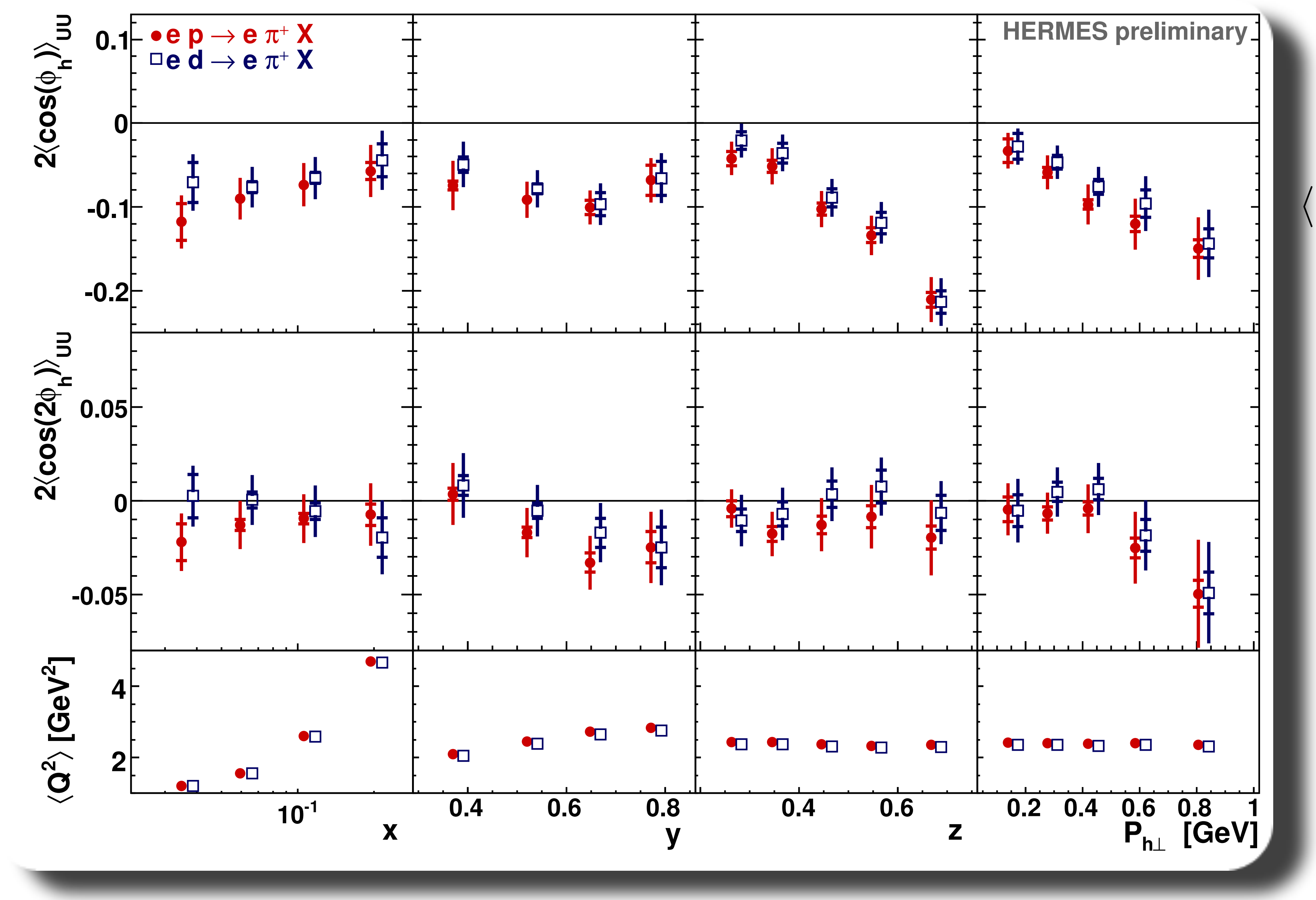
**Set 1:**

same sign for *u* and *d* quark

$$h^{\perp}_{1,u} > 0 \quad h^{\perp}_{1,d} > 0$$

# *Pions, Hydrogen vs. Deuterium*

Francesca Giordano



$\pi^+$

$$\langle \cos 2\phi_h \rangle^h_{target} = \frac{\sum_q e_q h^{\perp}_{1,q}(x) H^{\perp}_{1,q\rightarrow h}(z)}{\sum_q e_q f_{1,q}(x) D_{1,q}(z)}$$

**Set 1:**

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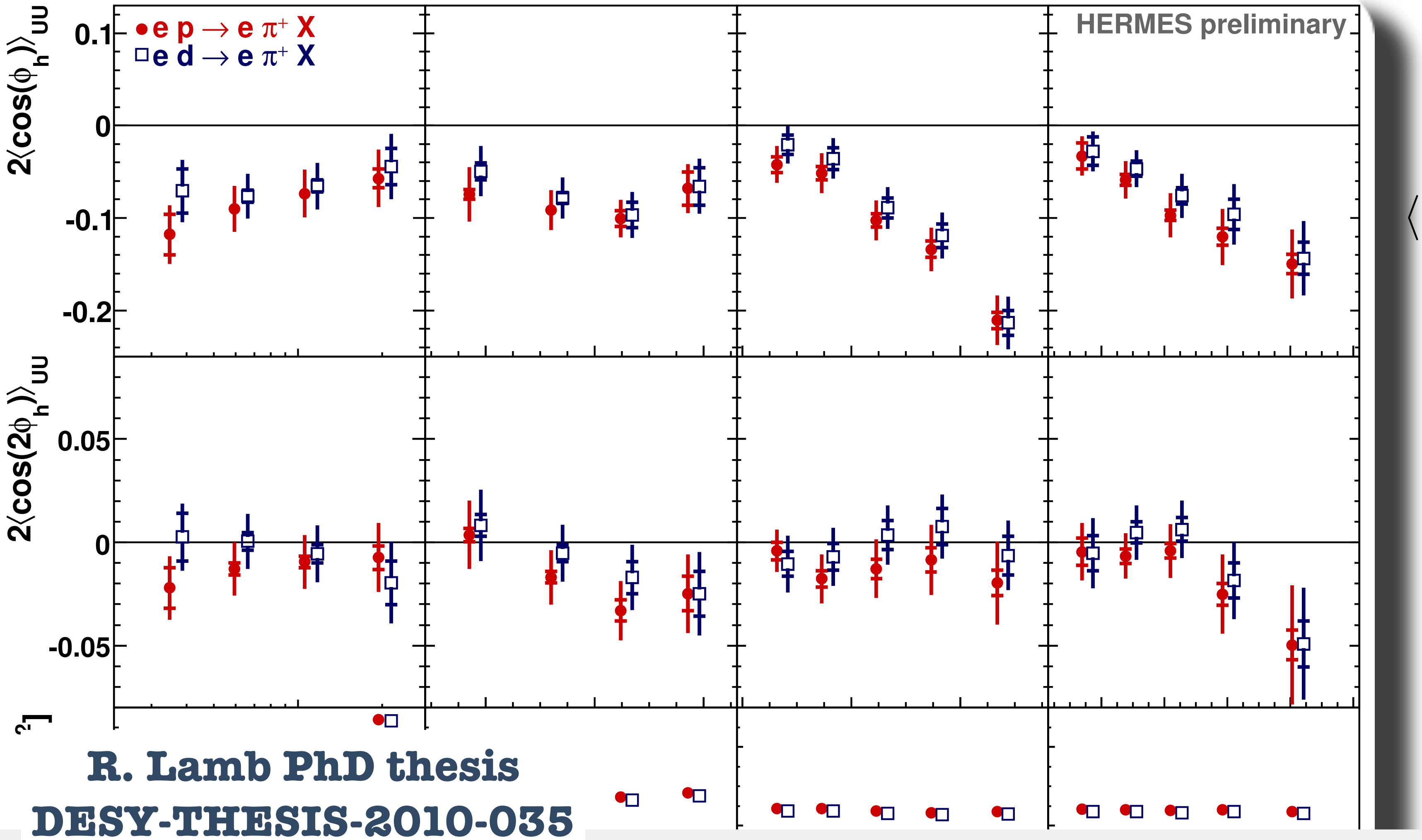
$h^{\perp}_{1,u} > 0 \quad h^{\perp}_{1,d} > 0$

**Set 2:**

opposite sign for *u* and *d* quark

$h^{\perp}_{1,u} > 0 \quad h^{\perp}_{1,d} < 0$

# *Pions, Hydrogen vs. Deuterium*



**R. Lamb PhD thesis**
**DESY-THESIS-2010-035**

$$\pi^+$$

$$\langle \cos 2\phi_h \rangle^h_{target} = \frac{\sum_q e_q h^\perp_{1,q}(x) H^\perp_{1,q\to h}(z)}{\sum_q e_q f_{1,q}(x) D_{1,q}(z)}$$

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**Set 2:**

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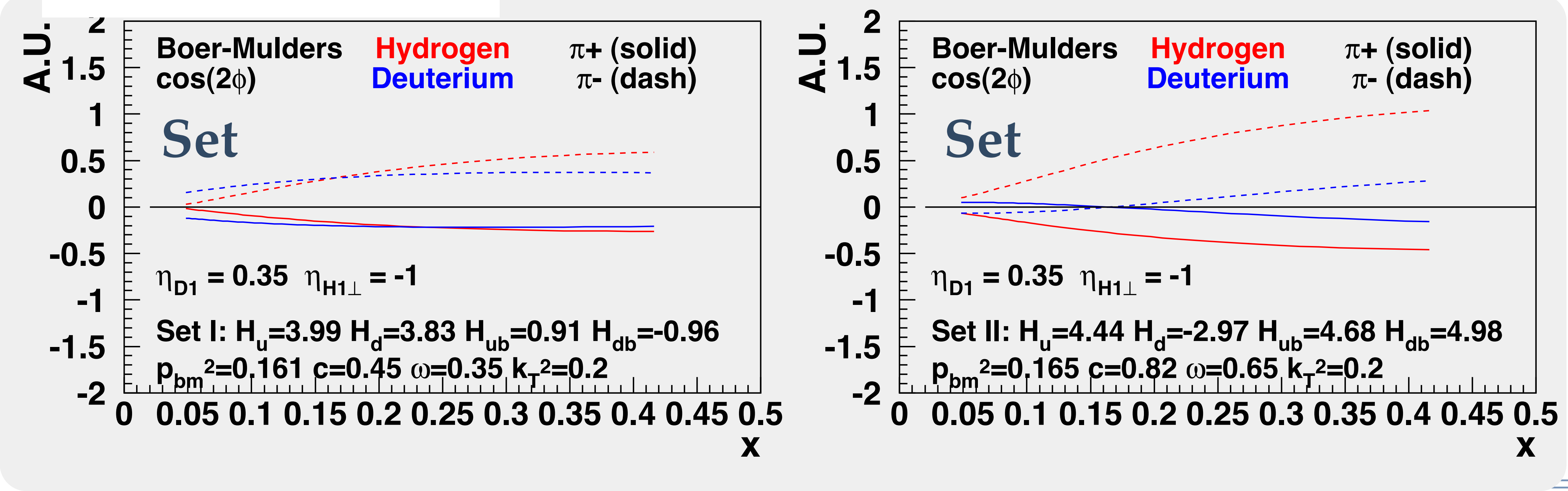
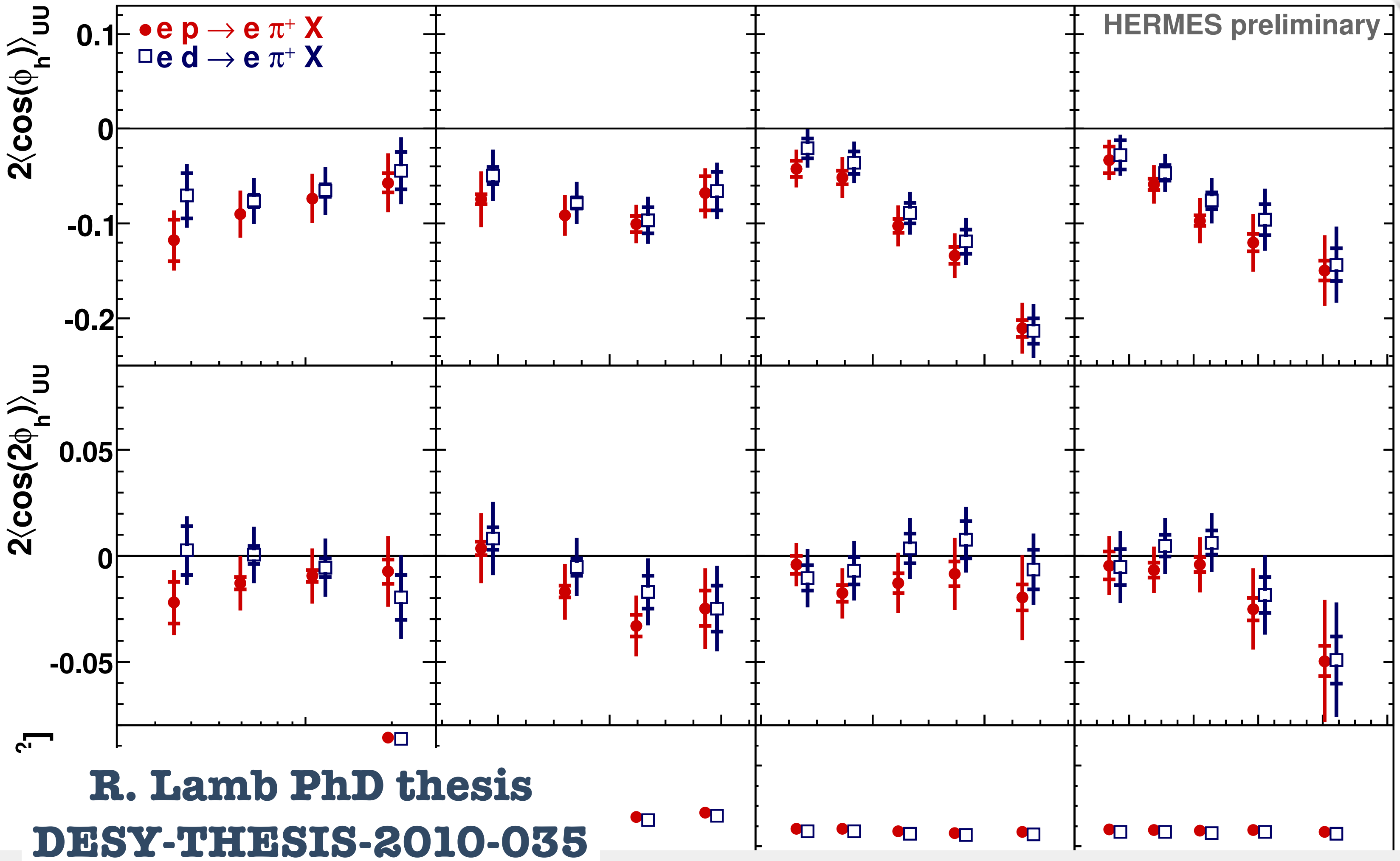


# *Pions, Hydrogen vs. Deuterium*



R. Lamb PhD thesis

DESY-THESIS-2010-035

$\pi^+$

$$\langle \cos 2\phi_h \rangle^h_{target} = \frac{\sum_q e_q h^\perp_{1,q}(x) H^\perp_{1,q\to h}(z)}{\sum_q e_q f_{1,q}(x) D_{1,q}(z)}$$

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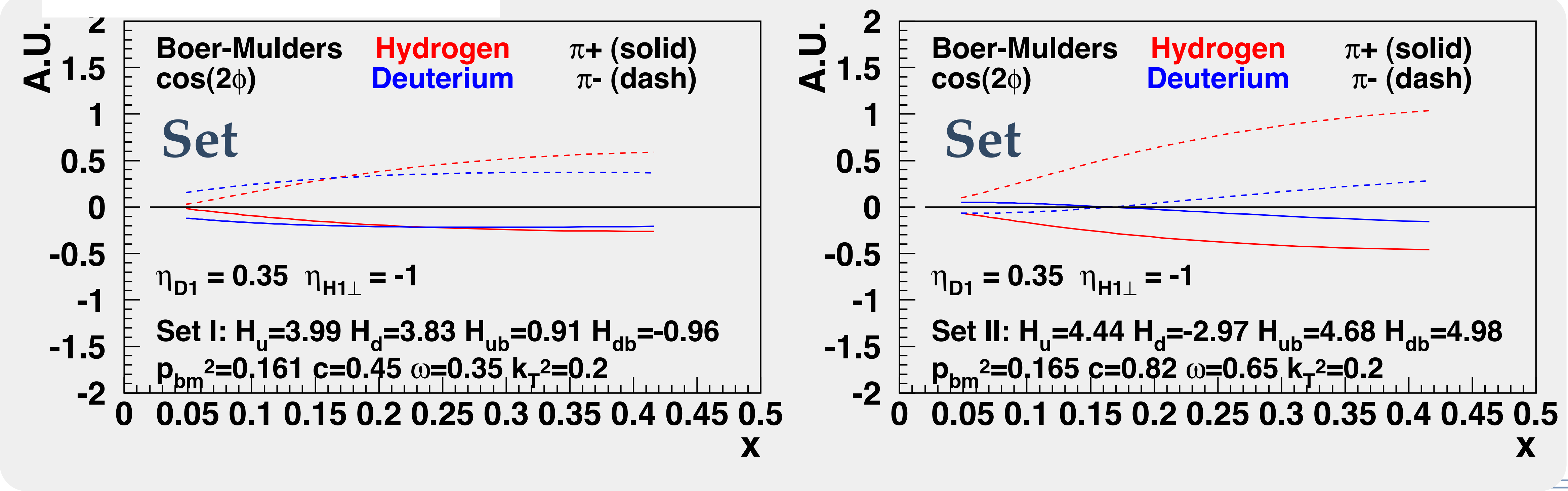
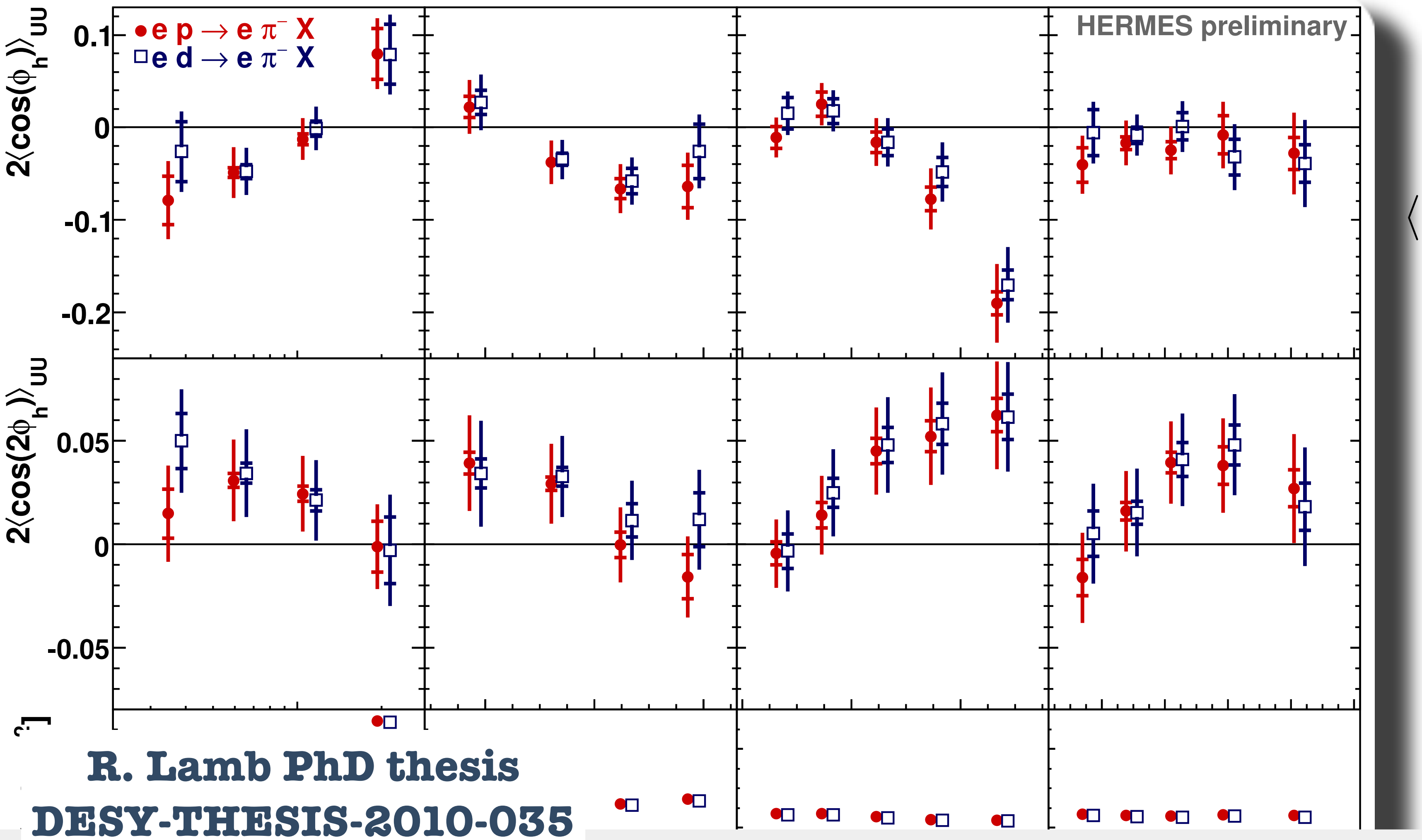
$h^\perp_{1,u} > 0 \quad h^\perp_{1,d} < 0$



# *Pions, Hydrogen vs. Deuterium*



**R. Lamb PhD thesis**
**DESY-THESIS-2010-035**

$\pi^{-}$

$$\langle \cos 2\phi_h \rangle^h_{target} = \frac{\sum_q e_q h^{\perp}_{1,q}(x) H^{\perp}_{1,q\to h}(z)}{\sum_q e_q f_{1,q}(x) D_{1,q}(z)}$$

**Set 1:**

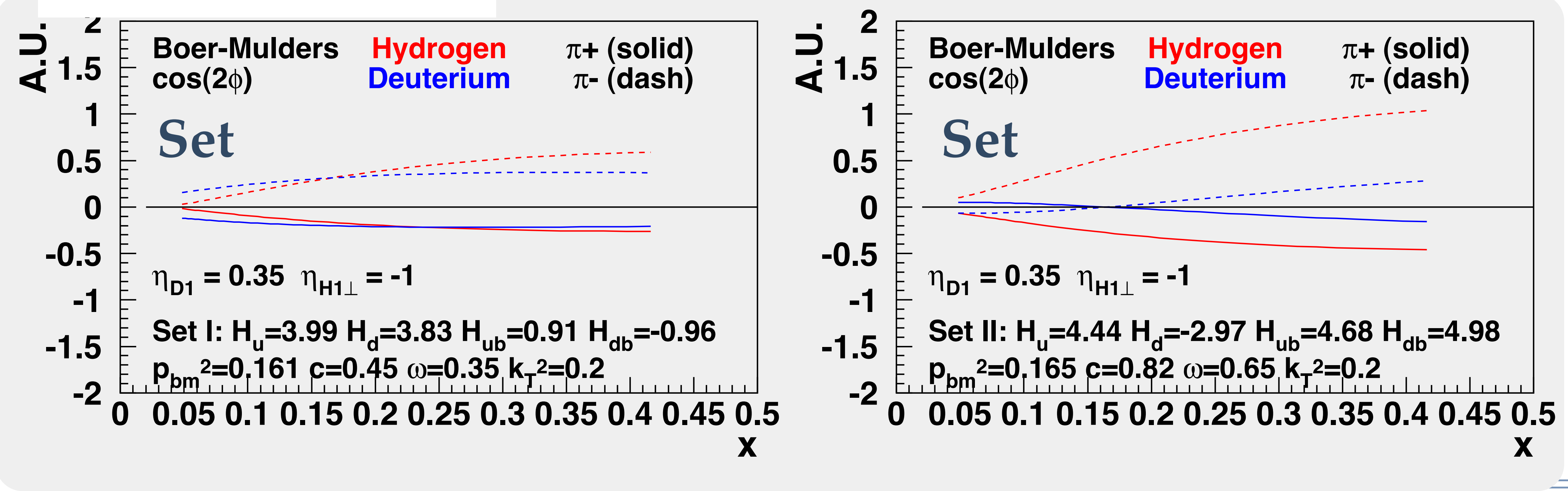
same sign for *u* and *d* quark

$h^{\perp}_{1,u} > 0 \quad h^{\perp}_{1,d} > 0$

**Set 2:**

opposite sign for *u* and *d* quark

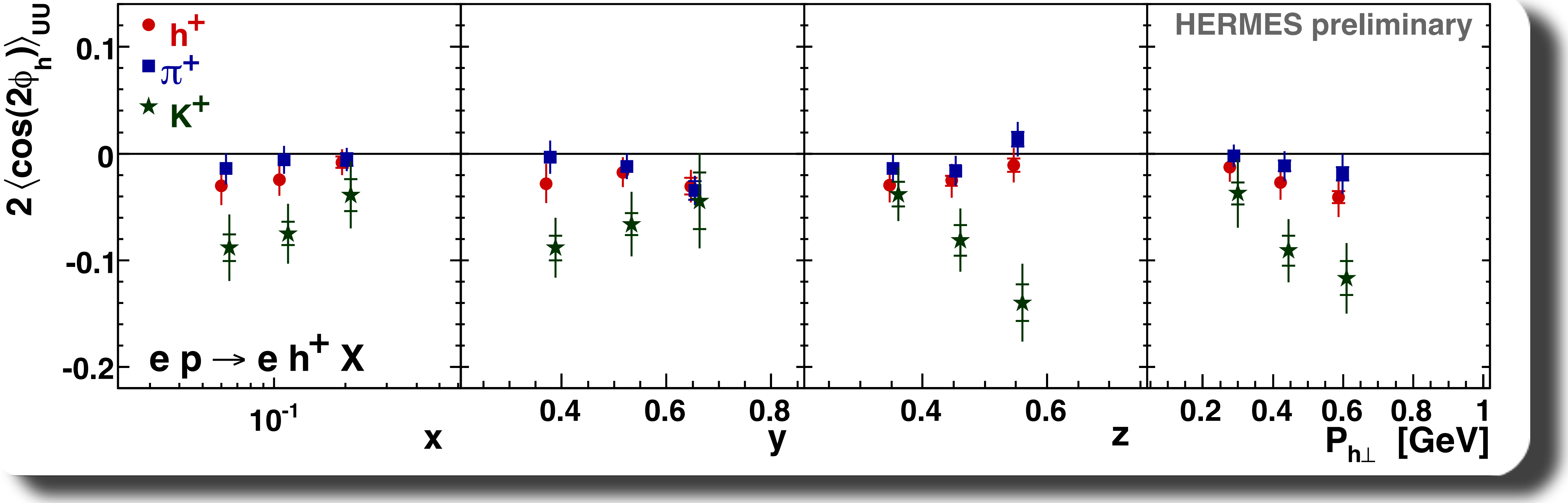
$h^{\perp}_{1,u} > 0 \quad h^{\perp}_{1,d} < 0$

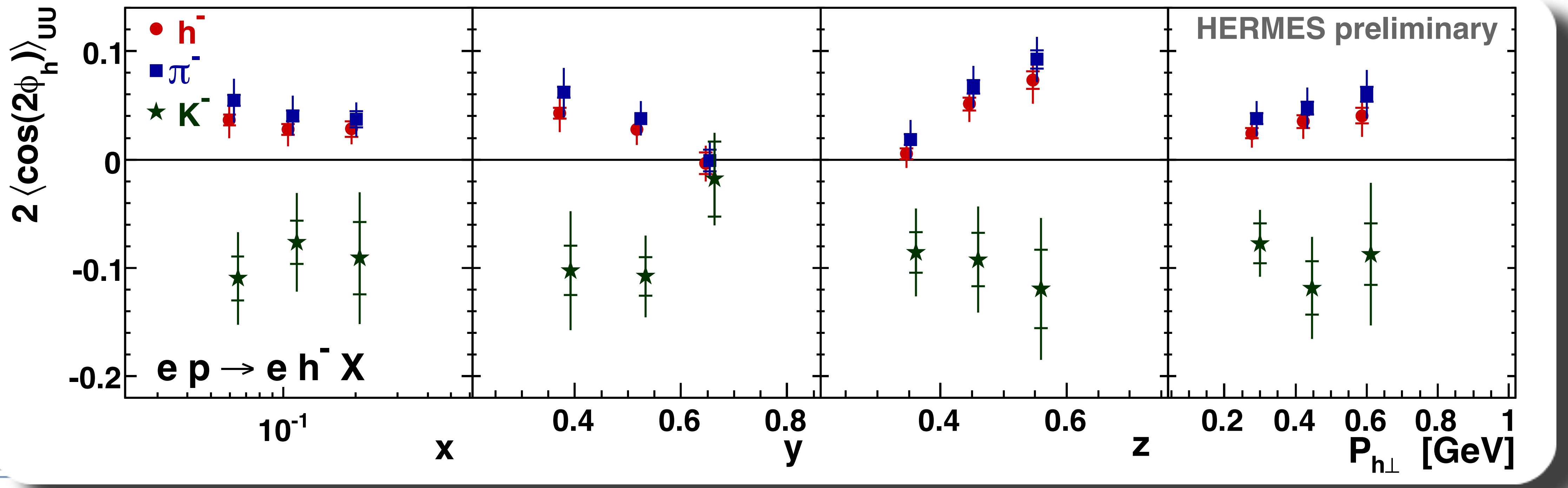


# *Pions, Hydrogen vs. Deuterium*

$\cos 2\phi_h$

kaons

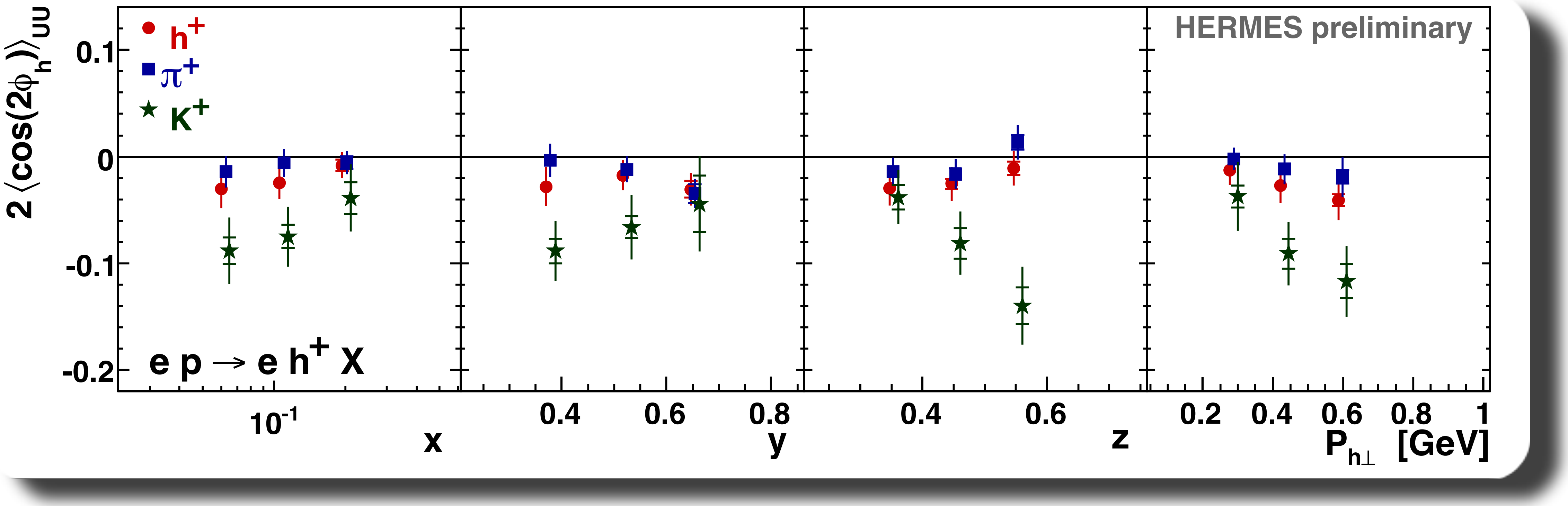




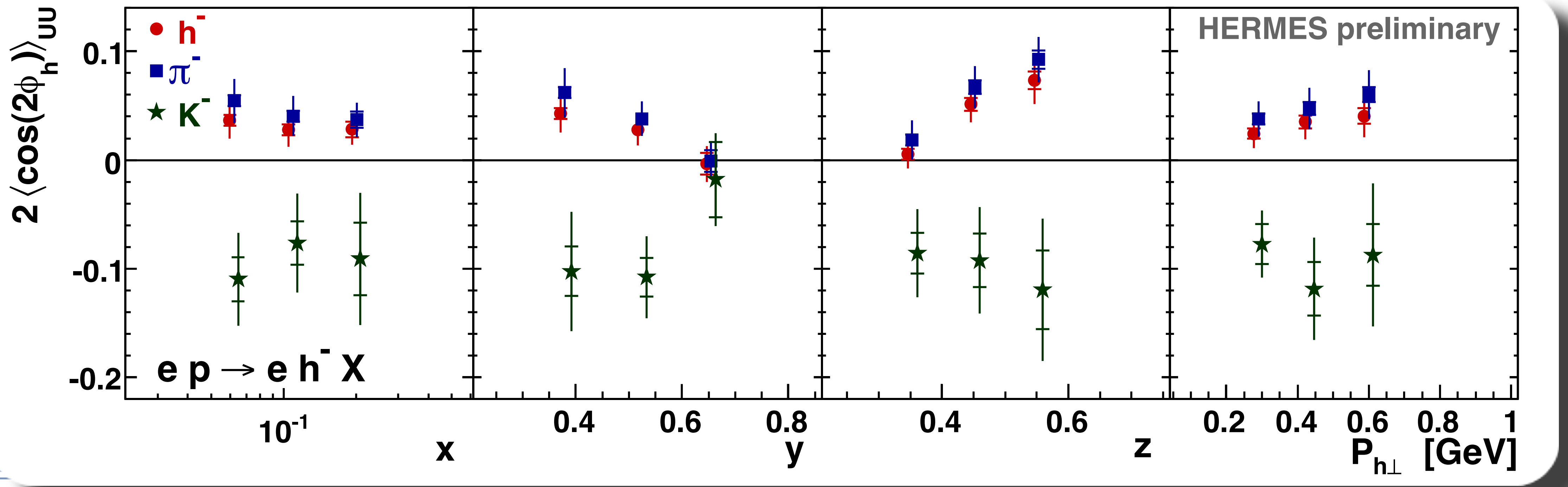
# *Kaons, Hydrogen*

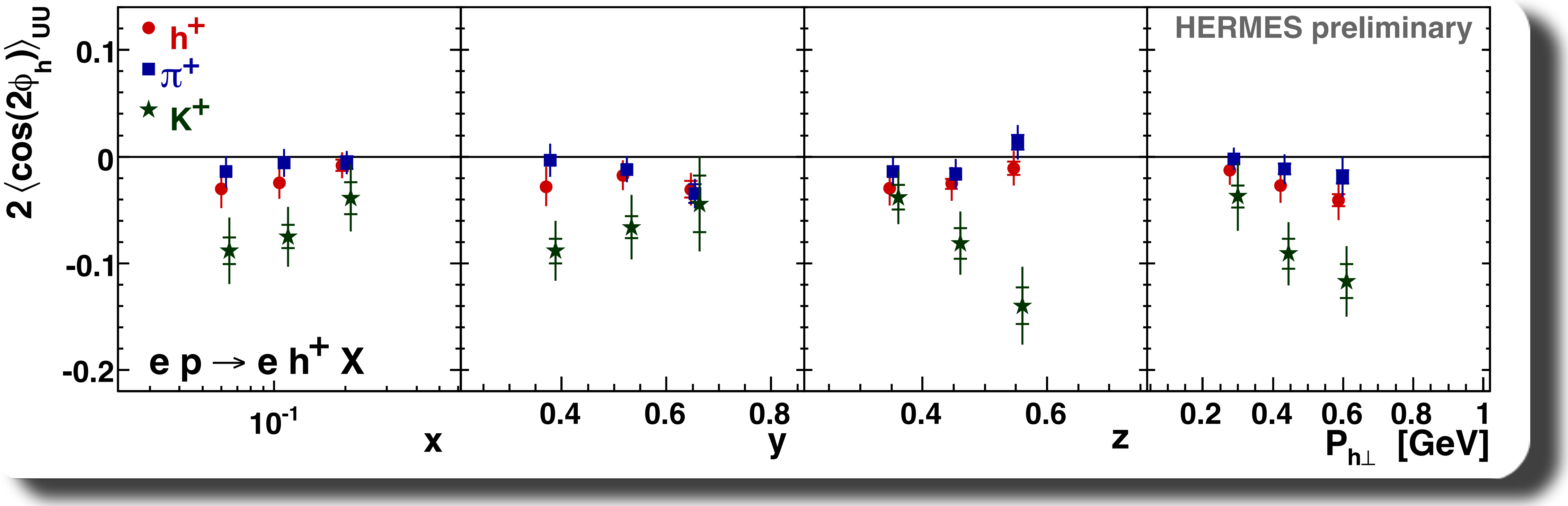
Francesca Giordano

# $\cos 2\phi_h$ kaons



$$\propto C[\underset{\text{Boer-Mulders}}{-h_1^\perp H_1^\perp} + \overset{\text{Cahn}}{\frac{\kappa_T^2}{Q^2} f_1 D_1} + ....]$$



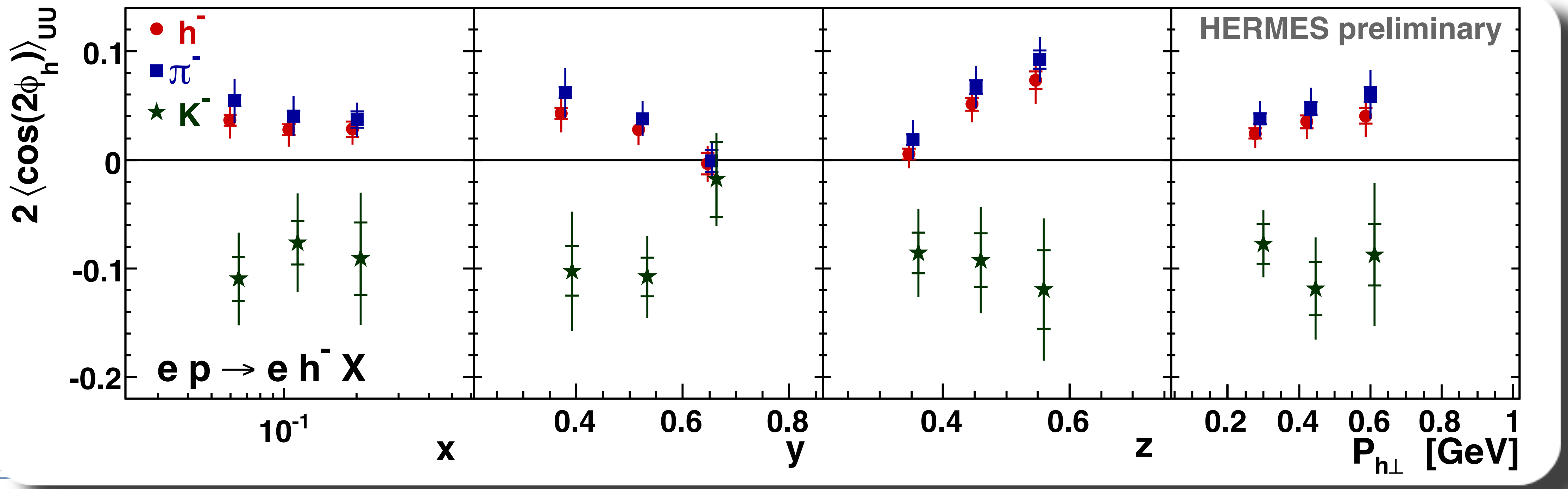


$\cos 2\phi_h$

kaons

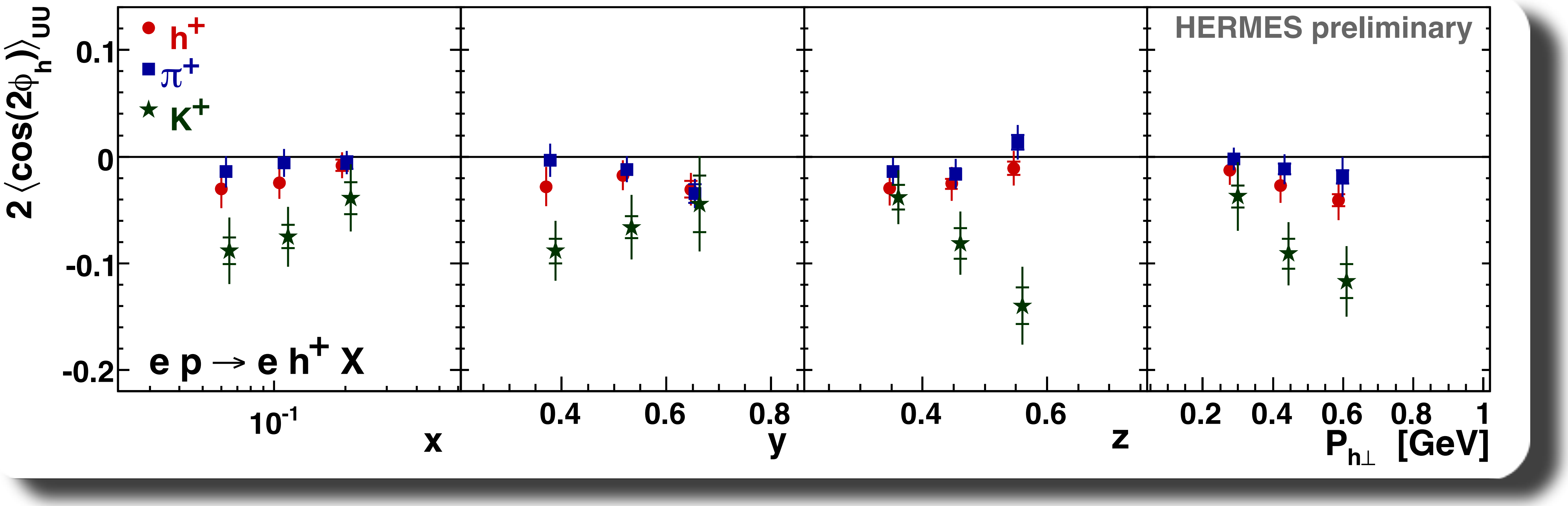
No model available!

$$\propto C[\underbrace{-h_1^\perp H_1^\perp}_{\text{Boer-Mulders}} + \overbrace{\frac{\kappa_T^2}{Q^2} f_1 D_1}^{\text{Cahn}} + ....]$$



## *Kaons, Hydrogen*

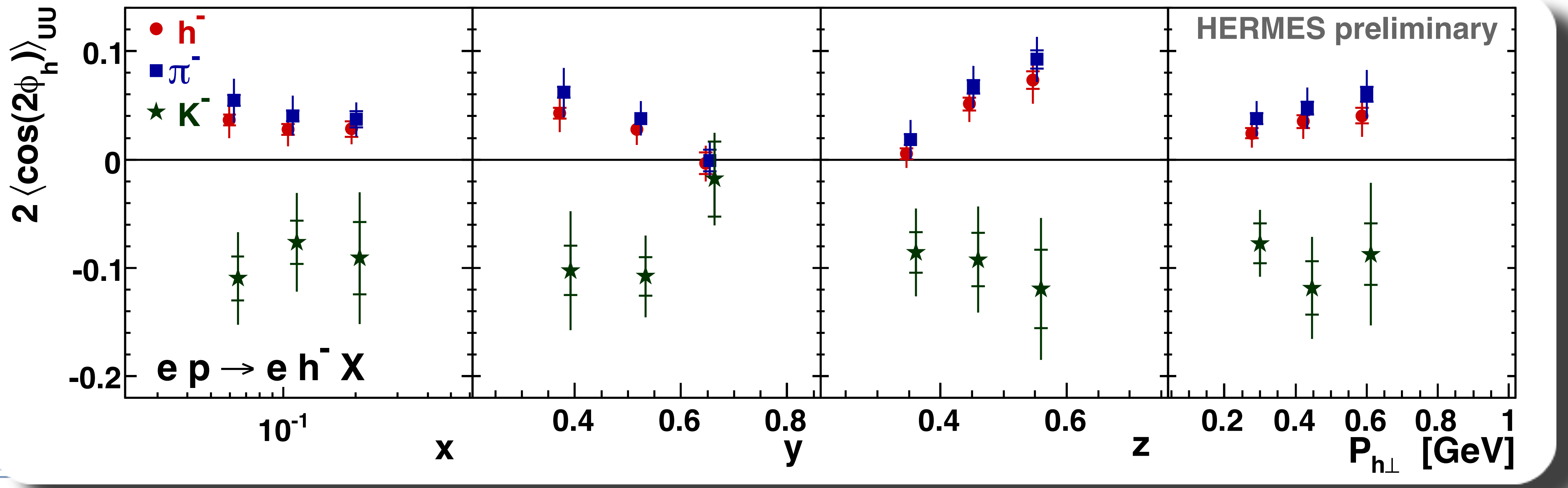
Francesca Giordano



## $\cos 2\phi_h$
## kaons

No model available!

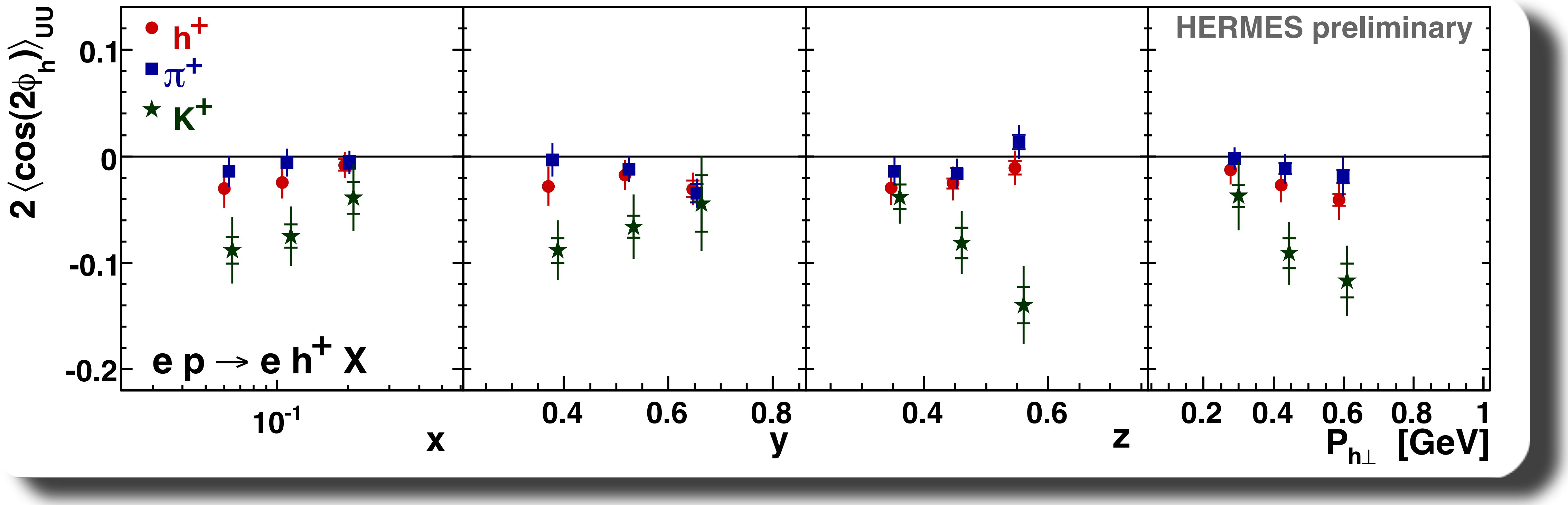
$$\propto C[\underbrace{-h_1^\perp H_1^\perp}_{\text{Boer-Mulders}} + \overset{\text{Cahn}}{\cancel{\frac{\kappa_T^2}{Q^2} f_1 D_1}} + ....]$$



# *Kaons, Hydrogen*

Francesca Giordano

## $\cos 2\phi_h$ kaons

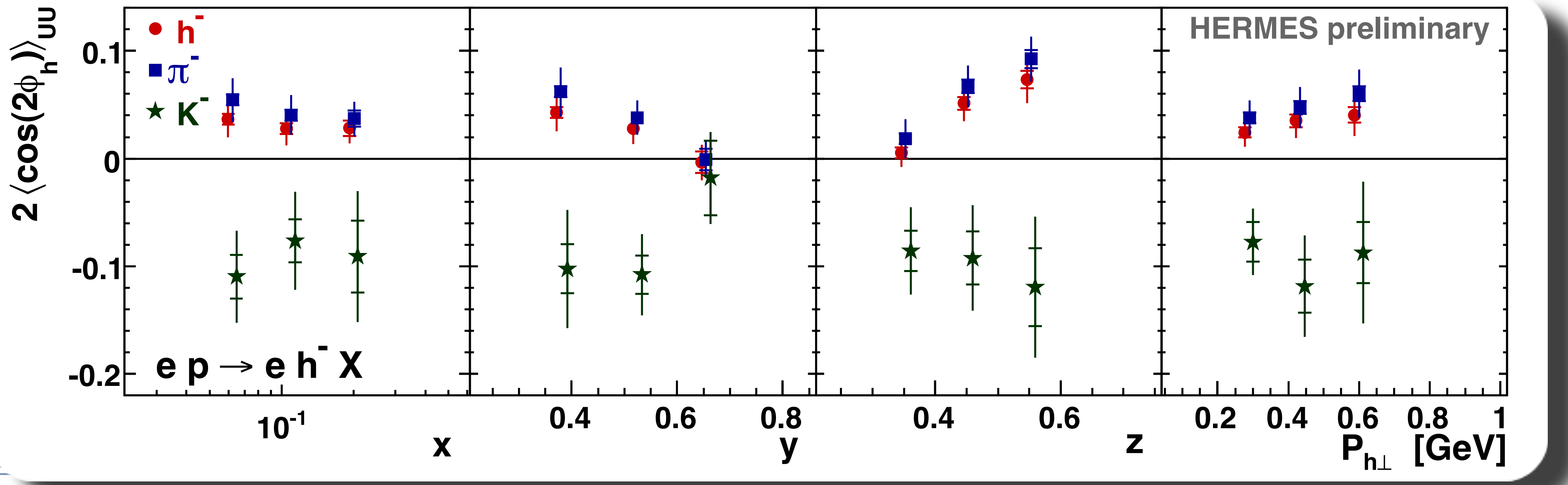


No model available!

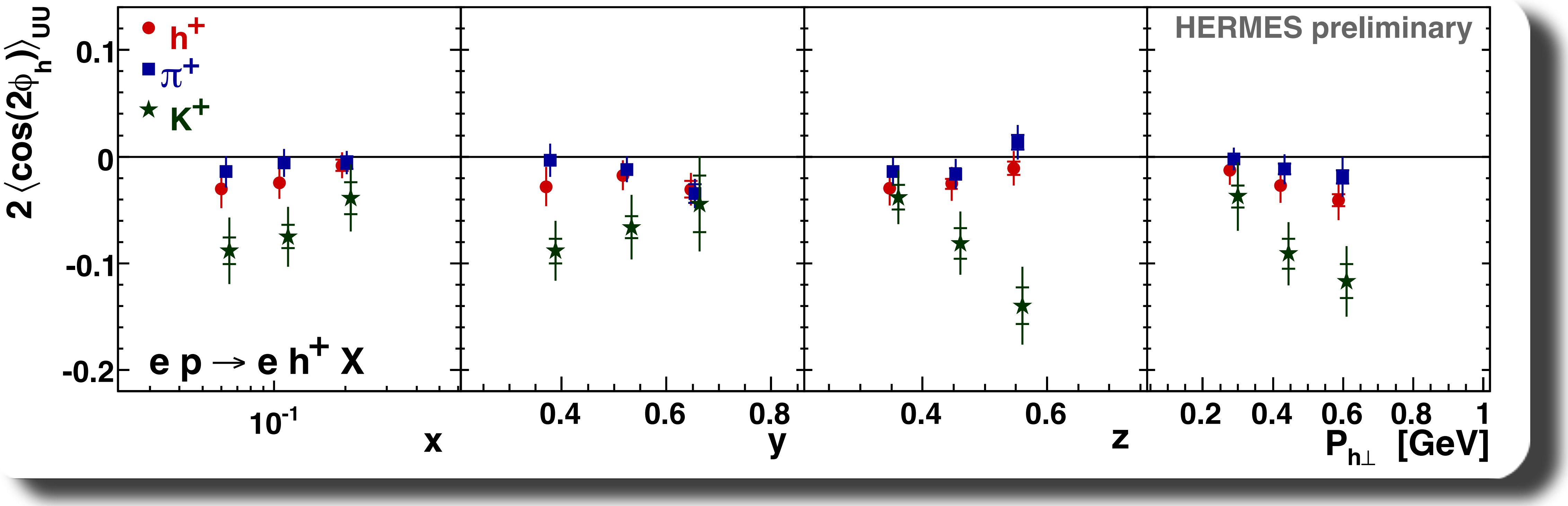
$u - dominance$

$$H_1^{\perp,u\rightarrow K^-} \overset{?}{\approx} H_1^{\perp,u\rightarrow K^+}$$

$$\propto C[\underset{\text{Boer-Mulders}}{-h_1^\perp H_1^\perp} + \overset{\text{Cahn}}{\cancel{\frac{\kappa_T^2}{Q^2} f_1 D_1}} + ....]$$



## *Kaons, Hydrogen*



$\cos 2\phi_h$

kaons

No model available!

$$\propto C[\underbrace{-h_1^\perp H_1^\perp}_{\text{Boer-Mulders}} + \overbrace{\cancel{\frac{\kappa_T^2}{Q^2} f_1 D_1}}^{\text{Cahn}} + ....]$$

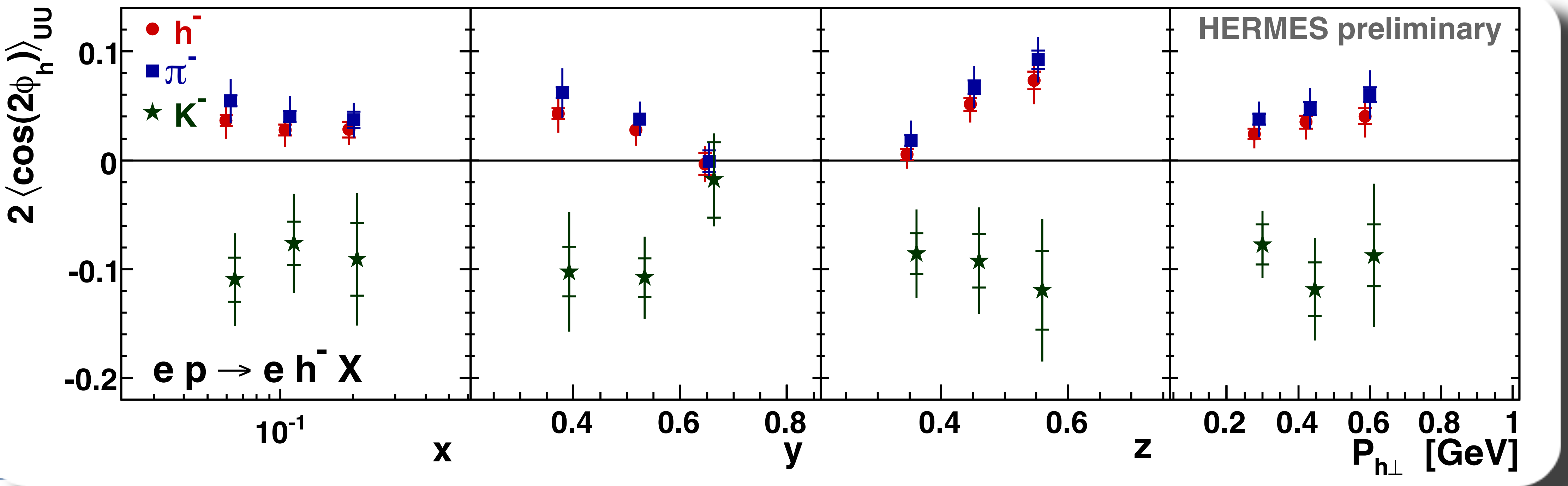
$u - dominance$

$$H_1^{\perp,u\to K^-} \overset{?}{\approx} H_1^{\perp,u\to K^+}$$

$K^-\{s\bar{u}\}$

⇩

$full\ sea\ object$



# *Kaons, Hydrogen*

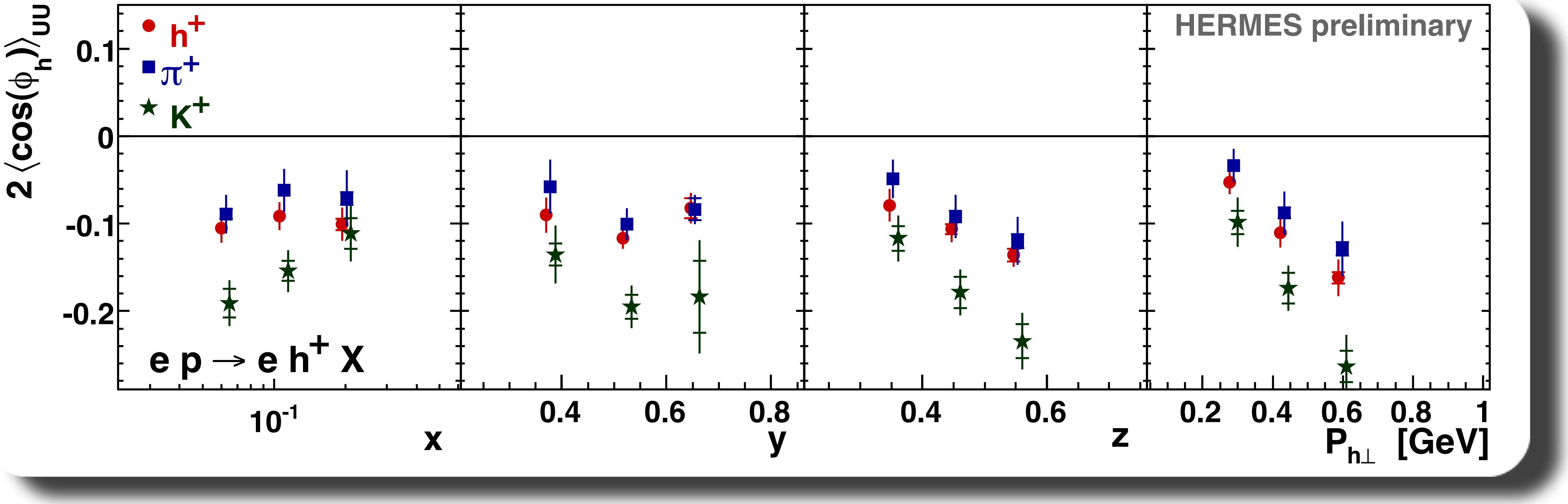
Francesca Giordano

$\cos \phi_h$

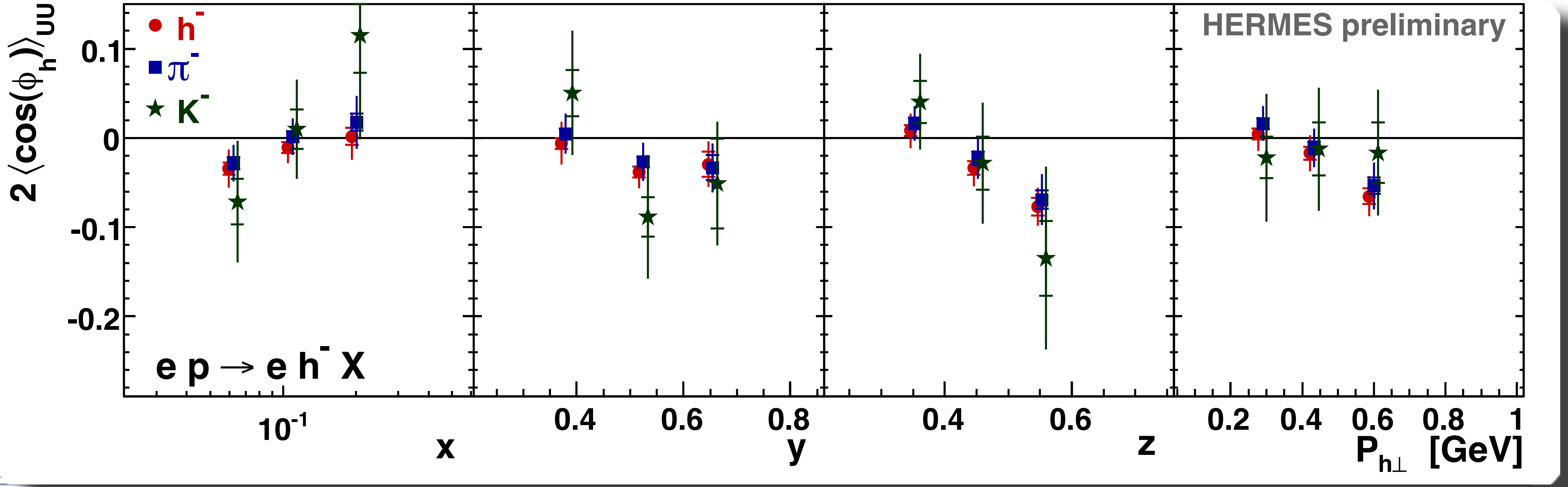
kaons

$\cos\phi_h$

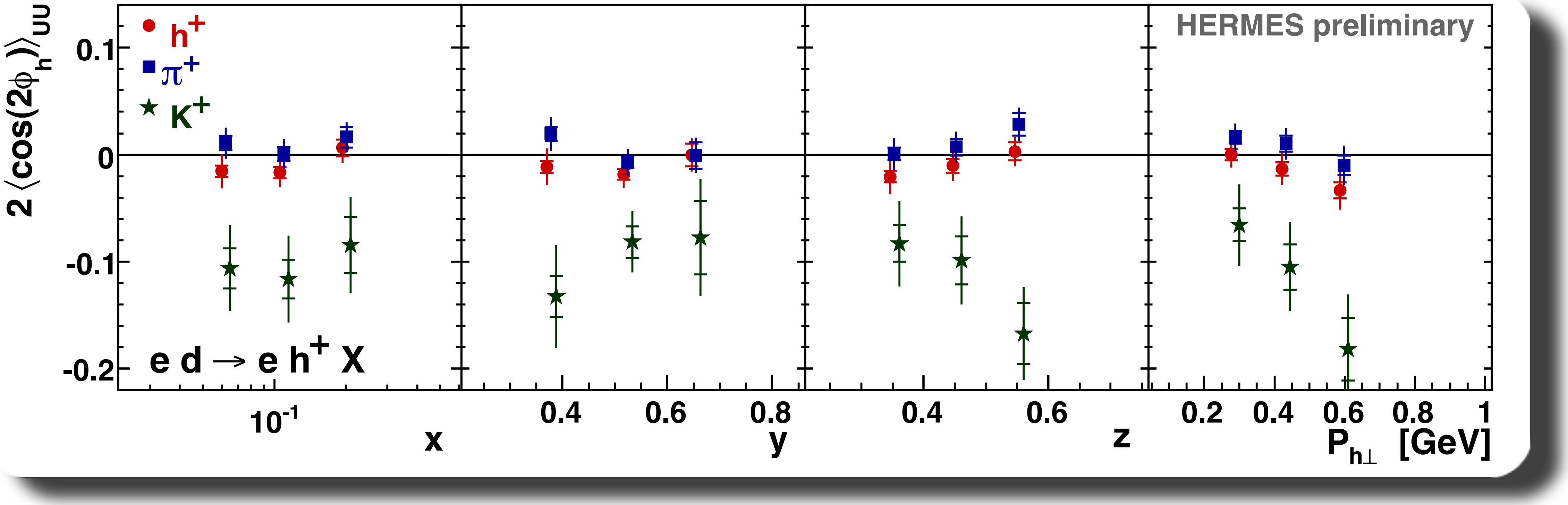
kaons



$$\propto \frac{2M}{Q} C[-h_1^\perp H_1^\perp - f_1 D_1 + ....]$$



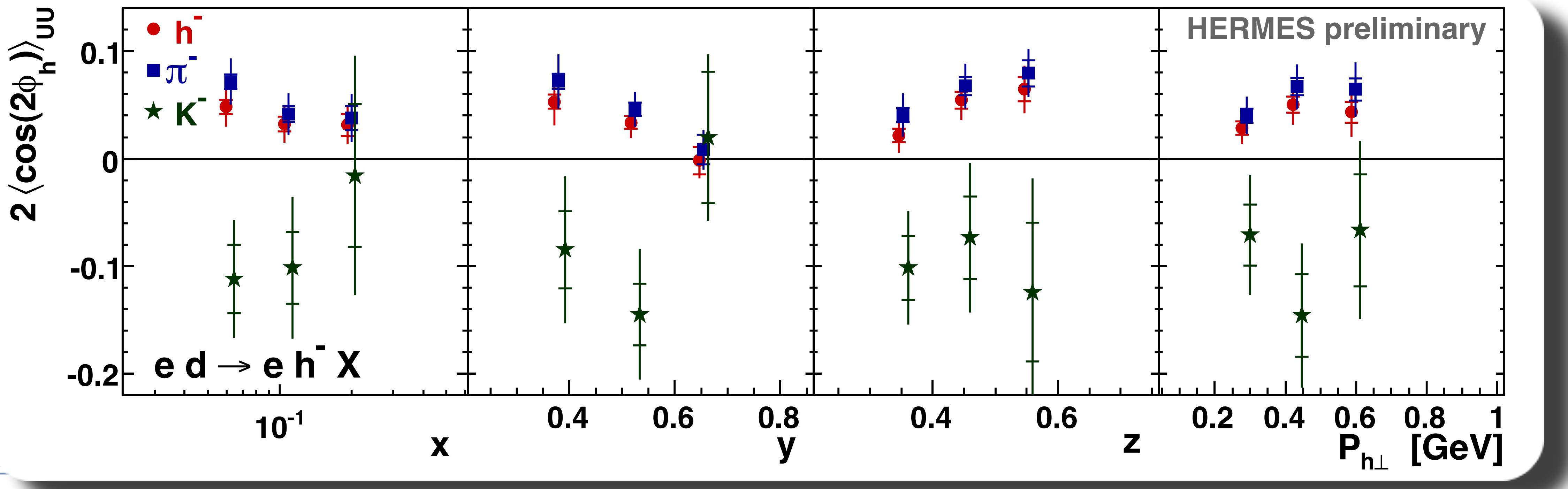
## *Kaons, Hydrogen*



$\cos 2\phi_h$

kaons

$$\propto C[-h_1^\perp H_1^\perp + \frac{\kappa_T^2}{Q^2} f_1 D_1 + ....]$$

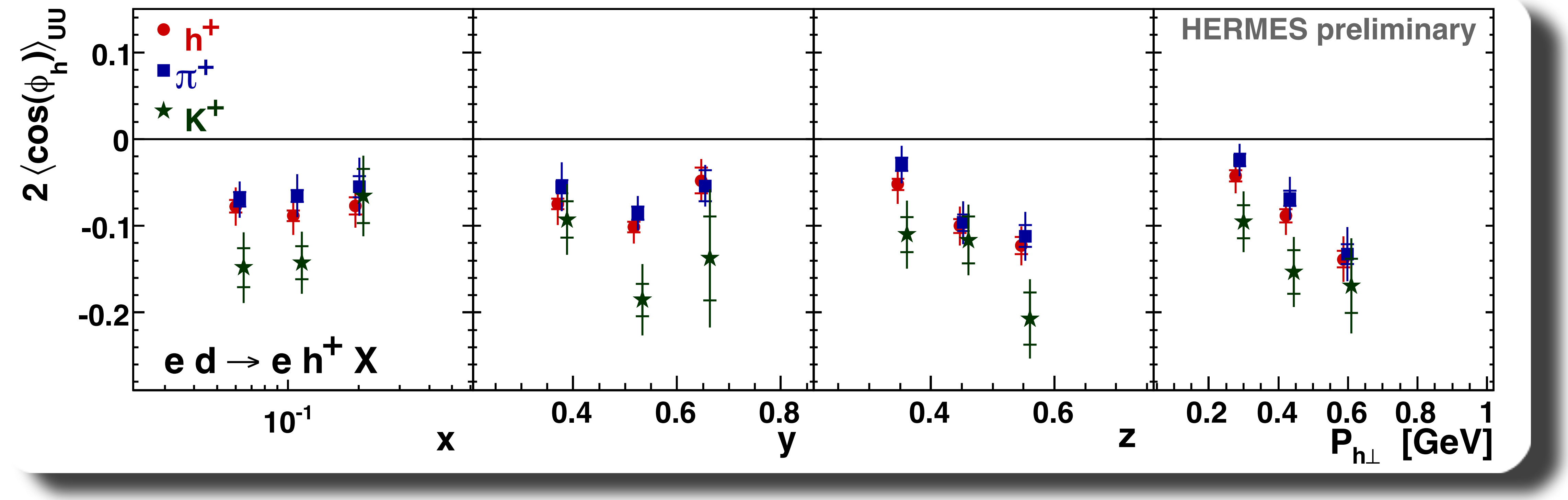


# *Kaons, Deuterium*

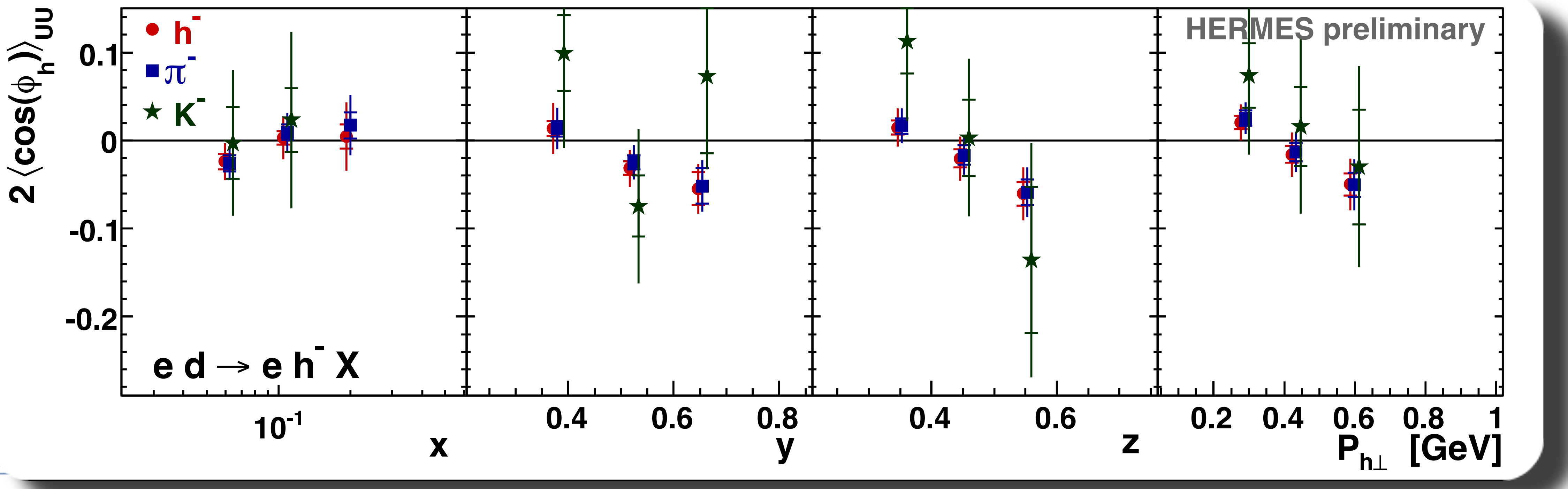
Francesca Giordano

$\cos\phi_h$

kaons



$$\propto \frac{2M}{Q} C[-h_1^\perp H_1^\perp - f_1 D_1 + ....]$$



# *Kaons, Deuterium*

- Differences between $\pi^+/\pi^-$:

  evidence of a non-zero Boer-Mulders function:
  confirms opposite sign for favored and unfavored pion Collins fragmentation functions

- Different behavior for $K^+/K^-$ with respect to pions:

  large signals and same sign for $\cos 2\phi_h$ modulation: indication of same sign for favored/unfavored strange Collins fragmentation functions?

- Similar results for deuterium & hydrogen data

  suggest a Boer-Mulders function with same sign for *u* and *d* quark

# Summary

To date hermes provides the most complete data set available!

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$\langle k_T^2 \rangle|_s$ $\langle k_T^2 \rangle|_u$

$\langle k_T^2 \rangle|_d$

# Summary

To date hermes provides the most complete data set available!

$\langle k_T^2\rangle|_s$ $\langle k_T^2\rangle|_u$

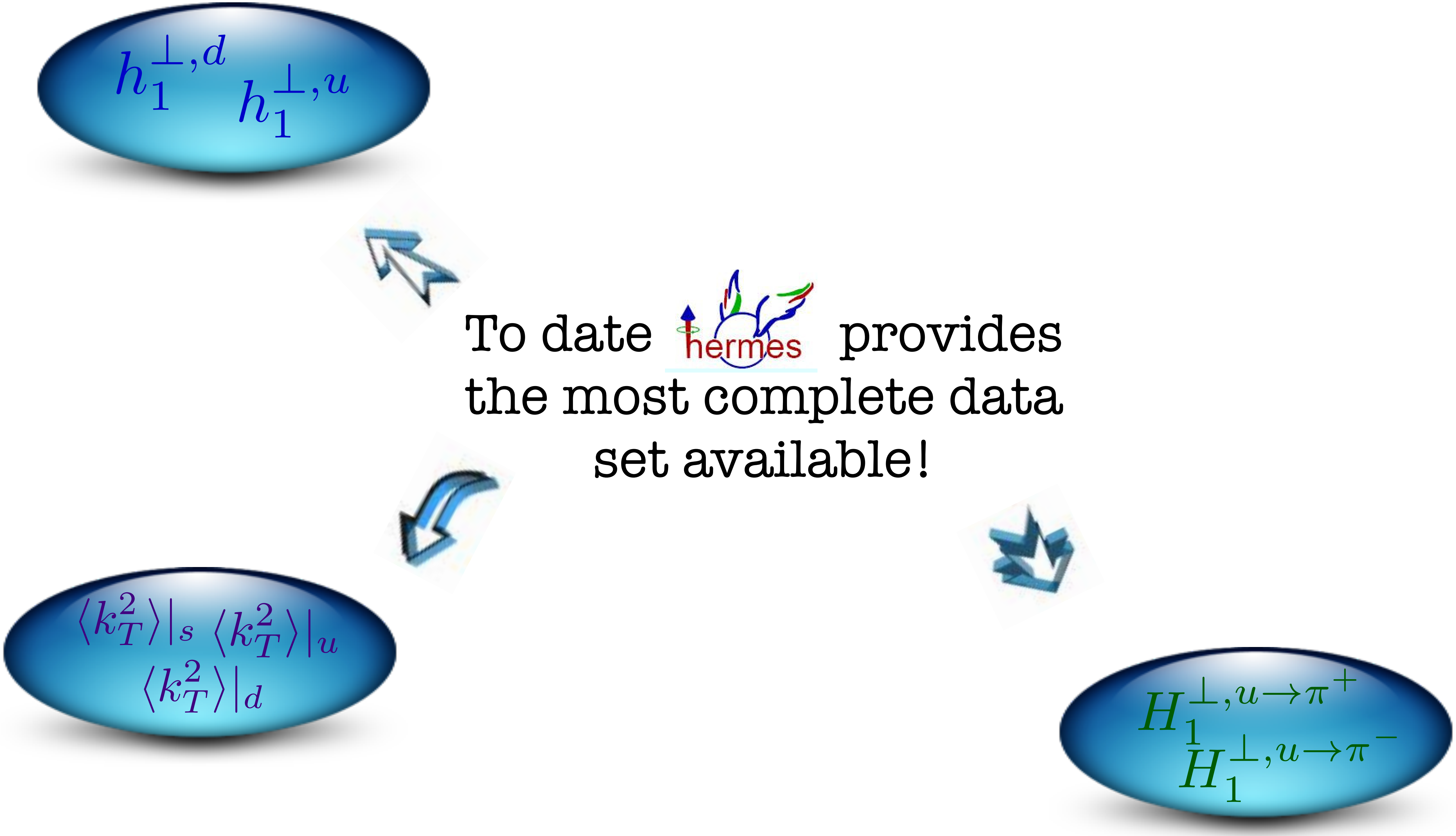
$\langle k_T^2\rangle|_d$

$H_1^{\perp,u\rightarrow\pi^+}$

$H_1^{\perp,u\rightarrow\pi^-}$

# Summary

# Summary



$h_1^{\perp,d}$ $h_1^{\perp,u}$

To date hermes provides the most complete data set available!

$\langle k_T^2\rangle|_s$ $\langle k_T^2\rangle|_u$
$\langle k_T^2\rangle|_d$

$H_1^{\perp,u\to\pi^+}$
$H_1^{\perp,u\to\pi^-}$

$h_1^{\perp,d}$ $h_1^{\perp,u}$

$H_1^{\perp,u\to K^+}$ $H_1^{\perp,u\to K^-}$

To date hermes provides the most complete data set available!

$\langle k_T^2\rangle|_s$ $\langle k_T^2\rangle|_u$ $\langle k_T^2\rangle|_d$

$H_1^{\perp,u\to \pi^+}$ $H_1^{\perp,u\to \pi^-}$

# Summary