High energy contribution to the Gerasimov - Drell - Hearn sum rule

XXXVII International Meeting on
Nuclear Physics

Bormio - January 25-31, 1999

A. Fantoni

LNF - INFN

Index of the talk

- The GDH sum rule
- The generalized GDH sum rule (extension for virtual photons)
- Results of first determination of DIS contribution to the generalized GDH sum rule
- Different high energy extrapolations
- Contribution of g_2
- Conclusions

The GDH sum rule

The GDH sum rule, derived in the sixties by Gerasimov and independently by Drell and Hearn, relates the anomalous contribution κ to the magnetic moment of the nucleon ($\kappa_{\rm p}$=1.79, $\kappa_{\rm n}$=−1.91) with the total absorption cross sections for circularly polarised real photons on polarised nucleons:

$$\int_0^\infty \left[\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)\right] \frac{d\nu}{\nu} = -\frac{2\pi^2\alpha}{M^2}\kappa^2$$

general dispersion relation for forward Compton scattering
+ low-energy theorem of Low
+ additional assumption that the spin-flip part of the Compton scattering amplitude goes to zero at infinite energy without any poles.

Theoretical predictions:
-204 μb p
-233 μb n

Never been tested directly

The generalised GDH sum rule

The integral can be generalised to the absorption of virtual photons with energy ν and squared four-momentum $-Q^2$:

$$I(Q^2) \equiv \int_{Q^2/2M}^{\infty} \left[\sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2)\right] \frac{d\nu}{\nu}$$

$$= \frac{8\pi^2\alpha}{M} \int_0^1 \frac{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)}{K} \frac{dx}{x}$$

g_1, g_2 polarised structure functions of the nucleon
x Bjorken variable
$K = \nu\sqrt{1+\gamma^2}$ flux factor of the virtual photons
$\gamma^2 = Q^2/\nu^2$

$I(Q^2)$ connects the GDH expectation for real photon with the Ellis-Jaffe (or Bjorken in the case of p-n) in the asymptotic region

The Q^2 dependence of $I(Q^2)$ connects the static ground state properties of the nucleon with its helicity structure as measured in inelastic scattering in the resonance and deep inelastic regions

$$I(Q^2) \equiv \int_{Q^2/2M}^{\infty} \Delta\sigma(\nu, Q^2) \frac{d\nu}{\nu}$$

where :

$$\Delta\sigma(\nu, Q^2) = \sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2) = \frac{8\pi^2\alpha}{M} \frac{A_1 F_1}{K}$$

$F_1 = F_2 \frac{1+\gamma^2}{2x(1+R)}$

$F_2 \rightarrow$ NMC parameterization

$R \rightarrow$ SLAC parameterization

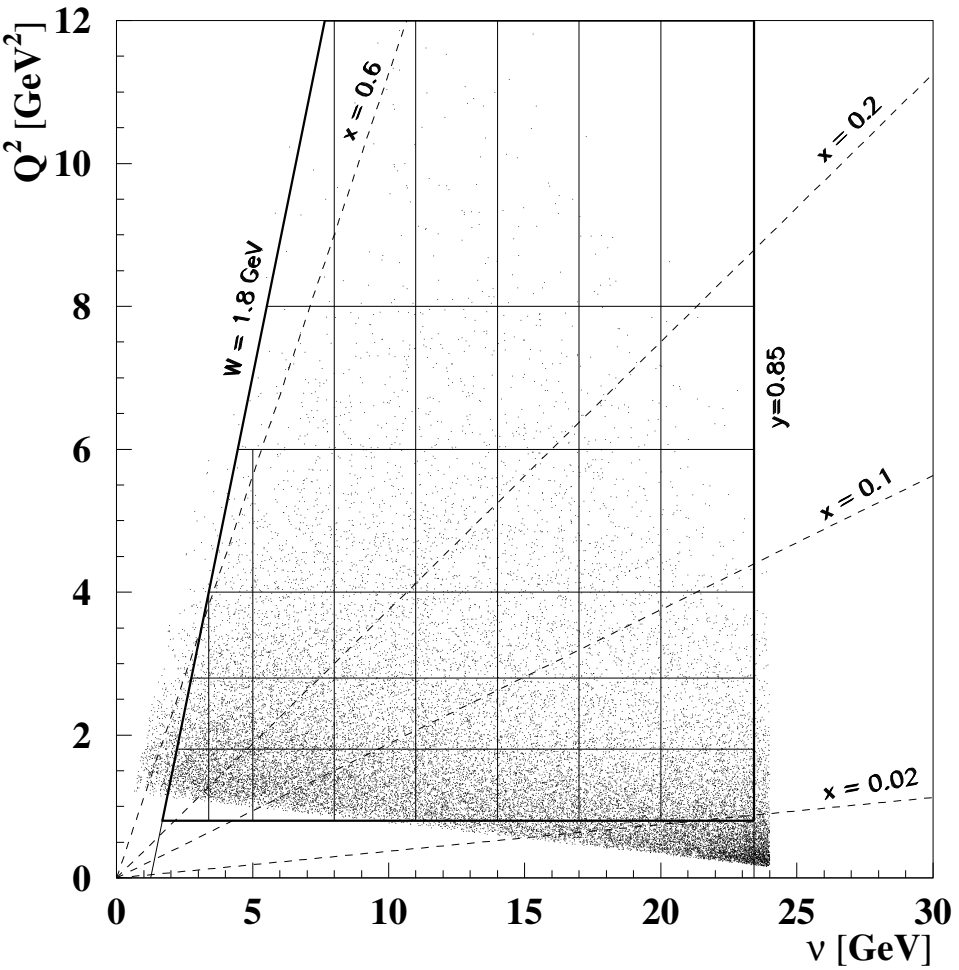
$A_1 = A_{\parallel}/D - \eta A_2$

$A_2 \rightarrow$ SLAC measurement

$K = \nu\sqrt{1+\gamma^2}$ flux factor of the virtual photons

HERMES kinematic plane

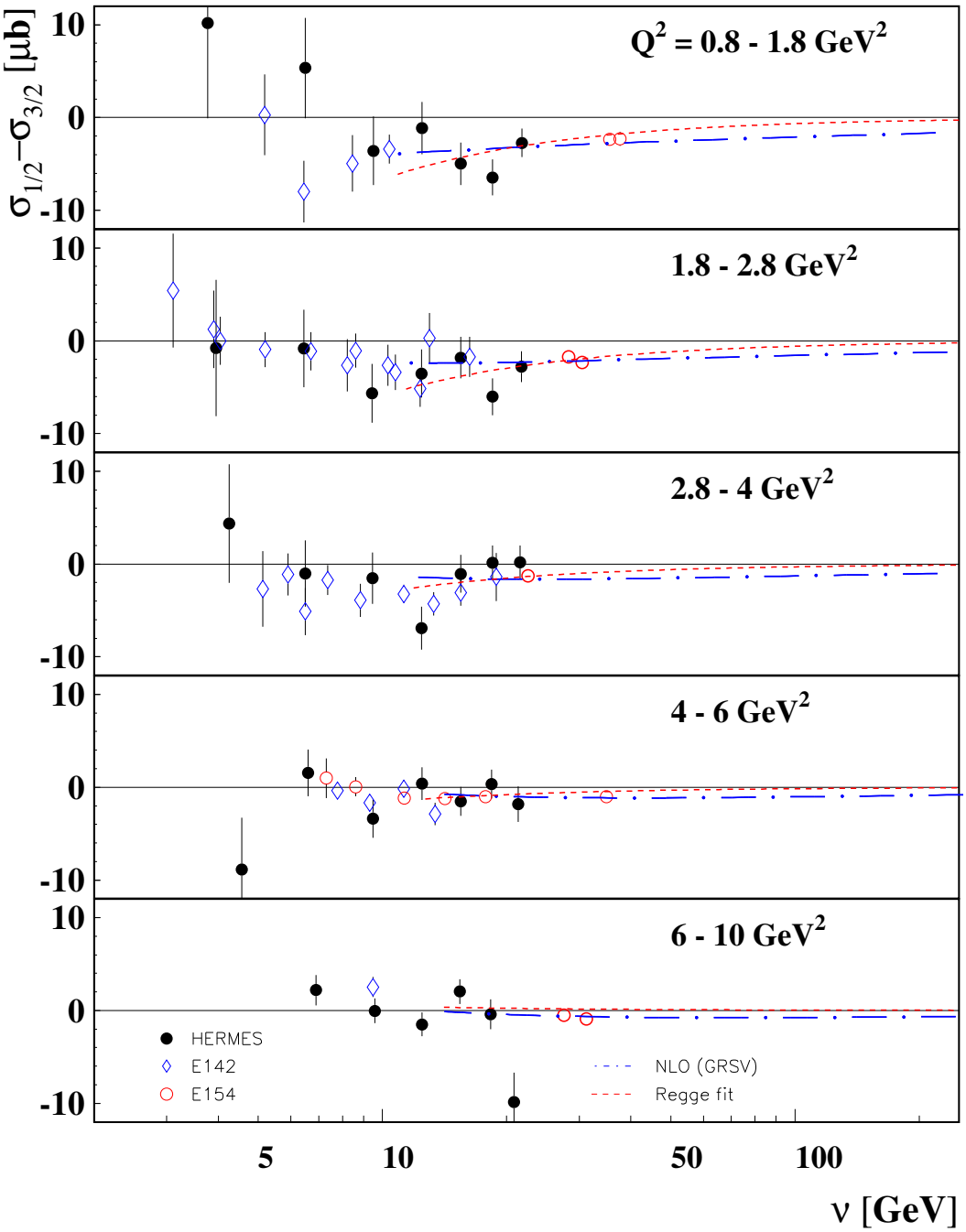
$E_{\rm beam}$=27.5 GeV
W $>$ 1.8 GeV, $\nu < 23.5$ GeV (y<0.85)
$\theta_{\rm min}$ = 40 mrad $\rightarrow$ $Q^2 \geq 0.8$ GeV^2
$\theta_{\rm max}$ = 220 mrad



$0.004 < x_{\rm Bj} < 0.8 \rightarrow g_1(x)$, Bjorken & E.J. sum rule;

$0.8 < Q^2 < 12$ GeV^2,
$2 < \nu < 23.5 \rightarrow \sigma_{1/2} - \sigma_{3/2}$, GDH sum rule

Neutron cross section difference

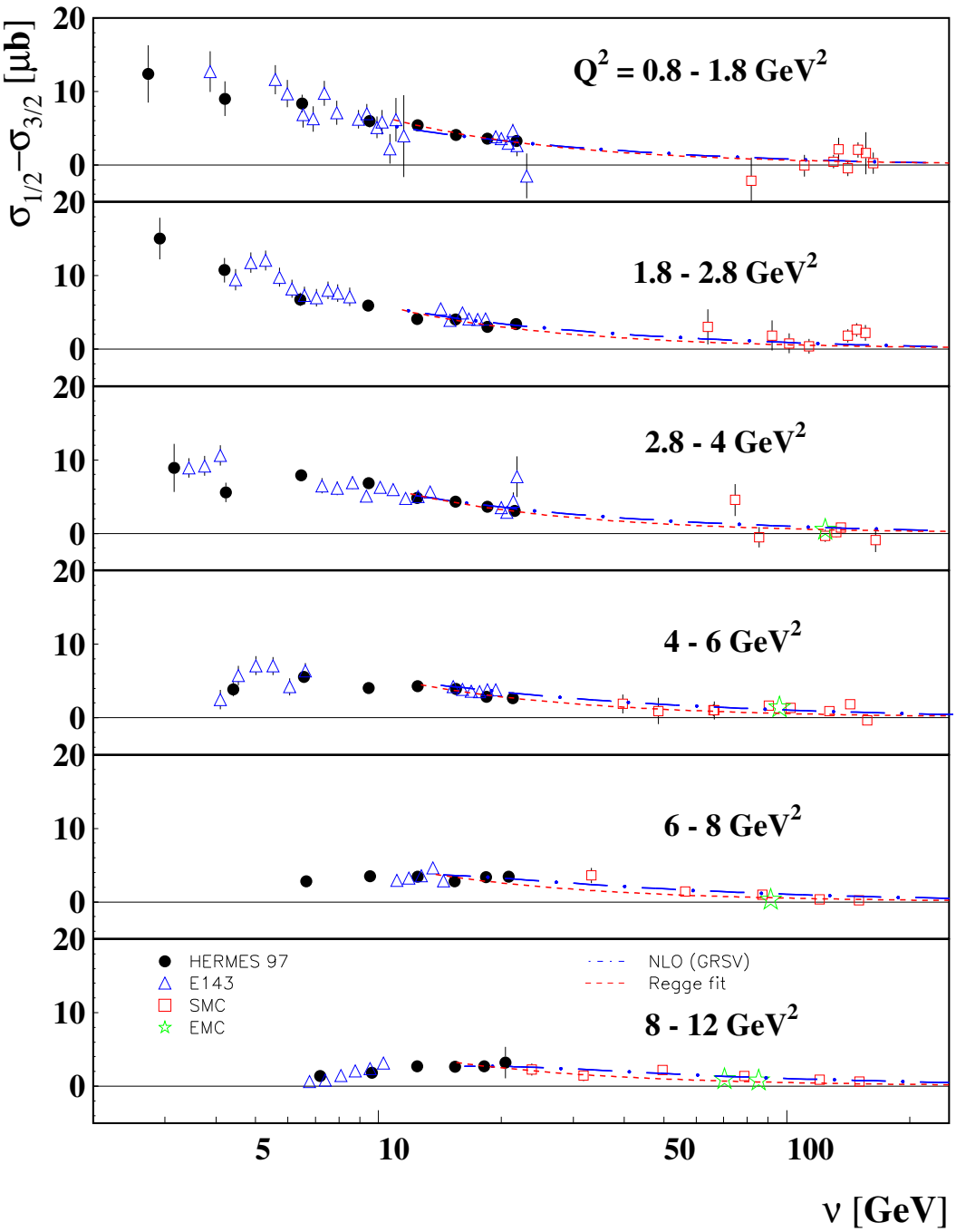


$$\sigma_{1/2} - \sigma_{3/2} = \frac{8\pi^2\alpha}{M}\,\frac{A_1 F_1}{K}$$

A_1^n from $A_{\parallel}^n$ (1995 data on ^{3}He target)

A. Fantoni, Bormio 99

Proton cross section difference



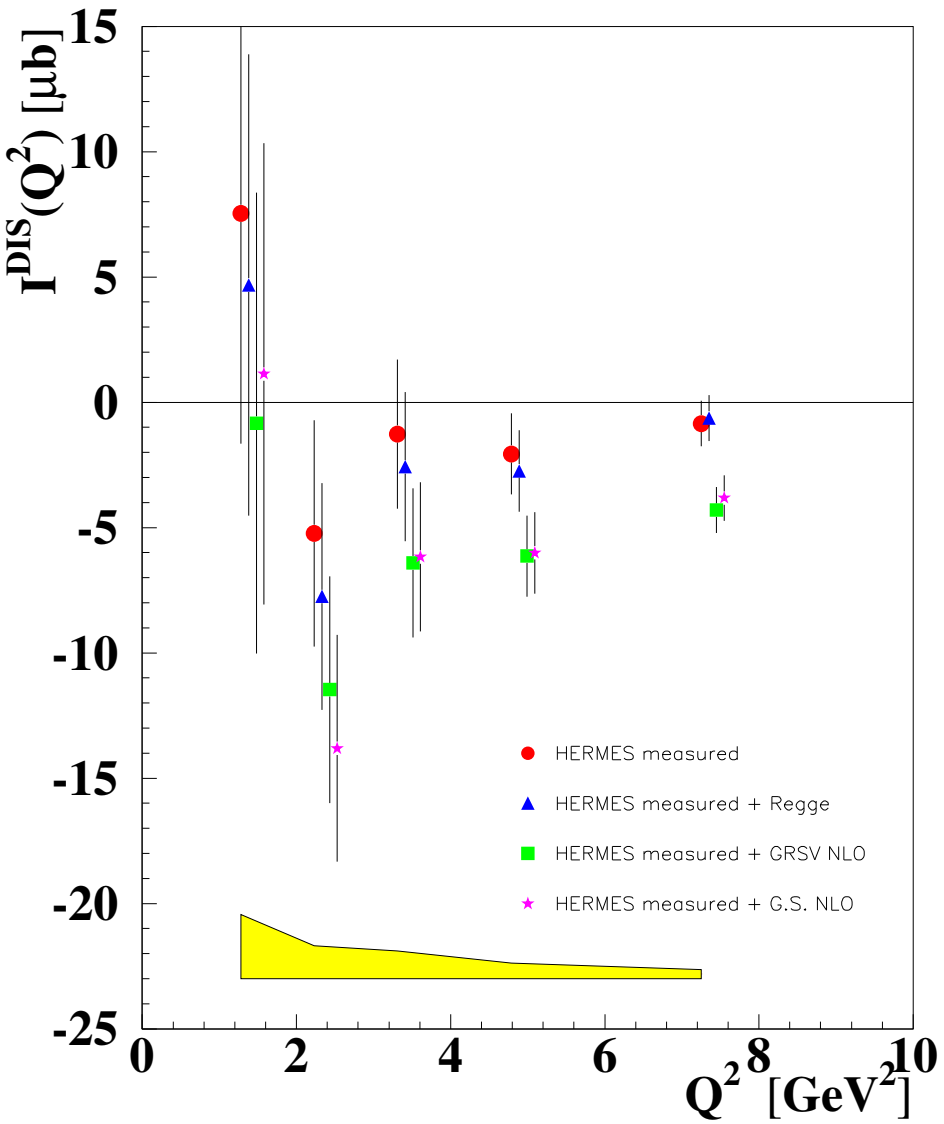
$$\sigma_{1/2} - \sigma_{3/2} = \frac{8\pi^2\alpha}{M}\,\frac{A_1 F_1}{K}$$

A_1^p from $A_{\parallel}^p$ (1997 data on ^{1}H target)

A. Fantoni, Bormio 99

Generalised GDH integral for the neutron

Data + different high energies extrapolations :



$$I^{\rm DIS}_{\rm meas}(Q^2) = \int_{\nu_0}^{23.5} [\sigma_{1/2} - \sigma_{3/2}] \frac{d\nu}{\nu}$$

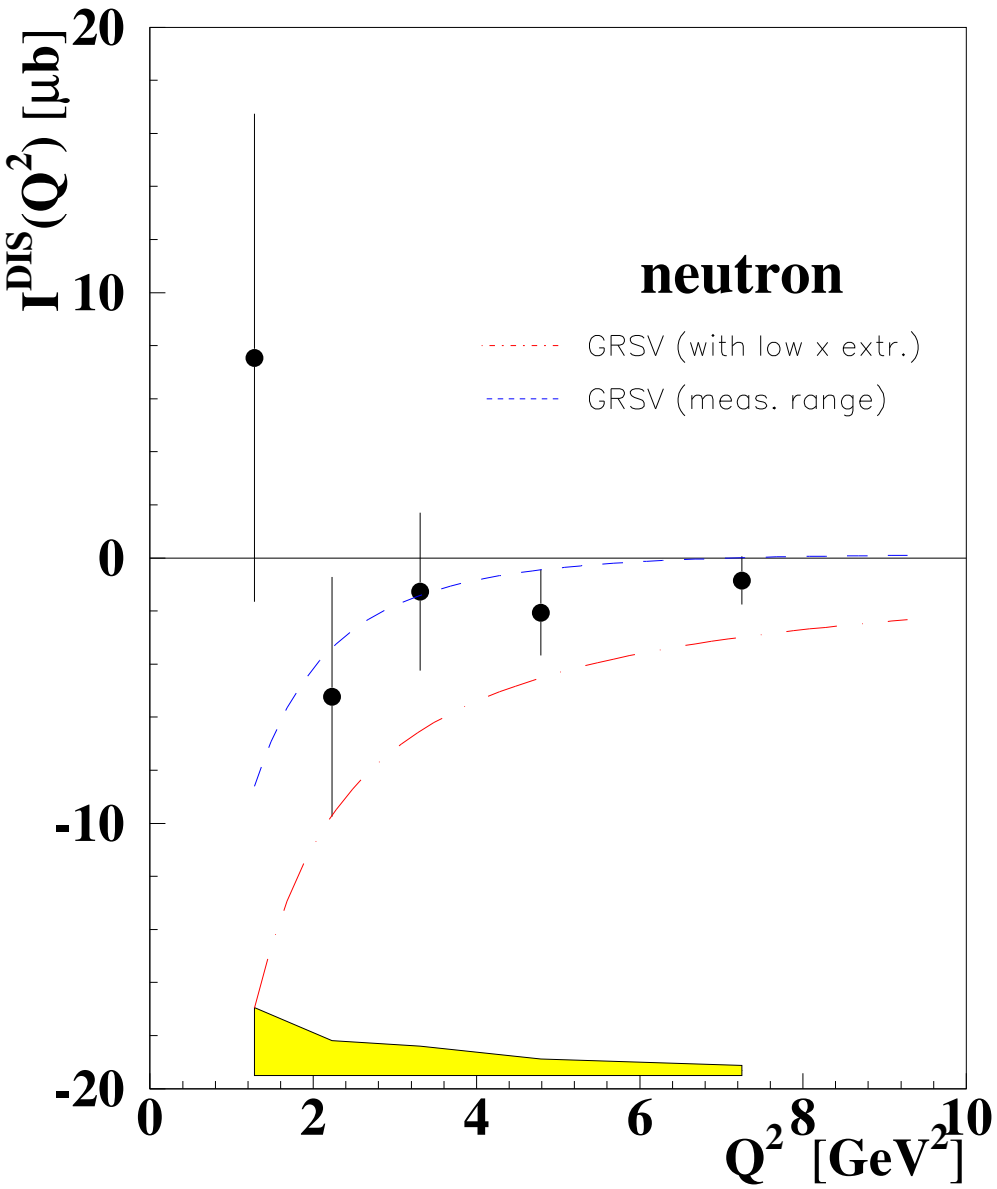
$$I^{\rm DIS}(Q^2) = \int_{\nu_0}^{\infty} [\sigma_{1/2} - \sigma_{3/2}] \frac{d\nu}{\nu}$$

$\nu_0 \rightarrow$ due to W=2 GeV cut

A. Fantoni, Bormio 99

Generalised GDH integral for the neutron

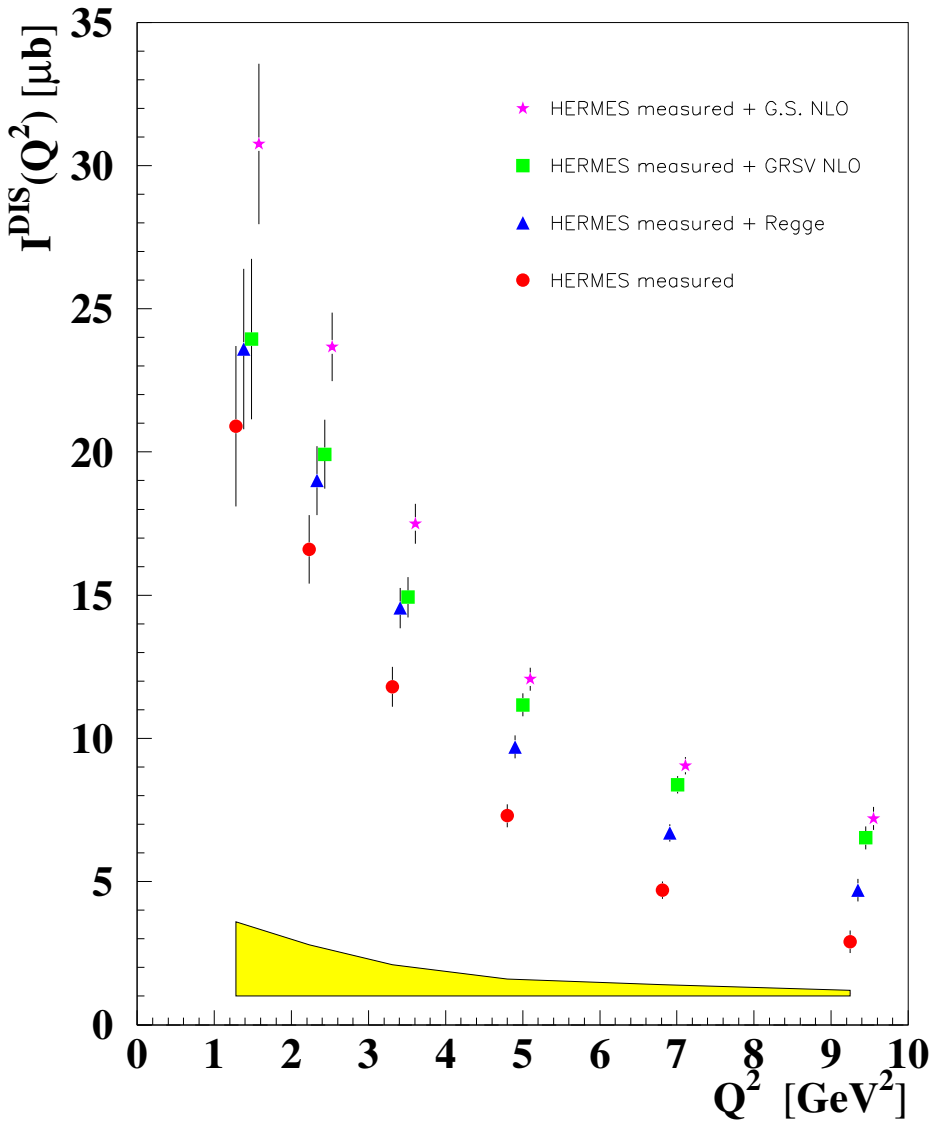
Comparison between data and NLO parameterisation :



The data are consistent with a QCD next-to-leading order fit based on previous deep-inelastic scattering data [M. Gluck *et al.*, *Phys. Rev.* **D 53 (1996) 6620**]

Generalised GDH integral for the proton

Data + different high energies extrapolations :



$$I^{\mathrm{DIS}}_{\mathrm{meas}}(Q^2) = \int_{\nu_0}^{23.5} [\sigma_{1/2} - \sigma_{3/2}] \frac{d\nu}{\nu}$$

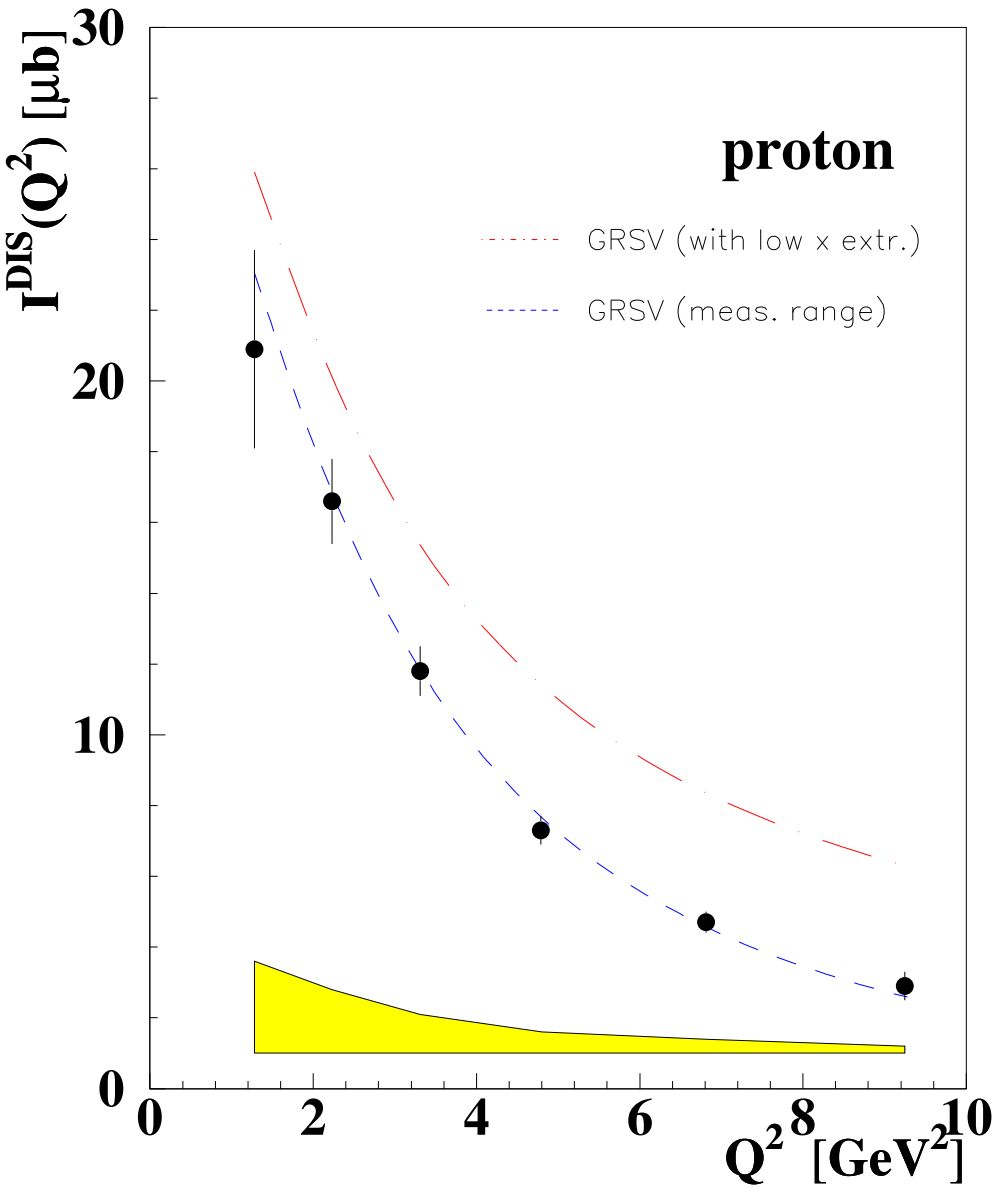
$$I^{\mathrm{DIS}}(Q^2) = \int_{\nu_0}^{\infty} [\sigma_{1/2} - \sigma_{3/2}] \frac{d\nu}{\nu}$$

$\nu_0 \rightarrow$ due to W=1.8 GeV cut

A. Fantoni, Bormio 99

Generalised GDH integral for the proton

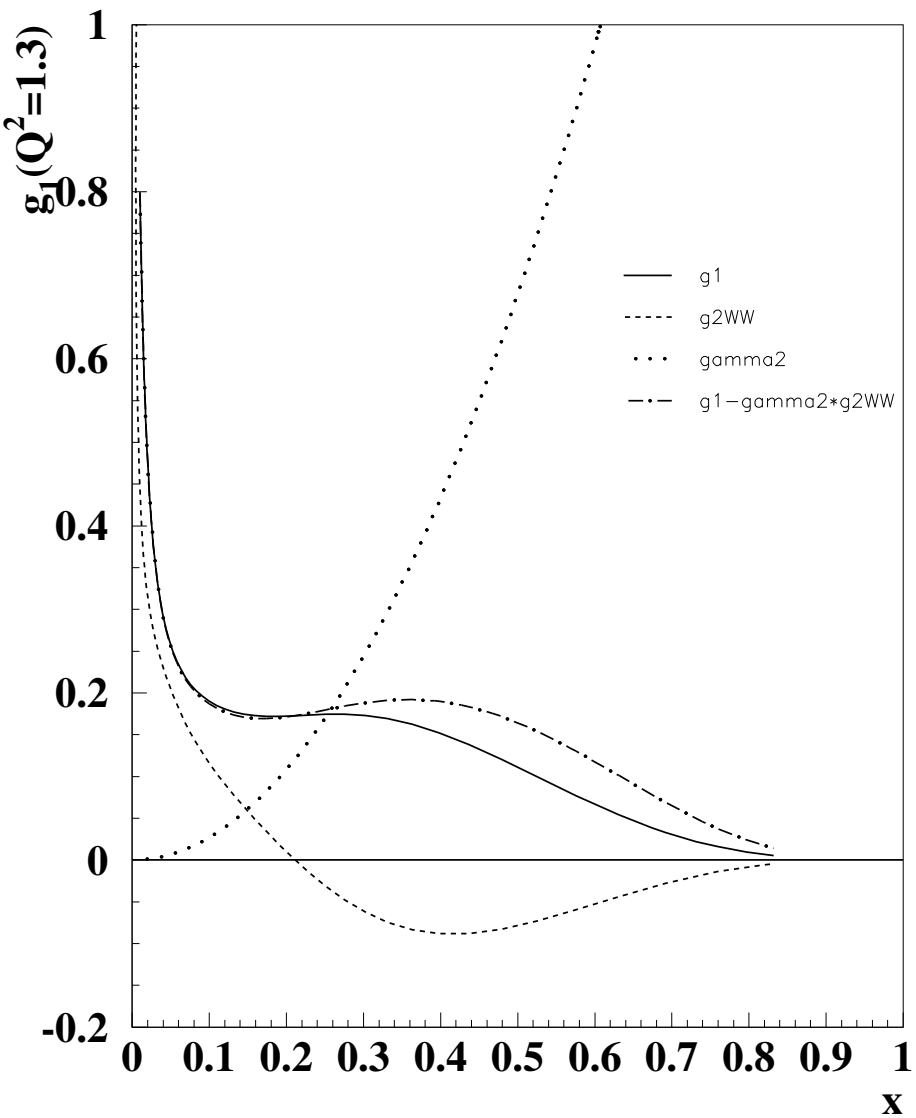
Comparison between data and NLO parameterisation :



The data are consistent with a QCD next-to-leading order fit based on previous deep-inelastic scattering data [M. Gluck *et al.*, Phys. Rev. **D 53** (1996) 6620]

Contribution of g_2 :

$$g_2^{\mathrm{WW}}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 g_1(y,Q^2)\frac{dy}{y}$$

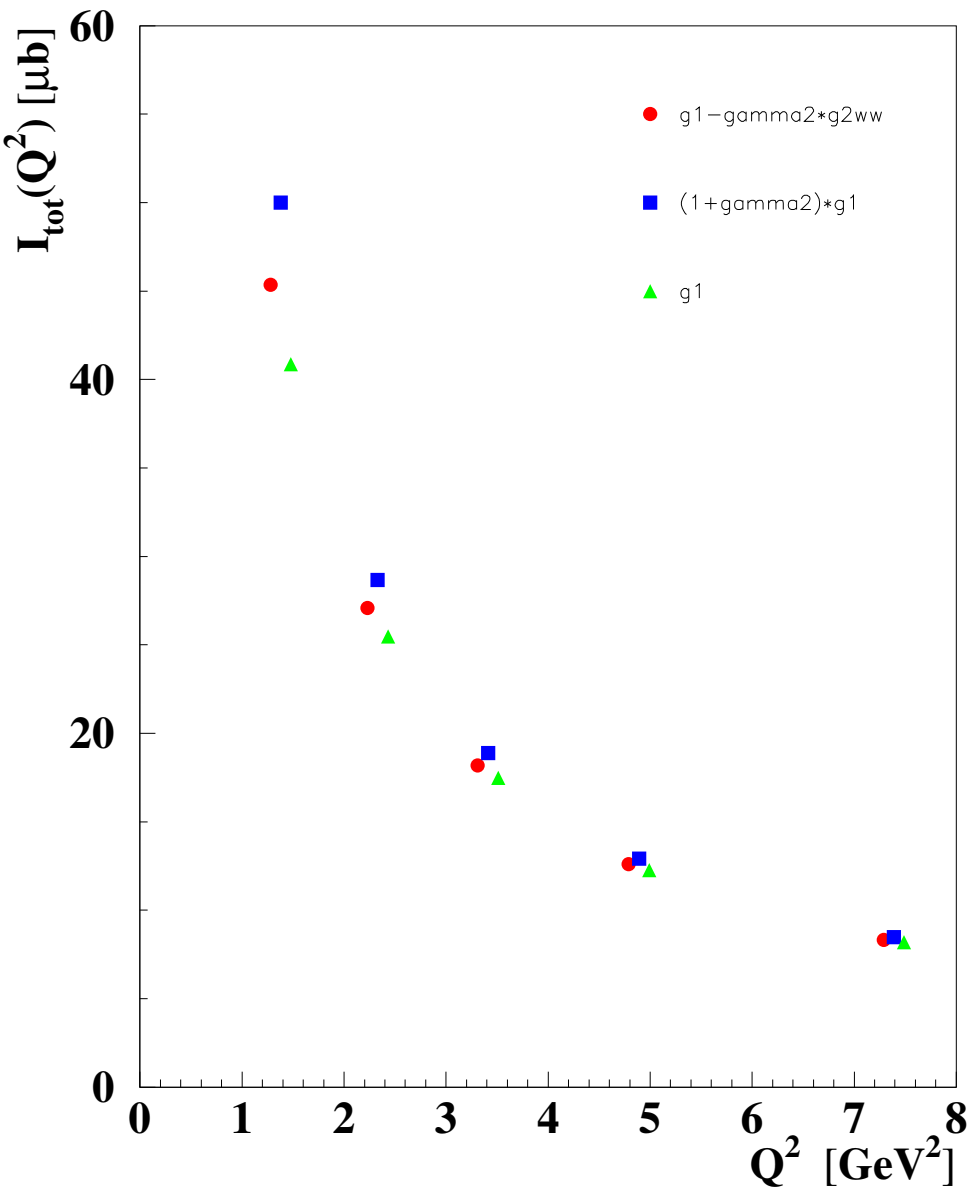


g_1^p from DGLAP NLO solution with G.S. parameterisation

[T. Gehrmann and W.J. Stirling, Phys. Rev. **D 53** (1996) 267, M. Hirai *et al.*, Comp. Phys. Comm. **108** (1998)]

$$I(Q^2) \equiv \int_{Q^2/2M}^{\infty} \left[\sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2)\right] \frac{d\nu}{\nu}$$

$$= \frac{8\pi^2\alpha}{M} \int_0^1 \frac{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)}{K} \frac{dx}{x}$$



The effect of g_2 is decreasing with Q^2 increasing:

20 % effect at $Q^2 = 1.3$ GeV^2

A. Fantoni, Bormio 99

Conclusions

- high energy part is missing in ALL predictions
- high energy part is missing in ALL proposed experiments
- HERMES provides first direct determination of $I^{\mathrm{DIS}}(Q^2)$
- proton : $I^{\mathrm{DIS}}_{\mathrm{HERMES}}(Q^2 = 1.3\ \mathrm{GeV}^2) = 21\ \mu\mathrm{b}$

$$\approx\ 1/10\ I_p^{\mathrm{GDH}}(0)$$

- For Q^2 above 1 GeV^2 the results are at least of the same order of magnitude as expectations for the resonance part of $I(Q^2)$
- Therefore the rapid excursion of $I(Q^2)$ towards the predicted negative values at Q^2=0 must occur below a Q^2 of about 1 GeV^2
- The data are consistent with a QCD next-to-leading order fit based on previous deep inelastic scattering data $\rightarrow$ higher twist effects are not significant even at the lowest measured Q^2