Fit on world data of inclusive proton DIS cross-sections

F_2 analysis

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DIS 2007, München

Features of the new fit

The fit presented has the following features

1. Based on the *ALLM functional form* which allows a very good description of the measured regions. It is a Regge-motivated approach constructed so that photoproduction data at $Q^2 = 0$ can be included.
2. *Includes new data* and covers 2740 data points. This is more than twice as much as used in the ALLM97 fit.
3. *Normalization uncertainties* are considered by an accurate method involving a penalty term in χ^2.
4. *Fit uncertainties* are determined.
   Covariance matrix provided for the first time.
5. *Self-consistent* with respect to the use of $R = \sigma_L/\sigma_T$.

Features of the new fit

Fits for QCD-inspired approach, 15 parameters

- first proposed by BCDMS
- A. Milsztajn et al., Z. Phys. C49 (1991) 527-542
- NMC, Phys. Lett. B364 (1995) 107-115
- SMC, Phys. Rev. D, Vol. 58 (1998) 112001

Fits for Regge-motivated approach, 23 parameters

- ALLM91: DESY-91068
  A parameterization of σ_T above the resonance region.
- ALLM97: hep-ph/9712415
  The ALLM parameterization of σ_{tot}, an update

F_2, R and cross-sections

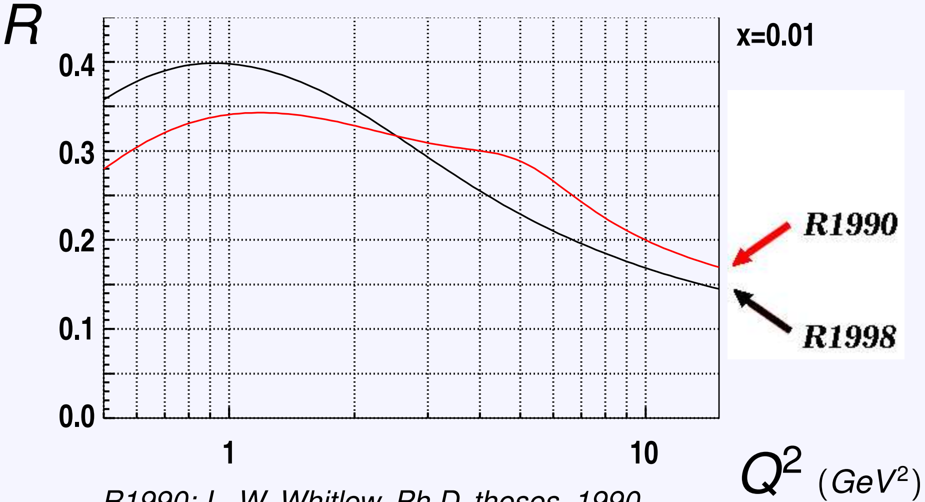
- The DIS cross-section in the 1-photon exchange approximation:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\frac{F_2}{x}\{1 - y - \frac{Q^2}{4E^2} + (1 - \frac{2m^2}{Q^2})\frac{y^2 + Q^2/E^2}{2(1+\mathbf{R})}\}$$

- F_2 can be related to the total cross-section $\sigma_{tot}=\sigma_T + \sigma_L$ by:

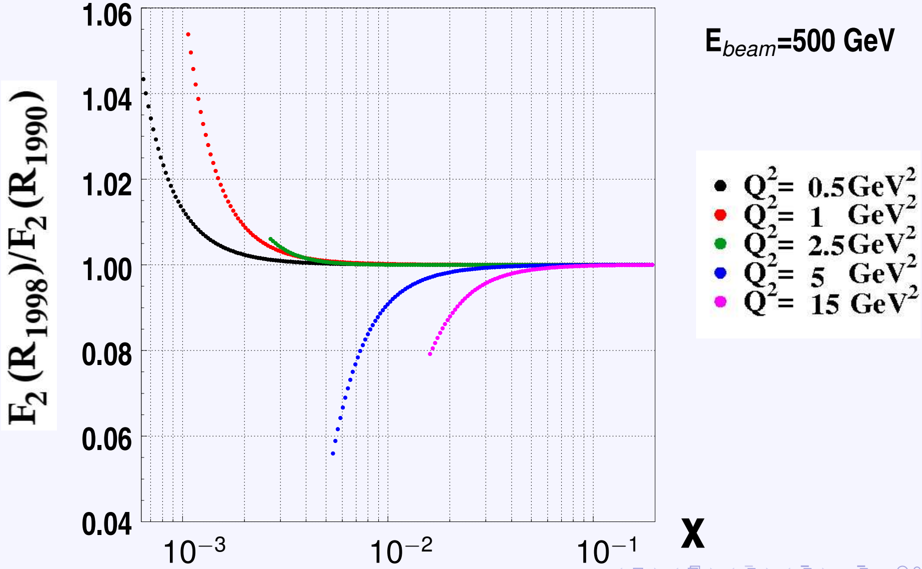
$$\sigma_{tot}(\gamma^* p) = \frac{4\pi\alpha}{Q^2(1-x)}\frac{Q^2 + 4M^2x^2}{Q^2}F_2(W^2, Q^2)$$

- Consistent treatment of **R** for all data sets



R1990: L. W. Whitlow, Ph.D. theses, 1990
R1998: Phys. Lett. B452: 194-200, 1999

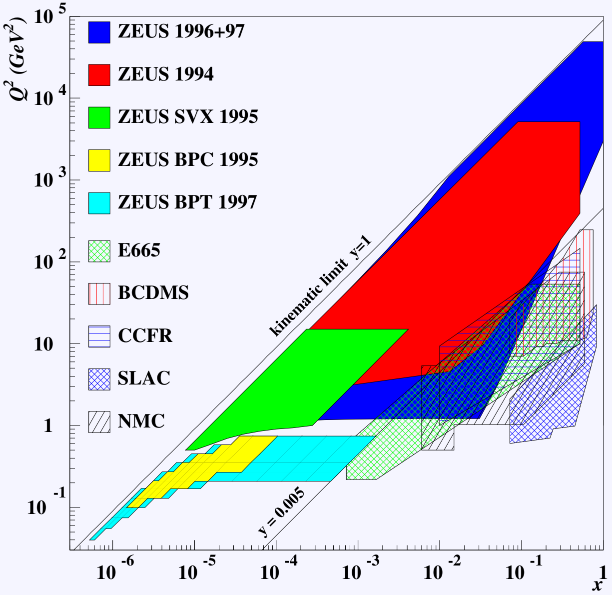
Effect of R on F_2



Experiments: F_2 measurements

Collaboration	Process		
		$E^{e,\mu}_{beam}(GeV)$	$E^{P}_{beam}(GeV)$
H1	$ep \rightarrow eX$	27.5	820, 920
ZEUS	$ep \rightarrow eX$	27.5	820, 920
SLAC	$ep \rightarrow eX$	4.50-20.00	-
NMC	$\mu p \rightarrow \mu X$	90, 120, 200, 280	-
BCDMS	$\mu p \rightarrow \mu X$	100, 120, 200, 280	-
E665	$\mu p \rightarrow \mu X$	470	-

$x - Q^2$ plane coverage



ZEUS, DIS 1999

The ALLM Parameterization

A 23-Parameter Regge-Motivated Approach

Pomeron exchange

$$F_2^{\mathcal{P}}(x, Q^2) = c_{\mathcal{P}}(t) x_{\mathcal{P}}^{a_{\mathcal{P}}(t)} (1 - x)^{b_{\mathcal{P}}(t)}$$

$$F_2(x, Q^2) = \frac{Q^2}{Q^2 + m_0^2} \left(F_2^{\mathcal{P}}(x, Q^2) + F_2^{\mathcal{R}}(x, Q^2) \right)$$

Reggeon exchange

$$F_2^{\mathcal{R}}(x, Q^2) = c_{\mathcal{R}}(t) x_{\mathcal{R}}^{a_{\mathcal{R}}(t)} (1 - x)^{b_{\mathcal{R}}(t)}$$

The functions in red and blue are formed by:

$$f(t) = f_1 + f_2 t^{f_3}$$

$$g(t) = g_1 + (g_1 - g_2)\left[\frac{1}{1 + t^{g_3}} - 1\right]$$

e.g. $a_{\mathcal{R}}(t) = a_{\mathcal{R}1} + a_{\mathcal{R}2} t^{a_{\mathcal{R}3}}$

$$\chi^2 = \sum_i^{n_{max}} \frac{[\sigma_i^{exp} - \sigma_i^{th}/(1 + \nu_{k(i)}\delta_{k(i)}^{norm})]^2}{\delta_{i,sta}^2 + \delta_{i,sys}^2} + \sum_k \nu_k^2.$$

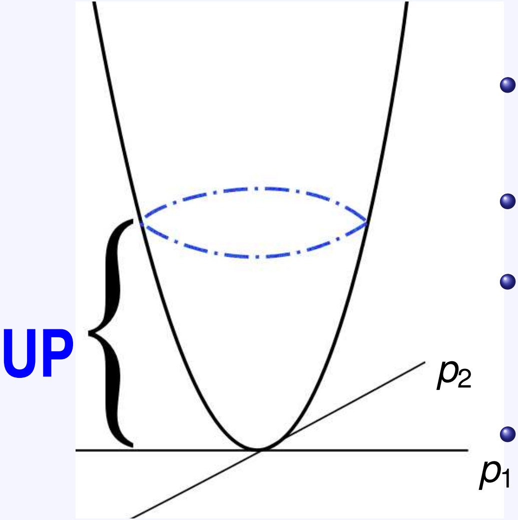
χ^2 -Minimization

- Introduce normalization parameters ν_k for the measurements k implemented by a **penalty term**.
- The normalization parameters ν_k perform a **normalization** according to normalization error $\delta_{k(i)}^{norm}$.
- The analytic solution of ν_k for a fixed set of model parameters can be obtained from $d\chi^2/d\nu_k = 0$, since ν_k are independent.

Error Propagation

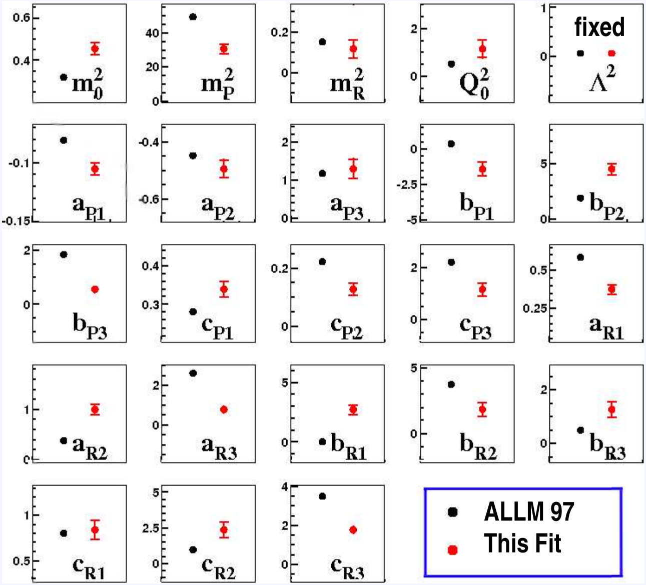
$$V(\sigma_{tot}(\mathbf{p}, x, Q^2)) = \Sigma_{i,j} cov^p_{i,j} \frac{d\sigma_{tot}(\mathbf{p}, x, Q^2)}{dp_i} \frac{d\sigma_{tot}(\mathbf{p}, x, Q^2)}{dp_j}$$

p : parameter vector
$cov^p_{i,j}$: covariance matrix for **p**
V : variance



- The *UP value* is the parameter in MINUIT by which uncertainties for parameters are defined.
- UP=1 defines a 1σ uncertainty for single parameters.
- UP$\approx$ npar corresponds to the 1σ uncertainty for the npar parameters to be simultaneously located inside the hypercontour.
- UP large: compensation for unknown systematic effects.

Model Parameters



χ^2 Contributions

Nr.	Exp	n	χ^2/n
1.	SLAC-E49a	98	0.48
2.	SLAC-E49b	187	1.15
3.	SLAC-E61	25	0.20
4.	SLAC-E87	94	0.68
5.	SLAC-E89a	72	1.11
6.	SLAC-E89b	98	0.99
7.	NMC 90 GeV	73	0.79
8.	NMC 120 GeV	65	1.50
9.	NMC 200 GeV	75	1.13
10.	NMC 280 GeV	79	0.94
11.	E665	91	1.06
12.	BCDMS 100	58	1.13
13.	BCDMS 120	62	0.73
14.	BCDMS 200	57	1.32
15.	BCDMS 280	52	1.11

16.	H1 lq 94a	37	0.35
17.	H1 lq 94b	156	0.62
18.	H1 SVX	44	0.48
19.	ZEUS 94	188	1.15
20.	ZEUS BPC	34	0.40
21.	ZEUS SVX	36	0.76
22.	ZEUS γp	1	2.44
23.	PDG γp	196	0.79
24.	ZEUS 96,97	242	0.75
25.	ZEUS 97	70	0.94
26.	H1 99,00	147	1.01
27.	H1 98,99	126	1.37
28.	H1 94,97	130	0.79
29.	H1 lq 96,97a	67	1.07
30.	H1 lq 96,97b	80	0.82
	total	**2740**	**0.94**

Normalization Parameters

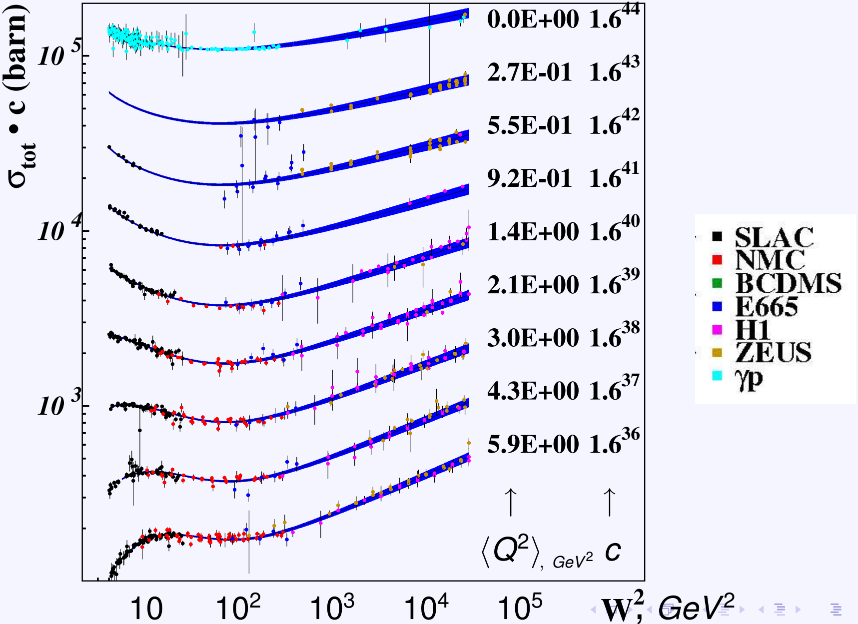
Nr.	Exp	δ_k^{nor}	ν_k
1.	SLAC-E49a	2.1	0.06
2.	SLAC-E49b	2.1	-0.28
3.	SLAC-E61	2.1	0.01
4.	SLAC-E87	2.1	0.07
5.	SLAC-E89a	2.1	1.31
6.	SLAC-E89b	2.1	0.17
7.	NMC 90 GeV	2.0	-0.37
8.	NMC 120 GeV	2.0	0.14
9.	NMC 200 GeV	2.0	-0.09
10.	NMC 280 GeV	2.0	-0.24
11.	E665	1.8	0.67
12.	BCDMS 100	3.0	-1.20
13.	BCDMS 120	3.0	0.03
14.	BCDMS 200	3.0	-1.09

Nr.	Experim.	δ_k^{nor}	ν_k
15.	BCDMS 280	3.0	-1.03
16.	H1 lq 94a	3.9	0.05
17.	H1 lq 94b	1.5	1.13
18.	H1 SVX	3.0	-3.02
19.	ZEUS 94	2.0	1.66
20.	ZEUS BPC	2.4	-1.28
21.	ZEUS SVX	3.0	-1.00
24.	ZEUS 9697	2.0	0.09
25.	ZEUS 97	2.0	-2.23
26.	H1 9900	1.5	-1.08
27.	H1 9899	1.8	-1.38
28.	H1 9497	1.5	-1.46
29.	H1 lq 9697a	1.7	1.77
30.	H1 lq 9697b	1.7	2.02

δ_k^{nor} : *normalization uncertainty in %* ν_k: *normaliz. parameter*

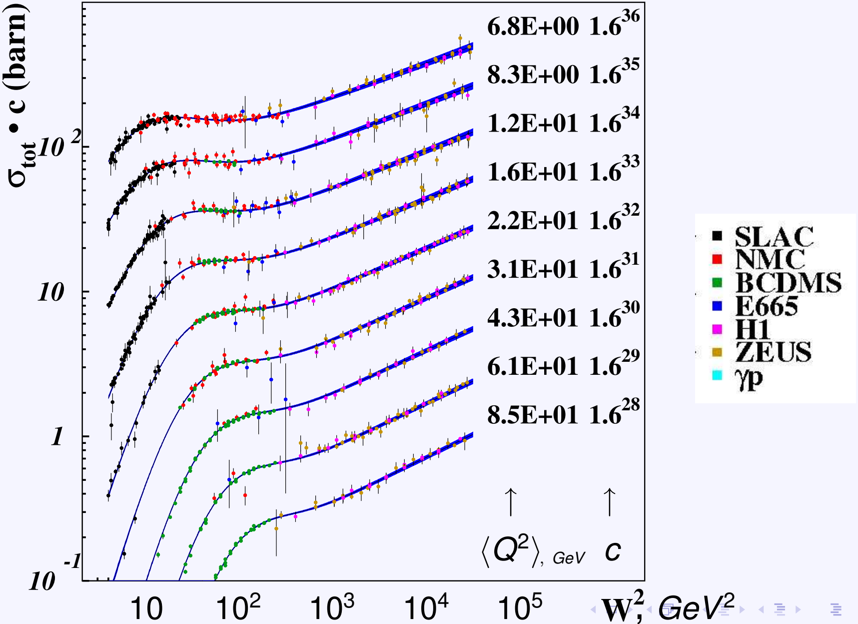
Fit Result and Data

$0\ GeV^2 < Q^2 < 6\ GeV^2$



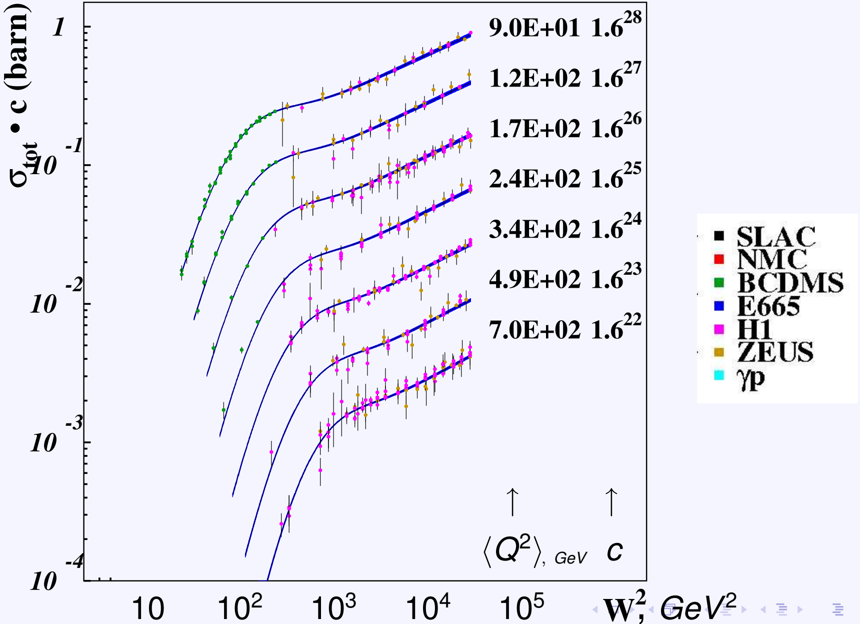
Fit Result and Data

$6\ GeV^2 < Q^2 < 90\ GeV^2$



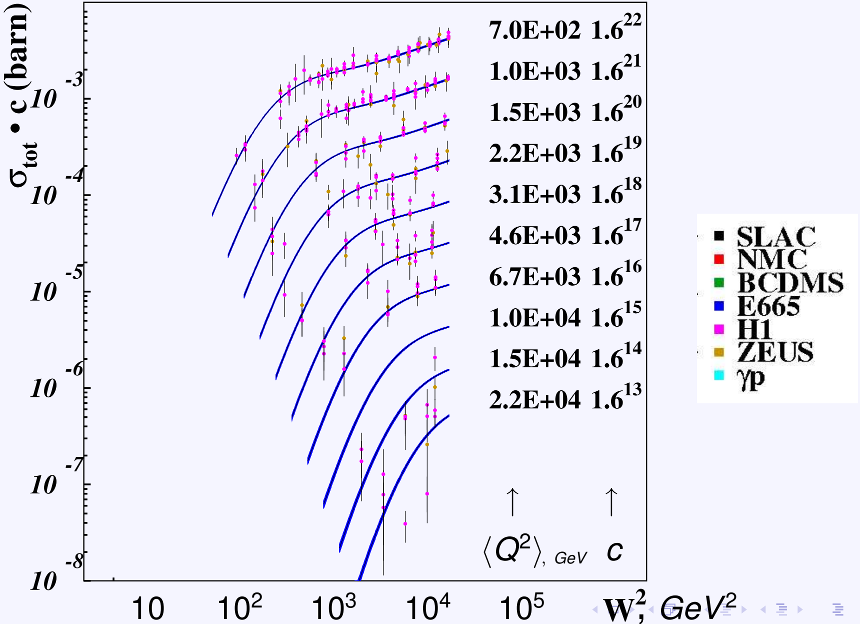
Fit Result and Data

$90\ GeV^2 < Q^2 < 700\ GeV^2$



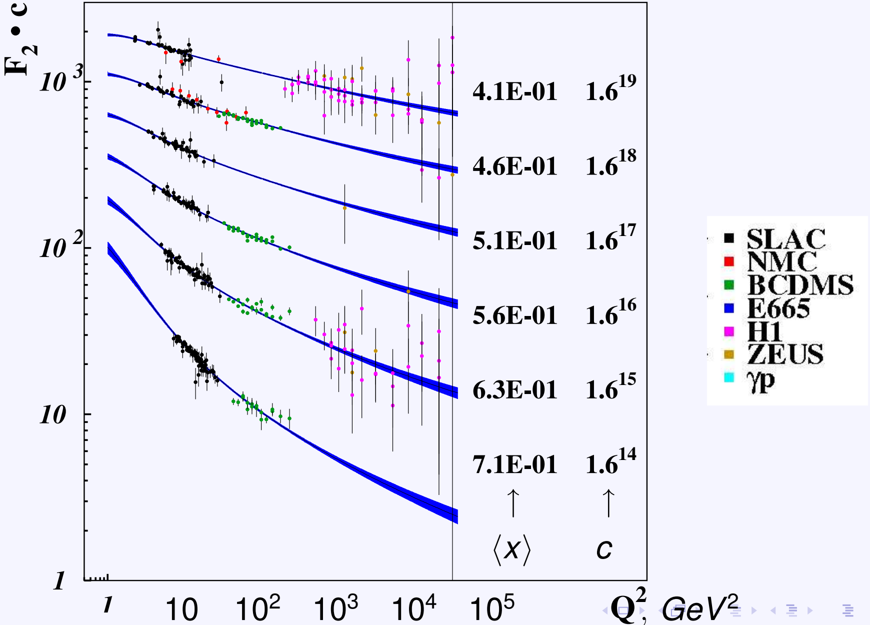
Fit Result and Data

$700\ GeV^2 < Q^2 < 30000\ GeV^2$



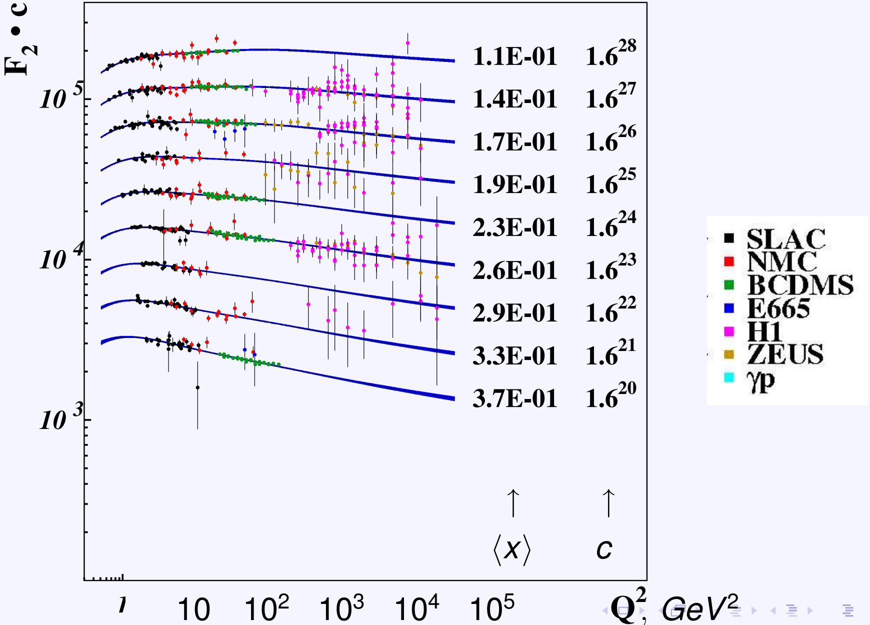
Fit Result and Data

$x_{bj} > 0.4\ GeV^2$



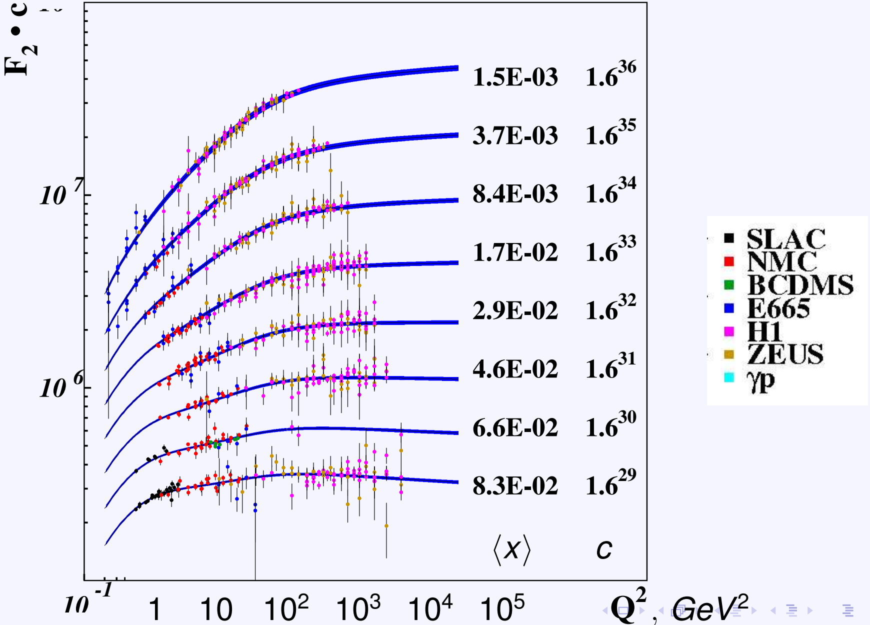
Fit Result and Data

$0.10 < x_{bj} < 0.40\ GeV^2$



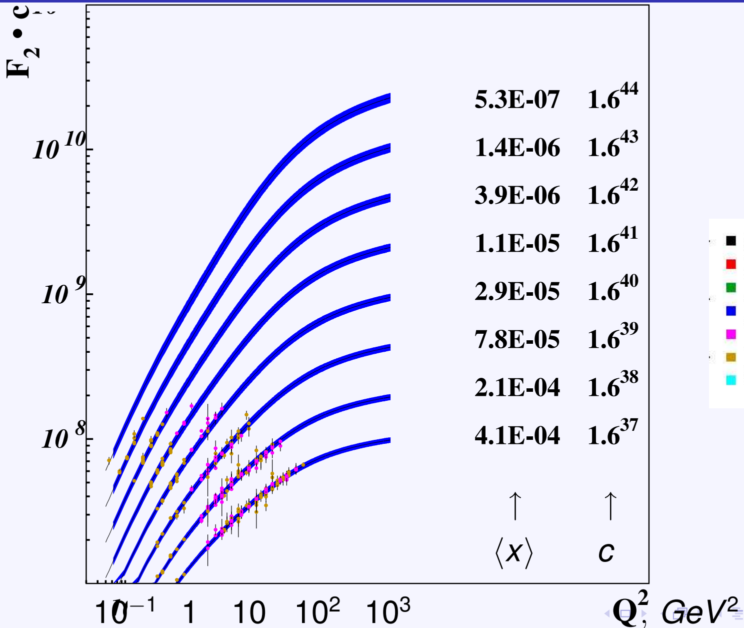
Fit Result and Data

$0.001 < x_{bj} < 0.1\ GeV^2$



Fit Result and Data

$x_{bj} < 0.001\ GeV^2$

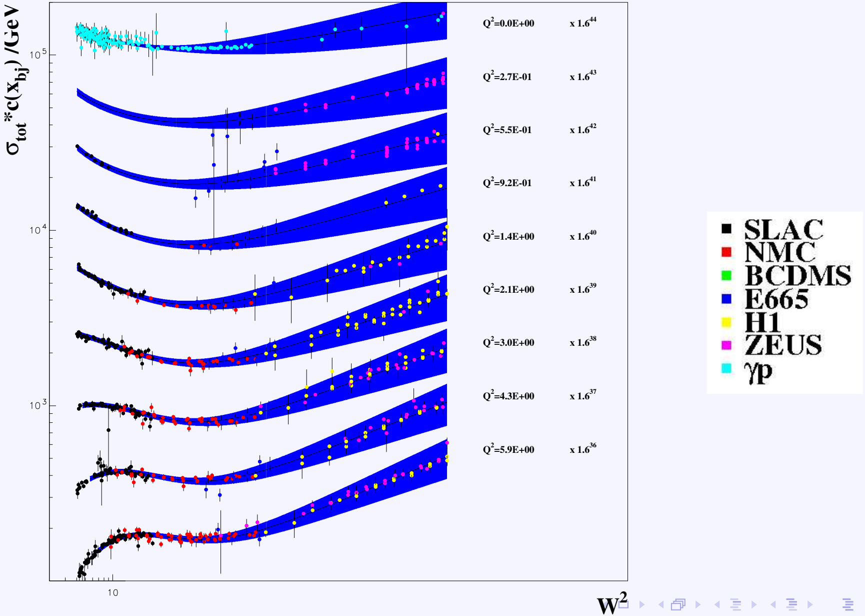


Summary

- Motivation: New fit on world data
- It reflects the most recent world knowledge on proton DIS cross sections.
- Useful for all extractions requiring F_2 as input.
- Relevant for MC at low Q^2.

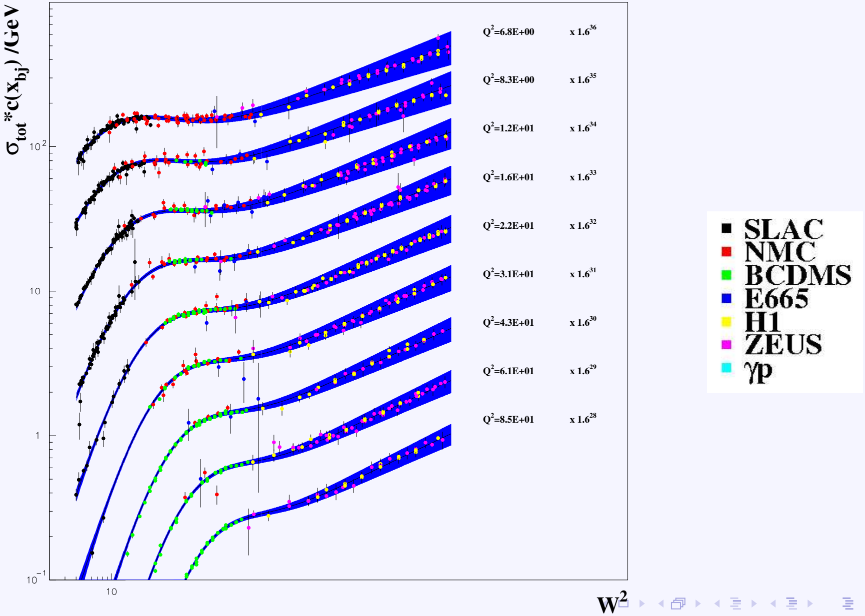
Plots

Data and Fit Result (UP=22), region 1



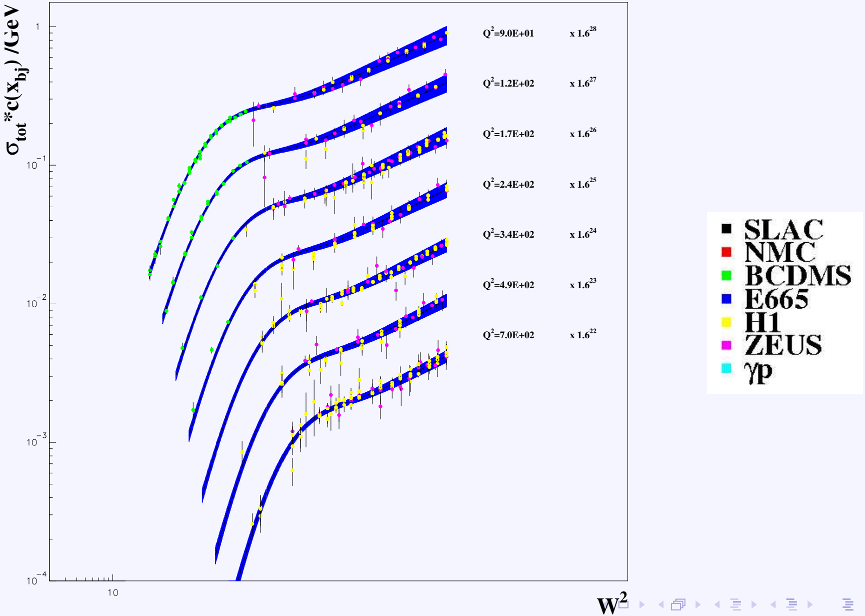
Plots

Data and Fit Result (UP=22), region 2



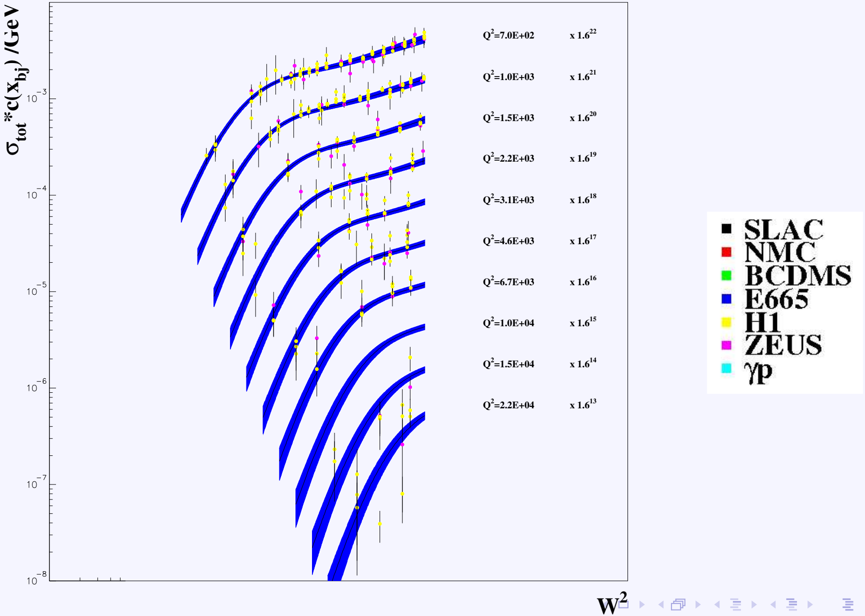
Plots

Data and Fit Result (UP=22), region 3



Plots

Data and Fit Result (UP=22), region 4



ALLM Parameterization

$$t = ln(\frac{ln(Q^2 + Q_0^2)/\Lambda^2}{ln(Q_0^2)/\Lambda^2})$$

$$\frac{1}{x_{\mathcal{P}}} = 1 + \frac{W^2 - M^2}{Q^2 + m_{\mathcal{P}}^2}$$

$$\frac{1}{x_{\mathcal{R}}} = 1 + \frac{W^2 - M^2}{Q^2 + m_{\mathcal{R}}^2}$$

Pname	ALLM91	ALLM97	this fit	single 1σ	simult.
m_0^2	0.3051	0.3198	0.45415$\pm$	0.02836	0.14261
$m_{\mathcal{P}}^2$	10.6760	49.4570	30.70720$\pm$	2.85200	4.38179
$m_{\mathcal{R}}^2$	0.2062	0.1505	0.11700$\pm$	0.04617	0.22918
Q_0^2	0.2779	0.5254	1.15117$\pm$	0.35841	1.91861
Λ^2	0.0653	0.0653	0.06527$\pm$	0.00000	0.00000
$a_{\mathcal{P}1}$	-0.0450	-0.0808	-0.10539$\pm$	0.00510	0.02573
$a_{\mathcal{P}2}$	-0.3641	-0.3641	-0.49519$\pm$	0.03068	0.16323
$a_{\mathcal{P}3}$	8.1709	1.1709	1.29187$\pm$	0.24412	1.29744
$b_{\mathcal{P}1}$	0.4922	0.3629	-1.41348$\pm$	0.47875	2.42369
$b_{\mathcal{P}2}$	0.5212	1.8917	4.50086$\pm$	0.51887	2.72348
$b_{\mathcal{P}3}$	3.5515	1.8439	0.55277$\pm$	0.11590	0.61635

Pname	ALLM91	ALLM97	this fit	single 1σ	simult.
$c_{\mathcal{P}1}$	0.2655	0.2806	0.33875 $\pm$	0.01941	0.09709
$c_{\mathcal{P}2}$	0.0486	0.2229	0.12725 $\pm$	0.02162	0.10828
$c_{\mathcal{P}3}$	1.0468	2.1980	1.15659 $\pm$	0.24584	1.26969
$a_{\mathcal{R}1}$	0.6041	0.5840	0.37495 $\pm$	0.03218	0.16062
$a_{\mathcal{R}2}$	0.1735	0.3788	0.99717 $\pm$	0.10170	0.53481
$a_{\mathcal{R}3}$	1.6181	2.6063	0.77456 $\pm$	0.11287	0.56784
$b_{\mathcal{R}1}$	1.2607	0.0114	2.71772 $\pm$	0.39336	1.93456
$b_{\mathcal{R}2}$	1.8362	0.3788	1.82806 $\pm$	0.53495	2.71407
$b_{\mathcal{R}3}$	0.8114	2.6063	1.26287 $\pm$	0.29756	1.48158
$c_{\mathcal{R}1}$	0.6764	0.8011	0.83915 $\pm$	0.10568	0.52170
$c_{\mathcal{R}2}$	0.4903	0.9730	2.35826 $\pm$	0.55228	2.93553
$c_{\mathcal{R}3}$	2.6628	3.4942	1.77238 $\pm$	0.20843	1.09286