Q^2–Dependence of the generalized GDH Integral for the Proton

Björn Seitz
Center for Subatomic Research
Dept. of Physics, University of Alberta, Edmonton

for

The HERMES collaboration

Contributed talk at CIPANP 2000, Quebec City, Canada

- Motivation
- Experimental setup and data analysis
- Results in the DIS and Resonance region
- Summary and Conclusion

Derivation of the GDH sum rule

Compton scattering amplitude in forward direction ($Q^2 = 0$):

$$T(\nu, \theta = 0) = \chi_f^\dagger [\underbrace{(\epsilon_{\mathrm{f}}^* \cdot \epsilon_{\mathrm{i}}) \cdot f(\nu)}_{\text{spin independent}} + \overbrace{\imath\sigma(\epsilon_{\mathrm{f}}^* \times \epsilon_{\mathrm{i}}) \cdot g(\nu)}^{\text{spin dependent}}]\chi_i$$

Low theorem (Lorentz and gauge invariance)

$$f(\nu) = -\frac{\alpha}{M} + (\alpha_N + \beta_N)\nu^2 + \mathcal{O}(\nu^4) \qquad (1)$$

$$g(\nu) = -\frac{\alpha\kappa_N^2}{2M^2}\nu + \gamma_0\nu^3 + \mathcal{O}(\nu^5) \qquad (2)$$

$$T_{3/2}(\nu) = f(\nu) - g(\nu) \qquad T_{1/2}(\nu) = f(\nu) + g(\nu)$$

Optical Theorem (Conservation of Probability)

$$Im[T_{3/2,1/2}(\nu)] = \frac{\nu}{4\pi}\sigma_{3/2,1/2}(\nu)$$

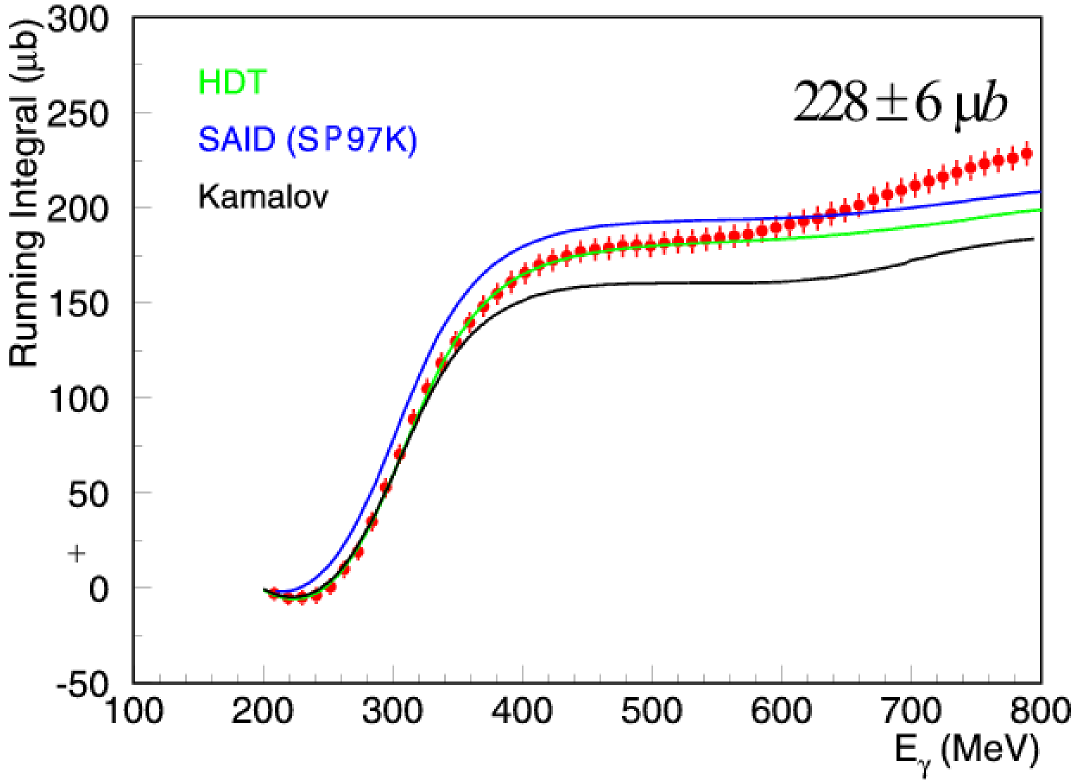
Unsubtracted Dispersion relation (Causality)

$$Re[g(\nu)] = \frac{2\nu}{\pi}\int_0^\infty \frac{\nu' d\nu'}{4\pi(\nu'^2 - \nu^2)} \frac{\sigma_{1/2} - \sigma_{3/2}}{2}$$

$$\boxed{\Rightarrow I = -\frac{2\pi^2\alpha\kappa_N^2}{M^2} = \int_0^\infty \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu} d\nu}$$

Measuring the GDH sum rule

GDH @ MAMI (preliminary) 0.2 GeV $\leq \nu \leq 0.8$ GeV



At the photon point:

Sum rule prediction: $I_p(Q^2 = 0) = -204\mu$b

with extrapolations (from SAID and Bianchi/Thomas):

$\Rightarrow$ close to GDH sum rule prediction

Extension of energy range needed:

$\Rightarrow$ ELSA, JLab, GRAAL, SPring8, (ELFE ?)

Generalisation of the GDH sum

Connect the photon point to DIS region

$$I_{GDH}(Q^2) = \int_{\nu_0}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu} d\nu \qquad \gamma^2 = \frac{Q^2}{\nu^2}$$

$$= \frac{8\pi^2\alpha}{M} \int_0^{x_0} \frac{g_1(x,Q^2) - \gamma^2 g_2(x,Q^2)}{\nu\sqrt{1+\gamma^2}} \frac{dx}{x}$$

Real photon limit predicted by GDH

$$I(Q^2=0) = -\frac{2\pi^2\alpha}{M^2}\kappa^2 = -204\mu\text{b}$$

High energy limit ($\gamma \ll 1$)

$$I(Q^2) = \frac{16\pi^2\alpha}{Q^2}\int_0^1 g_1(x)dx = \frac{16\pi^2\alpha}{Q^2}\Gamma_1$$

Measured (E 143): $\Gamma_1(5\ \text{GeV}^2) = 0.129 \pm 0.010$

$$I(5\ \text{GeV}^2) = 11.6 \pm 0.9\mu\text{b}$$

Explore transition region between pQCD and nucleon resonance excitations

Data Analysis

- $1.2\ \text{GeV}^2 \le Q^2 \le 12\ \text{GeV}^2$ in 6 bins
- $1.0\ \text{GeV}^2 \le W^2 \le 4.2\ \text{GeV}^2$ (resonance region) and $4.2\ \text{GeV}^2 \le W^2 \le 45\ \text{GeV}^2$ (DIS region)

$$\begin{aligned} I_{GDH}(Q^2) &= \int_{\nu_0}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu} d\nu \\ &= \frac{8\pi^2\alpha}{M} \int_0^{x_0} \frac{A_1 F_1}{\nu\sqrt{1+\gamma^2}} \frac{dx}{x} \end{aligned}$$

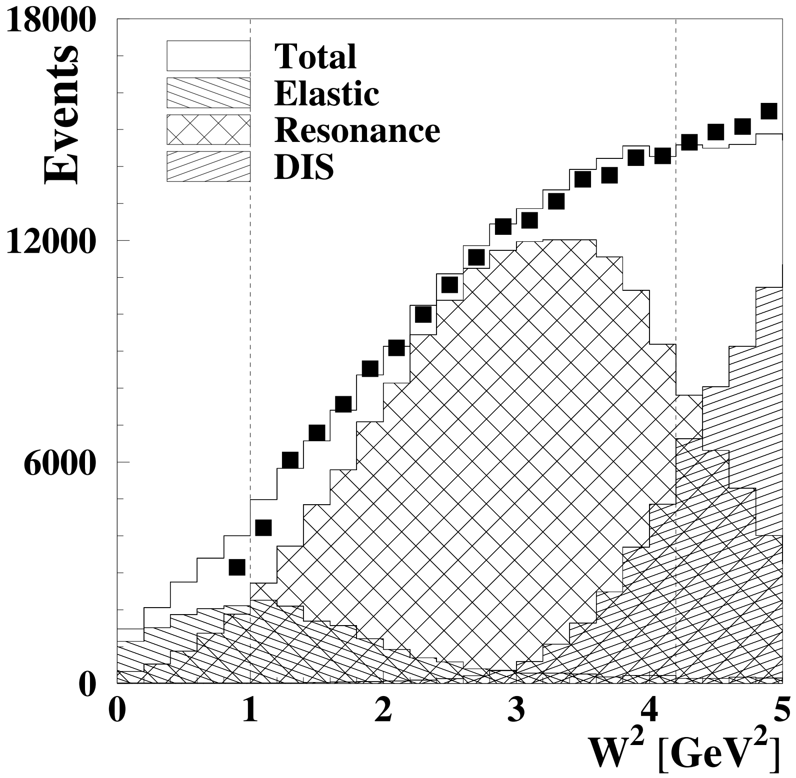
$$A_1 = \frac{A_\parallel}{D} - \eta A_2 \qquad\qquad A_\parallel = \frac{N^- L^+ - N^+ L^-}{N^- L_p^+ + N^+ L_p^-}$$

Systematical uncertainties

- Beam and target polarization: 5.3%
- Smearing effects: < 5.5 %
- Uncertainty in F_2: 2%
- for resonance region: assume $A_2 = 0$

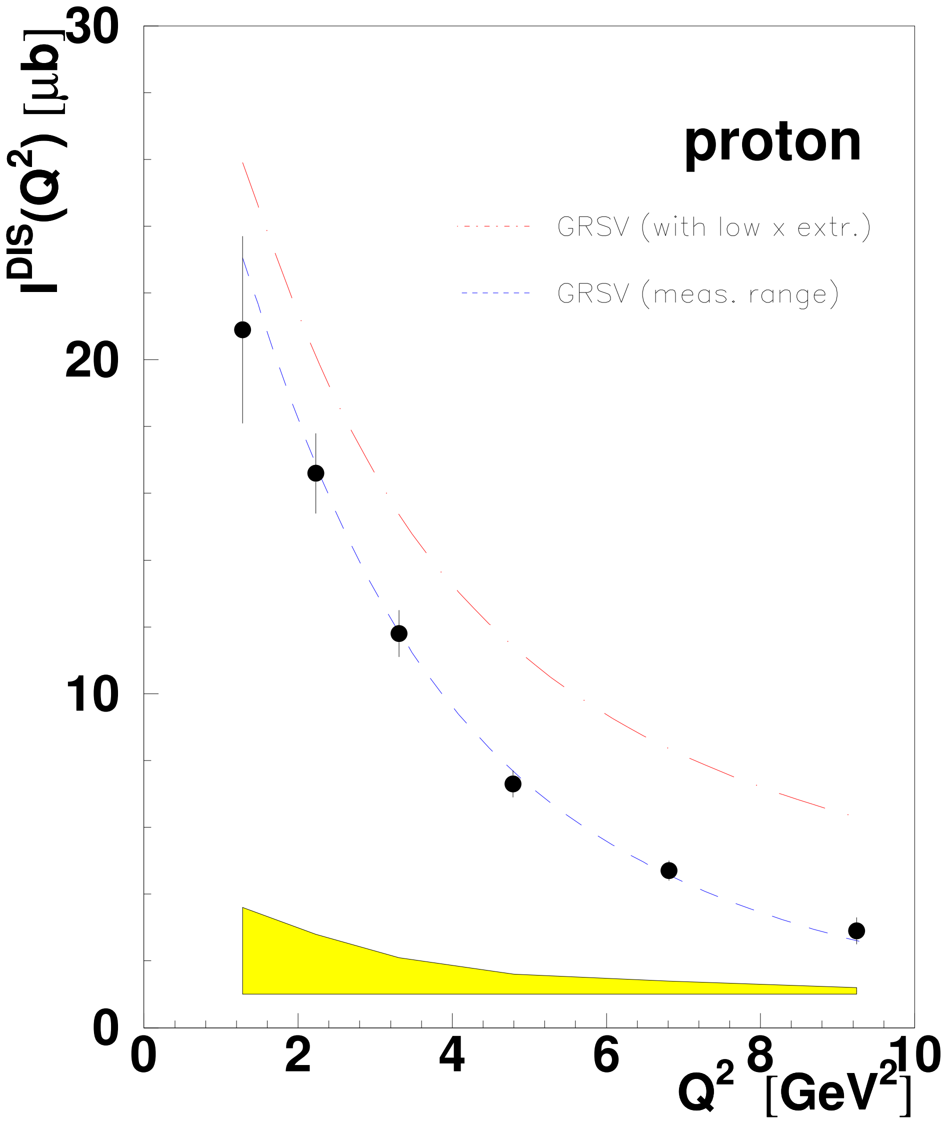
$\Rightarrow$ Ignorance of A_2: Dominant uncertainty in the resonance region up to 17%

Monte Carlo studies in the resonance region



- Parametrisation of F_2 taken from A. Bodek et al., Phys. Rev. **D 20** (1979) 1471; R=0.18
- Smearing effects found to be small
- Good agreement between data and Monte Carlo
- Energy resolution $\approx 300\text{MeV} \Rightarrow$ No separation of resonances
- Contamination to the resonance region from elastic and DIS regions amount to 8% and 9% resp.

Generalized GDH integral in the DIS region

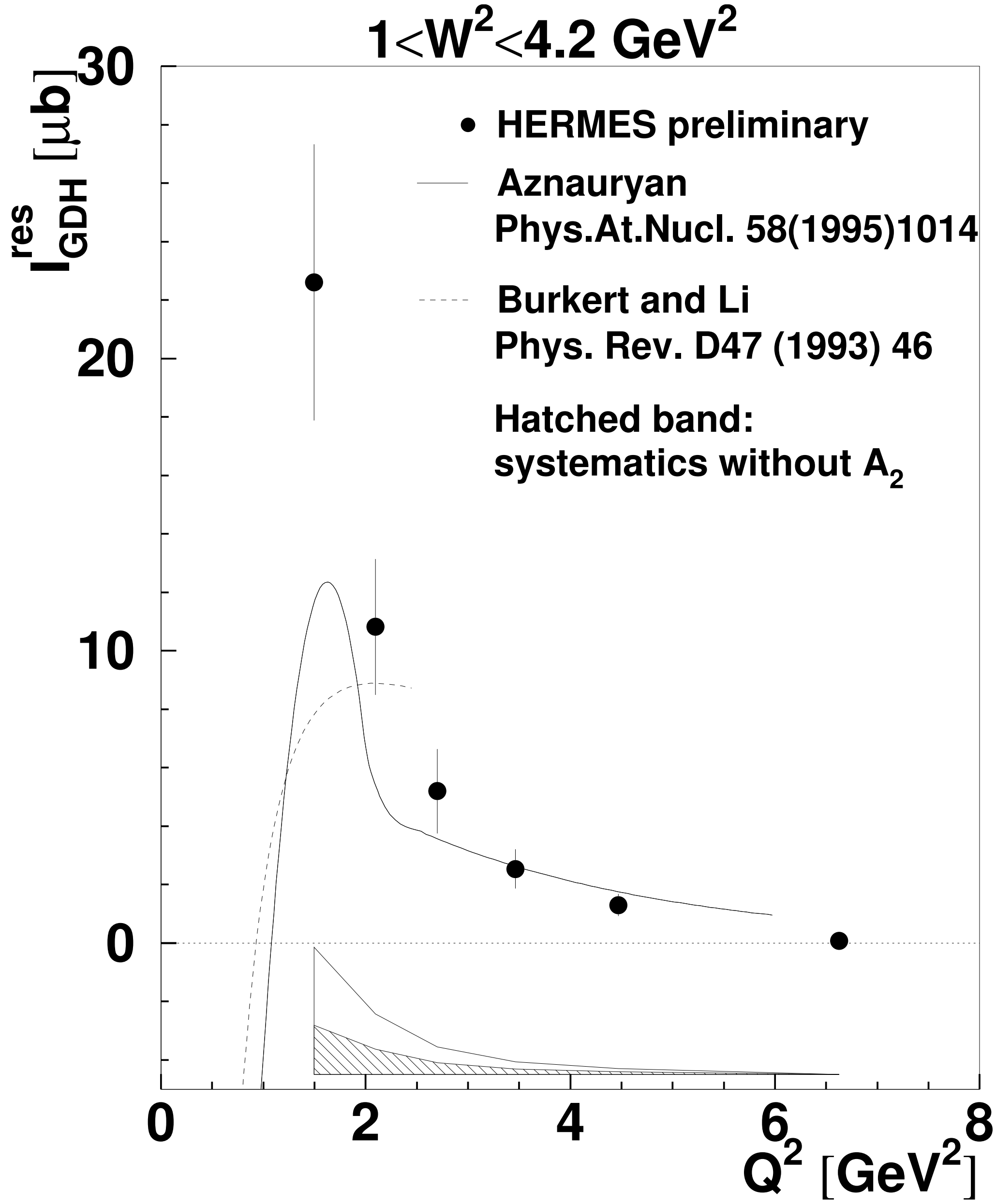


$$I(Q^2) = \int_{\nu_0}^{23.5} \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu} d\nu \qquad I(Q^2) = \int_{\nu_0}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu} d\nu$$

NLO fit: M. Glück et al. Phys. Rev **D 53** (1996) 6620)

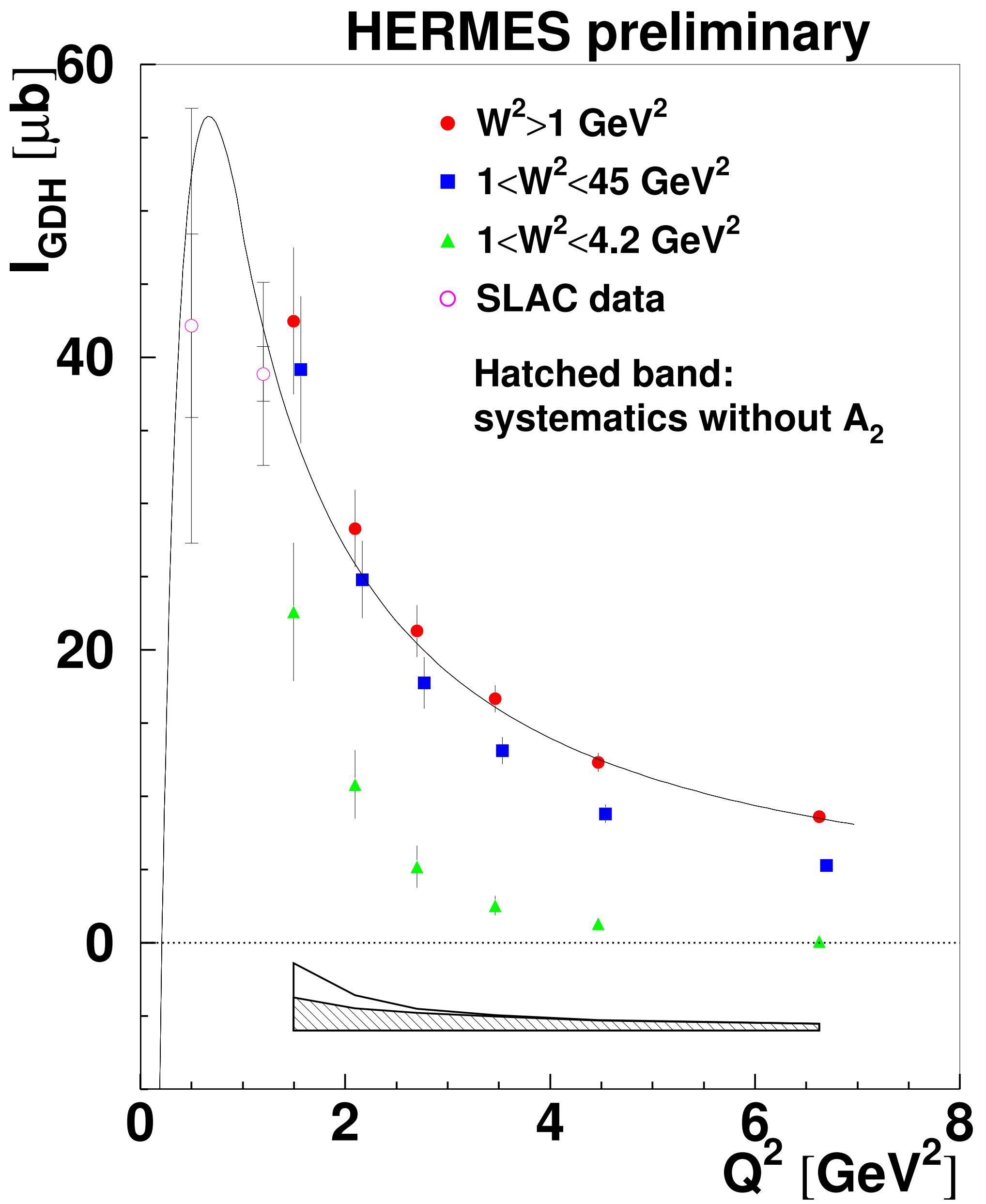
data:K. Ackerstaff et al. Phys. Lett. **B 444** (1998) 531

Generalized GDH integral in the resonance region



$$I^{res}_{meas}(Q^2) = \int_{\nu_0}^{\nu_{max}} \frac{\sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2)}{\nu} d\nu$$

Combined result



- **Solid line:** J. Soffer, O. V. Teryaev: Phys. Rev. **D 51** (1995) 25)
- Extrapolation for $W^2 > 45$ GeV^2: 3.5 μb (following: N. Bianchi, E. Thomas, Phys. Lett. **B 450** (1999) 439)

Summary & Conclusion

- The generalized GDH sum connects the real photon point and the DIS region
- Q^2 evolution shows the influence of resonance form factors and higher twist as a deviation from Q^{-2} behaviour
- HERMES provides consistent data sample both in the DIS and the resonance region

- Existing models in the measured Q^2 range do not describe the data in the resonance region
- DIS contribution of the same order as the resonance contribution $\Rightarrow$ can not be neglected
- No sign of a turnover in HERMES Q^2 range
- No significant contribution of higher twist terms observed

A lot of work has been done, what is left ?

- Data on the neutron $\rightarrow$ HERMES results to come
- Extended energy range at the photon point
- Measurements at $Q^2 < 1$ GeV^2 $\Rightarrow$ JLab