



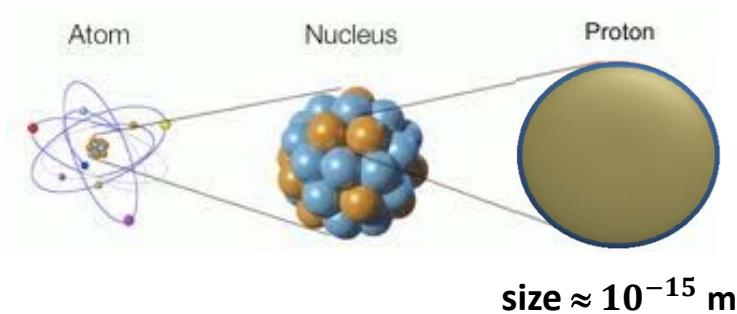
# Transverse Momentum Distributions: an experimental update

Luciano L. Pappalardo

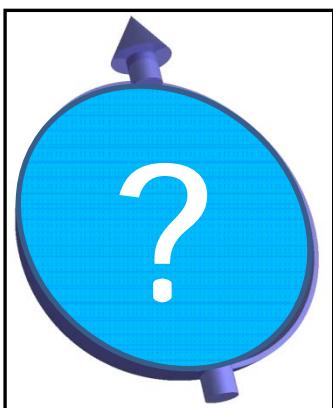
INFN & University of Ferrara

QNP2012 - Palaiseau (France) - April 16-20 2012

# Looking deeply into the proton

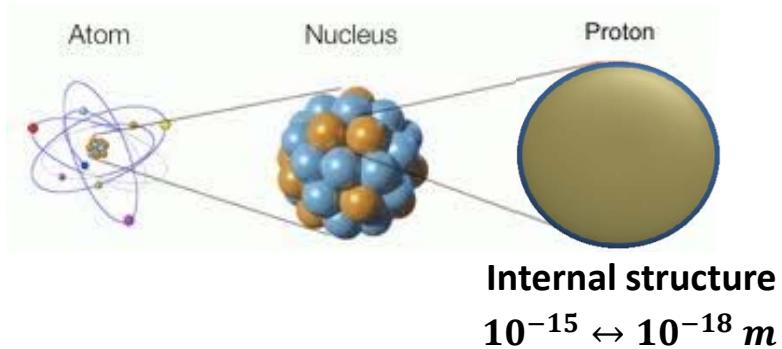


< 1960

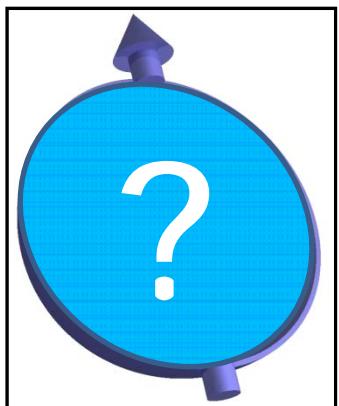


- Elementary particle?

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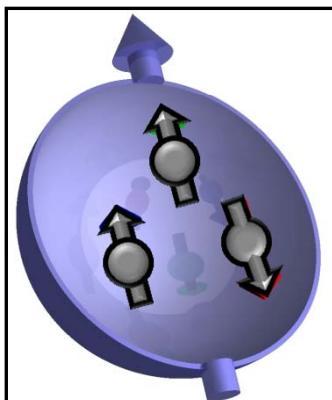


< 1960



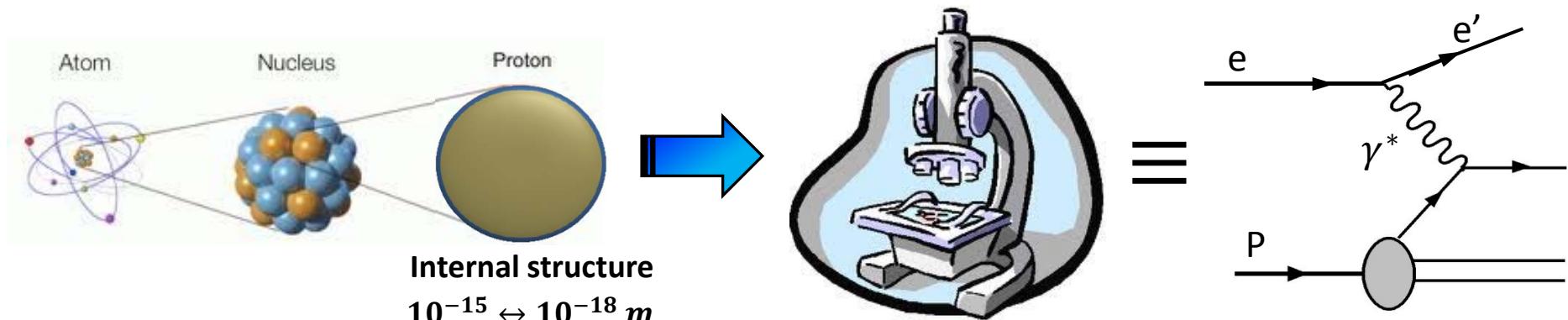
- Elementary particle?

1964

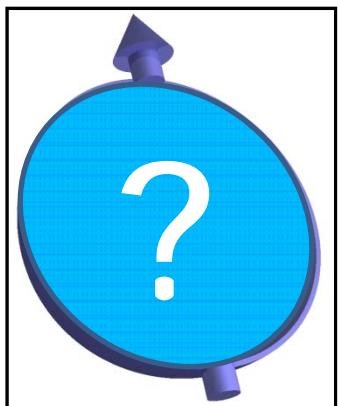


- Quark hypothesis  
(Gell-Mann - Zweig)

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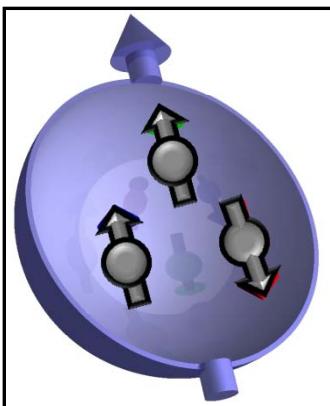


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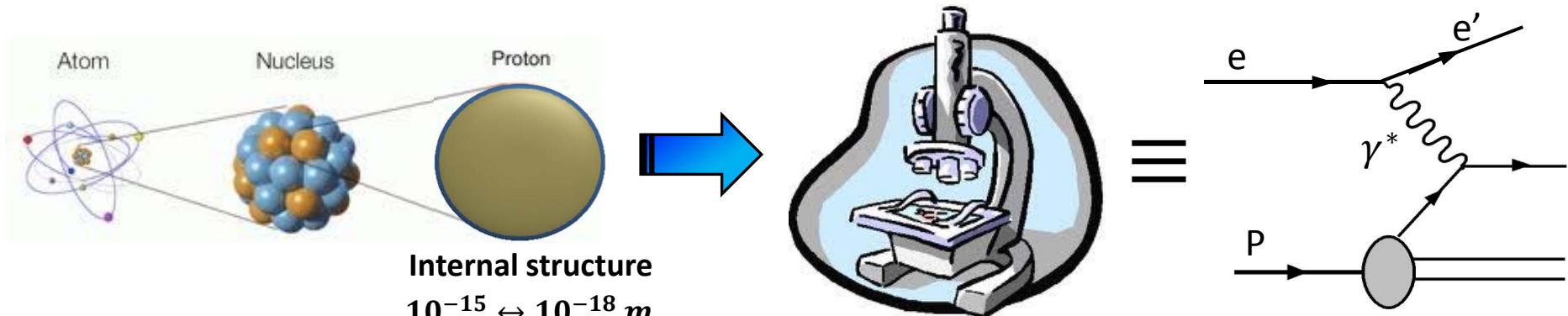
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1964-1969

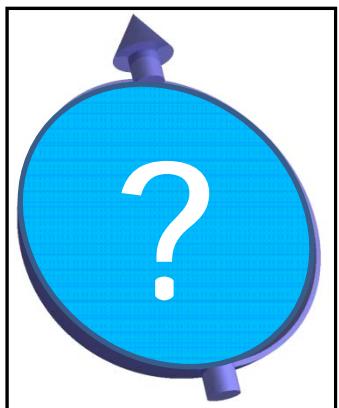


- Quark hypothesis  
(Gell-Mann - Zweig)
- Scaling at SLAC ('69)
- Parton Model  
(Faynman, Bjorken)

# Looking deeply into the proton

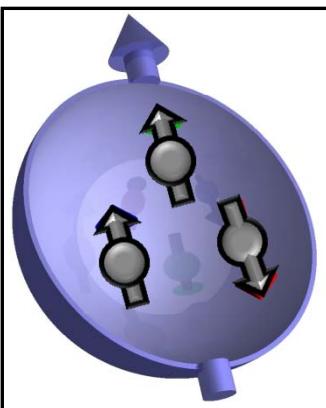


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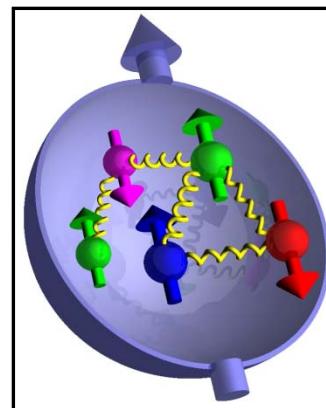
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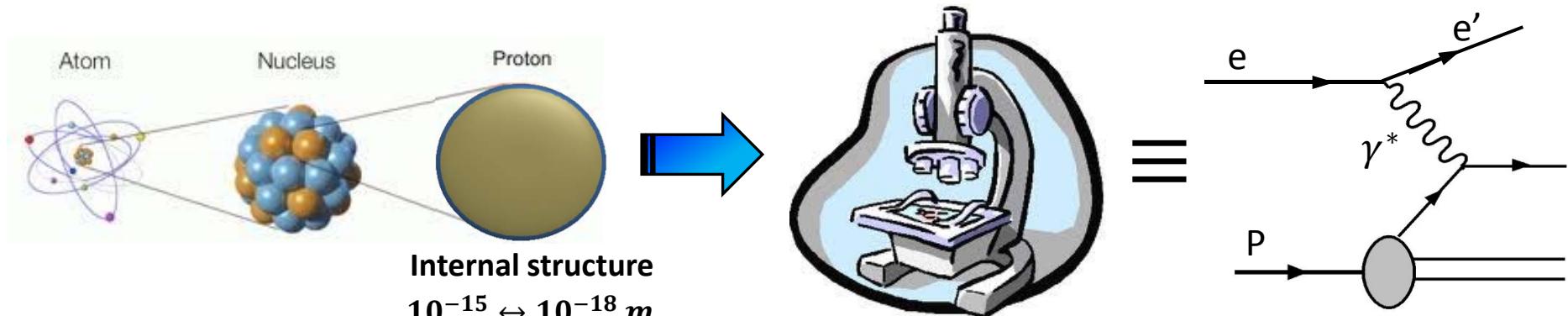
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1972-1973

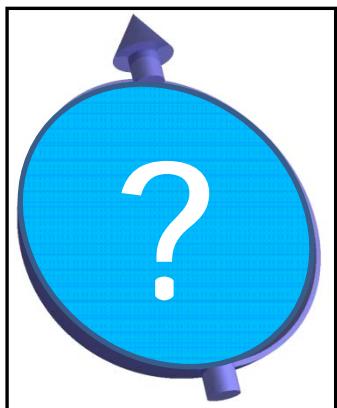


- QCD lagrangian  
- colors, sea quarks,  
gluons  
- discovery of gluons  
(PETRA '73)

# Looking deeply into the proton

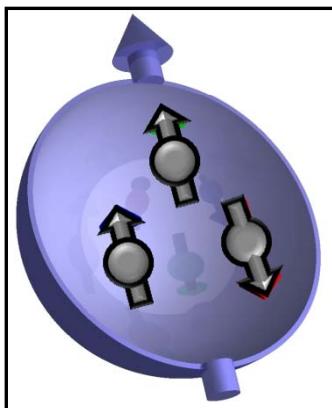


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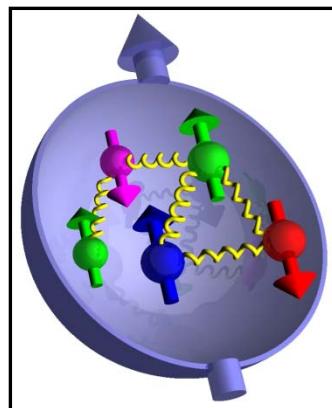
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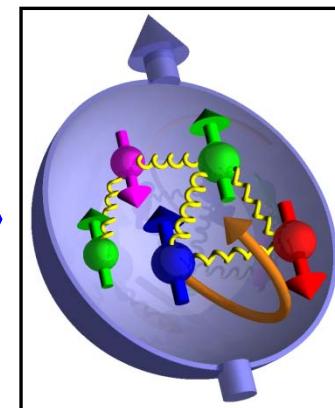
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1972-1973



- QCD lagrangian
- **colors**, sea quarks, gluons
- **discovery of gluons**  
(PETRA '73)

> 1988



- EMC experiment
- the **spin crisis**
- $\frac{1}{2} = \Delta\Sigma + \Delta G + L_{q,g}$
- quest for  $\Delta G$  &  $L_{q,g}$

# What is the final goal?

Understand the **full phase-space distribution of the partons**:

- Where are they located ?  $\rightarrow x, y, z \equiv r$
  - How do they move ?  $\rightarrow p_x, p_y, p_z \equiv x, p_T$
- Wigner function**

...but  $\Delta x \Delta p \geq \frac{\hbar}{2}$   $\rightarrow$  no simultaneous knowledge of momentum and position!

$$W(x, p_T, r)$$



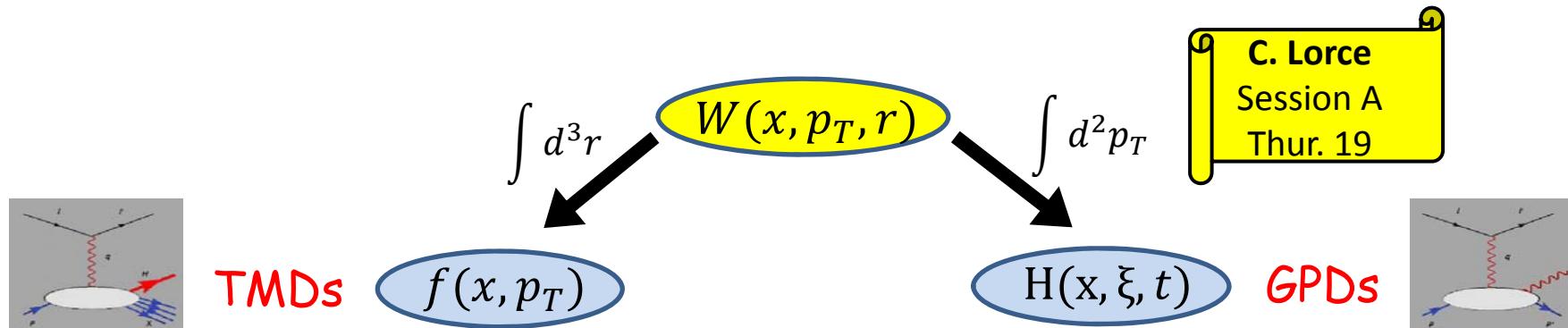
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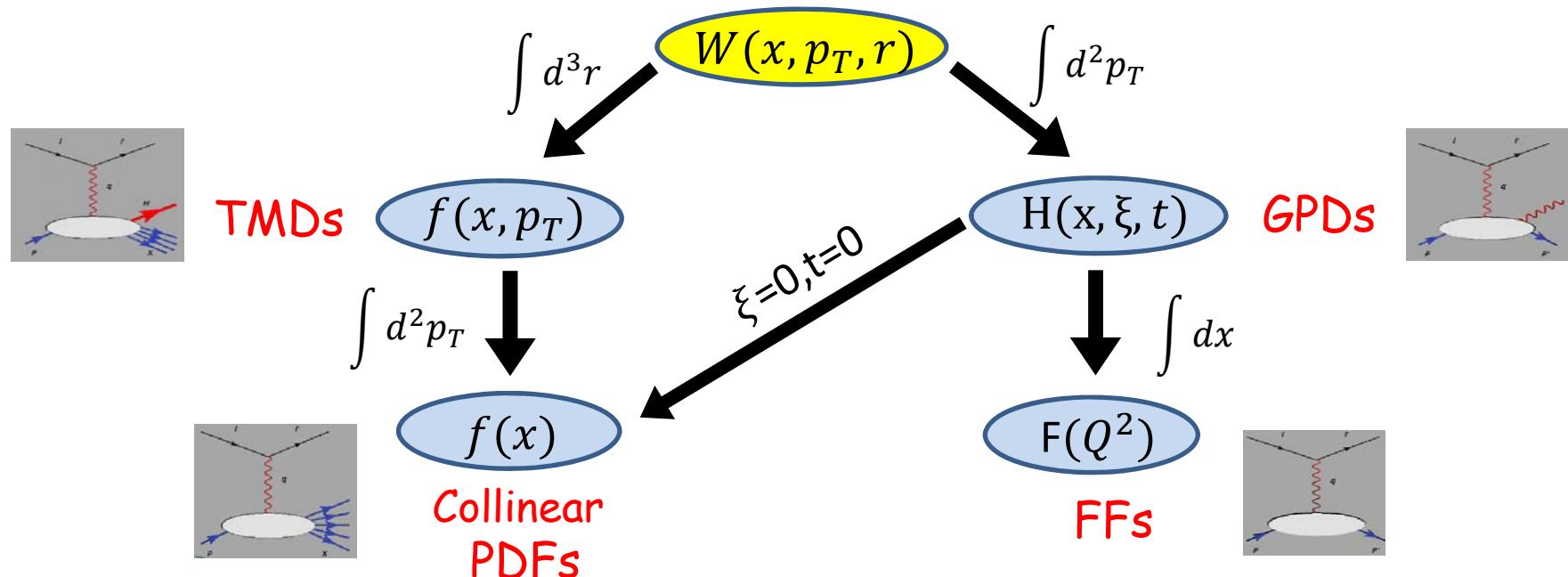
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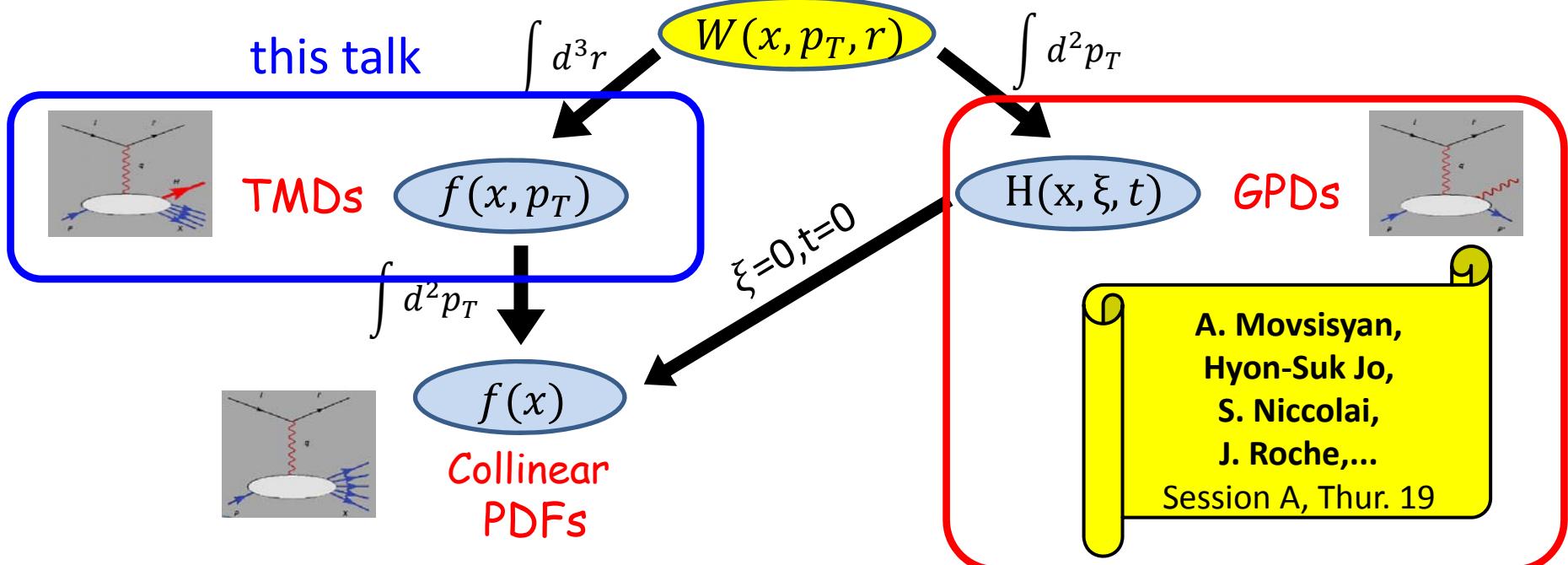
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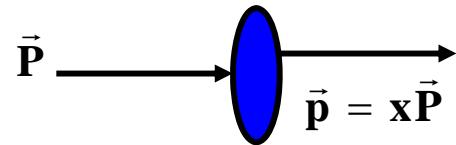
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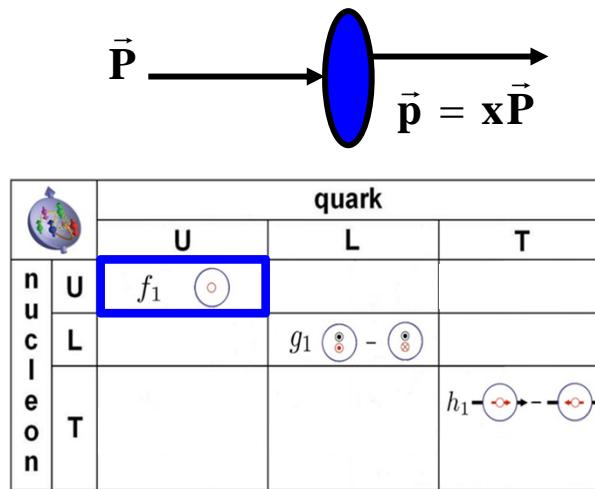
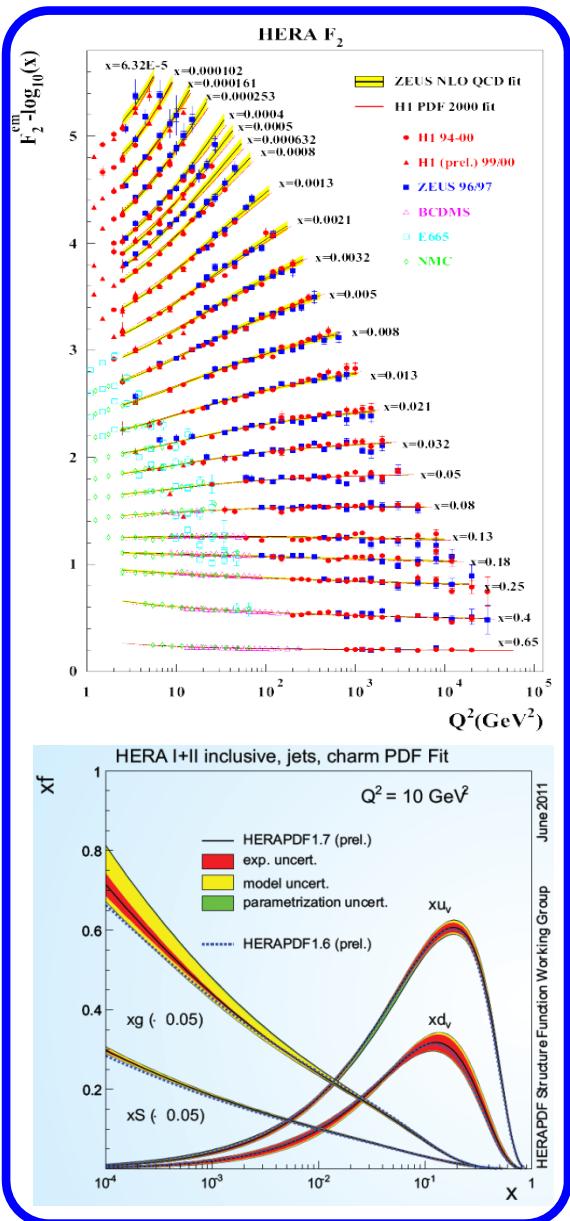


# The nucleon collinear structure

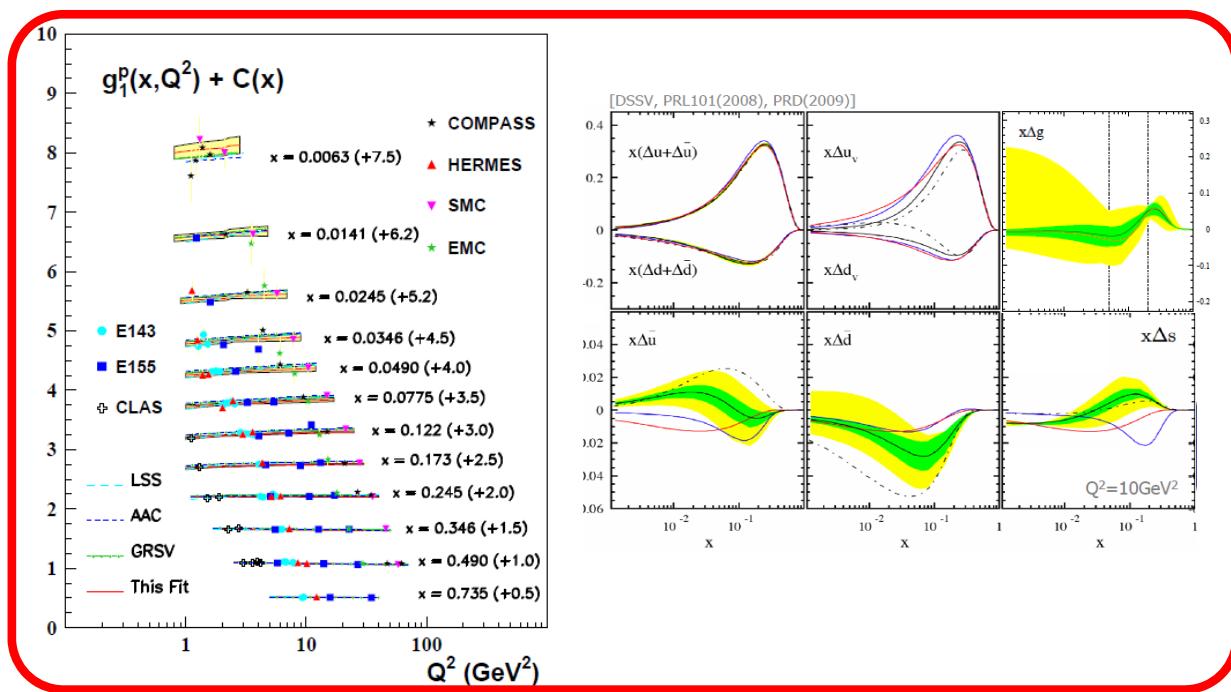
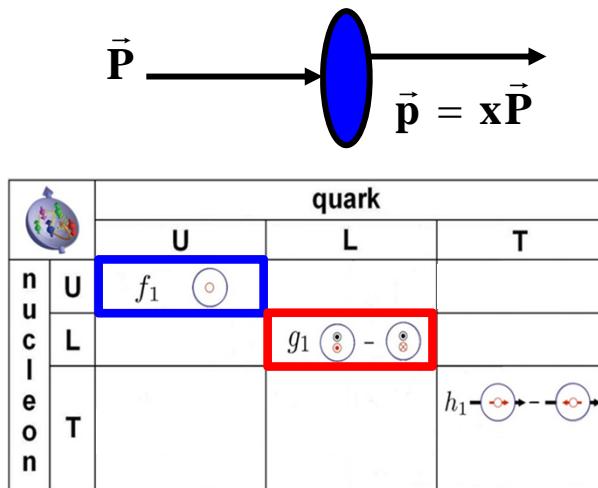
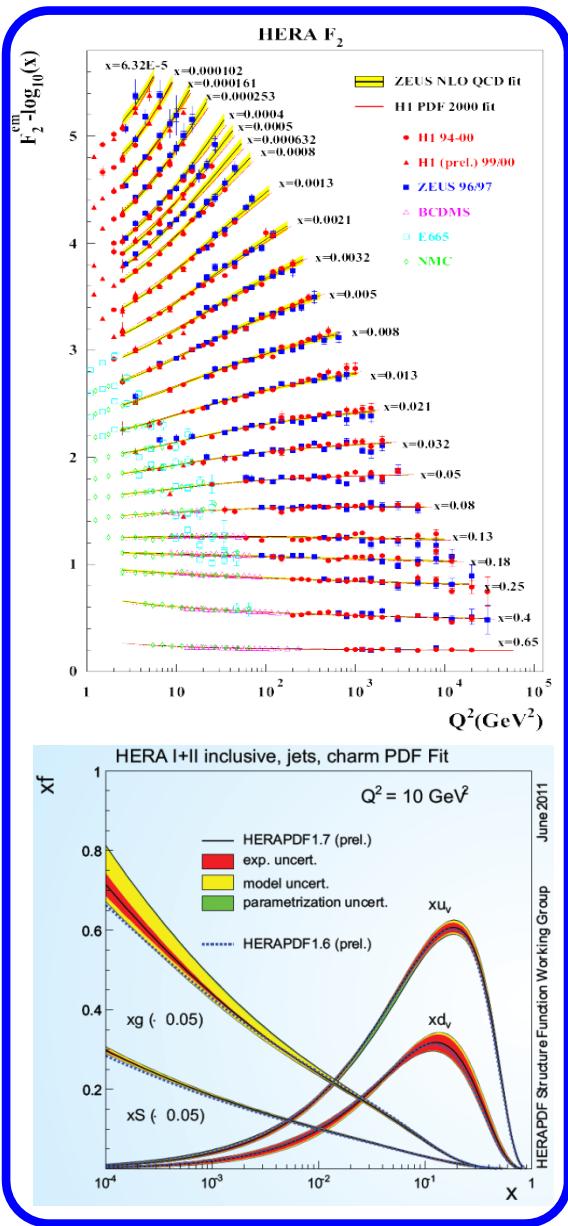


		quark		
		U	L	T
n u c l e o n	U	$f_1$		
	L		$g_1$ -	
	T			$h_1$

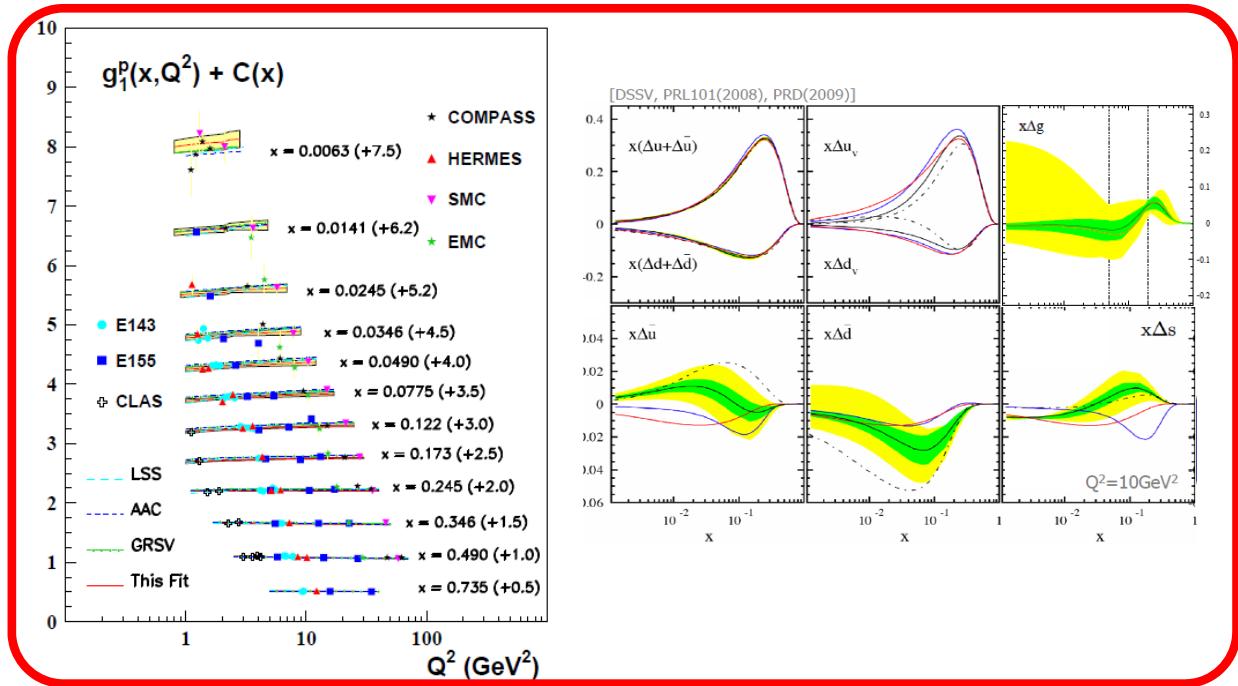
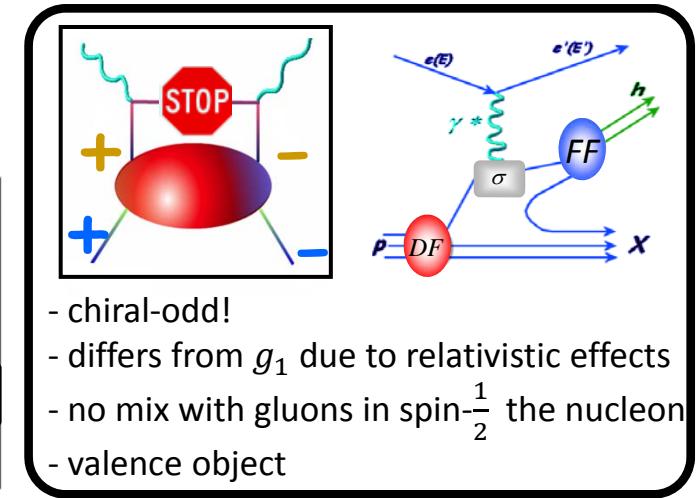
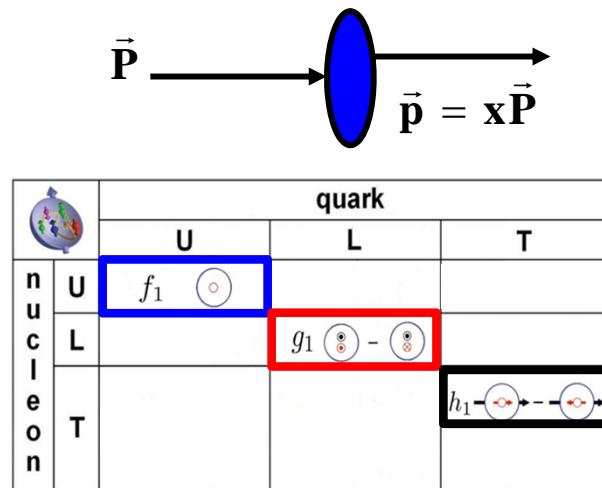
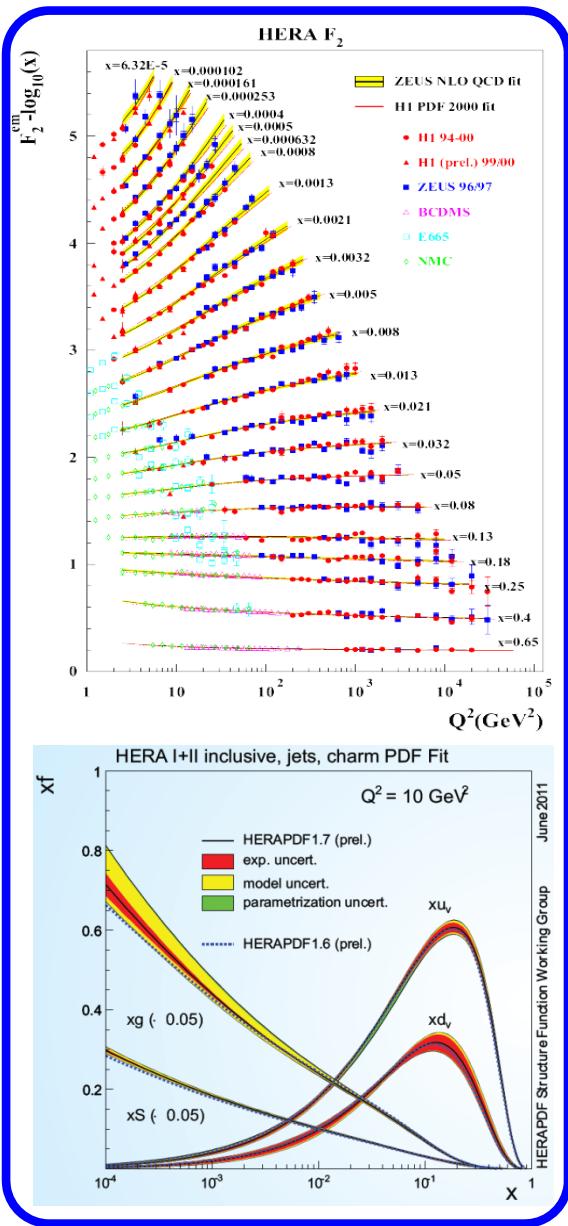
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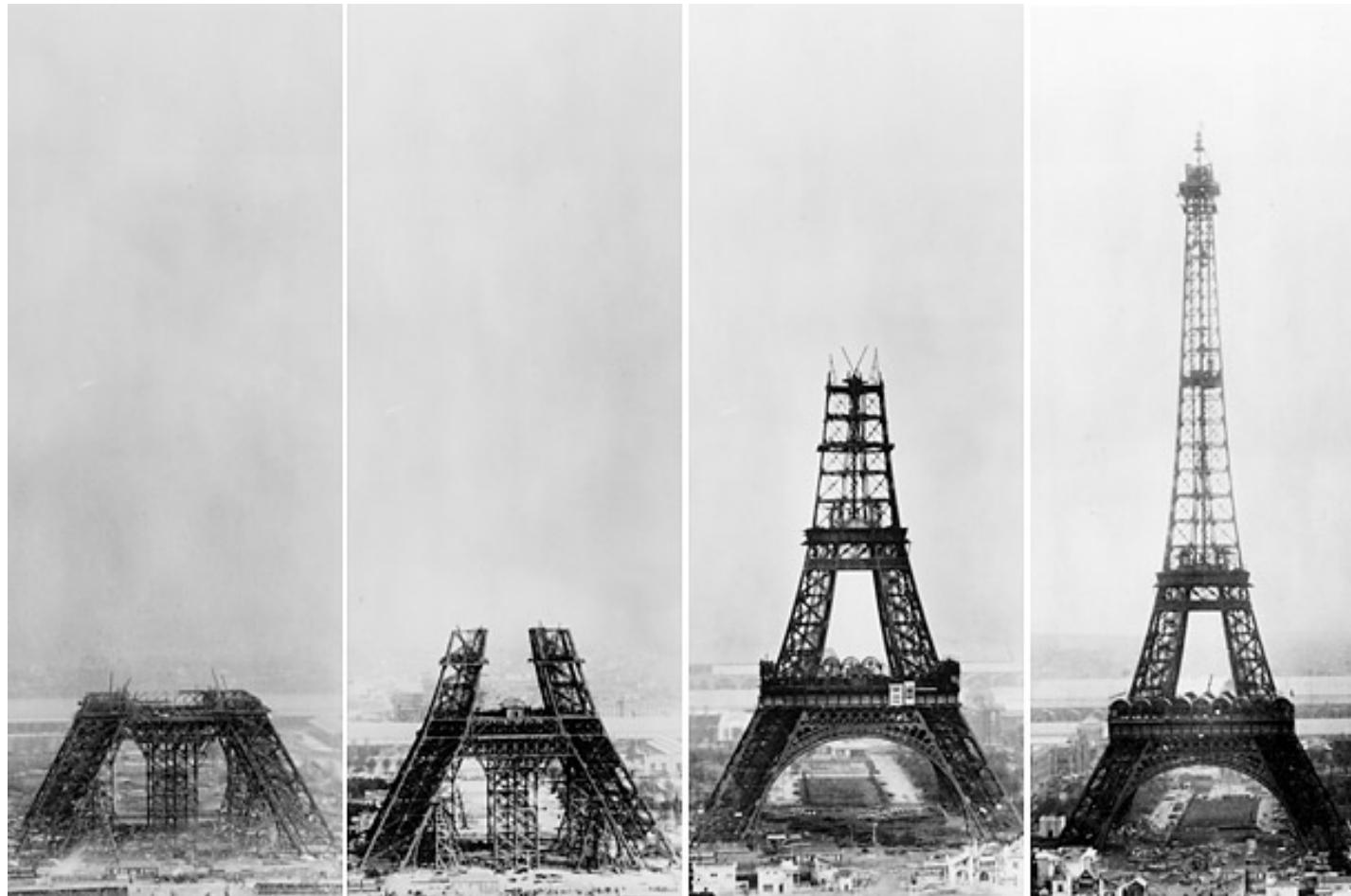
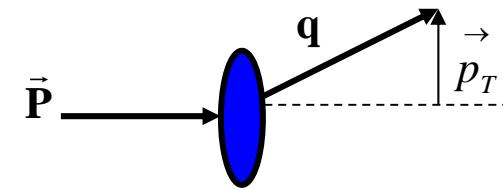
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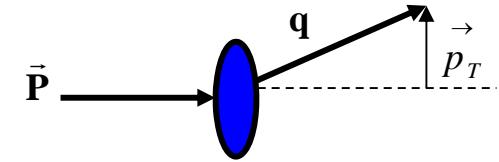
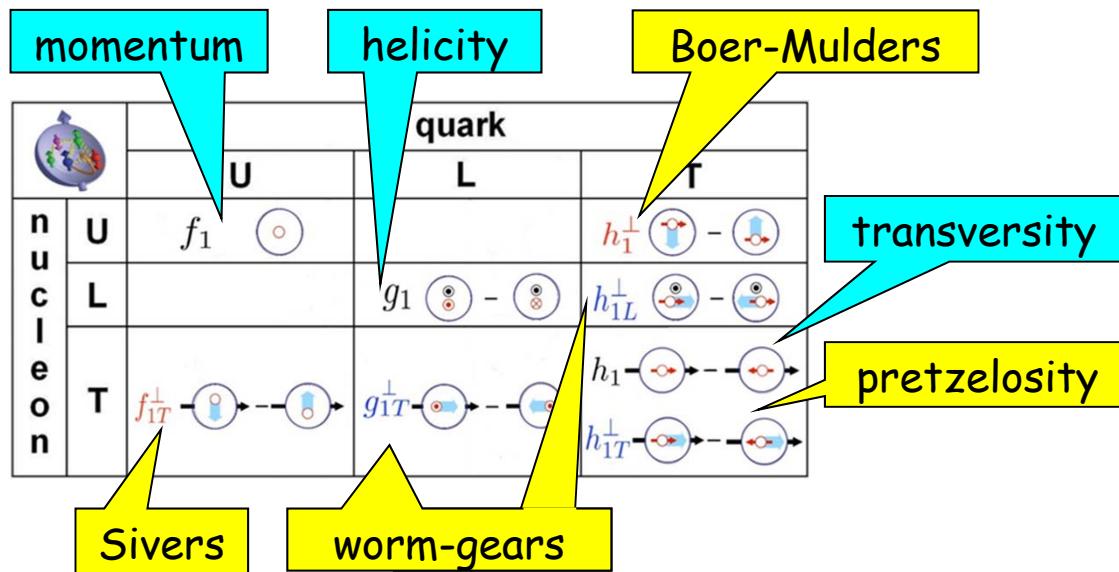
# The nucleon collinear structure



...now let's go transverse!

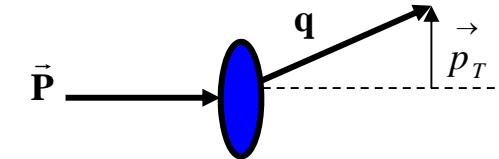
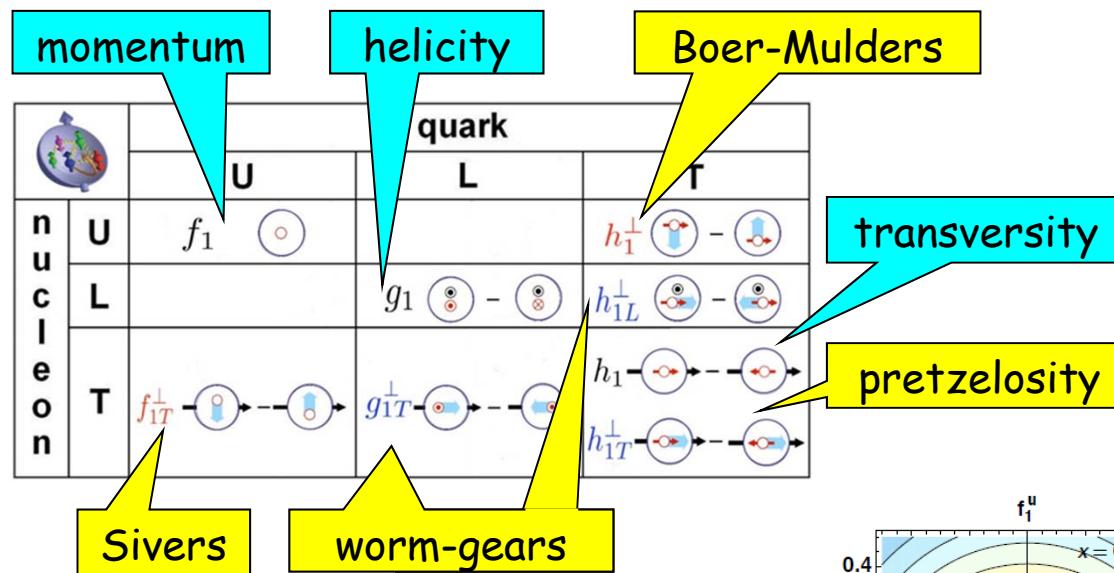


# The non-collinear structure of the nucleon

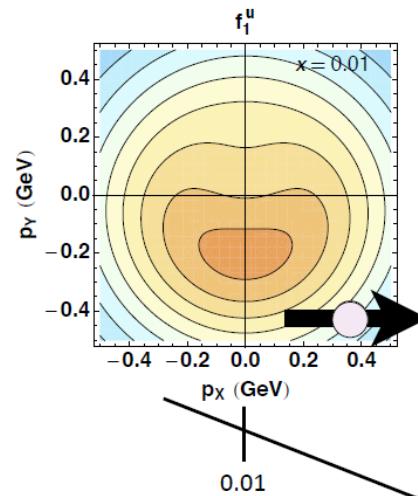
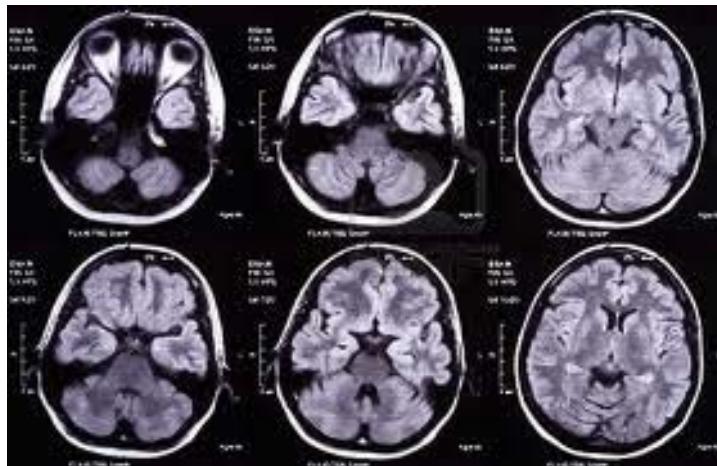


- TMDs depend on  $p_T$
- Vanish when integrated over  $p_T$
- Describe correlations between  $p_T$  and quark or nucleon spin (**spin-orbit correlations**)

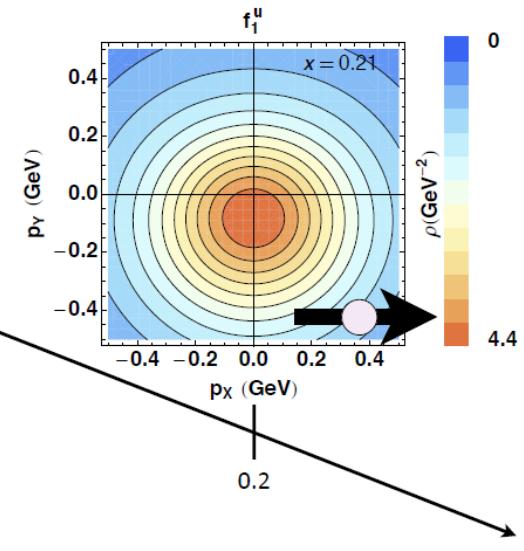
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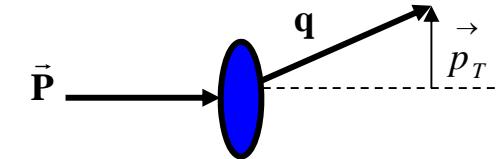
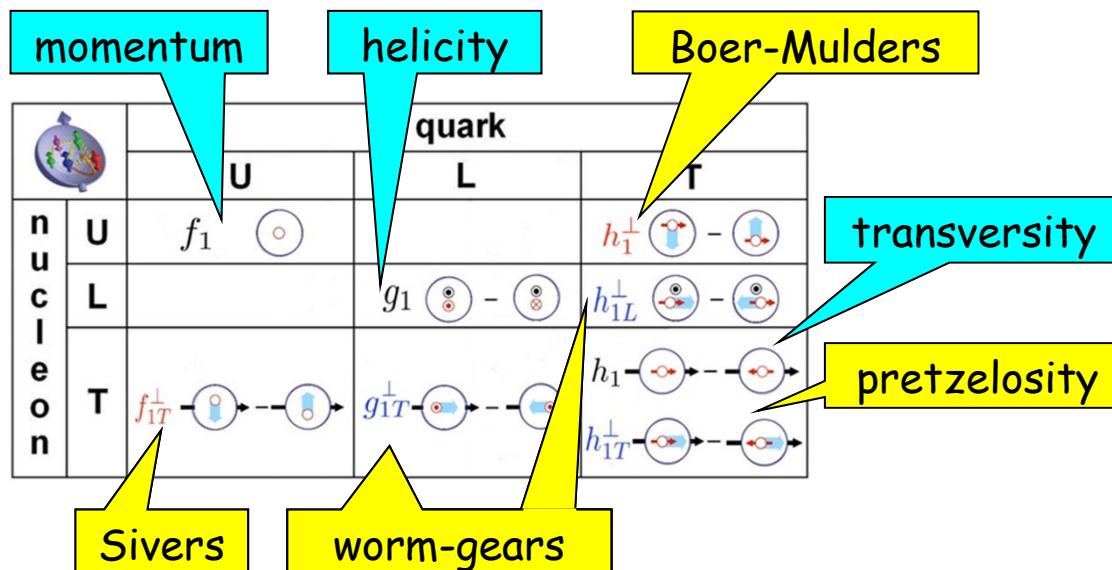
- TMDs depend on  $p_T$
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- Provide a **3-dim picture** of the nucleon in momentum space (**nucleon tomography**)



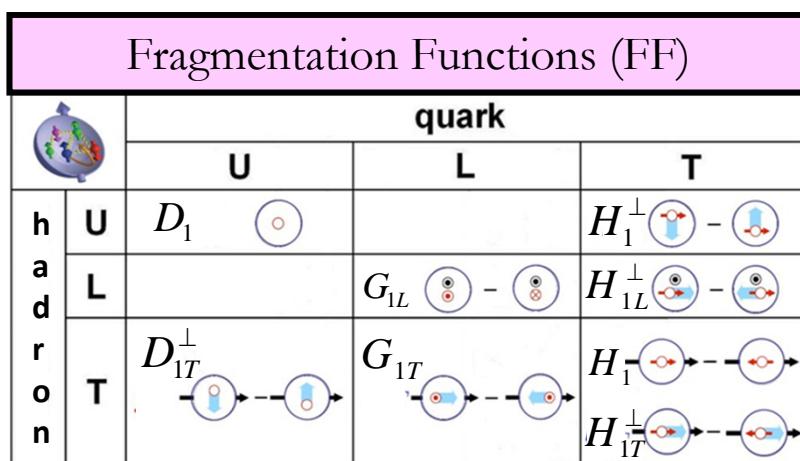
Based on model calculation  
A.B., Conti, Guagnelli, Radici, arXiv:1003.1328



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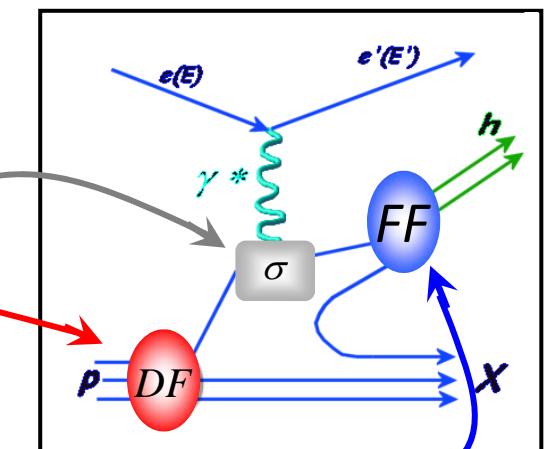


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- Mostly studied in **SIDIS**

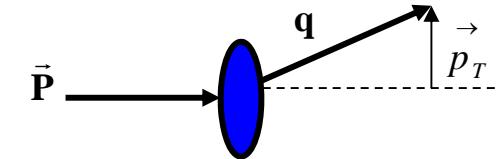
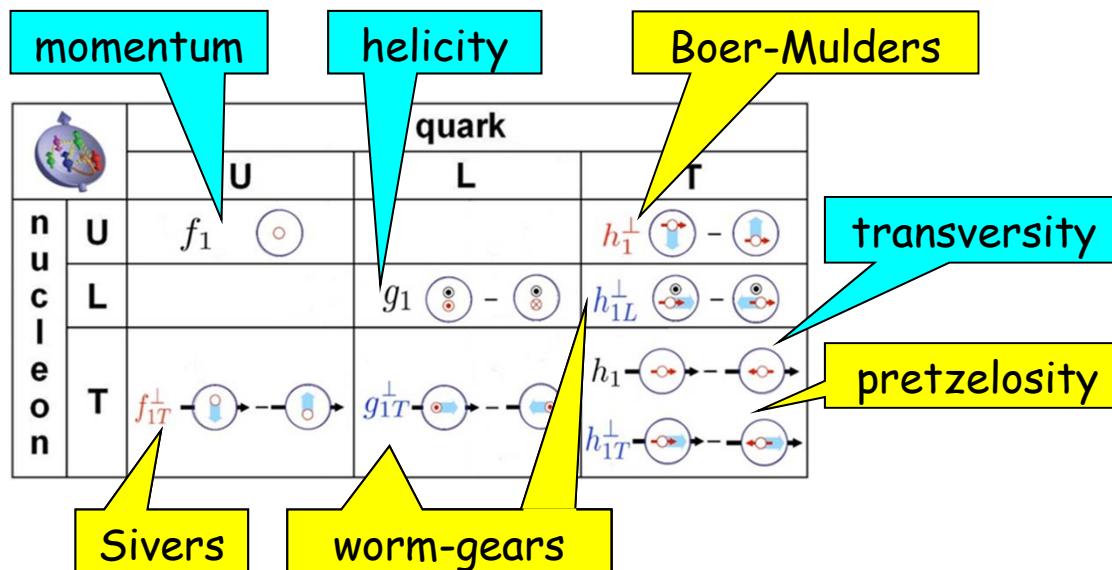


**Factorization**  $\rightarrow$   
(key result of QCD!)

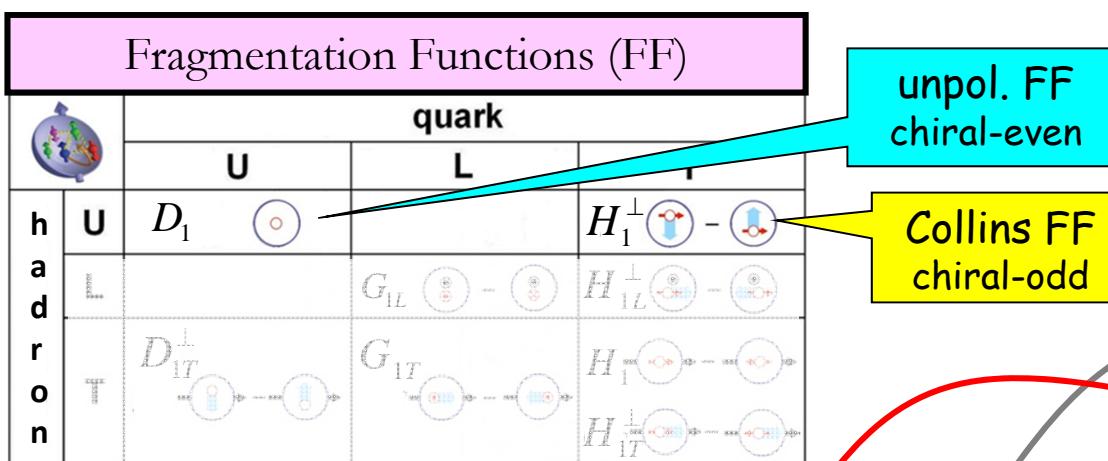
$$\sigma^{ep \rightarrow e h X} = \sum_q DF \otimes \sigma^{eq \rightarrow eq} \otimes FF$$



# The non-collinear structure of the nucleon

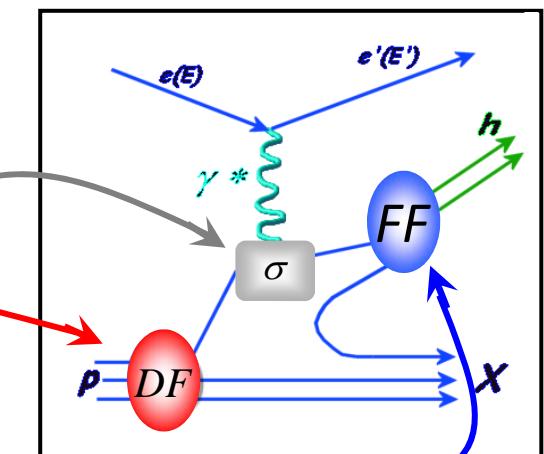


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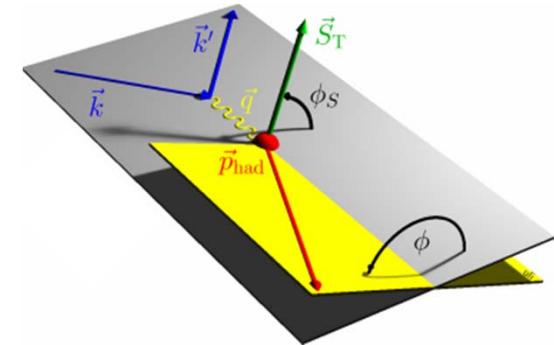
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# The SIDIS cross-section

$$\begin{aligned}
\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \\
\left\{ \begin{aligned}
& \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\
& \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \\
+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\
+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\
+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\
+ S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\
& + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\
& + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\
& \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \\
+ S_T \lambda_l \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\
& + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\
& \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \Big\}
\end{aligned} \right.$$



# The SIDIS cross-section

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$$\left\{ \begin{array}{l} [F_{UU,T} + \epsilon F_{UU,L} \\ + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)}] \end{array} \right.$$

unpolarized

$$+ \lambda_l [\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)}]$$

beam polarization

$$+ S_L [\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)}]$$

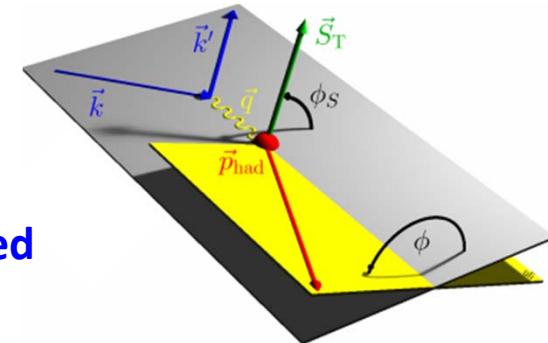
target polarization

$$+ S_L \lambda_l [\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)}]$$

$$\begin{aligned} + S_T & [\sin(\phi - \phi_S) (F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}) \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)}] \end{aligned}$$

beam and target polarization

$$\begin{aligned} + S_T \lambda_l & [\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)}] \} \end{aligned}$$



# Transversity

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{aligned} & \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{aligned} \right.$$

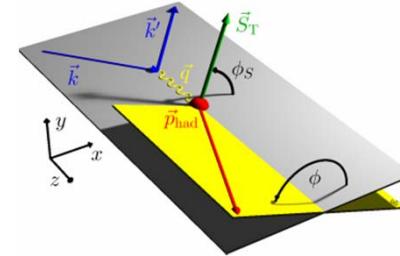
$$+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$\boxed{+ S_T \left[ \begin{aligned} & \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \end{aligned} \right]}$$

$$\left. + S_T \lambda_l \left[ \begin{aligned} & \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \end{aligned} \right] \right\}$$



Describes probability to find transversely polarized quarks in a transversely polarized nucleon

## Distribution Functions

		quark		
		U	L	T
nucleon	U	$f_1$		-
	L		-	-
	T	-	-	-
	T	$f_{1T}^\perp$	-	-

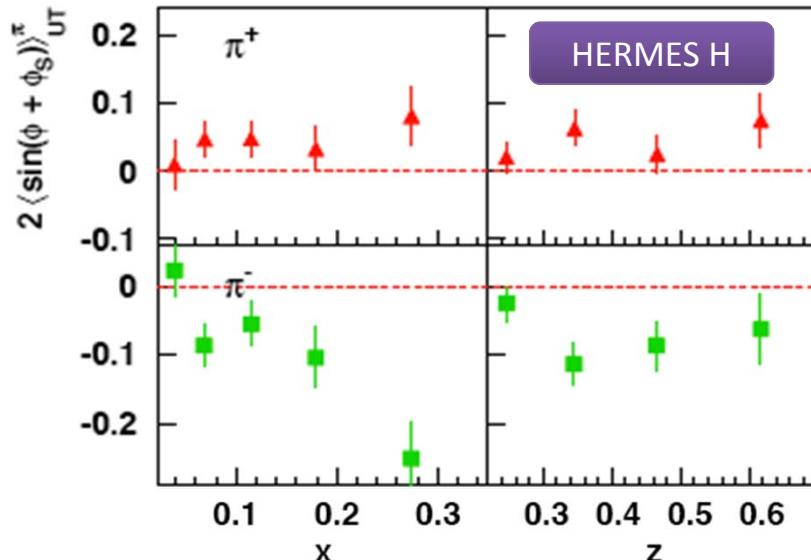
## Fragmentation Functions

		quark		
		U	L	T
h	U	$D_1$		-

# Transversity

$$A_{UT}^{\sin(\phi + \phi_S)} \propto \frac{\sum_q e_q^2 h_1^q(x, p_T^2) \otimes H_1^{q,\perp}(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)}$$

*Phys. Rev. Lett. 94 (2005) 012002*



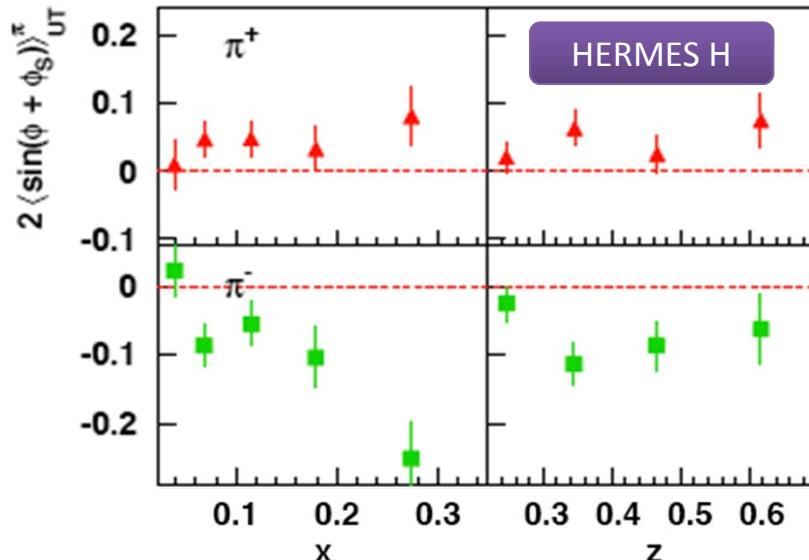
- First evidence by HERMES (05)
- Transv. pol. H target
- Limited statistics (2002-2003)
- Non-zero Collins amplitudes
- Non-zero transversity & Collins FF

$$H_1^{\perp, \text{unfav}}(z) \approx -H_1^{\perp, \text{fav}}(z)$$

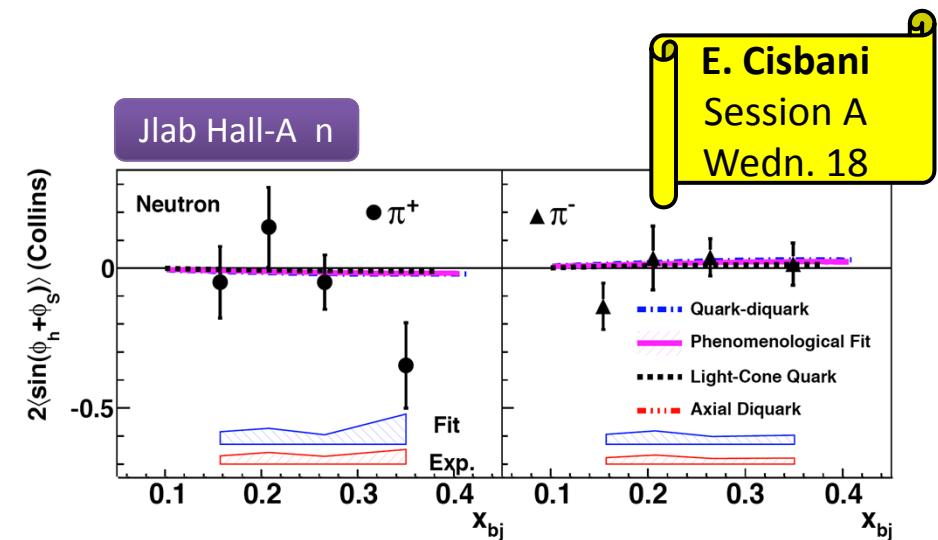
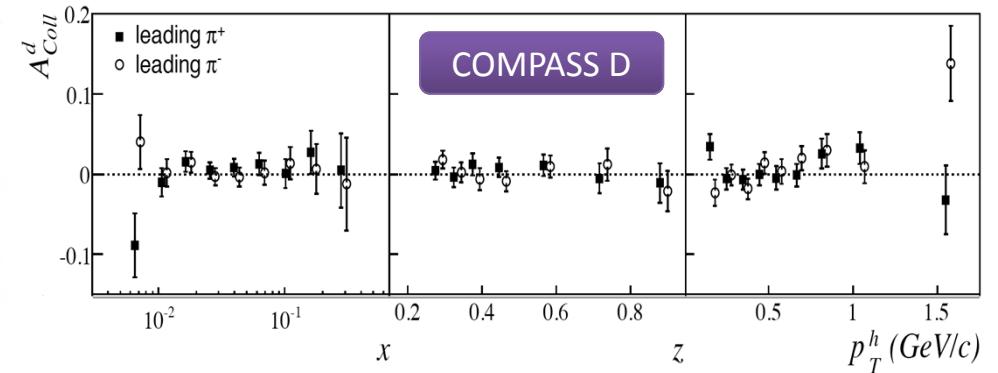
# Transversity

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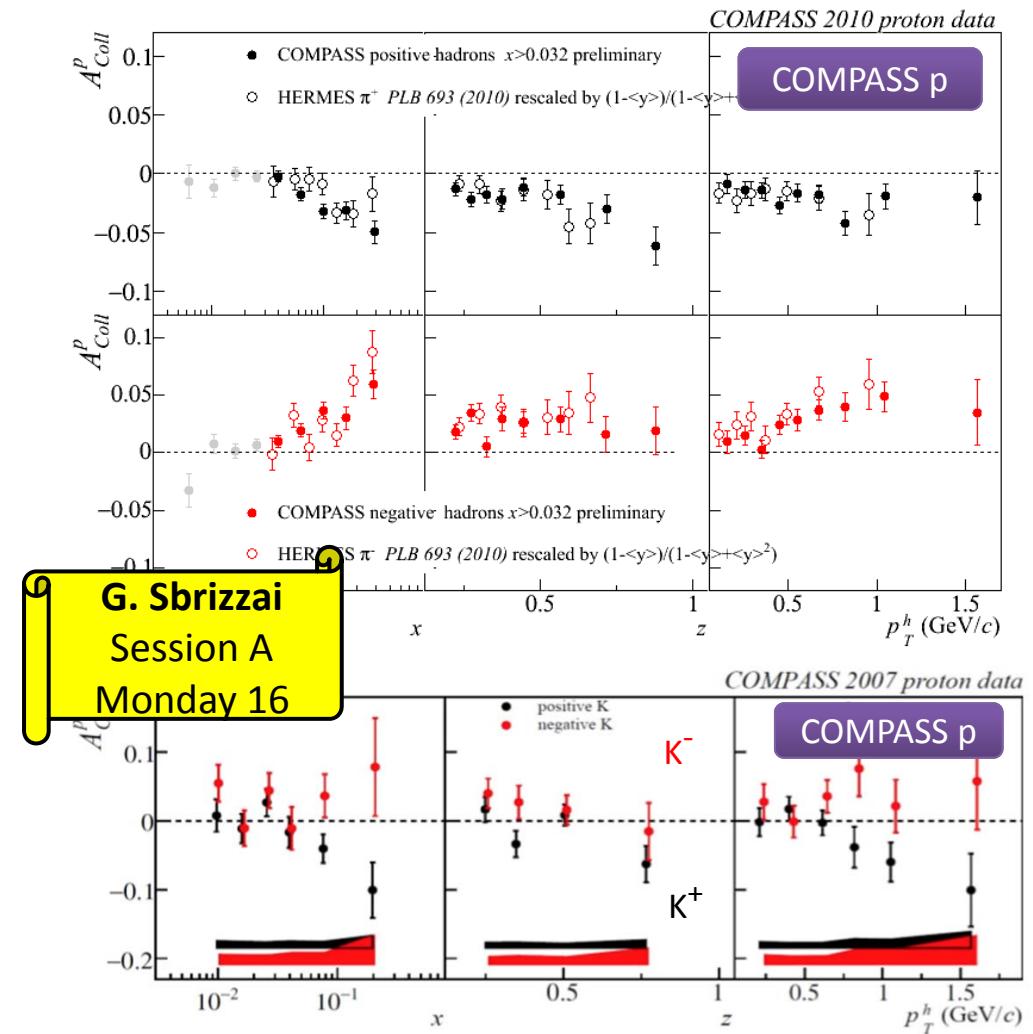
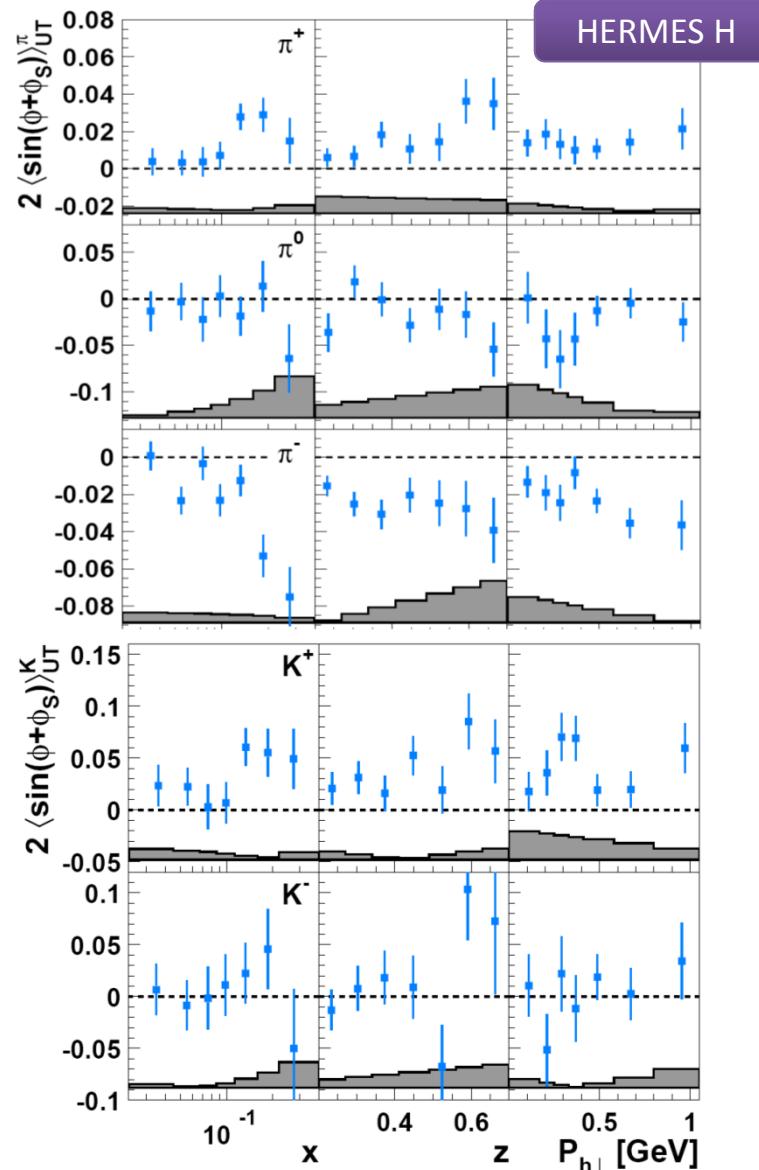
*Phys. Rev. Lett. 94 (2005) 012002*



- First evidence by HERMES (05)
  - Transv. pol. H target
  - Limited statistics (2002-2003)
  - Non-zero Collins amplitudes
  - Non-zero transversity & Collins FF
- $H_1^{\perp, \text{unfav}}(z) \approx -H_1^{\perp, \text{fav}}(z)$
- Consistent with COMPASS zero amplitude on Deuterium



# Transversity

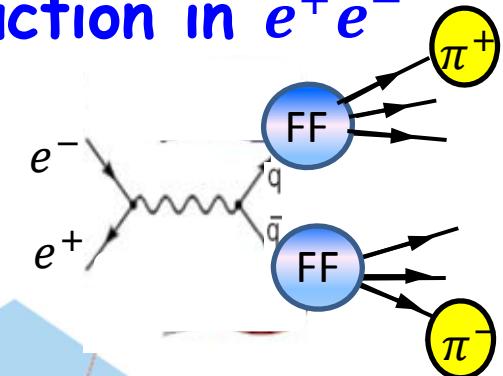


**COMPASS  $\pi$  results on p consistent with HERMES!**

- $K^-$  amplitudes are not in agreement
- statistics for kaons relatively poor
- **Need new data (e.g. from JLab @12Gev)**

# Collins FF from inclusive hadron-pair production in $e^+e^-$

Describes correlation between quark polarization and observed hadron transverse momentum

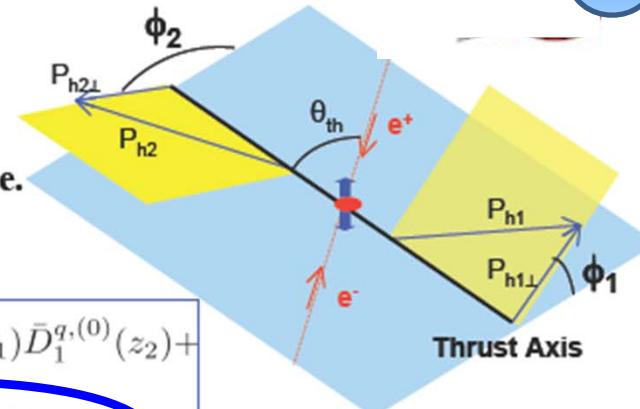


$\phi_1 + \phi_2$  or Thrust RF

$\theta$ : angle between the  $e^+e^-$  axis and the thrust axis;  
 $\phi_{1,2}$ : azimuthal angles between  $P_{h1(h2)}$  and the scattering plane.

All quantities in  $e^+e^-$  center of mass

$$\begin{aligned} \frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d\phi_1 d\phi_2} = & \sum_{q,\bar{q}} \frac{3\alpha^2}{Q^2} \frac{e_q^2}{4} z_1^2 z_2^2 \left[ (1 + \cos^2 \theta) D_1^{q,(0)}(z_1) \bar{D}_1^{q,(0)}(z_2) + \right. \\ & \left. + \sin^2(\theta) \cos(\phi_1 + \phi_2) H_1^{\perp,(1),q}(z_1) \bar{H}_1^{\perp,(1),q}(z_2) \right] \end{aligned}$$



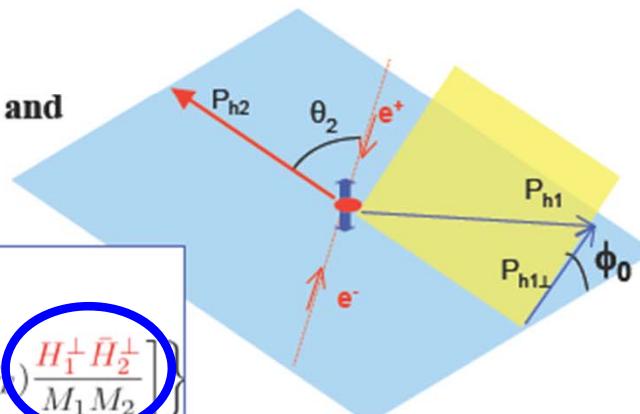
..... N

$2\phi_0$  or  $P_{h2}$  RF

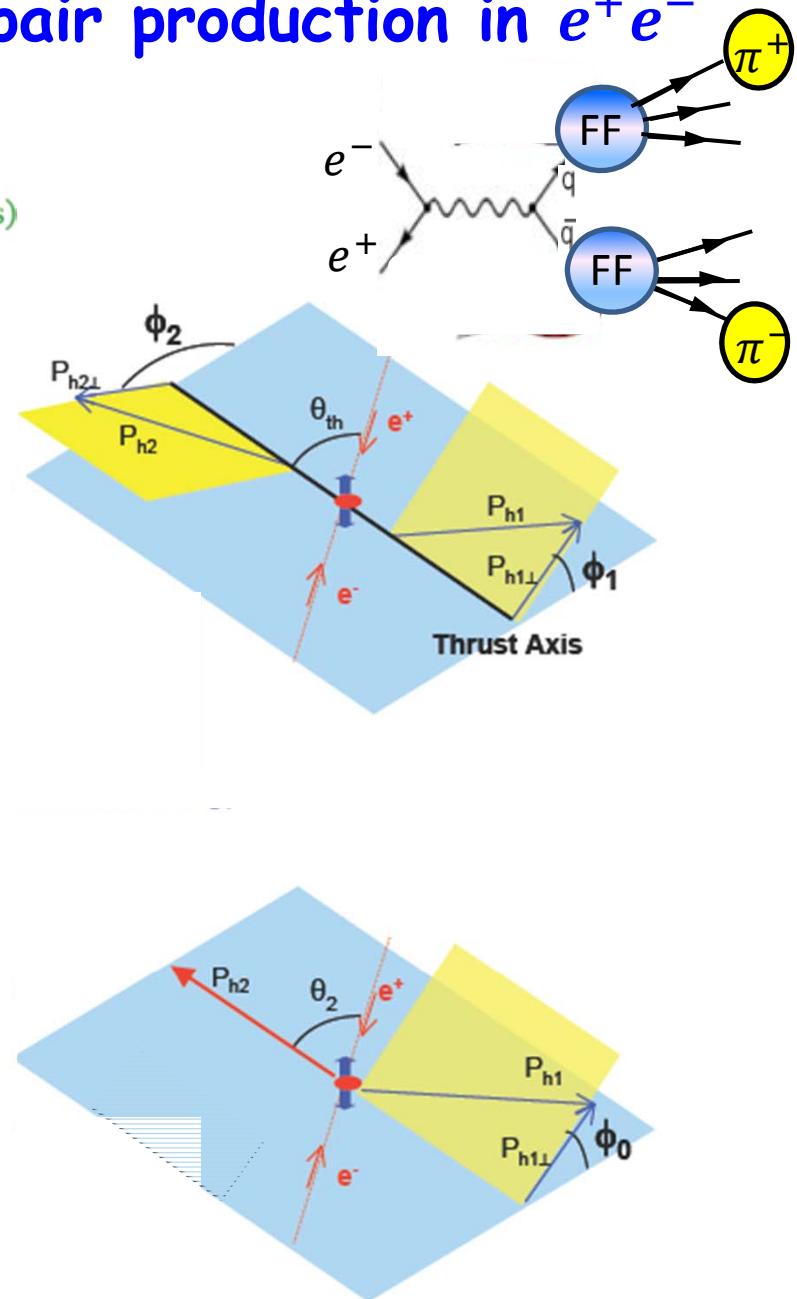
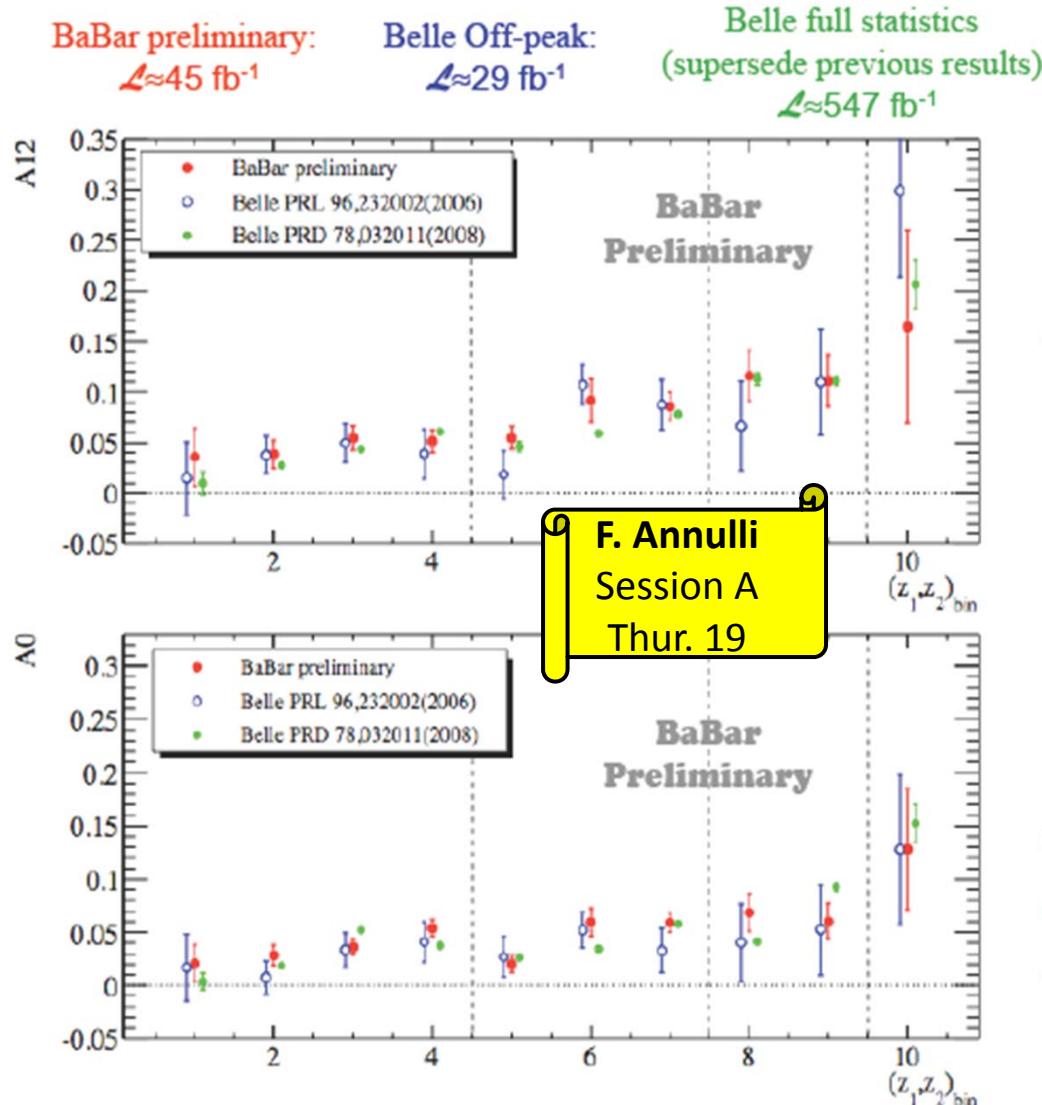
$\theta_2$ : angle between the  $e^+e^-$  axis and  $P_{h2}$ ;  
 $\phi_0$ : angle between the plane spanned by  $P_{h2}$  and the  $e^+e^-$  axis, and the direction of  $P_{h1}$  perpendicular to  $P_{h2}$ .

All quantities in  $e^+e^-$  center of mass

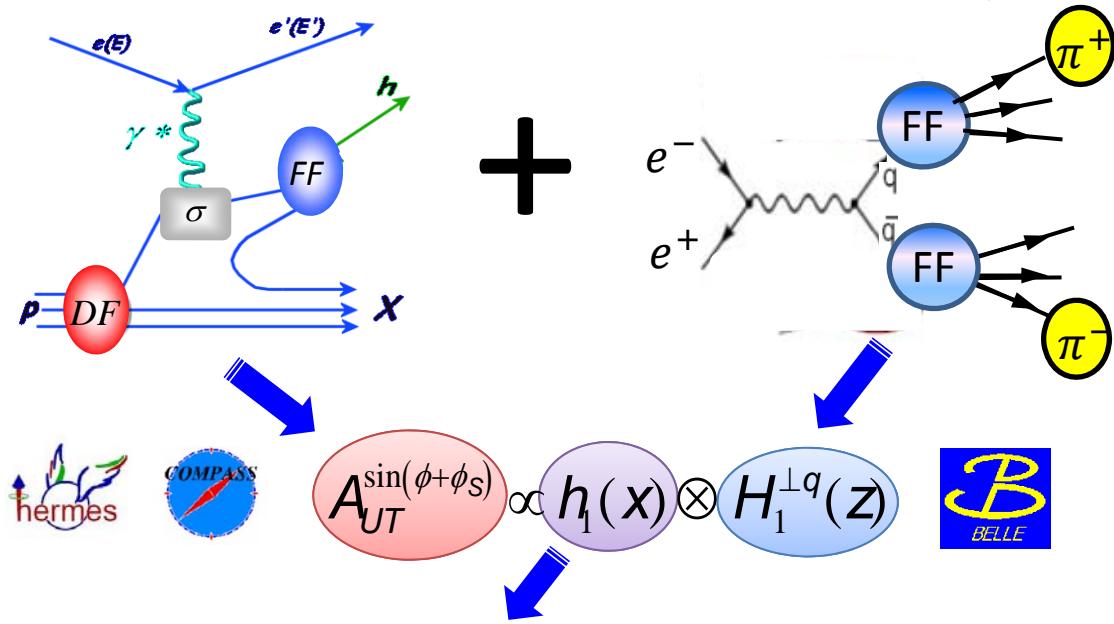
$$\begin{aligned} \frac{d\sigma(e^+e^- \rightarrow h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2 \vec{q}_T} = & \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \left\{ A(y) \mathcal{F}[D_1 \bar{D}_2] + \right. \\ & \left. + B(y) \cos(2\phi_0) \mathcal{F} \left[ (2\hat{h} \cdot \vec{k}_T \hat{h} \cdot \vec{p}_T - \vec{k}_T \cdot \vec{p}_T) \frac{H_1^\perp \bar{H}_2^\perp}{M_1 M_2} \right] \right\} \end{aligned}$$



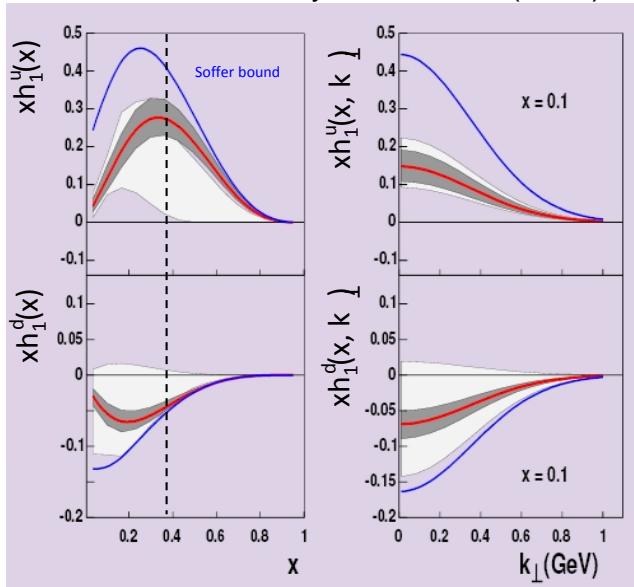
# Collins FF from inclusive hadron-pair production in $e^+e^-$



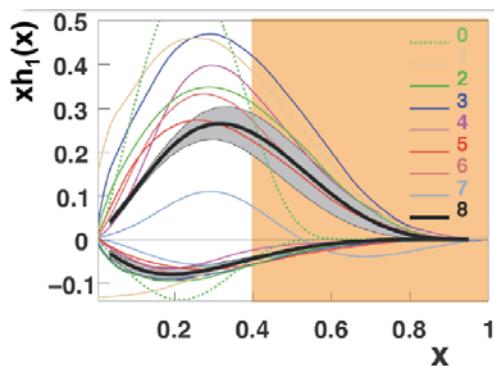
# First extraction of Transversity



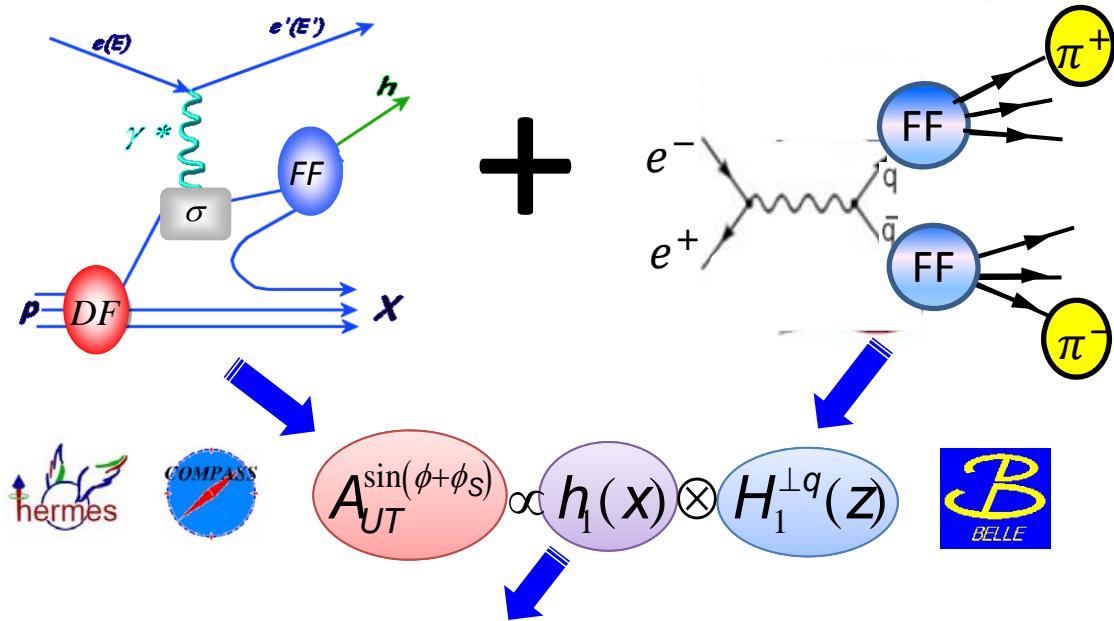
Anselmino et al. Phys. Rev. D 75 (2007)



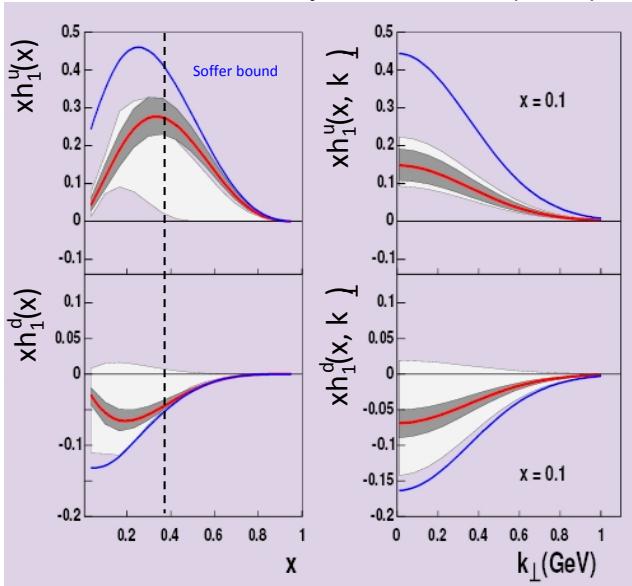
0. chiral color-dielectric model [Barone et al., PLB 390 (97)]
1. Soffer bound [Soffer et al., PRD 65 (02)]
2.  $h_1 = g_1$  [Korotov et al., EPJC 18 (01)]
3. Chiral quark-soliton model [Schweitzer et. al., PRD 64 (01)]
4. chiral-quark soloiton model [Wakamatsu, PLB 509 (01)]
5. light-cone constituent quark model [Pasquini et al., PRD 72 (05)]
6. quark-diquark model [Cloet, Bentz, Thomas, PLB 659 (08)]
7. quark-diquark model [Bacchetta, Conti, Radici, PRD 78 (08)]
8. Parametrization [Anselmino et al., arXiv 0807.0173]



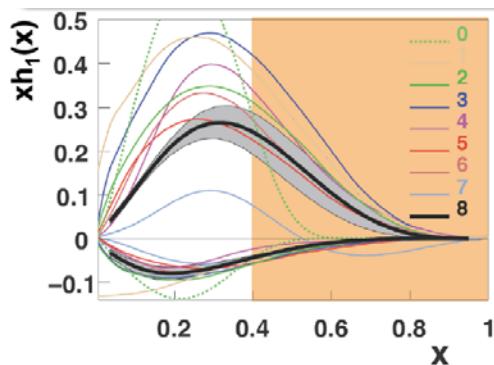
# First extraction of Transversity



Anselmino et al. Phys. Rev. D 75 (2007)

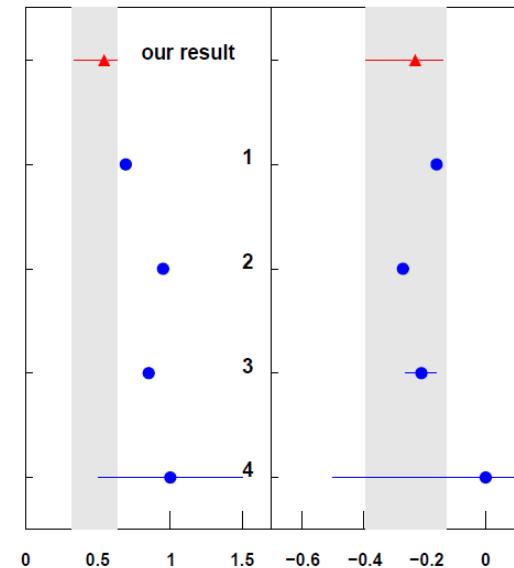


0. chiral color-dielectric model [Barone et al., PLB 390 (97)]
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## Tensor charge

$$\delta q = \int_0^1 dx \left[ h_1^q(x) - \bar{h}_1^q(x) \right]$$



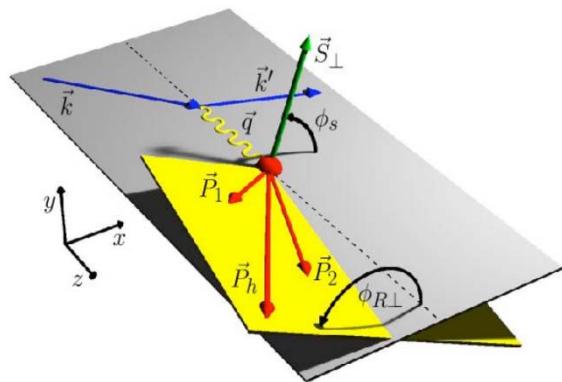
- 1: Quark-diquark model
- 2: Chiral quark soliton model
- 3: Lattice QCD
- 4: QCD sum rules

$$\delta u = 0.54^{+0.09}_{-0.22} \quad \delta d = -0.23^{+0.09}_{-0.16}$$

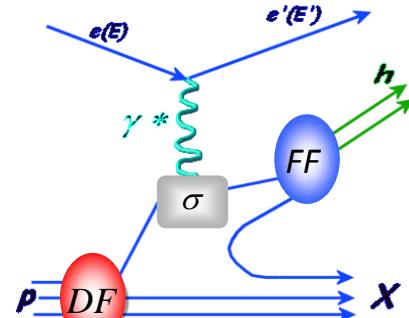
M. Anselmino et al [hep-ph:0812.4366](https://arxiv.org/abs/hep-ph/0812.4366)

**Need data in valence region  
( $x > 0.4$ ) -> JLab @ 12 GeV**

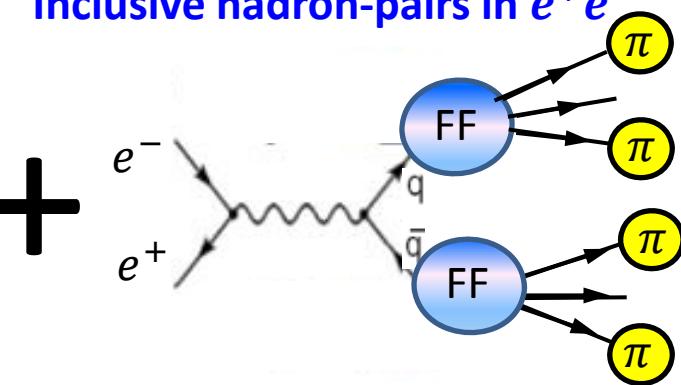
# Transversity



## 2h Semi-Inclusive DIS



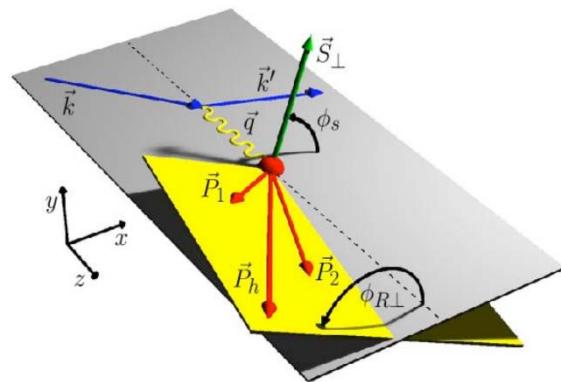
## Inclusive hadron-pairs in $e^+e^-$



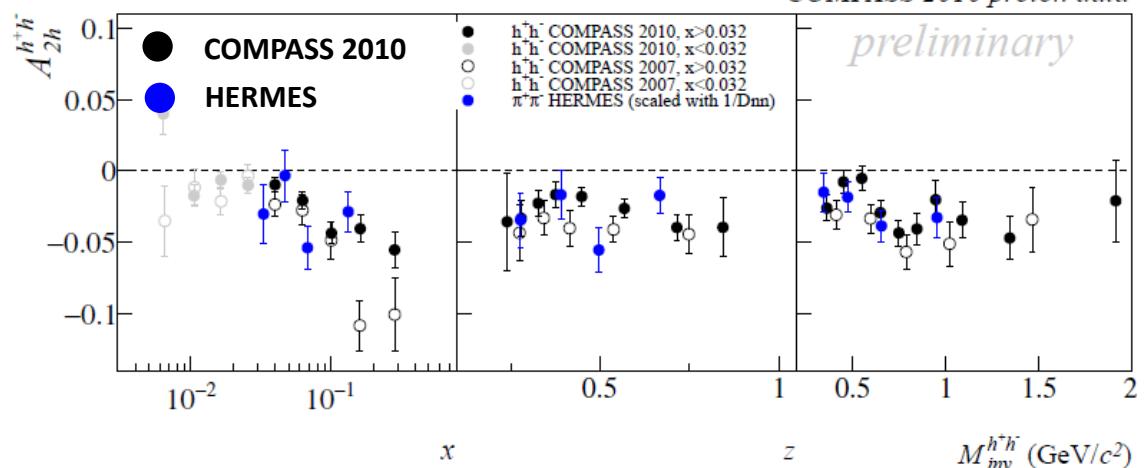
$$A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta} \propto \frac{\sum_q e_q^2 h(x, Q^2) H_1^\triangleleft(z M_h^2, Q^2)}{\sum_q e_q^2 f_1(x, Q^2) D_1^\triangleleft(z M_h^2, Q^2)}$$

- Survives integration over transverse momentum
- Collinear factorization (simple product)
- DGLAP evolution

# Transversity

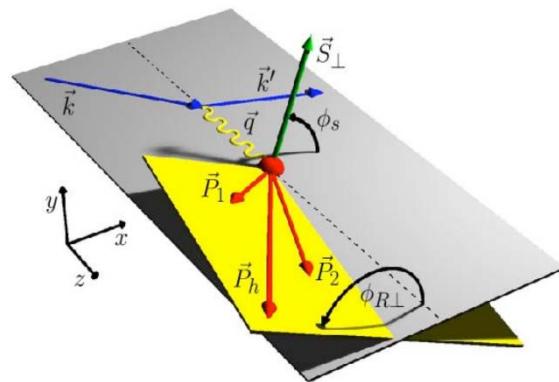


$$A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta} \propto \frac{\sum_q e_q^2 h(x, Q^2) H_1^\triangle(z M_h^2, Q^2)}{\sum_q e_q^2 f_1(x, Q^2) D_1^\triangle(z M_h^2, Q^2)}$$



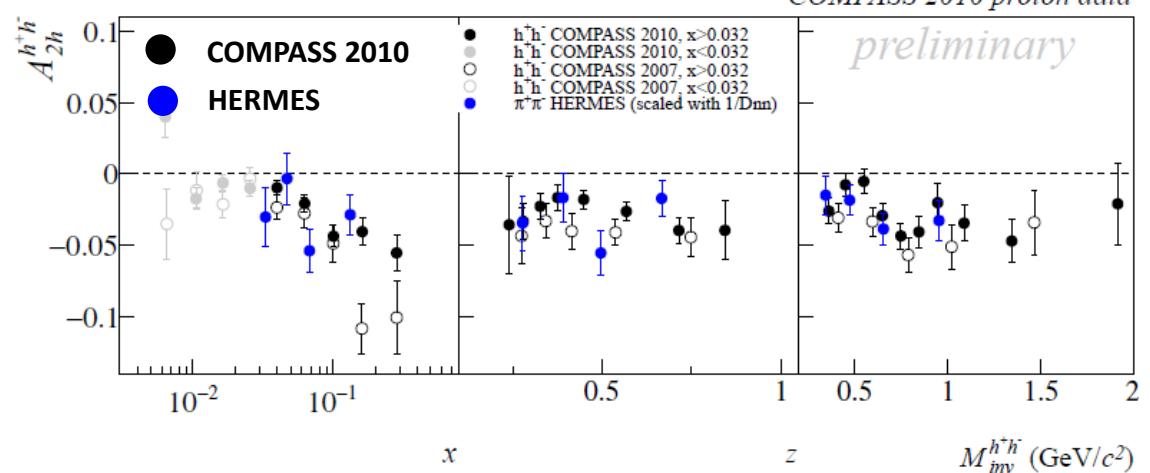
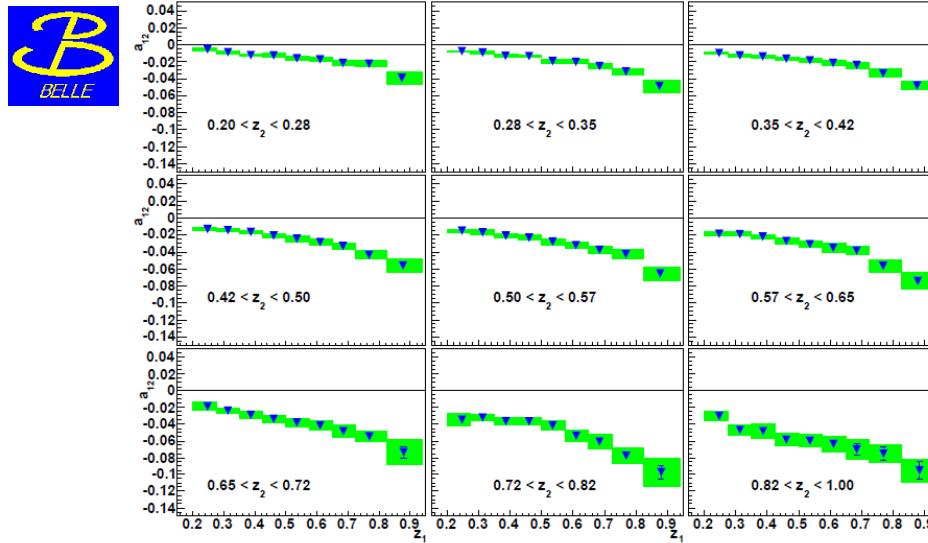
- Survives integration over transverse momentum
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- DGLAP evolution
- Large asymmetries @ HERMES & COMPASS

# Transversity



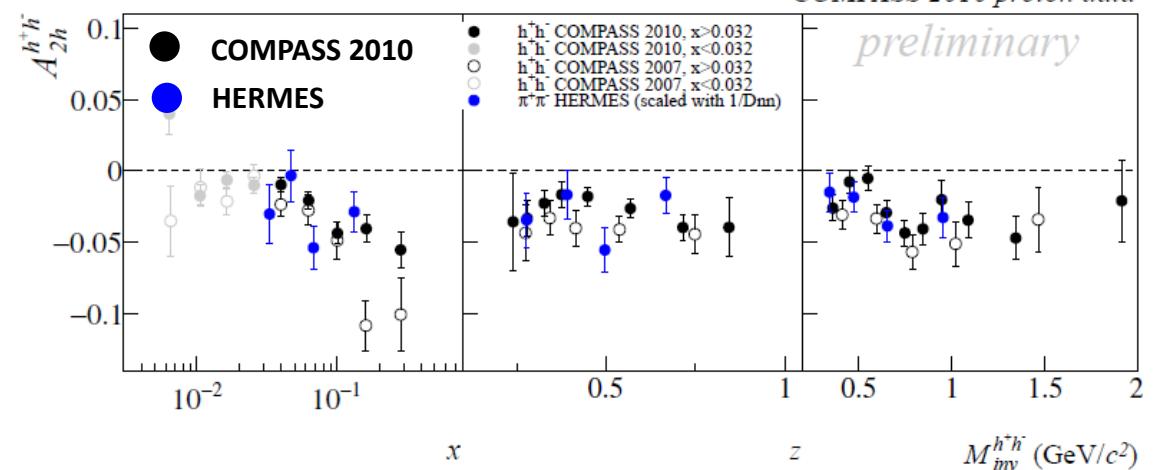
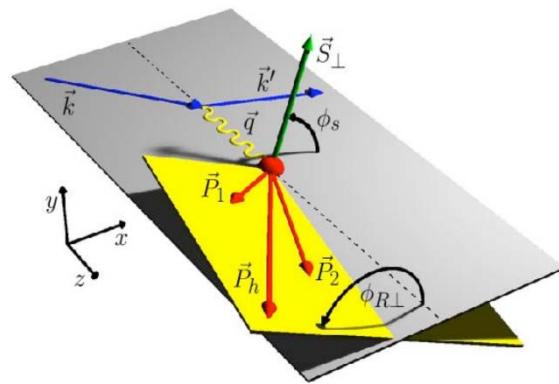
$$A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta} \propto \frac{\sum_q e_q^2 h_1(x, Q^2) H_1^\leftarrow(z M_h^2, Q^2)}{\sum_q e_q^2 f_1(x, Q^2) D_1^\leftarrow(z M_h^2, Q^2)}$$

Phys.Rev.Lett.107:072004,2011



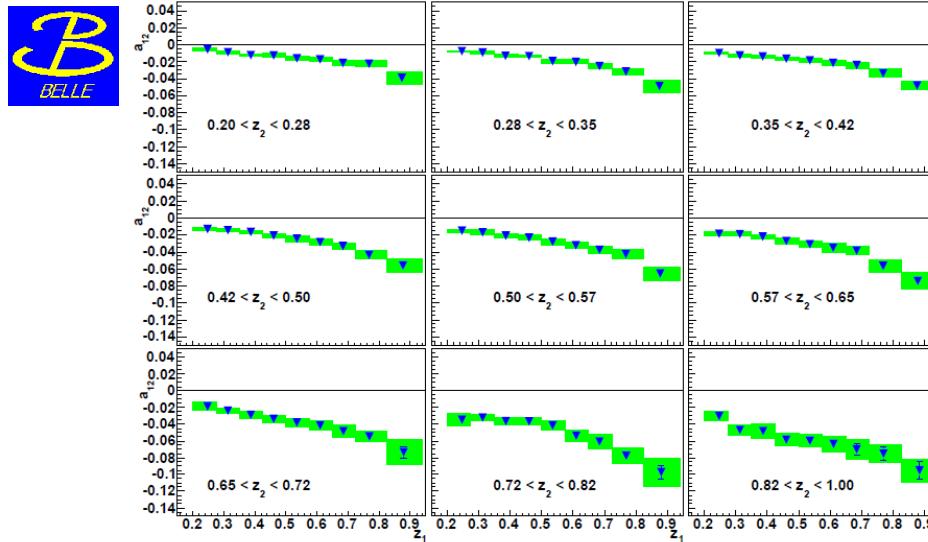
- Survives integration over transverse momentum
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- Large asymmetries @ HERMES & COMPASS
- **H1 $^\leftarrow$  chiral-odd, measured at BELLE**

# Transversity



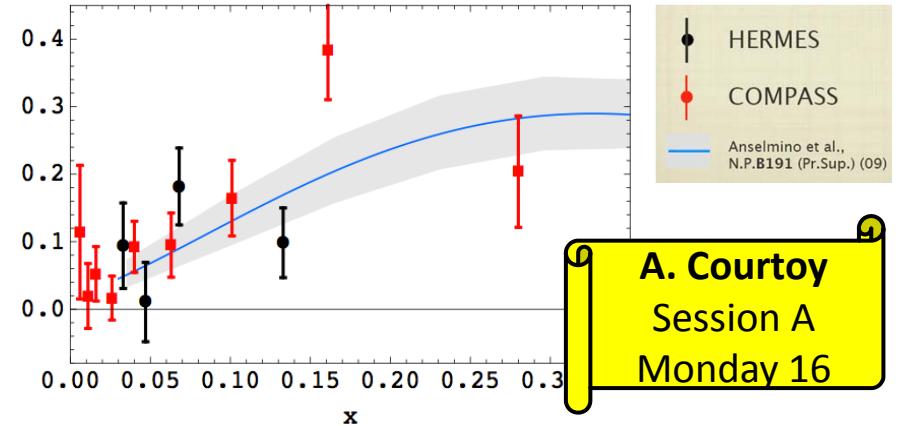
$$A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta} \propto \frac{\sum_q e_q^2 h_1(x, Q^2) H_1^\Delta(z M_h^2, Q^2)}{\sum_q e_q^2 f_1(x, Q^2) D_1^\Delta(z M_h^2, Q^2)}$$

Phys.Rev.Lett.107:072004,2011



- Survives integration over transverse momentum
- Collinear factorization (simple product)
- DGLAP evolution
- Large asymmetries @ HERMES & COMPASS
- $H1^\Delta$  chiral-odd, measured at BELLE
- Independent extraction of transversity!

$$x h_1^u(x) - \frac{x}{4} h_1^d(x) \quad \text{Bacchetta et al., PRL 107 (11)}$$



# Sivers function

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{aligned} & \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{aligned} \right.$$

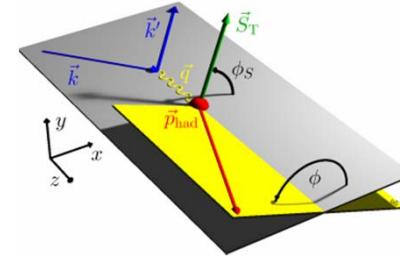
$$+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$\left. \begin{aligned} & + S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \end{aligned} \right\}$$

$$\left. \begin{aligned} & + S_T \lambda_l \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \right\}$$



Describes correlation between quark transverse momentum and nucleon transverse polarization

## Distribution Functions

		quark		
		U	L	T
nucleon	U	$f_1$		-
	L			-
	T		-	-
	T	-	-	-

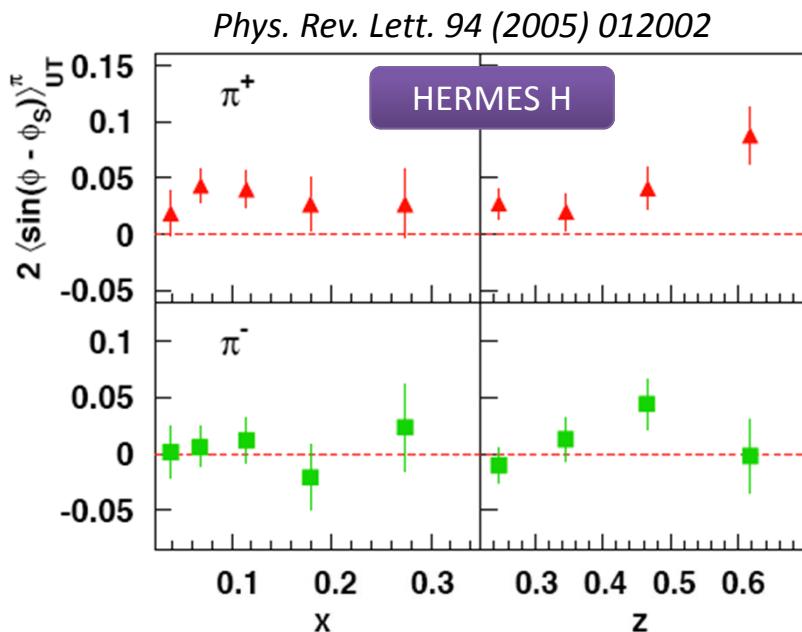
## Fragmentation Functions

		quark		
		U	L	T
h	U	$D_1$		-

# Sivers function

$$A_{UT}^{\sin(\phi + \phi_S)} \propto -\frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x, p_T^2) \otimes D_1^q(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)}$$

First evidence by HERMES (2005)



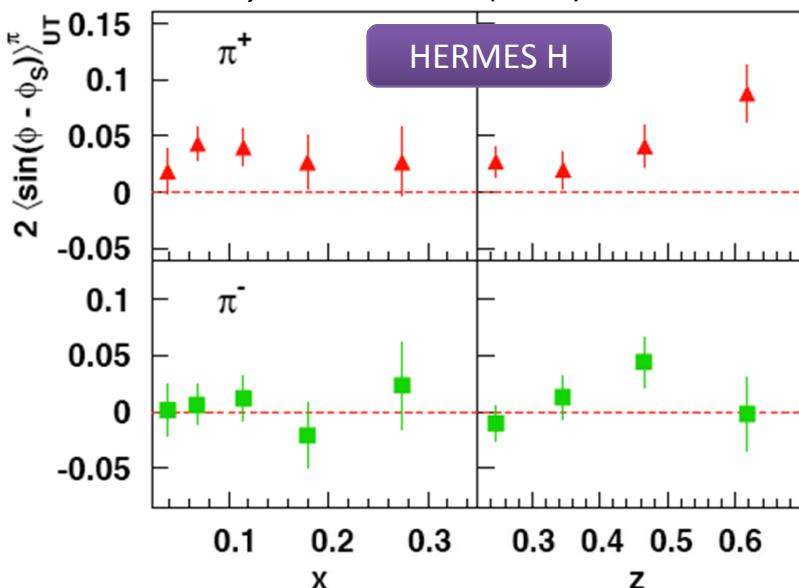
- Transv. pol. H target
- Limited statistics (2002-2003)
- **Non-zero Sivers amplitudes for  $\pi^+$**   
- Non-zero Sivers function !!

# Sivers function

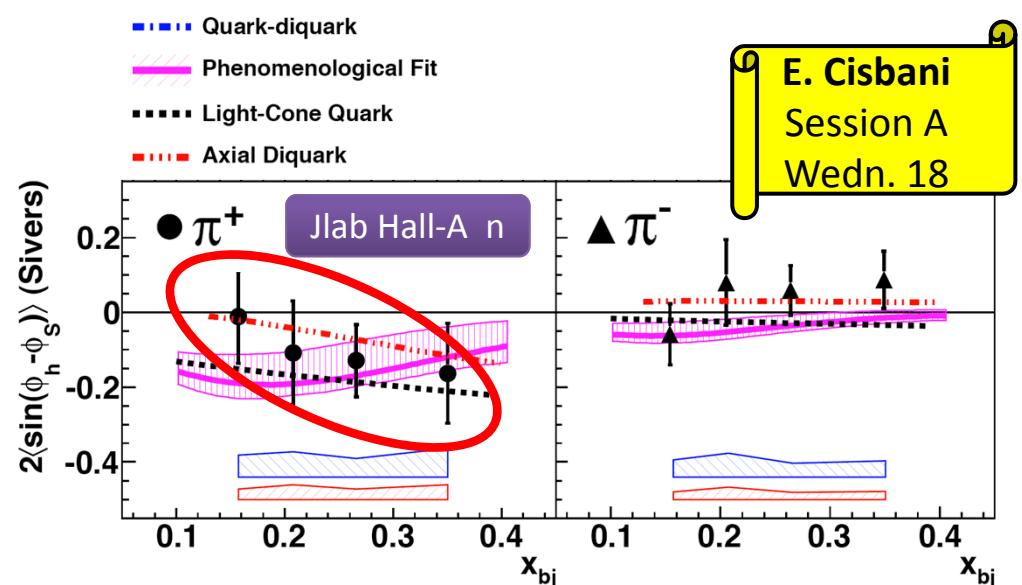
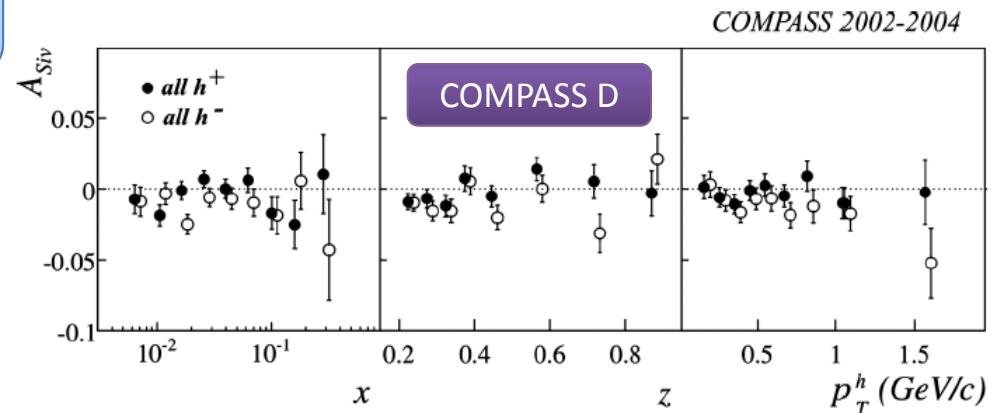
$$A_{UT}^{\sin(\phi + \phi_S)} \propto -\frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x, p_T^2) \otimes D_1^q(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)}$$

First evidence by HERMES (2005)

Phys. Rev. Lett. 94 (2005) 012002

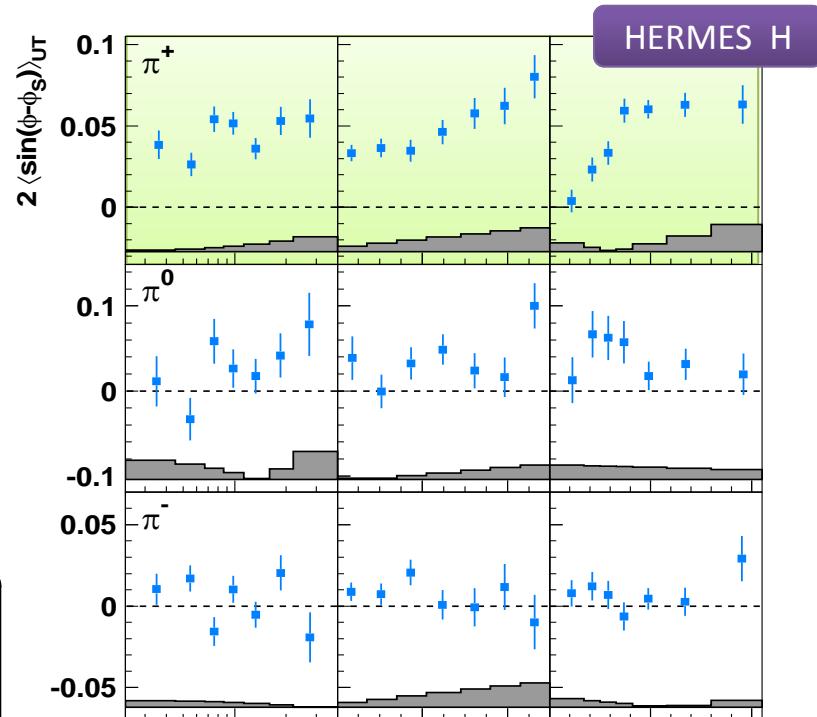
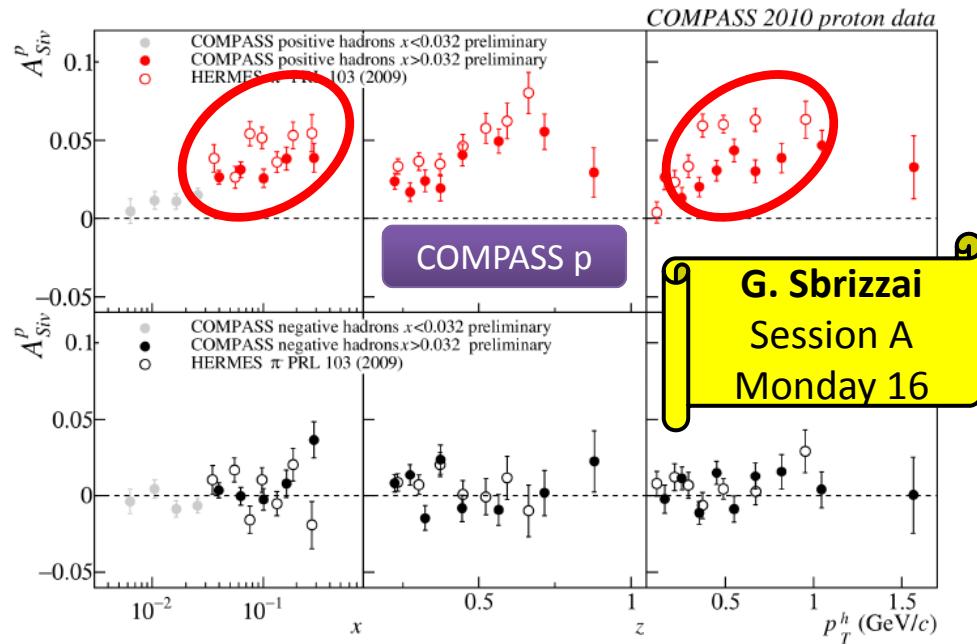


- Transv. pol. H target
- Limited statistics (2002-2003)
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- Non-zero Sivers function !!



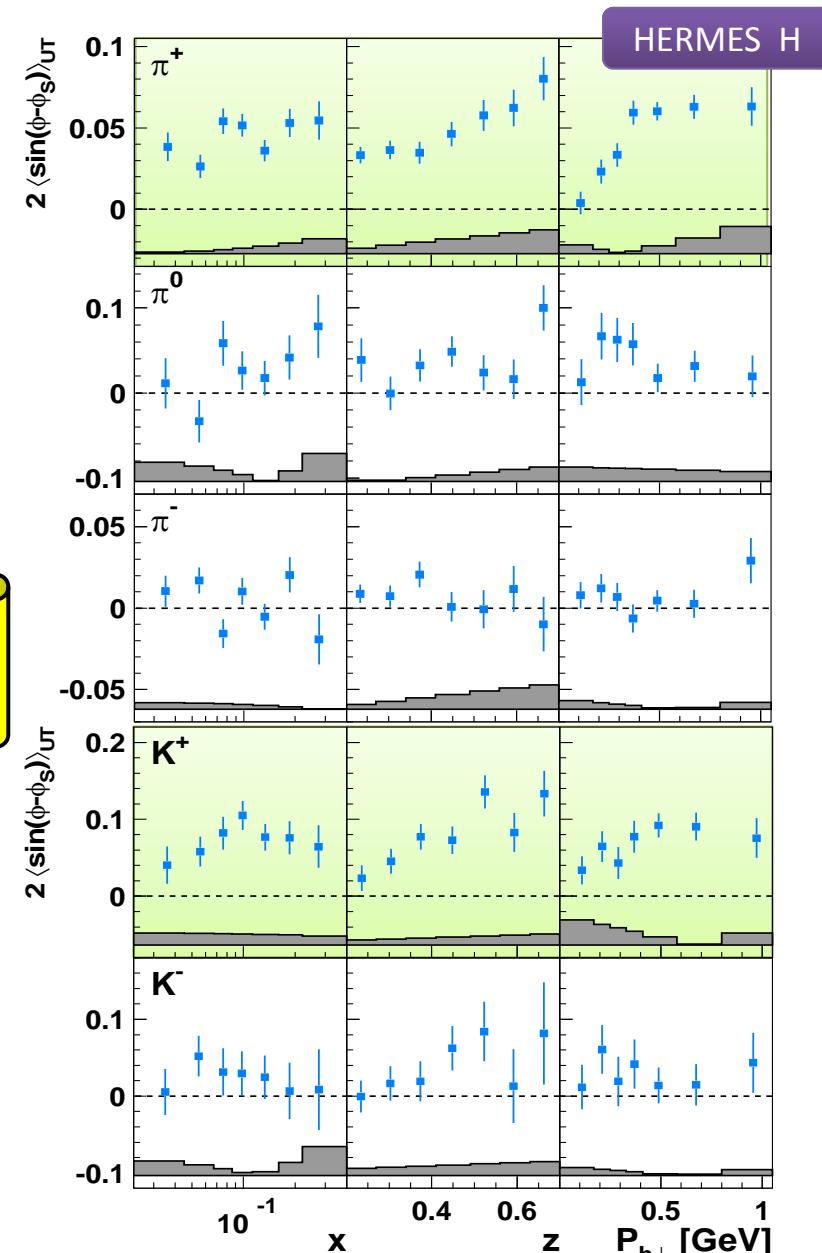
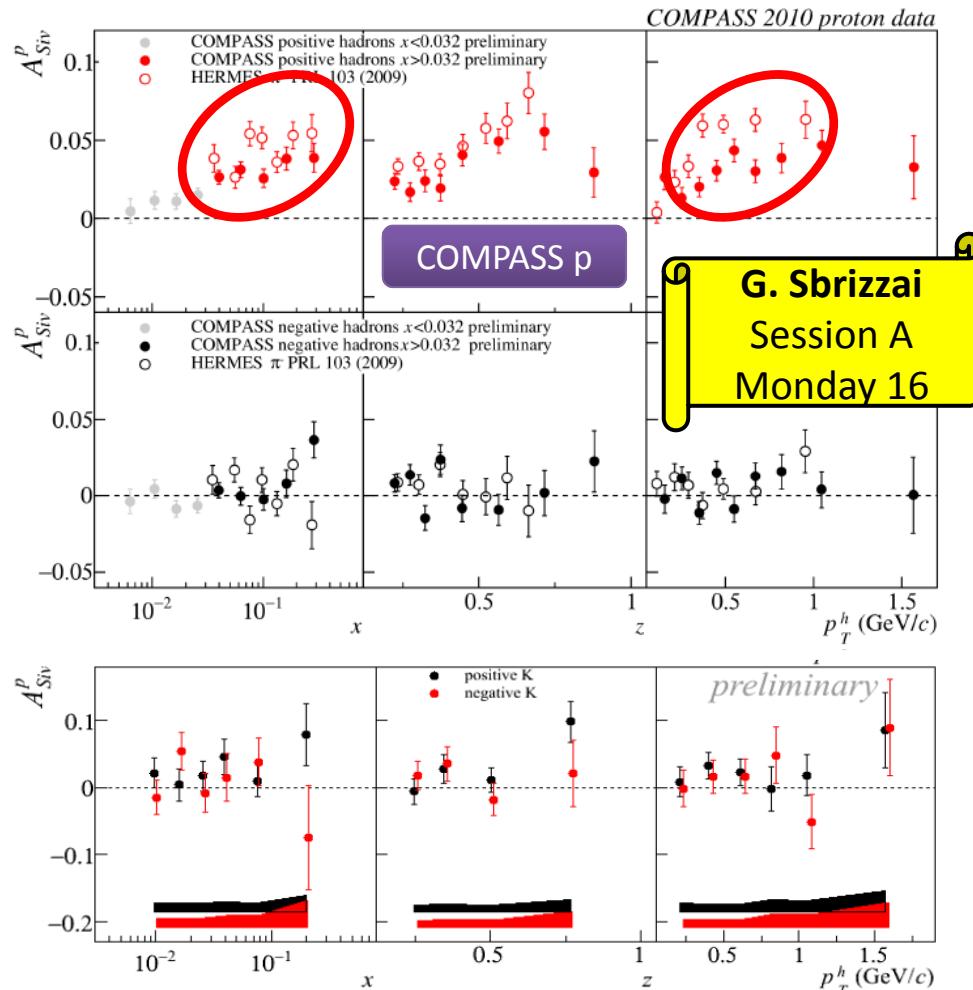
# Sivers function

- COMPASS amplitudes smaller than HERMES
- New studies: difference reduces substantially in the low  $y$  region ( $0.05 < y < 0.1$ )
- TMD evolution? (Anselmino)



# Sivers function

- COMPASS amplitudes smaller than HERMES
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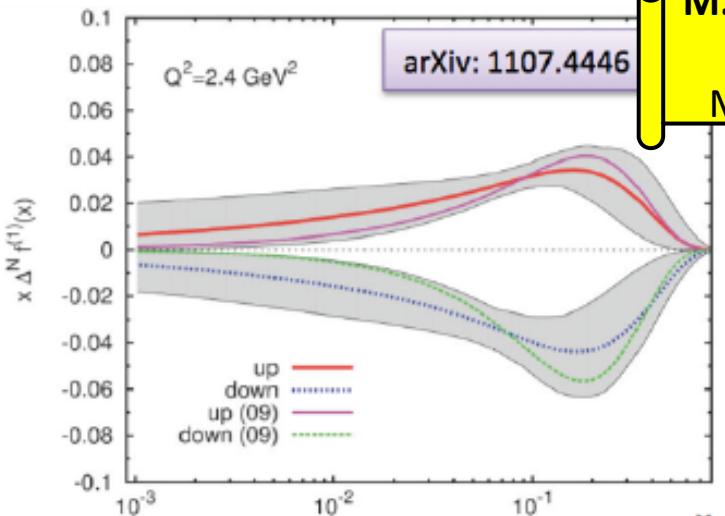
$\pi^+ \equiv |ud\rangle, K^+ \equiv |us\rangle \rightarrow$  role of sea quarks?

# Extracting the Sivers function

Torino

$$A_{UT}^{\sin(\Phi - \Phi_s)} \propto f_{1T}^\perp(x) \otimes D_1^q(z)$$

**Fit to COMPASS d, HERMES p + unpol FF**



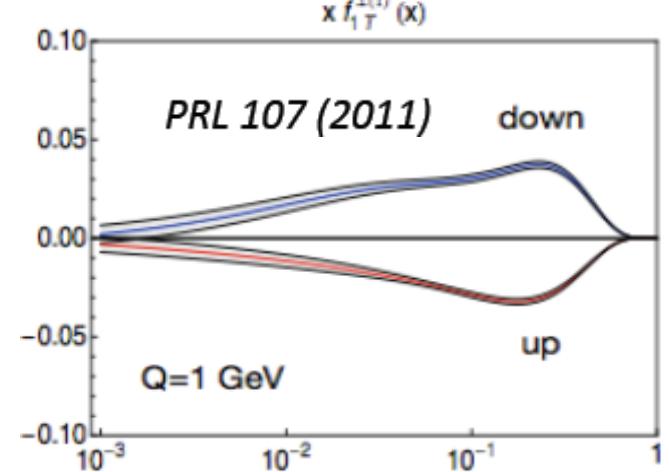
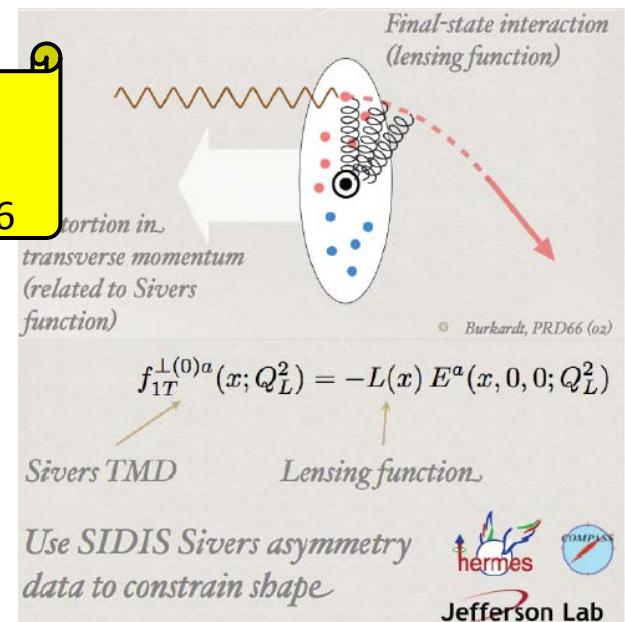
New approach: TMD evolution!

Extraction of Sivers through **Bessel weighting** avoids convolution!

L. Gamberg  
Session A  
Monday 16

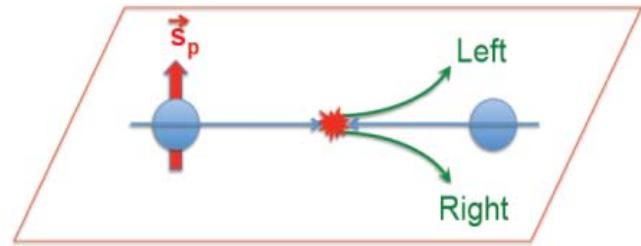
M. Radici  
Session A  
Monday 16

Pavia



Also put constraints on GPD  $E_q$  and quark OAM!!

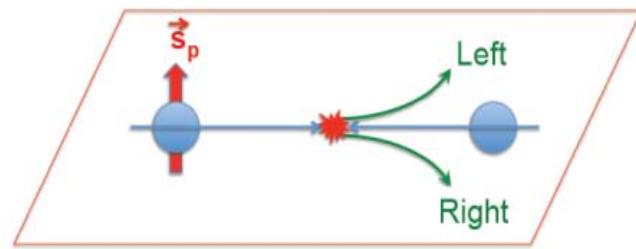
# Transversity & Sivers in pp scattering



$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

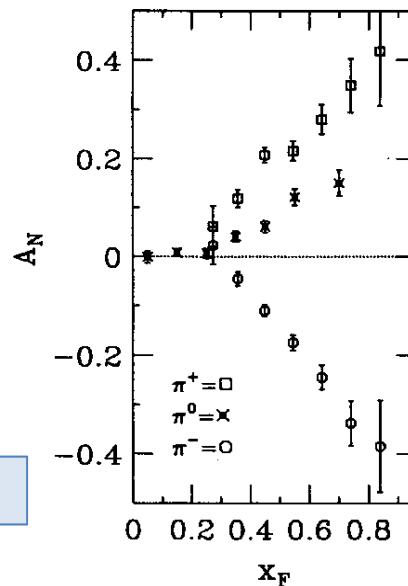
Naive (collinear) pQCD predicts  $A_N \approx 0$

# Transversity & Sivers in pp scattering



$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

Naive (collinear) pQCD predicts  $A_N \approx 0$



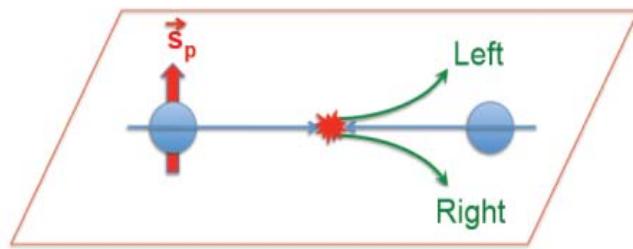
In '90s E-704 exp. @ Fermilab reported large  $A_N$  asymmetries (up to 40%!!)

$$x_F \sim \langle z \rangle P_{jet} / P_L$$

Possible explanations:

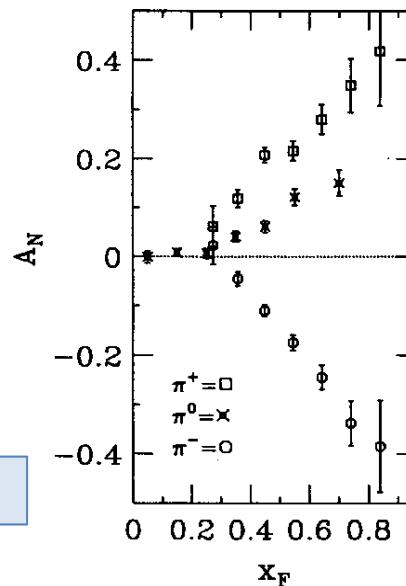
- **Collins effect** (Transv. x Collins FF)
- **Sivers effect** (orbital motion of quarks)
- **Twist-3 effects**
- Combination of above

# Transversity & Sivers in pp scattering



$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

Naive (collinear) pQCD predicts  $A_N \approx 0$

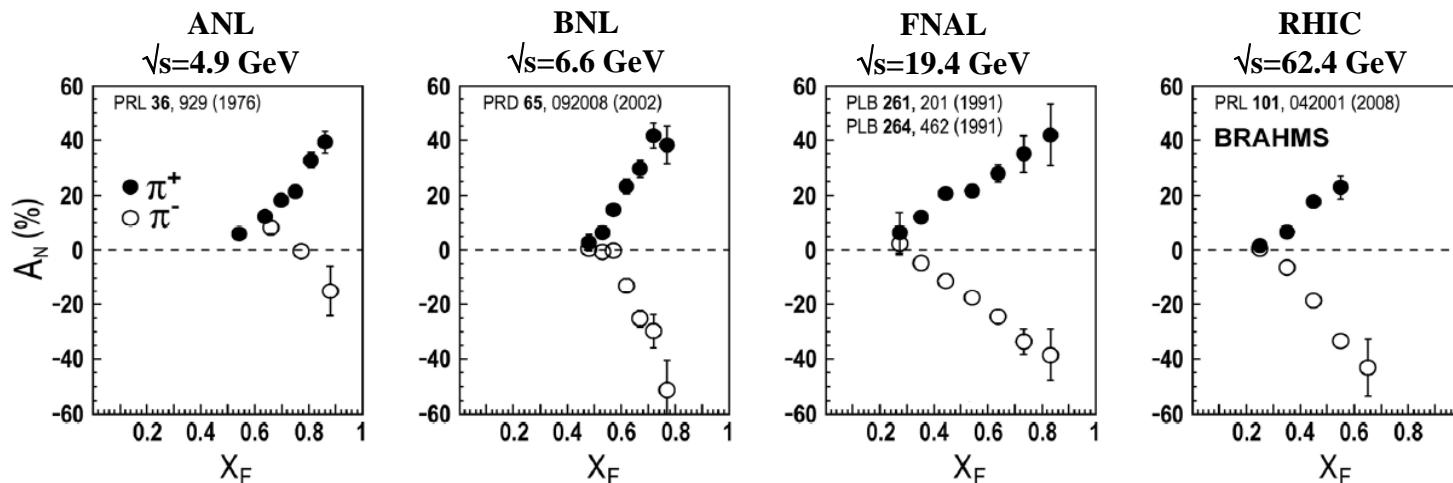


In '90s E-704 exp. @ Fermilab reported large  $A_N$  asymmetries (up to 40%!!)

$$x_F \sim \langle z \rangle P_{jet}/P_L$$

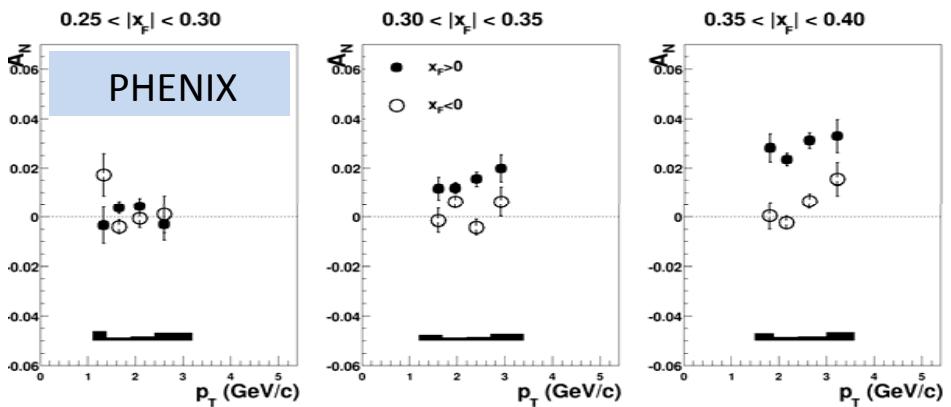
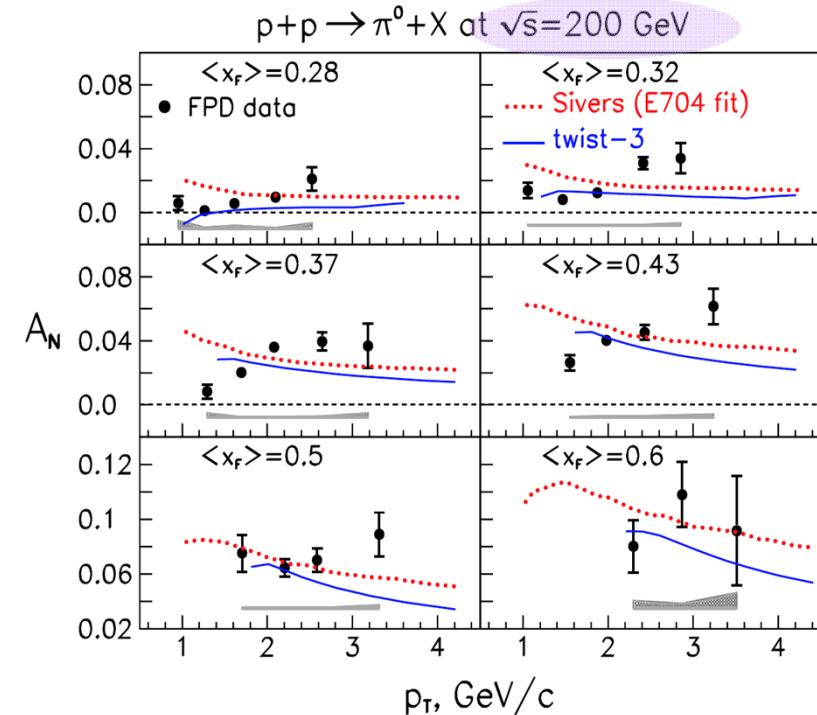
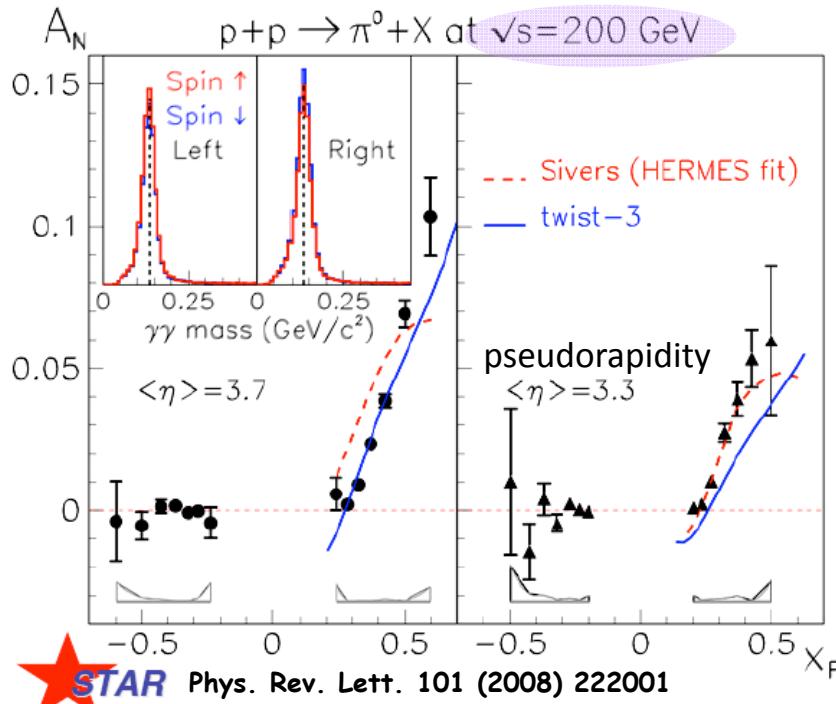
Possible explanations:

- **Collins effect** (Transv. x Collins FF)
- **Sivers effect** (orbital motion of quarks)
- **Twist-3** effects
- Combination of above



- Reproduced by various exps. over 35 years!
- **persistent with energy!**

# Transversity & Sivers in pp scattering: RHIC



- Large asymmetries measured by STAR, PHENIX and BRAHMS
- No strong dependence on  $\sqrt{s}$  from 19.4 to 200 GeV
- Admixture of Transversity, Sivers and Twist-3 effects?

## Boer-Mulders function: DY

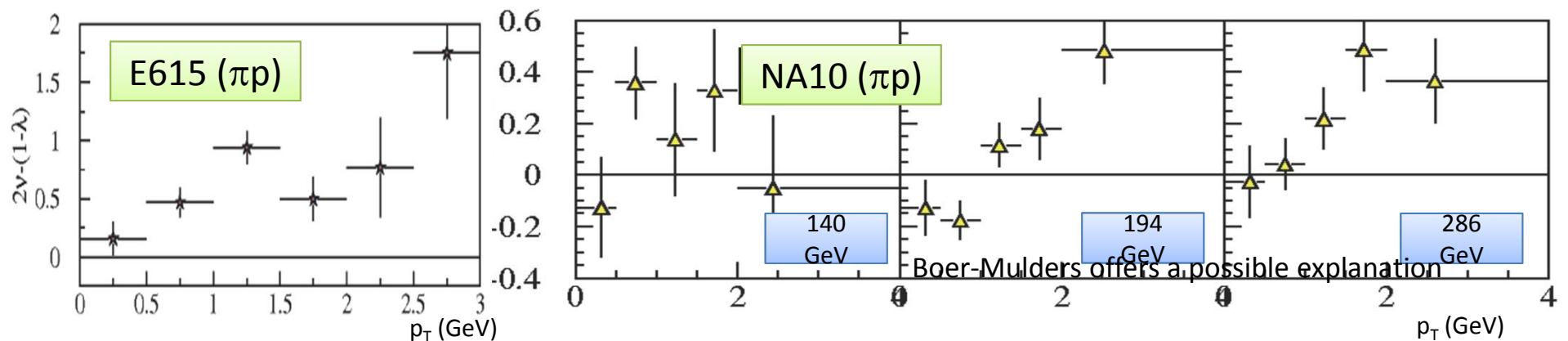
$$\frac{d\sigma^{hp \rightarrow e\bar{e}X}}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$

$$(1 - \lambda) - 2\nu = 0 \text{ Lam-Tung rel.}$$

# Boer-Mulders function: DY

$$\frac{d\sigma^{hp \rightarrow eeX}}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$

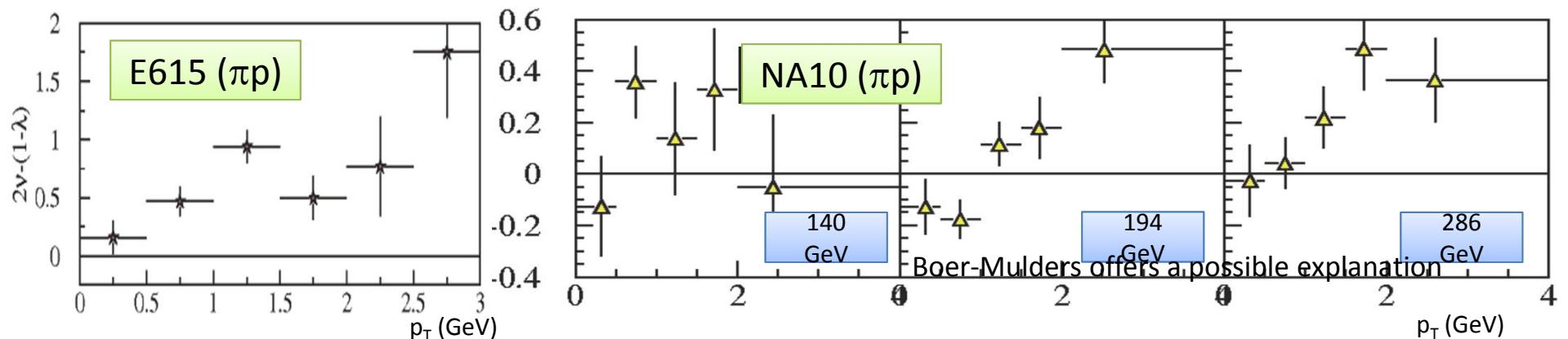
$(1 - \lambda) - 2\nu = 0$  Lam-Tung rel.  
violated in pion-induced DY!



# Boer-Mulders function: DY

$$\frac{d\sigma^{hp \rightarrow eeX}}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$

$(1 - \lambda) - 2\nu = 0$  Lam-Tung rel.  
violated in pion-induced DY!

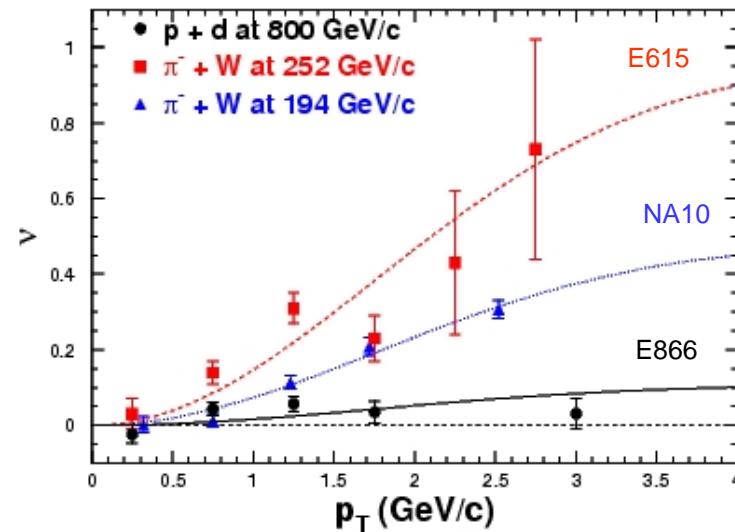


Naive parton model:

$$d\sigma/d\Omega \propto 1 + \cos^2 \theta \Rightarrow \nu \approx 0$$

Boer-Mulders effect offers an explanation:

$$\nu \approx h_{1q}^\perp \times h_{1\bar{q}}^\perp$$



# Boer-Mulders function: SIDIS

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right)$$

$$\left\{ \begin{array}{l} F_{UU,T} + \epsilon F_{UU,L} \\ + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \end{array} \right.$$

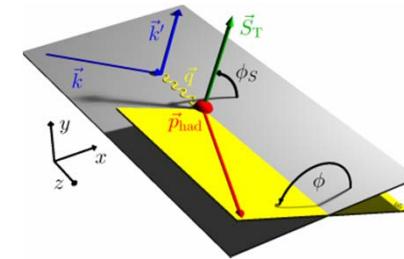
$$+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \quad \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{\text{UL}}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{\text{UL}}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{\text{LL}} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{\text{LL}}^{\cos(\phi)} \right]$$

$$+ S_T \left[ \sin(\phi - \phi_S) \left( F_{\text{UT,T}}^{\sin(\phi - \phi_S)} + \epsilon F_{\text{UT,L}}^{\sin(\phi - \phi_S)} \right) \right. \\ \left. + \epsilon \sin(\phi + \phi_S) F_{\text{UT}}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{\text{UT}}^{\sin(3\phi - \phi_S)} \right. \\ \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{\text{UT}}^{\sin(\phi_S)} \right. \\ \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{\text{UT}}^{\sin(2\phi - \phi_S)} \right]$$

$$+ S_T \lambda_l \left[ \sqrt{1 - \epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \sqrt{2\epsilon(1 - \epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} + \sqrt{2\epsilon(1 - \epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right] \right\}$$



Describes correlation between quark transverse momentum and transverse spin in unpolarized nucleon

## Distribution Functions

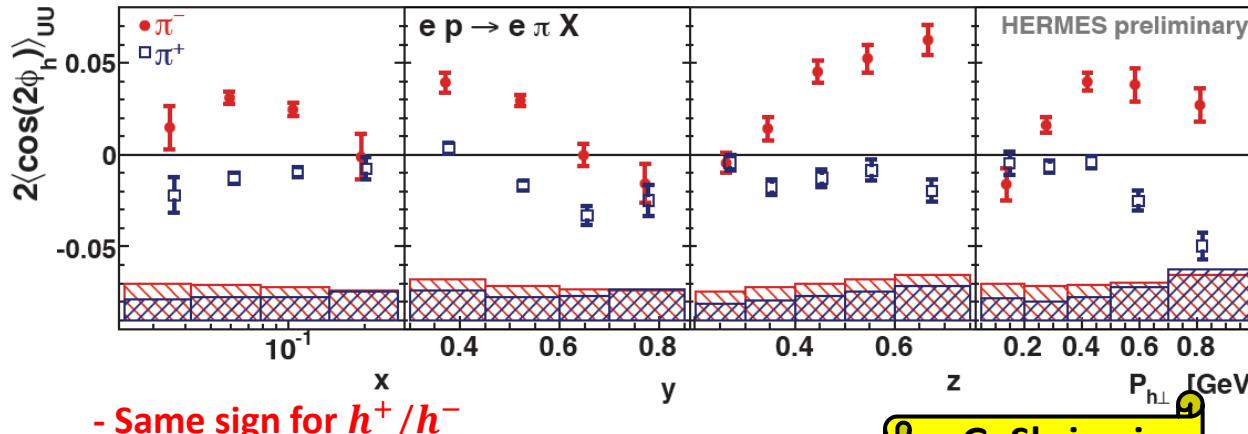
		quark		
		U	L	T
nucleon	U	$f_1$		$h_1^\perp$ -
	L		$g_1$ -	$h_{1L}^\perp$ -
	T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_{1T}^\perp$

## Fragmentation Functions

	quark		
	U	L	T
h	U	$D_1$	
			$H_1^\perp$  - 

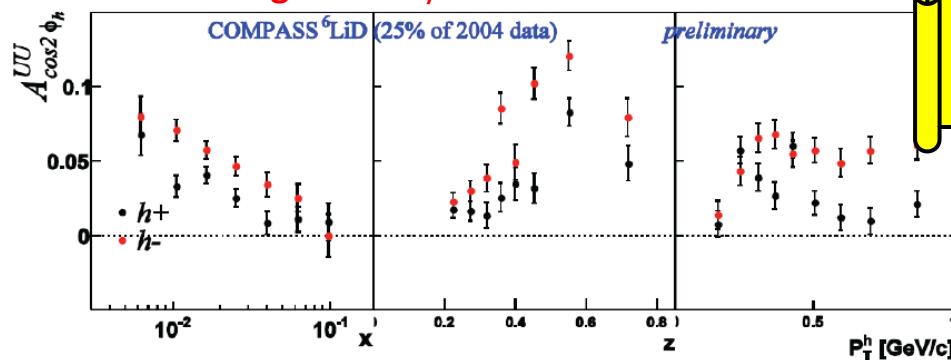
# Boer-Mulders function: SIDIS ( $\cos 2\phi$ )

$$\sigma_{UU}^{\cos(2\phi)} \propto h_1^\perp \otimes H_1^\perp + [f_1 \otimes D_1 + \dots] / Q^2$$



- Opposite signs for  $\pi^+/\pi^-$
- Similar results on H & D target  
→ same sign for u and d?
- $K^+/K^-$  amplitudes are larger and of same sign

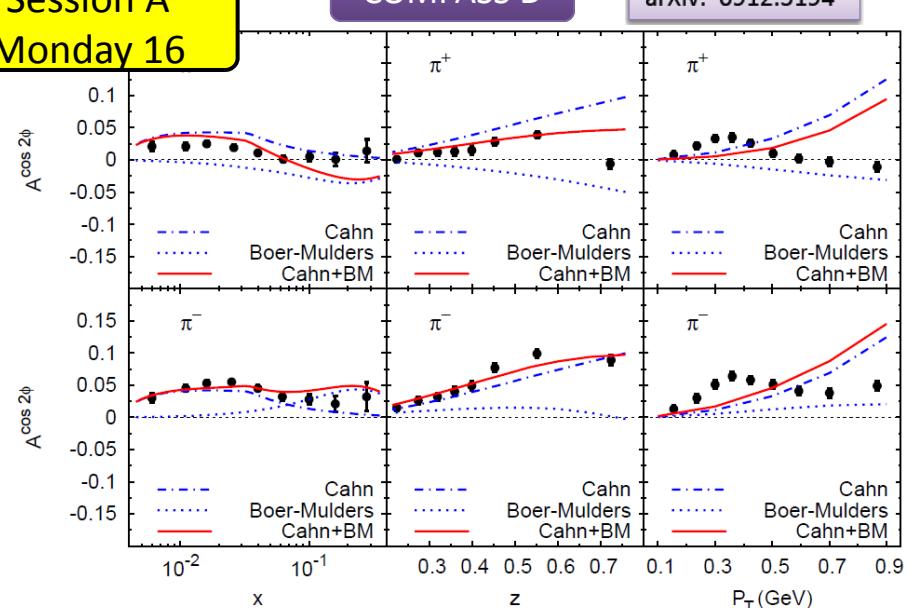
- Same sign for  $h^+/h^-$



G. Sbrizzai  
Session A  
Monday 16

COMPASS D

arXiv: 0912.5194



- Amplitudes are significant
- Clear evidence of BM effect
- Strong dependence on kinematic variables
- Issue on data consistency for  $h^+$
- New 2-dim extraction from COMPASS!
- Discrepancy with theory: uncert. on Chan + HT

# Worm-gear $h^{\perp}_{1L}$

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{array}{l} \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\ \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{array} \right.$$

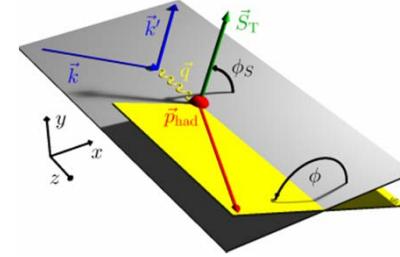
$$+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$\begin{aligned} + S_T & \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \end{aligned}$$

$$\begin{aligned} + S_T \lambda_l & \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \} \end{aligned}$$



Describes the probability to find transversely polarized quarks in a longitudinally polarized nucleon

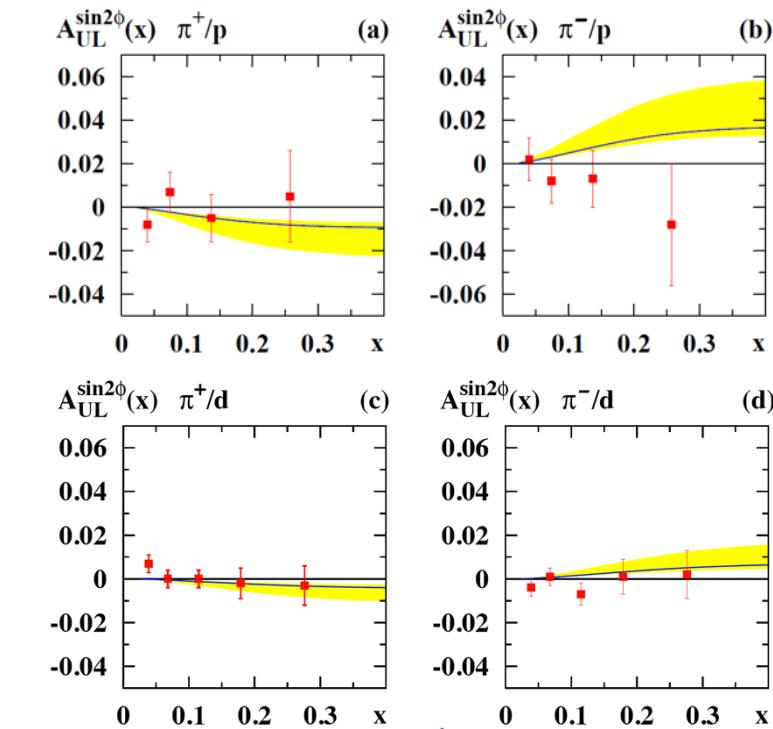
## Distribution Functions

		quark		
		U	L	T
n	U	$f_1$		
u	L			
c	T			
u	U	$f_{1T}^{\perp}$		
c	L			
e	T	$f_{1T}^{\perp}$		
n	U	$g_1$		
u	L			
c	T	$g_{1T}^{\perp}$		
u	U	$h_1$		
c	L			
e	T	$h_{1T}^{\perp}$		

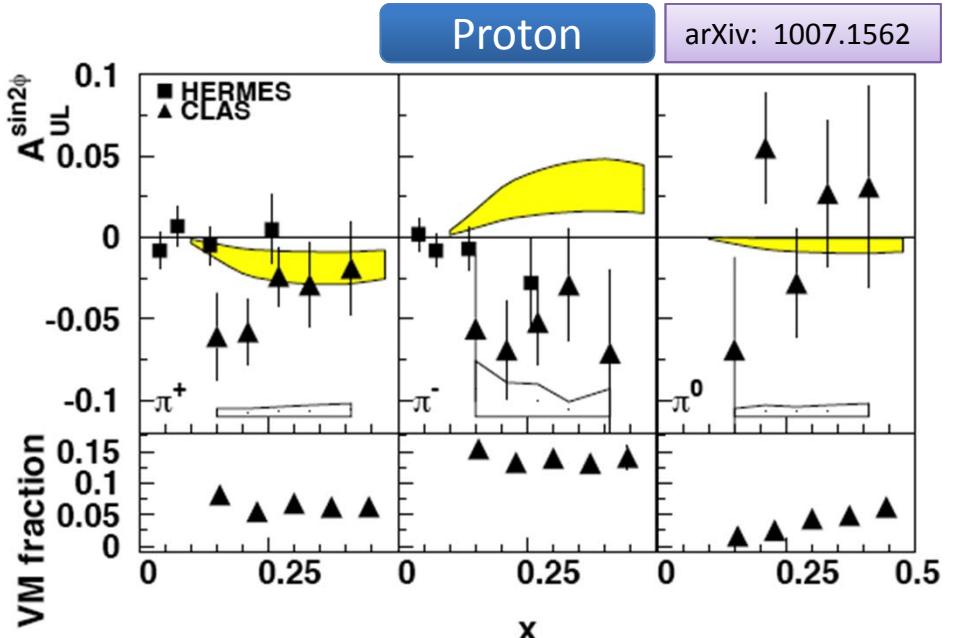
## Fragmentation Functions

		quark		
		U	L	T
h	U	$D_1$		

# Worm-gear $h^\perp_{1L}$

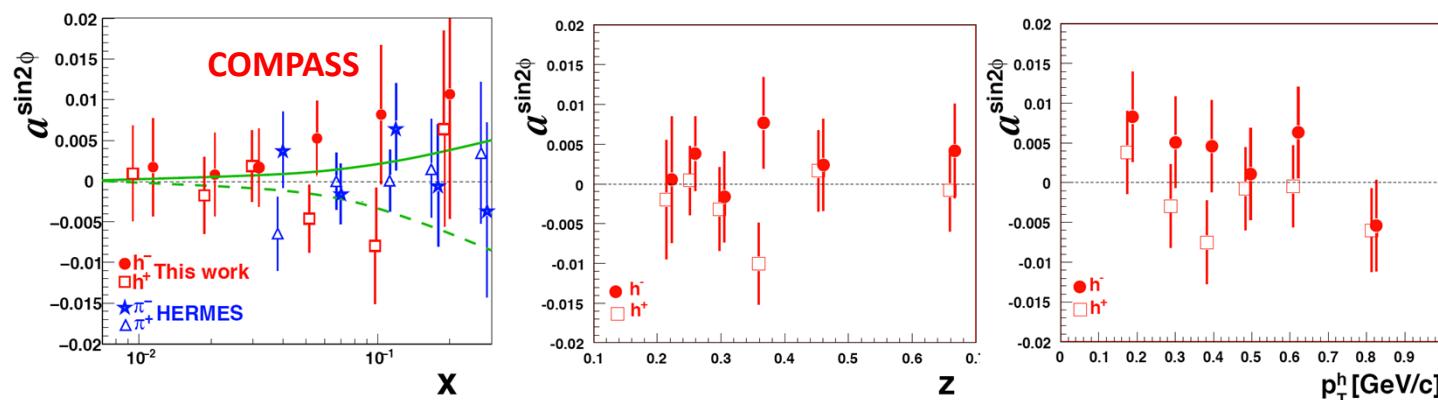


arXiv: 0902.0689



arXiv: 1007.1562

- Consistent with zero @ HERMES & COMPASS
- Significant amplitude @ CLAS!



Deuteron

arXiv: 1003.4549

G. Sbrizzai  
Session A  
Monday 16

# Worm-gear $g^{\perp}_{1T}$

$$\begin{aligned}
 \frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \\
 \left\{ \begin{aligned} & \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \\ + \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \right. \\ + S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\ + S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\ + S_T \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \\ + S_T \lambda_l \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \end{aligned} \right\}
 \end{aligned}$$

Describes the probability to find longitudinally polarized quarks in a transversely polarized nucleon!

- The only TMD that is both **chiral-even** and **naïve-T-even**
- requires interference between wave funct. components that differ by 1 unit of OAM

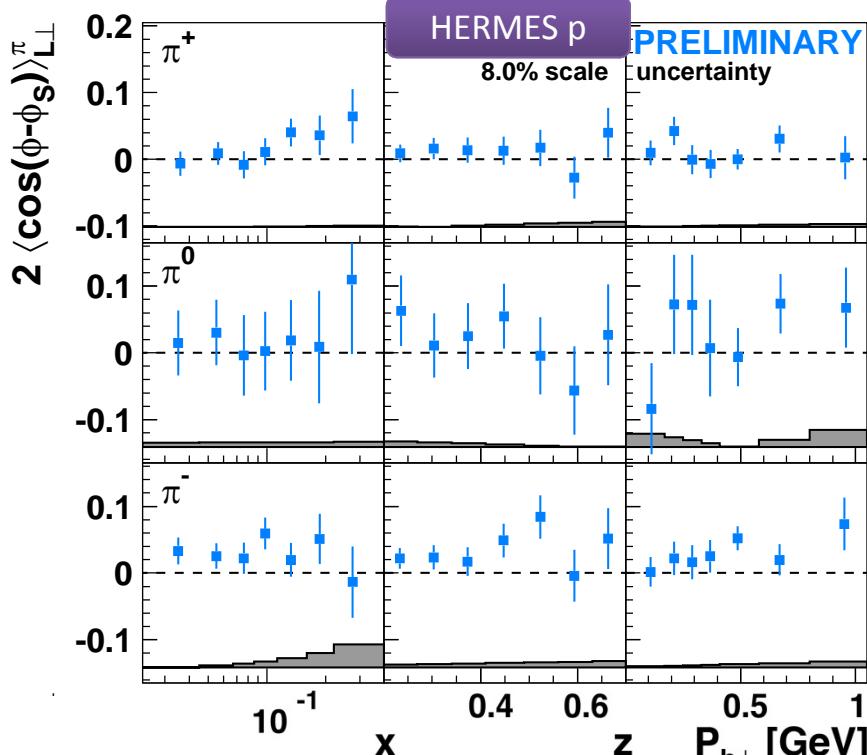
Distribution Functions

		quark		
		U	L	T
nucleon	U	$f_1$		$h_1^\perp$
	L		$g_1$ -	$h_{1L}^\perp$
	T	$f_{1T}^\perp$ -		$h_1$ -
	T		$g_{1T}^\perp$ -	$h_{1T}^\perp$ -

Fragmentation Functions

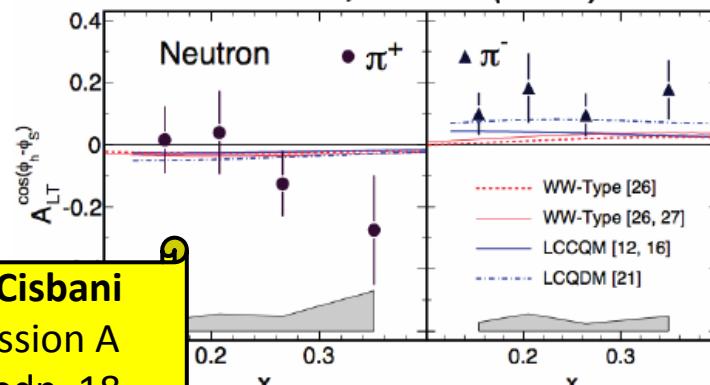
		quark		
		U	L	T
h	U	$D_1$		$H_1^\perp$

# Worm-gear $g_{1T}^{\perp}$

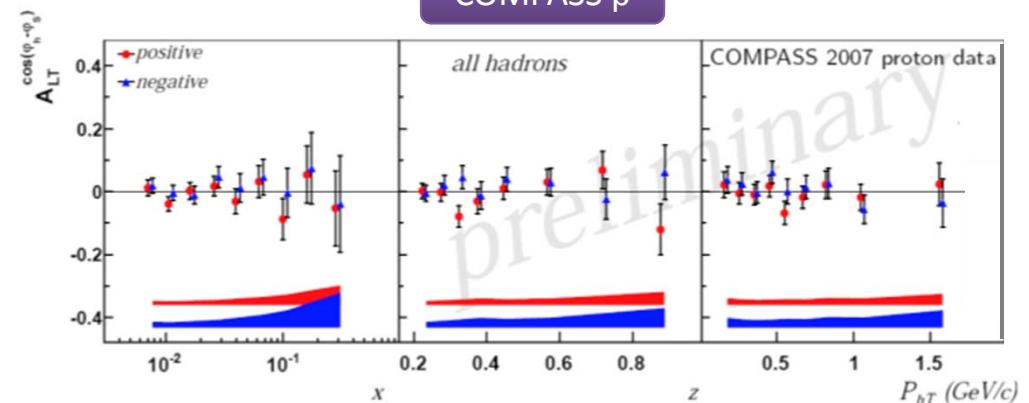


Jlab Hall-A n

Jlab n, PRL108(2012)



E. Cisbani  
Session A  
Wedn. 18



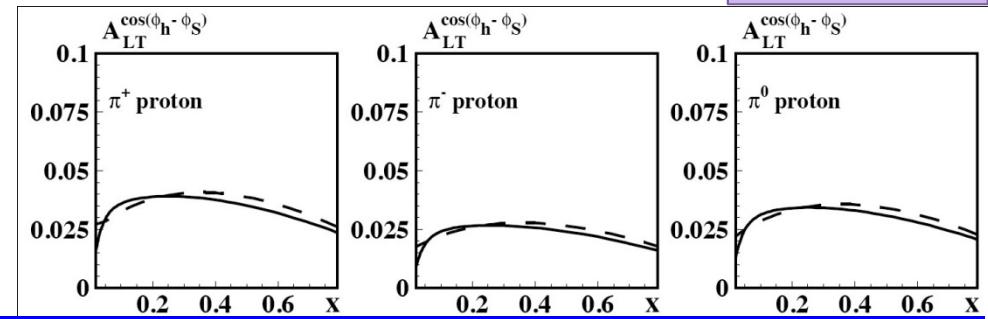
Statistics not enough to investigate relations supported by many theoretical models:

$$g_{1T}^q = -h_{1L}^{\perp q} \quad (\text{supported by Lattice QCD and first data})$$

$$g_{1T}^{q(1)}(x) \stackrel{WW\text{-type}}{\approx} x \int_x^1 \frac{dy}{y} g_1^q(y) \quad (\text{Wandura-Wilczek type approximation})$$

From constituent quark model:

arXiv: 0903.1271



# Pretzelosity

$$\begin{aligned}
 \frac{d\sigma^h}{dx dy d\phi_S dz d\phi dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \\
 \left\{ \begin{aligned} & \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \\ + \lambda_l & \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\ + S_L & \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\ + S_L \lambda_l & \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\ + S_T & \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \\ + S_T \lambda_l & \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \end{aligned} \right\}
 \end{aligned}$$

Describes correlation between quark transverse momentum and transverse spin in a transversely pol. nucleon

- Sensitive to the D-wave component
- Sensitive to **non-spherical shape** of the nucleon
- Kinematically suppressed w.r.t. Collins and Sivers by a factor  $P_{h\perp}^{-2}$

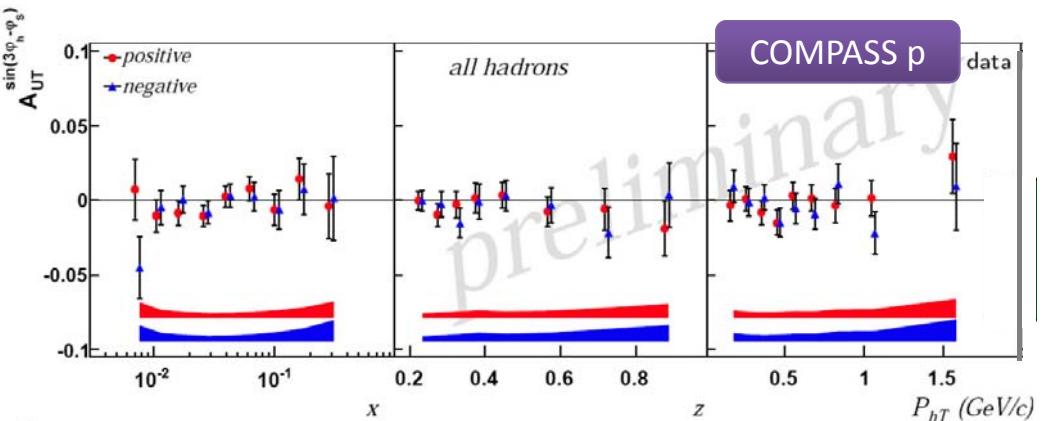
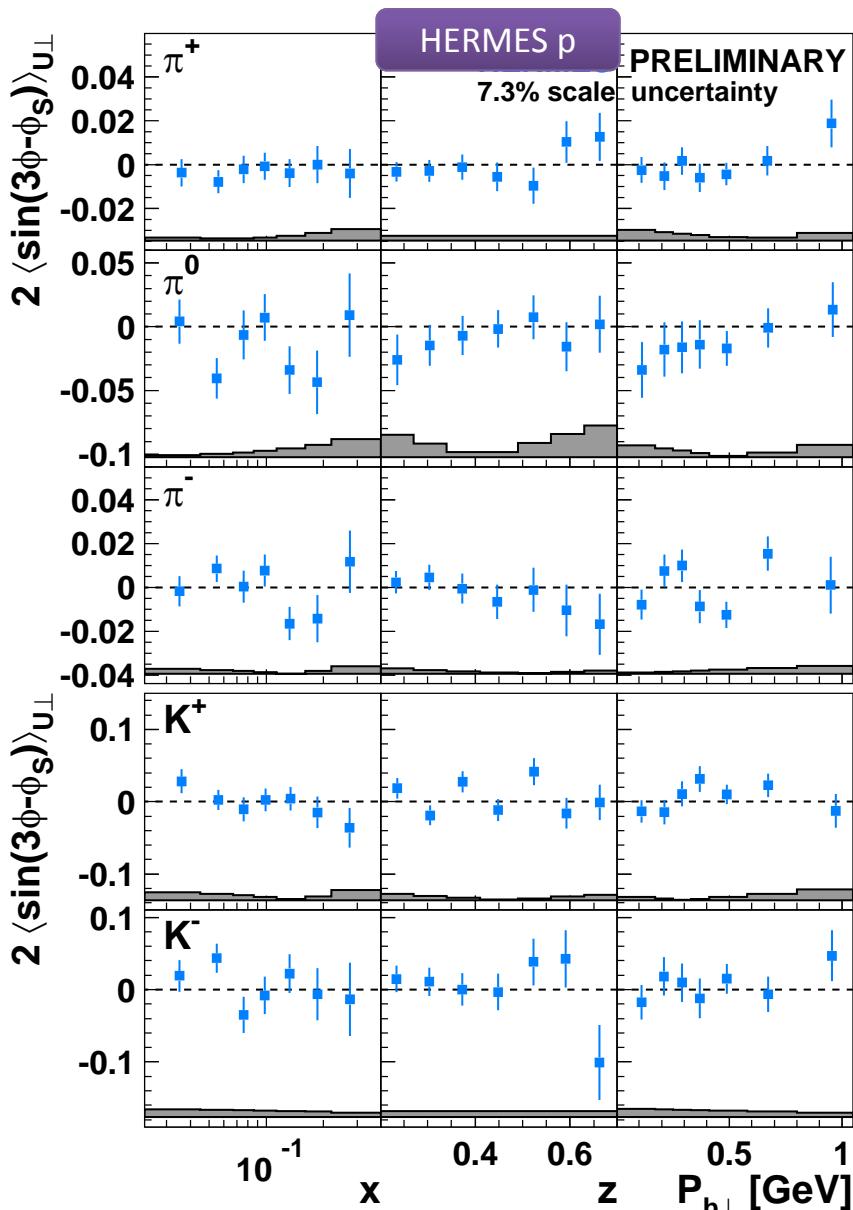
## Distribution Functions

		quark		
		U	L	T
nucleon	U	$f_1$		$h_1^\perp$
	L		$g_1$	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_{1T}$

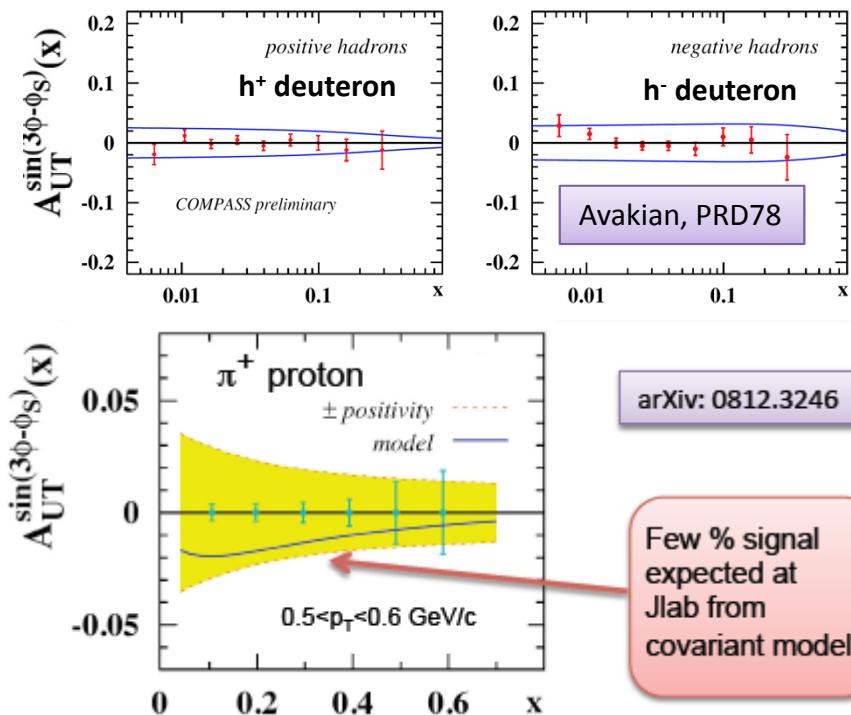
## Fragmentation Functions

		quark		
		U	L	T
h	U	$D_1$		$H_1^\perp$

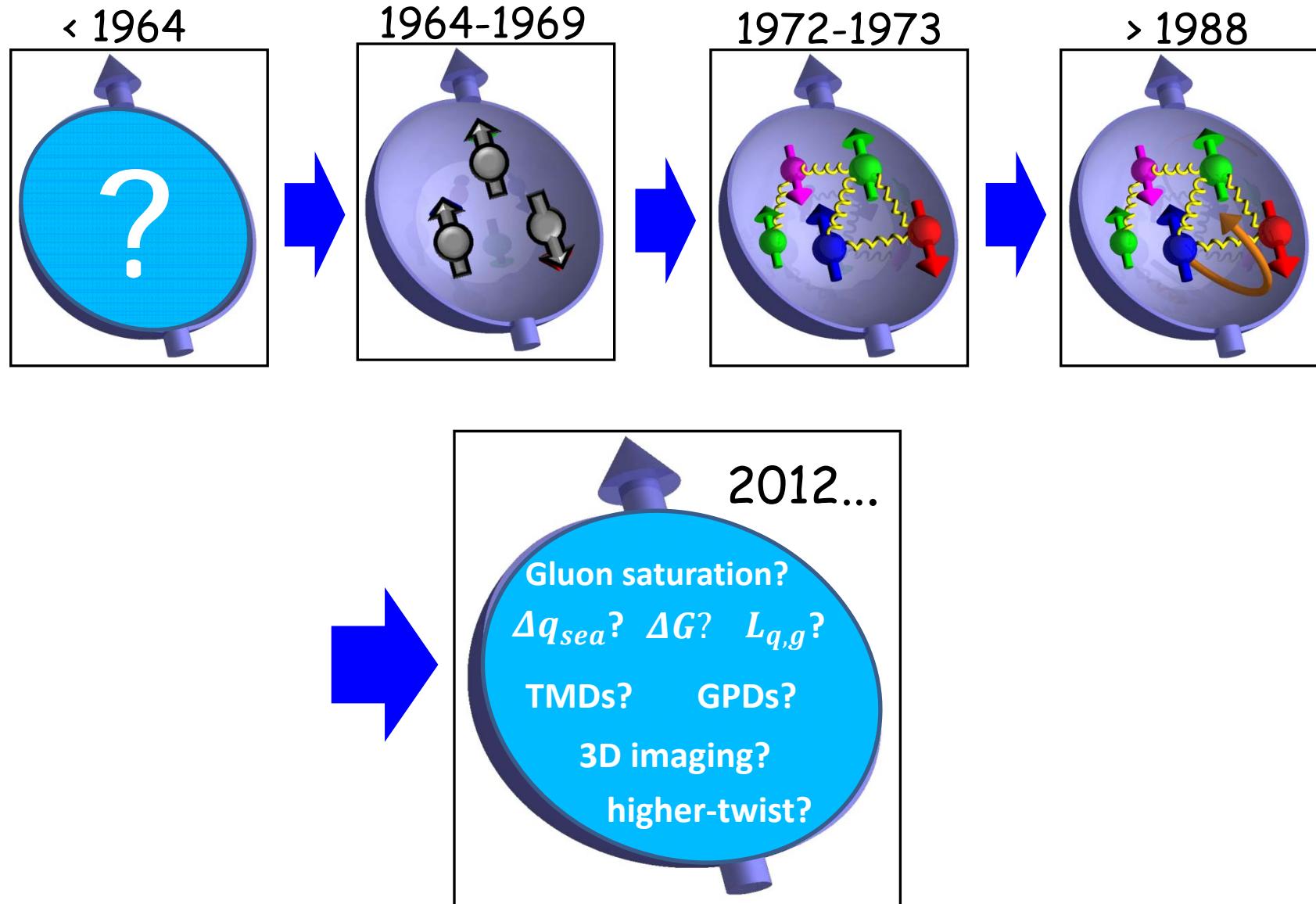
# Pretzelosity



- Proton and deuteron data consistent with zero
- Statistical power of existing data is not enough to observe significant signals



# Looking deeply into the proton



# The treasure map!

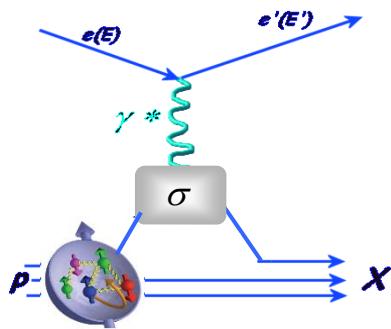


# Back-up

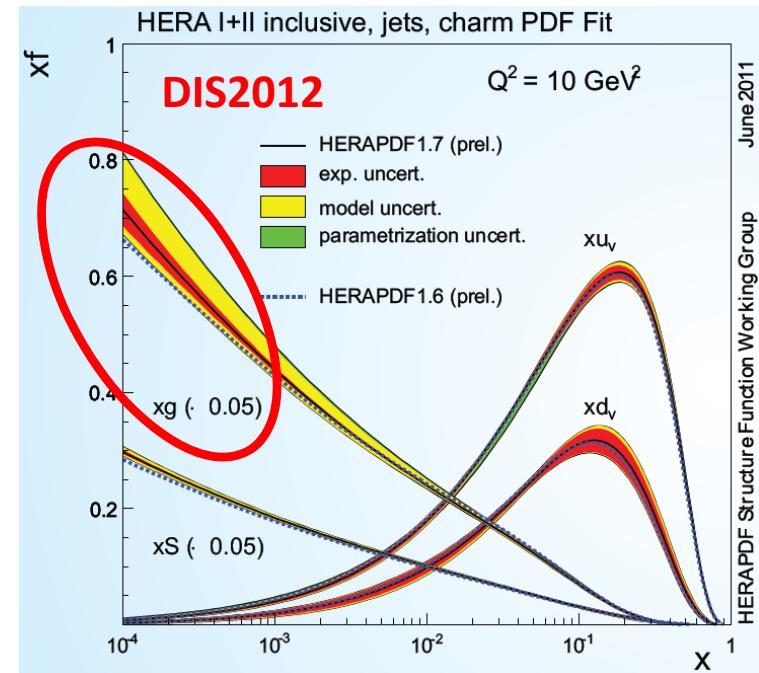
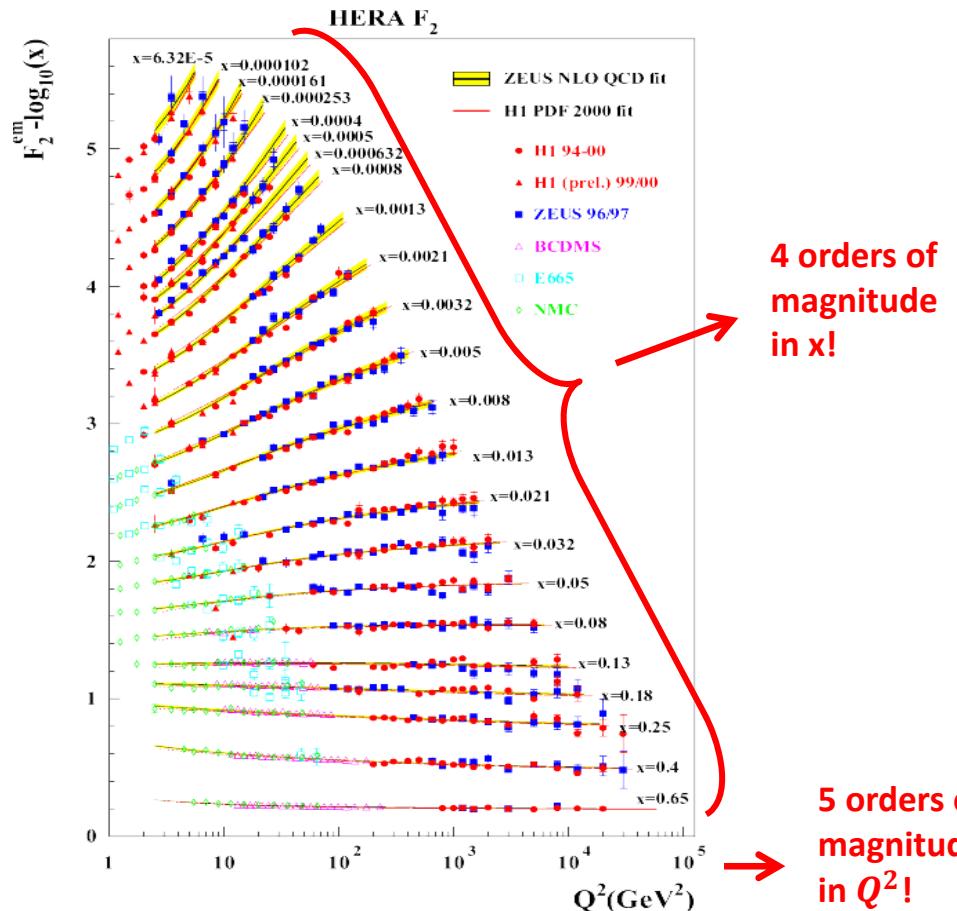
# The nucleon collinear structure: momentum

	U	L	T
U	$f_1$		
L			
T			

$$f_1(x) = q^+(x) + q^-(x)$$



- 40 years of **inclusive DIS** experiments
- fixed target exp. & HERA collider
- **very precise data!**
- wide kinematic range in  $x$  and  $Q^2$
- prediction of  $Q^2$  dependence → triumph of QCD!

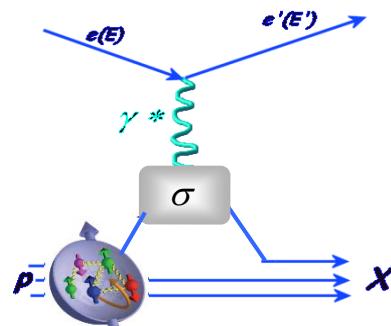


very good knowledge of longitudinal momentum structure of the nucleon!  
...but gluons should saturate!

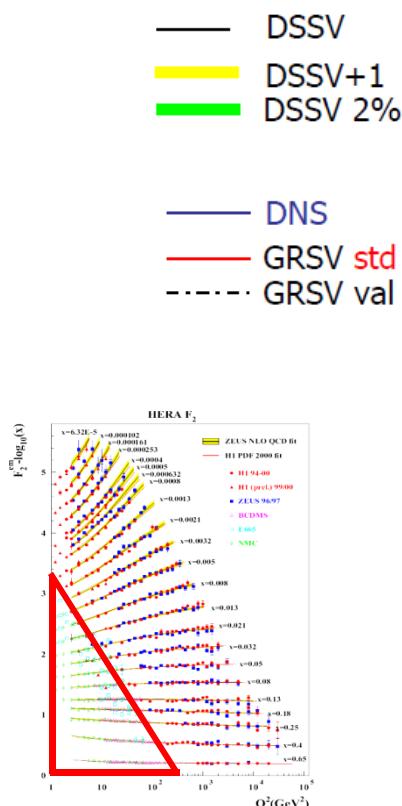
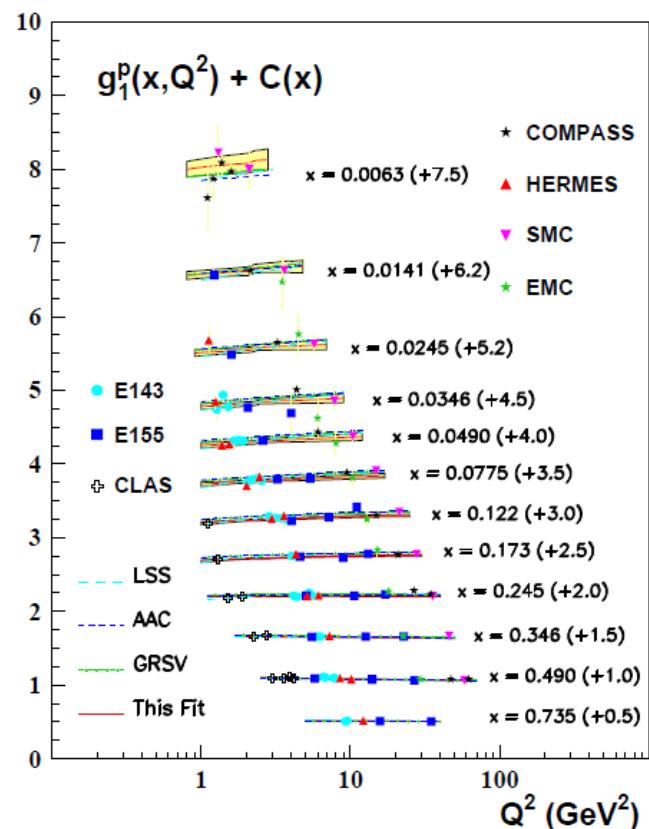
# The nucleon collinear structure: helicity

	U	L	T
U			
L		$g_{1L}$	
T			

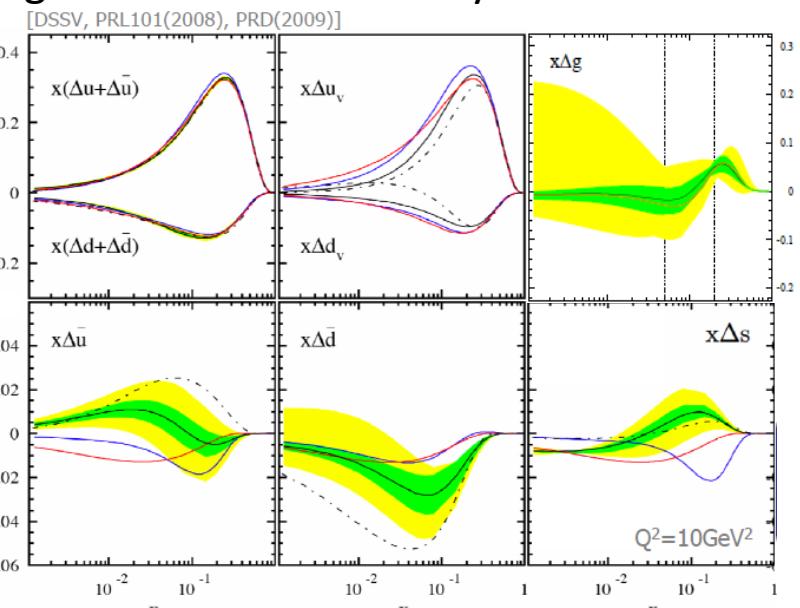
$g_1(x) = q^+(x) - q^-(x)$



- **inclusive DIS** (+ semi-inclusive → flavor tagging)
- Fixed target experiments
- polarized beams and target → challenging
- relatively limited kinematic coverage in  $x$  and  $Q^2$



global NLO fit of helicity distributions



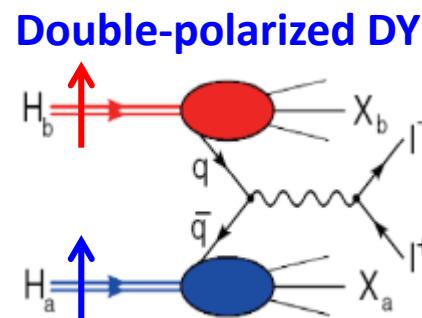
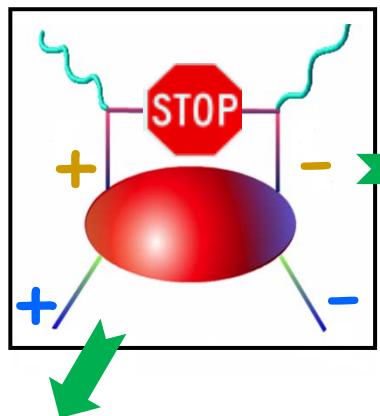
good knowledge of helicity distrib. of valence quarks  
Sea quarks and gluon helicity still poorly constrained...

# The nucleon collinear structure: transversity

	U	L	T
U			
L			
T			$h_1$

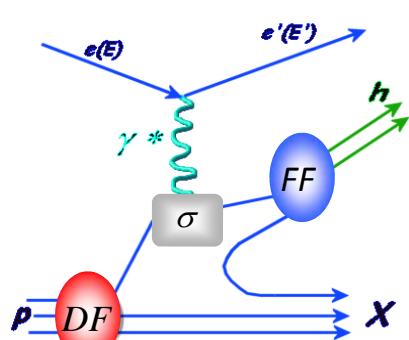
$h_1(x) = q^\uparrow(x) - q^\downarrow(x)$

Chiral-odd!!!



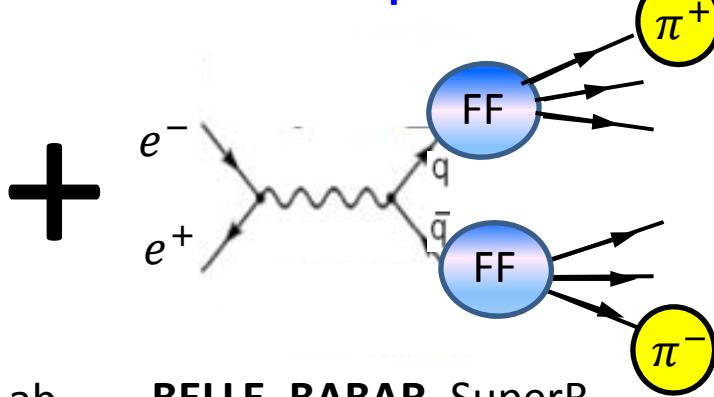
Golden process!  
...but challenging:  
- very small X-section  
- huge background  
- polarizing  $\bar{p}$  beams  
Future exp. RHIC, CERN, FAIR

Semi-Inclusive DIS



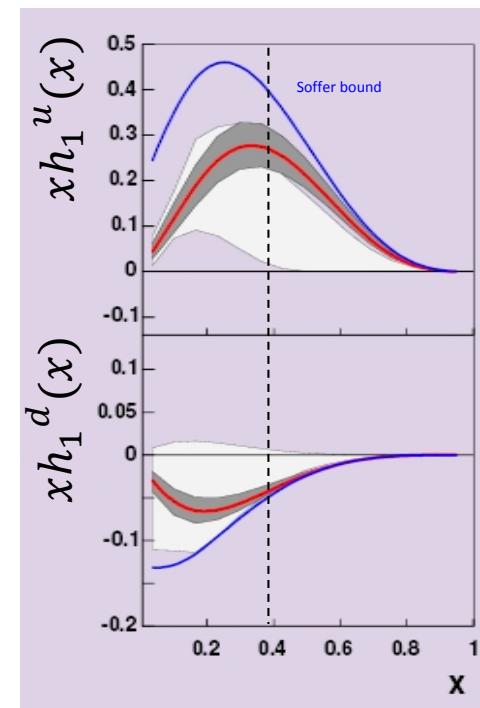
HERMES, COMPASS, JLab,...

Inclusive hadron-pair in  $e^+e^-$

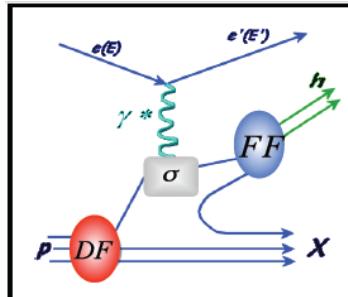


BELLE, BABAR, SuperB,...

- Very recent: first evidence (2005), first extraction (2007)!
- limited coverage in  $x$  ( $< 0.4$ )
- more data in valence region (JLab) to constrain tensor charge
- sea quark transversity completely unconstrained



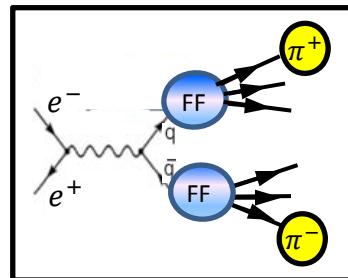
# Reactions and experiments



SIDIS: rich phenomenology, the most explored so far

SIDIS

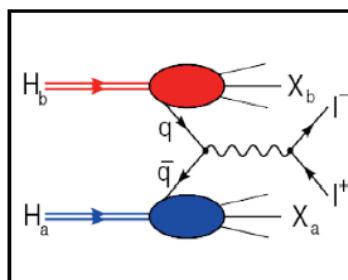
$$\sigma^{ep \rightarrow ehX} = \sum_q (DF) \otimes \sigma^{eq \rightarrow eq} \otimes (FF)$$



$e^+e^-$ : B-factories as powerful fragmentation laboratories

$e^+e^-$

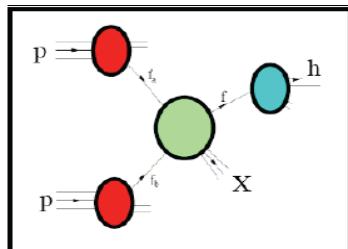
$$\sigma^{ee \rightarrow hhX} = \sum_q \sigma^{qq \rightarrow ee} \otimes (FF) \otimes (FF)$$



DY: challenging for experiments (only unpolarized so far)

DY

$$\sigma^{pp \rightarrow eeX} = \sum_q (DF) \otimes (DF) \otimes \sigma^{qq \rightarrow ee}$$



Hadron reactions: challenging for theory (ISI + FSI)

pp

$$\sigma^{pp \rightarrow hX} = \sum_q (DF) \otimes (DF) \otimes \sigma^{qq \rightarrow qq} \otimes (FF)$$



# Boer-Mulders function: SIDIS

$$\frac{d^5\sigma}{dx dy dz d\phi_h dP_{h\perp}} \propto \left\{ F_{UU,T} + \varepsilon F_{UU,L} + 2\sqrt{\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos 2\phi_h F_{UU}^{\cos 2\phi_h} \right\}$$

Twist-2:  $d\sigma_{UU}^{\cos 2\phi} \propto \cos 2\phi \cdot \sum_q e_q^2 I \left[ \frac{2(\hat{P}_{h\perp} \cdot \vec{k}_T)(\hat{P}_{h\perp} \cdot \vec{p}_T) - \vec{k}_T \cdot \vec{p}_T}{MM_h} h_1^\perp H_1^{\perp q} \right]$

Boer & Mulders PRD 57 (1998)

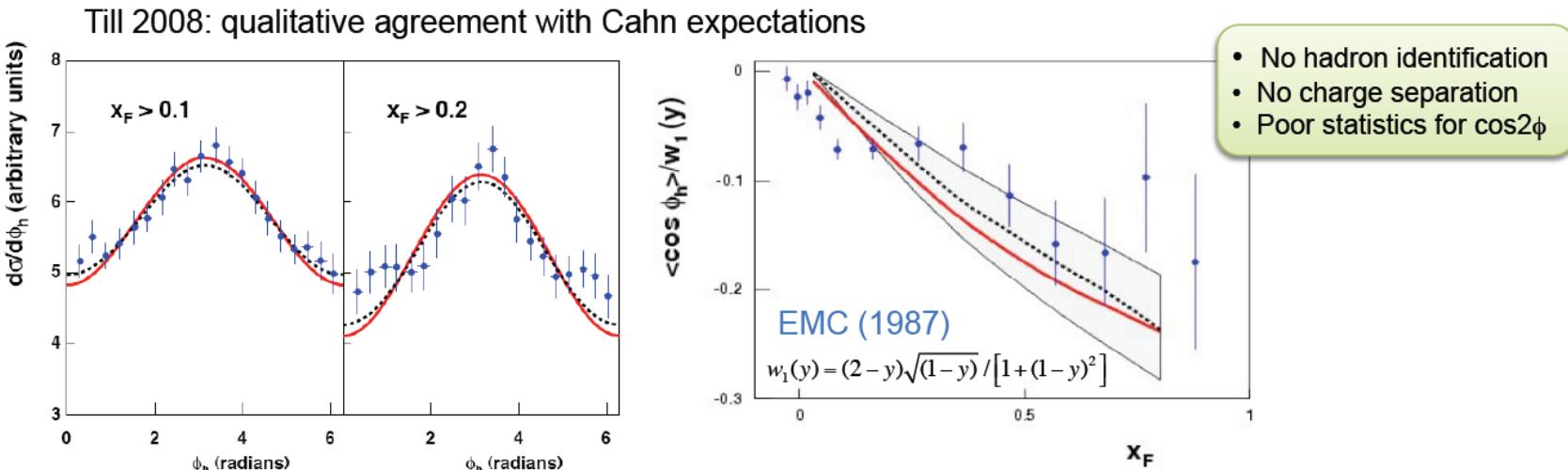
**Boer-Mulders effect**

Pure kinematic effect due to transverse momentum of partons in the nucleon

**Cahn effect**

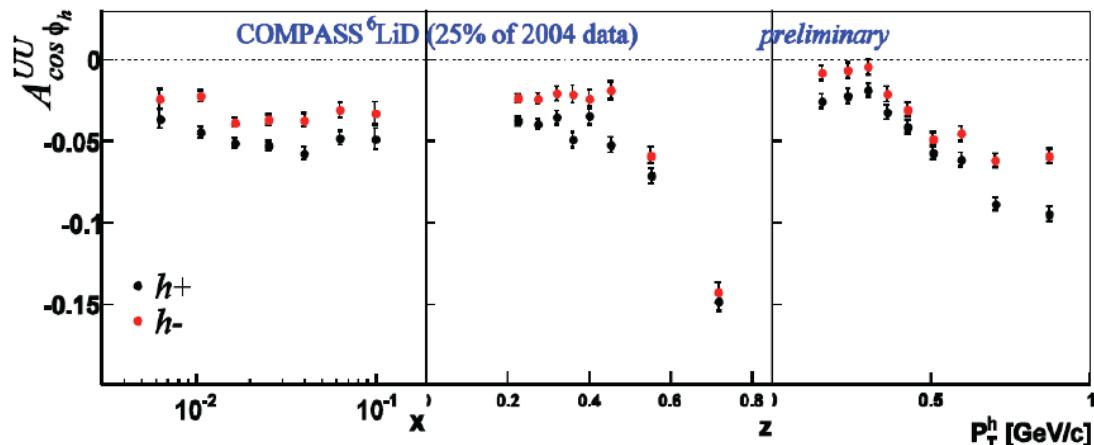
Cahn PLB 78 (1978)

Twist-3:  $d\sigma_{UU}^{\cos\phi} \propto \cos\phi \cdot \sum_q e_q^2 \frac{2M}{Q} I \left[ -\frac{(\hat{P}_{h\perp} \cdot \vec{p}_T)}{M_h} x h_1^\perp H_1^{\perp q} - \frac{(\hat{P}_{h\perp} \cdot \vec{k}_T)}{M} x f_1 D_1 + \dots \right]$

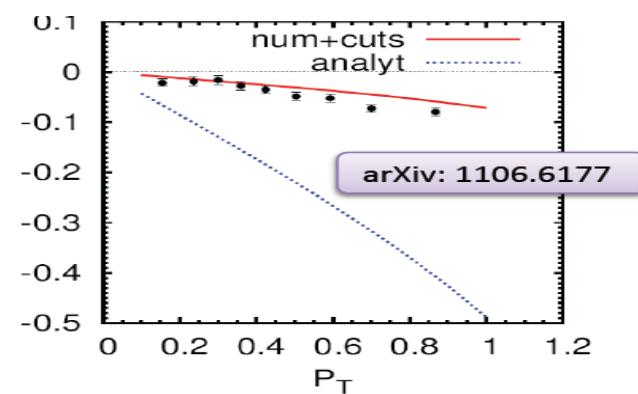
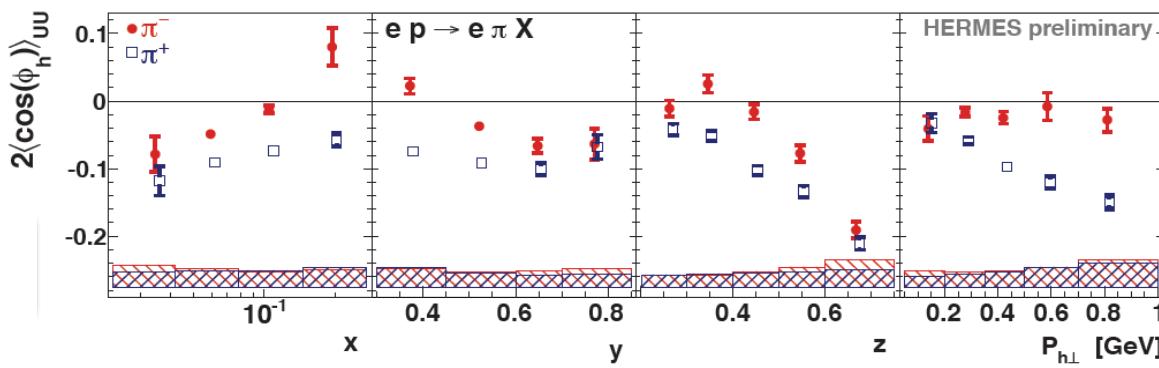
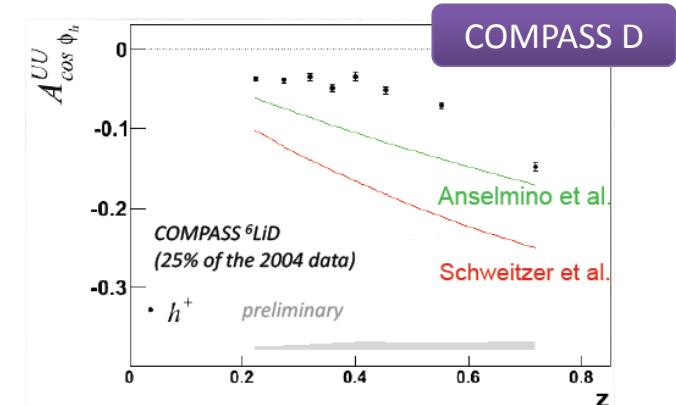


# Boer-Mulders function: SIDIS ( $\cos\phi$ )

$$\sigma_{UU}^{\cos(\phi)} \propto [f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp + \dots] / Q$$



- No dependence on hadron charge is expected
- Difference between  $h^+/h^-$  due to Boer-Mulders term
- Predictions overestimate data
- BM or twist-3?



# Unpolarized TMDs

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right)$$

$$\left\{ \begin{array}{l} [F_{UU,T} + \epsilon F_{UU,L} \\ + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)}] \end{array} \right.$$

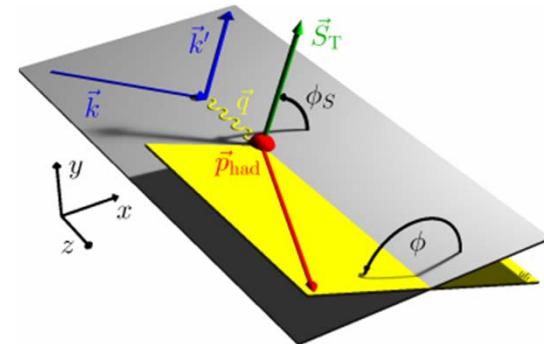
$$+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$\begin{aligned} + S_T & \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \end{aligned}$$

$$\begin{aligned} + S_T \lambda_l & \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \} \end{aligned}$$



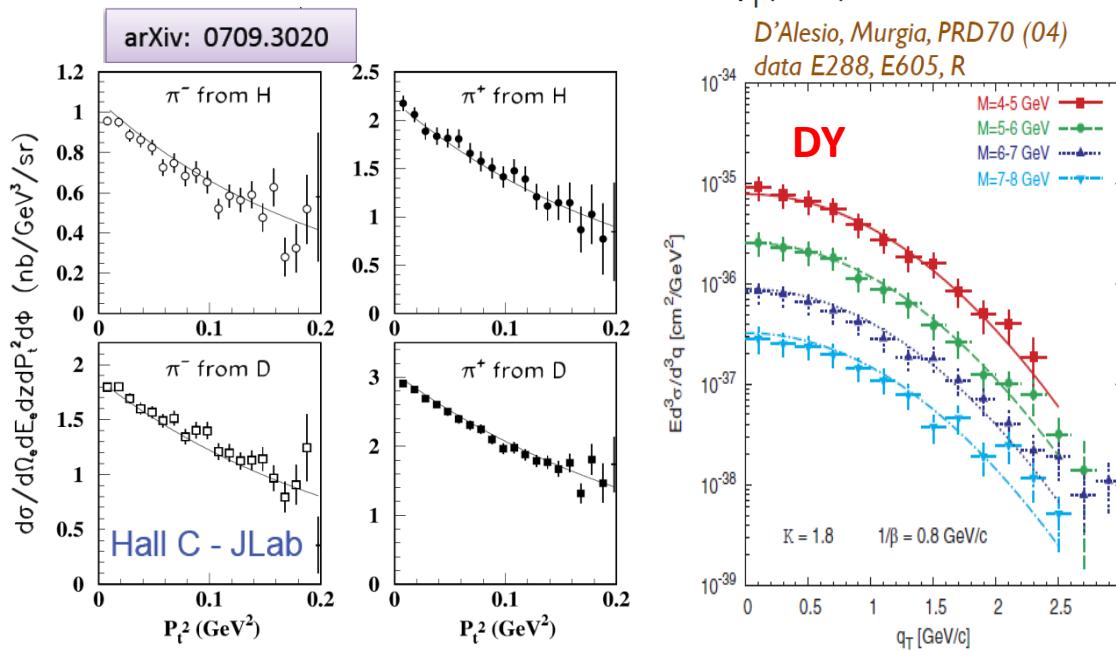
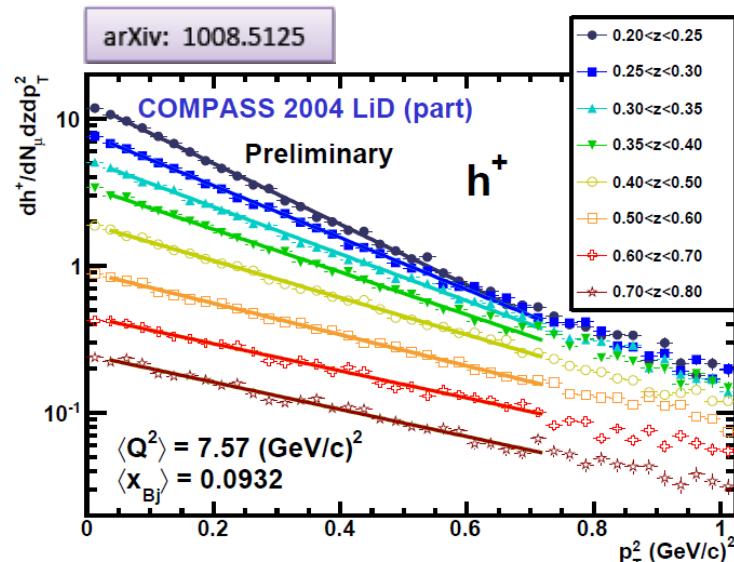
Distribution Functions

		quark		
		U	L	T
nucleon	U	$f_1$		$h_1^\perp$ -
	L		$g_1$ -	$h_{1L}^\perp$ -
	T	$f_{1T}^\perp$ -	$g_{1T}^\perp$ -	$h_1$
				$h_{1T}^\perp$

Fragmentation Functions

		quark		
		U	L	T
h	U	$D_1$		$H_1^\perp$ -

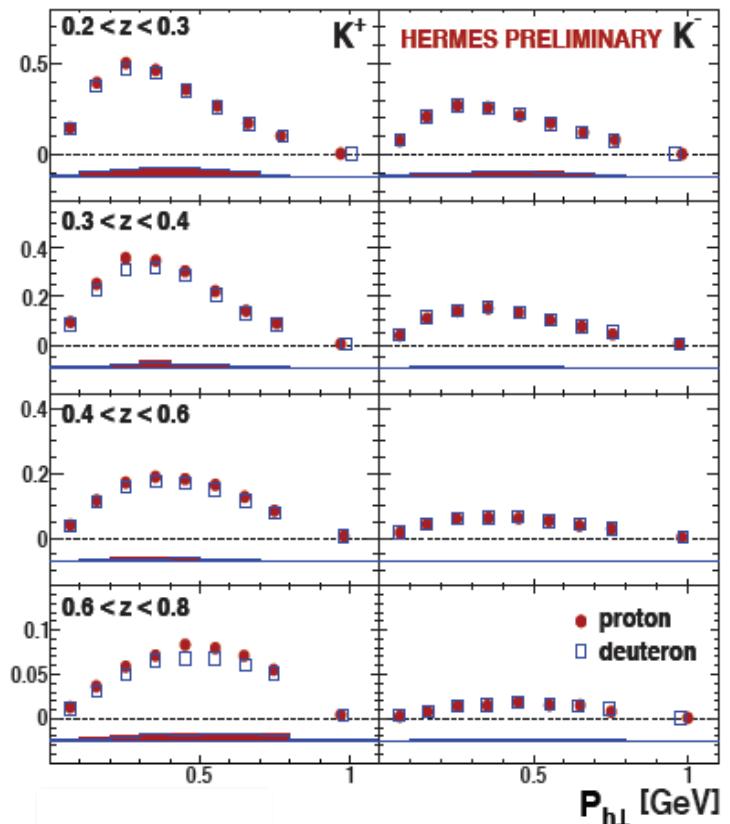
# Unpolarized TMDs: $P_{h\perp}$ -unintegrated distributions



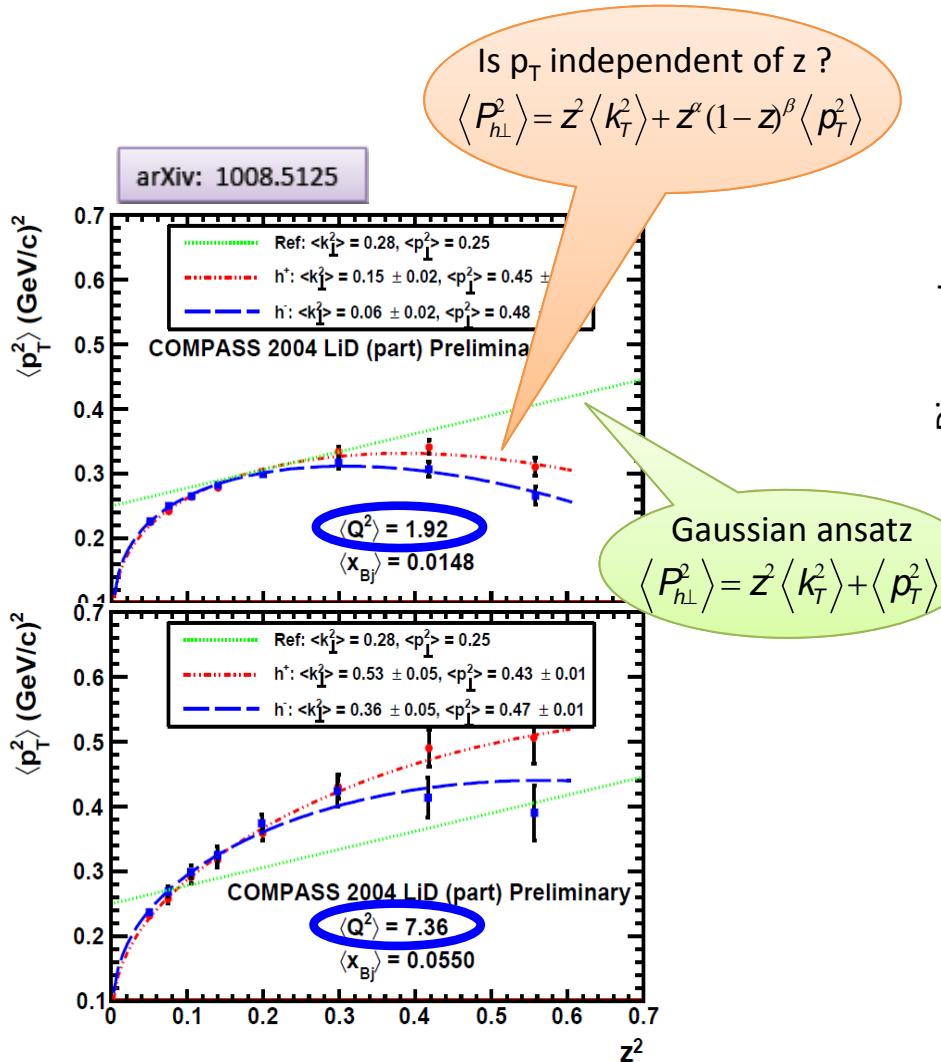
$$M_N^h = \frac{1}{N_N^{DIS}(Q^2)} \frac{dN_N^h(z, Q^2)}{dz} = \frac{\sum_q e_q^2 \int dx f_{1q}(x, Q^2) D_{1q}^h(z, Q^2)}{\sum_q e_q^2 \int dx f_{1q}(x, Q^2)}$$

disentangling the  $z$  and  $P_{h\perp}$  dependencies allows to access intrinsic transverse momenta  $p_T$  and  $k_T$ :  
 $\langle P_{h\perp}^2 \rangle = z^2 \langle k_T^2 \rangle + \langle p_T^2 \rangle \rightarrow$  3D analysis ( $x, z, P_{h\perp}$ )

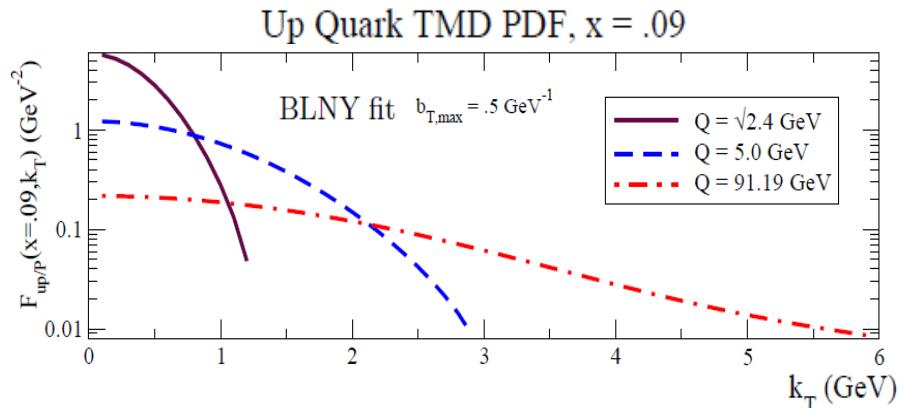
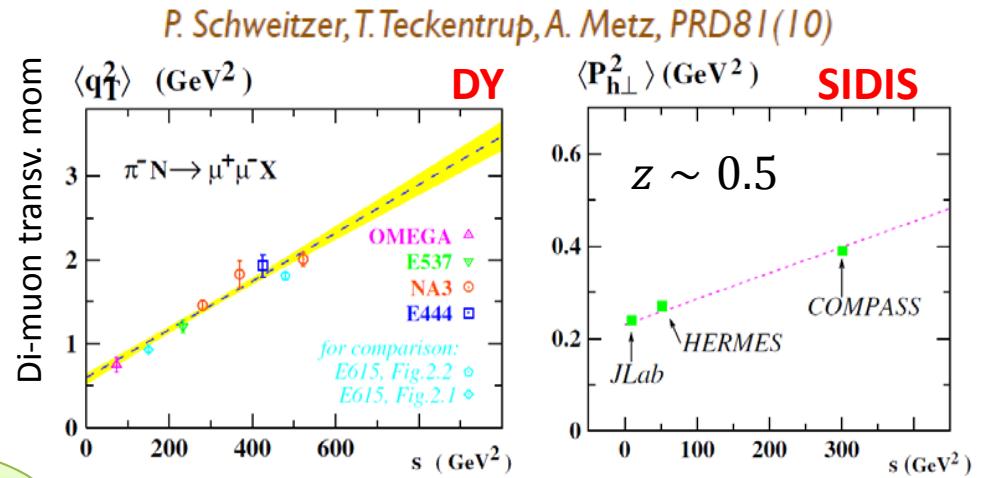
## Hadron multiplicities



# Unpolarized TMDs: energy dependence & evolution



- Hint of z-dependence
- Hint of flavour dependence
- $Q^2$ -dependence



S.M. Aybat & T.C. Rogers arXiv: 1101.5057v2  
"QCD evolution has a strong quantitative effect on TMDs"

# Helicity TMDs

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{aligned} & \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{aligned} \right.$$

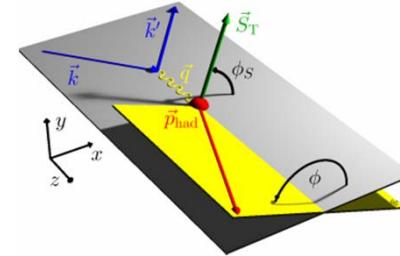
$$+ \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$\begin{aligned} + S_T & \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \end{aligned}$$

$$\begin{aligned} + S_T \lambda_l & \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \} \end{aligned}$$



Describes probability to find longitudinally polarized quarks in a longitudinally polarized nucleon

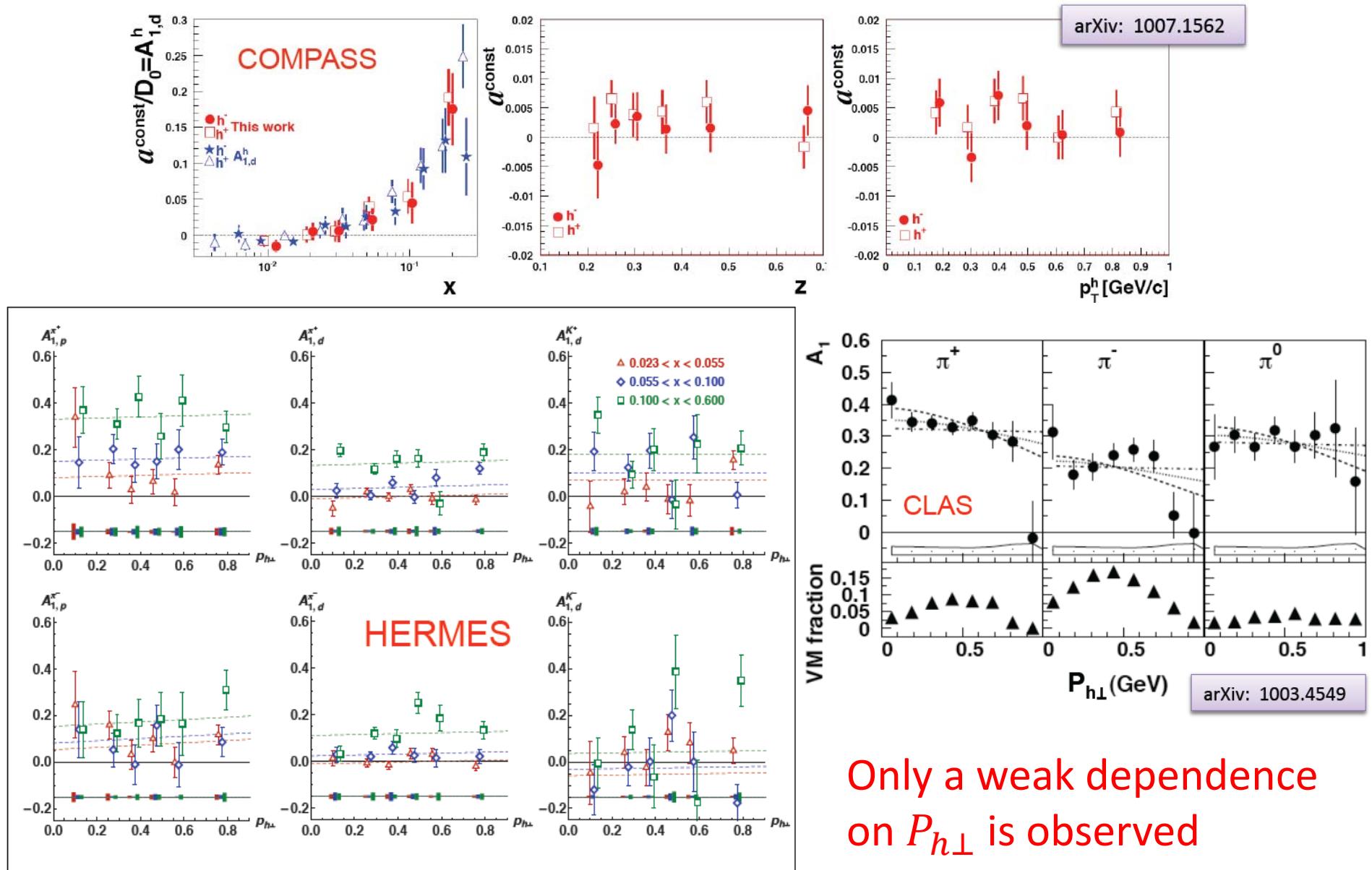
## Distribution Functions

		quark		
		U	L	T
nucleon	U	$f_1$		-
	L		-	-
	T	-	-	-
	T	-	-	-

## Fragmentation Functions

		quark		
		U	L	T
h	U	$D_1$		-

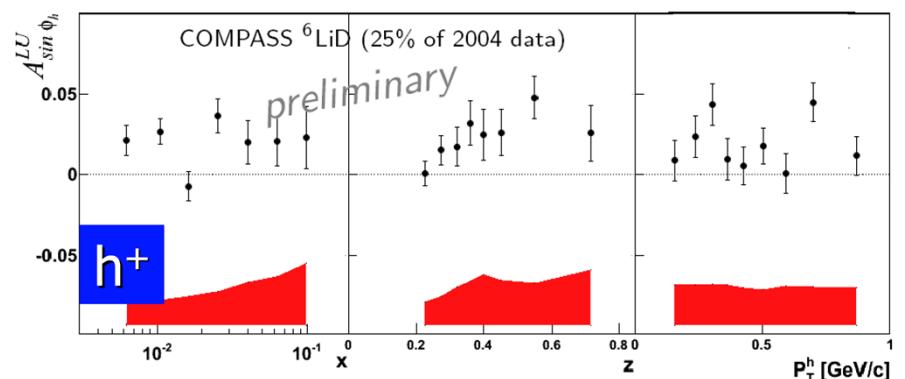
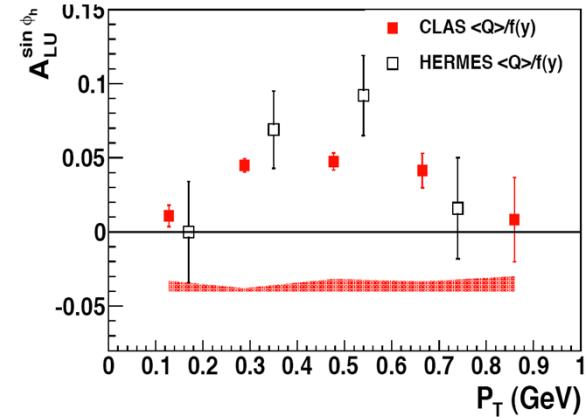
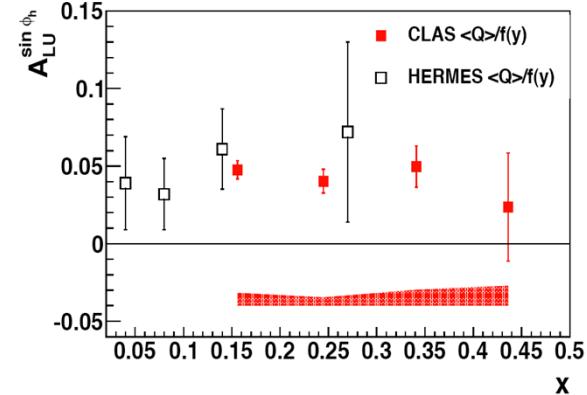
# Helicity TMDs: $P_{h\perp}$ -unintegrated $A_{LL}$ DSAs



# Higher-twist

$$\begin{aligned}
\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \\
& \left\{ \begin{aligned} & \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{aligned} \right. \\
& + \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\
& + S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\
& + S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\
& + S_T \left[ \begin{aligned} & \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \end{aligned} \right] \\
& + S_T \lambda_l \left[ \begin{aligned} & \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \end{aligned} \right]
\end{aligned}$$

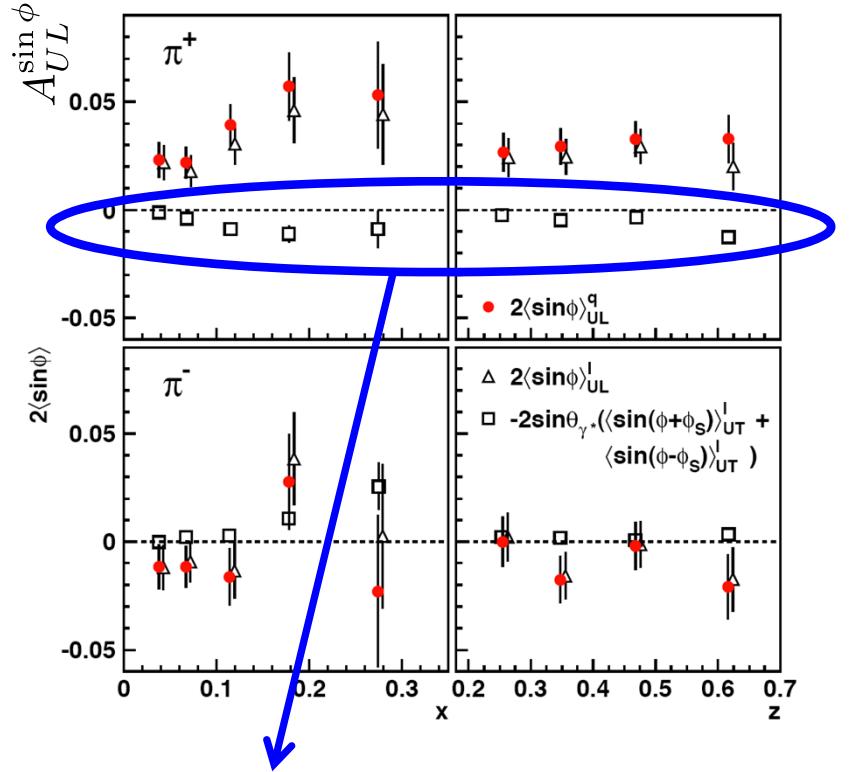
$$\sigma_{LU}^{\sin(\phi)} \propto [e \otimes H_1^\perp + g^\perp \otimes D_1 + \dots] / Q$$



# Higher-twist

$$\begin{aligned}
\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \\
\left\{ \begin{aligned} & \left[ F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \\ + \lambda_l \left[ \sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \end{aligned} \right. \\
+ S_L \left[ \sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\
+ S_L \lambda_l \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\
+ S_T \left[ \begin{aligned} & \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \end{aligned} \right] \\
+ S_T \lambda_l \left[ \begin{aligned} & \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \end{aligned} \right] \end{aligned} \Big\}$$

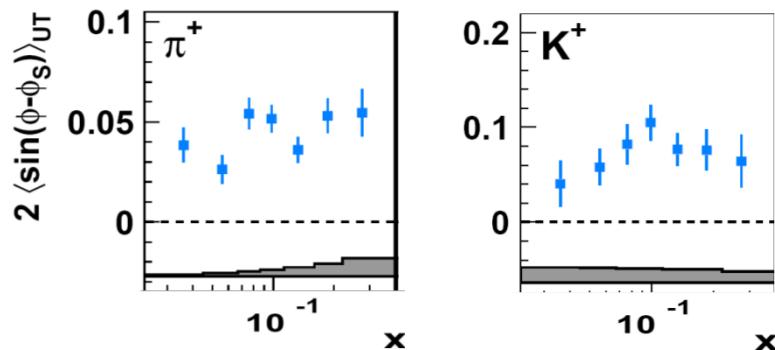
$$\sigma_{UL}^{\sin(\phi)} \propto [h_L \otimes H_1^\perp + f_L^\perp \otimes D_1 + \dots] / Q$$



Collins and Sivers contributions are small  
 $\rightarrow$  **Measured asymmetry is a genuine higher-twist effect**

# Sivers kaons amplitudes: open questions

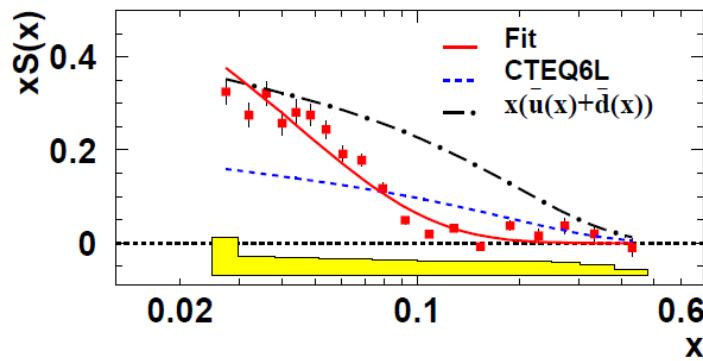
$\pi^+/\bar{K}^+$  production dominated by u-quarks, but:



?

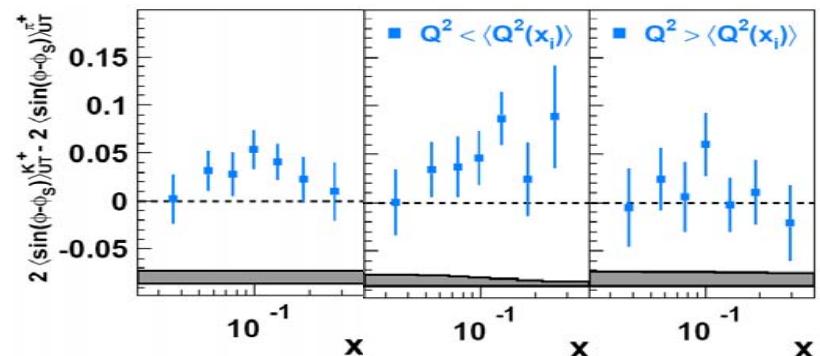
$$\pi^+ \equiv |ud\rangle, \quad K^+ \equiv |u\bar{s}\rangle \rightarrow$$

different role of various sea quarks ?

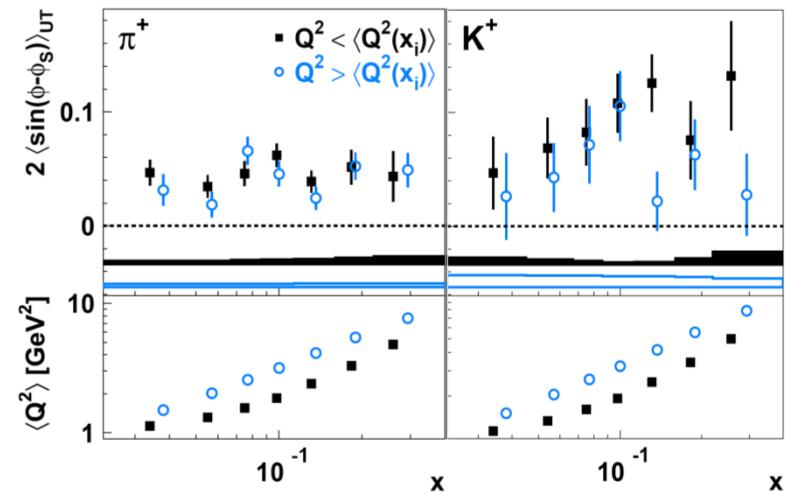


?

impact of different  $k_T$  dependence of FFs in the convolution integral

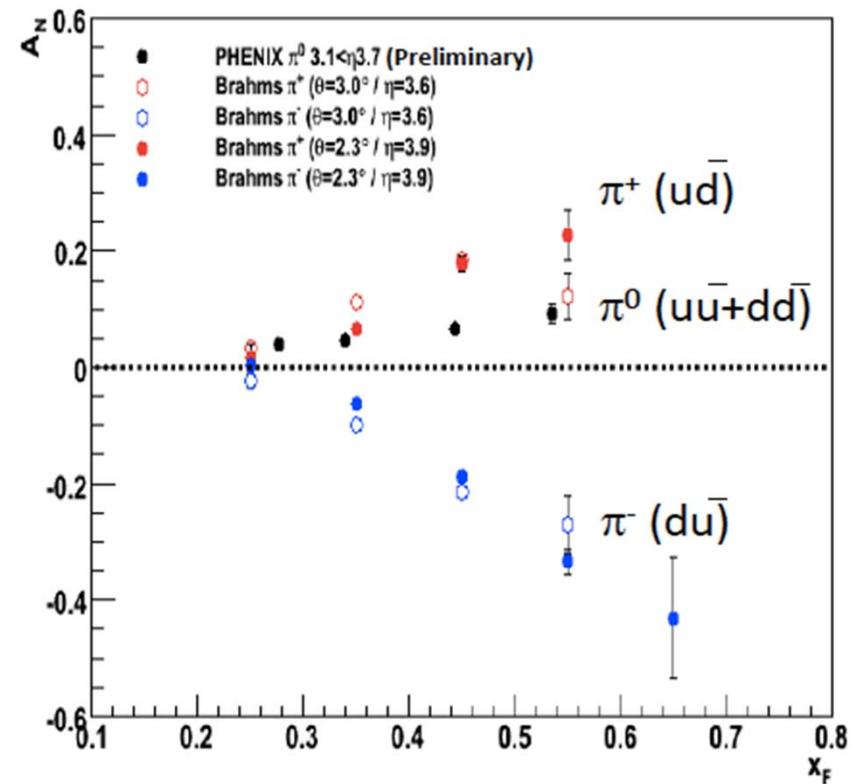
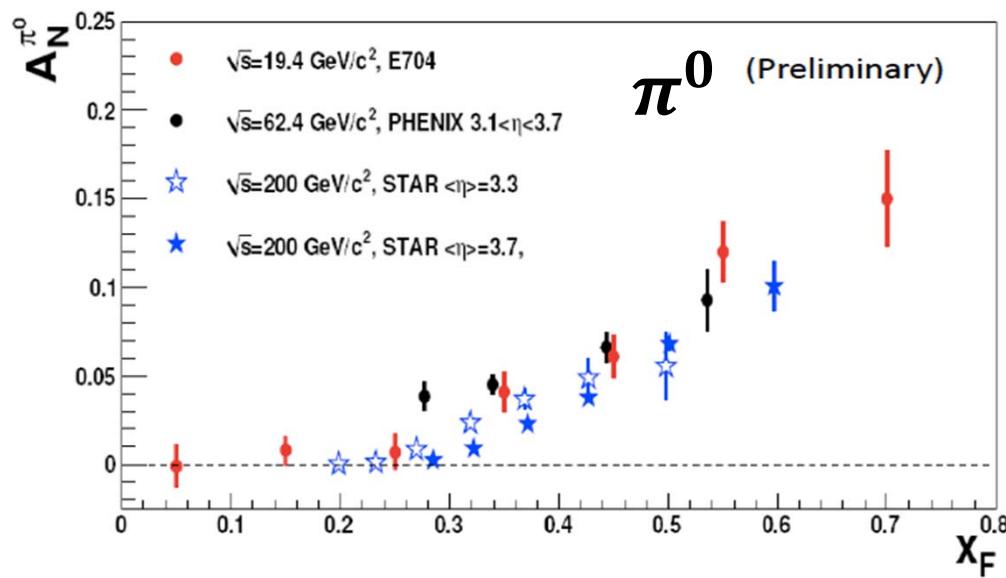


only in low- $Q^2$  region significant  
(90% C.L.) deviation is observed



Each x-bin divided into two  $Q^2$  bins  
Higher-twist contrib. for kaons?

# Transversity & Sivers in pp scattering: RHIC



- Large asymmetries measured by STAR, PHENIX and BRAHMS
- No strong dependence on  $\sqrt{s}$  from 19.4 to 200 GeV
- Spread of data probably due to different acceptance in pseudorapidity and/or  $p_T$
- Could be due to admixture of Transversity, Sivers and Twist-3 effects