The Q^2–Dependence of the Generalized GDH Sum Rule on the Proton

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GDH sum rule for real and virtual photons

Derivation of the GDH sum rule

Compton scattering amplitude in forward direction ($Q^2=0$):

$$T(\nu,\theta=0)=\chi_f^\dagger[\underbrace{(\epsilon_f^*\cdot\epsilon_i)\cdot f(\nu)}_{\text{spin independent}}+\overbrace{i\sigma(\epsilon_f^*\times\epsilon_i)\cdot g(\nu)}^{\text{spin dependent}}]\chi_i$$

Low theorem (Lorentz and gauge invariance):

$$f(\nu) = -\frac{\alpha}{M}+(\alpha_N+\beta_N)\nu^2+\mathcal{O}(\nu^4)$$
$$g(\nu) = -\frac{\alpha\kappa_N^2}{2M^2}\nu+\gamma_0\nu^3+\mathcal{O}(\nu^5)$$

$T_{3/2}(\nu)=f(\nu)-g(\nu)$ $\qquad$ $T_{1/2}(\nu)=f(\nu)+g(\nu)$

Optical Theorem: $\quad Im[T_{3/2,1/2}(\nu)]=\frac{\nu}{4\pi}\sigma_{3/2,1/2}(\nu)$

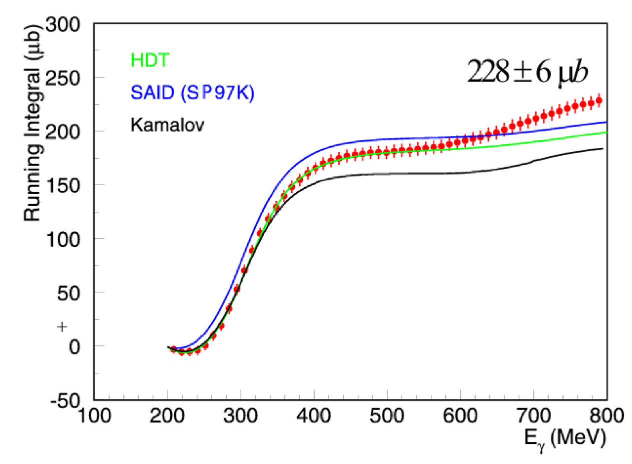
Unsubtracted Dispersion relation (Causality):

$$Re[g(\nu)]=\frac{2\nu}{\pi}\int_0^\infty\frac{\nu' d\nu'}{4\pi(\nu'^2-\nu^2)}\frac{\sigma_{1/2}-\sigma_{3/2}}{2}$$

$$\Rightarrow I_{GDH}=-\frac{2\pi^2\alpha\kappa_N^2}{M^2}=\int_0^\infty\frac{\sigma_{1/2}-\sigma_{3/2}}{\nu}d\nu$$

Measurement with Real Photons

GDH @ MAMI (preliminary): $\quad$ Energy range 0.2 GeV $\leq\nu\leq0.8$ GeV



Note: different sign convention in Nuclear Physics ($-I_{GDH}$ shown)
At the photon point ($Q^2=0$):

Sum rule prediction: $I_{GDH}(Q^2=0)=-204\mu$b

with extrapolations (from SAID and Bianchi/Thomas):
⇒ close to GDH sum rule prediction
Extension of energy range needed for complete evaluation:
⇒ Measurement at ELSA, JLab, GRAAL, SPring8, (ELFE ?)

Generalisation of the GDH sum

Connect the photon point to the DIS region

$$I_{GDH}(Q^2)=\int_{\nu_0}^\infty\frac{\sigma_{1/2}-\sigma_{3/2}}{\nu}d\nu \qquad \gamma^2=\frac{Q^2}{\nu^2}$$
$$=\frac{8\pi^2\alpha}{M}\int_0^{x_0}\frac{g_1(x,Q^2)-\gamma^2 g_2(x,Q^2)}{\nu\sqrt{1+\gamma^2}}\frac{dx}{x}$$

Real photon limit ($Q^2=0$) predicted by GDH

$$I_{GDH}(Q^2=0)=-\frac{2\pi^2\alpha}{M^2}\kappa^2=-204\mu\text{b}$$

High energy limit ($\gamma\ll1$)

$$I_1(Q^2)=\frac{16\pi^2\alpha}{Q^2}\int_0^1 g_1(x)dx=\frac{16\pi^2\alpha}{Q^2}\Gamma_1$$

Expect (if higher twist is small): simple $1/Q^2$ dependence

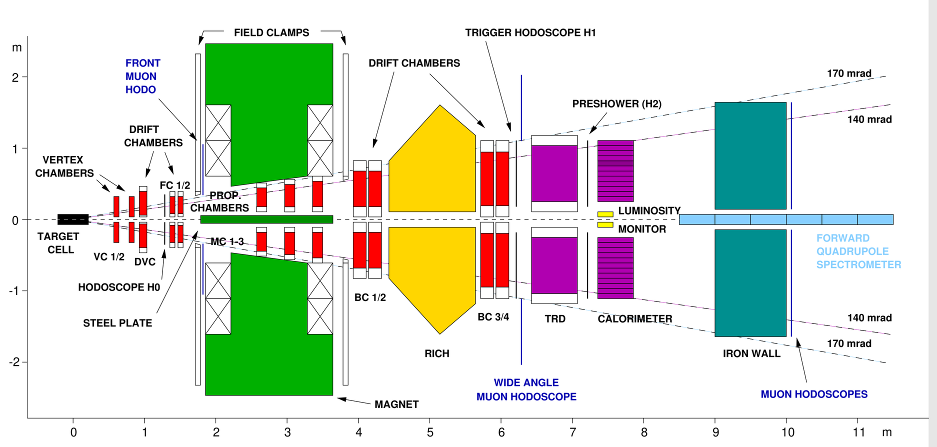
Measured (E 143): $\quad\Gamma_1(5\text{ GeV}^2)=0.129\pm0.010$

$\qquad I_1(5\text{ GeV}^2)=11.6\pm0.9\mu$b

Explore *transition region* between
pQCD and nucleon resonance excitations

The Measurement – Detector and Analysis

The Spectrometer



- uses HERA lepton beam ($e^\pm$) $E_{beam}=27.5$ GeV, $P_e=0.55\pm0.02$
  Beam polarisation is measured by two independent polarimeters
- internal gas target ($\vec{H}$, also $\vec{D},He,N,Ne,Kr$) $P_{\vec{H}}=0.88\pm0.04$
- Integrated Luminosity for this analysis: 70 pb^{-1}
- Kinematical range
  $0.02\leq x\leq0.8$ for $Q^2\geq1$GeV^2 and $W\geq2$ GeV
- Tracking
  57 tracking chamber planes, $\delta P/P<1.3$ %, $\delta\theta\leq0.6$ mrad
- Particle ID
  Čerenkov – Detector (since 1998 RICH), TRD, Preshower, Leadglass Calorimeter
  ⇒ for Leptons: efficiency $\epsilon\approx0.99$, contamination $<1\%$

Data Analysis

- 1.2 GeV$^2\leq Q^2\leq12$ GeV^2 in 6 bins
- 1.0 GeV$^2\leq W^2\leq4.2$ GeV^2 (resonance region)
  and 4.2 GeV$^2\leq W^2\leq45$ GeV^2 (DIS region)

$$I_{GDH}(Q^2)=\int_{\nu_0}^\infty\frac{\sigma_{1/2}-\sigma_{3/2}}{\nu}d\nu=\frac{8\pi^2\alpha}{M}\int_0^{x_0}\frac{A_1F_1}{\nu\sqrt{1+\gamma^2}}\frac{dx}{x}$$

$$A_1=\frac{A_\parallel}{D}-\eta A_2 \qquad F_1=F_2\cdot\frac{1+\gamma^2}{2x(1+R)}$$

$$A_\parallel=\frac{N^-L^+-N^+L^-}{N^-L_p^++N^+L_p^-}$$

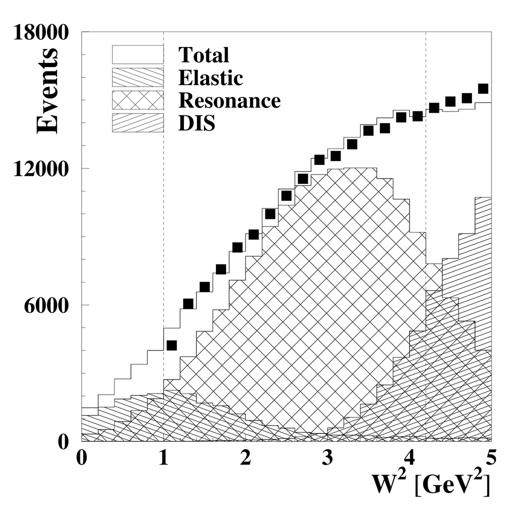
η, D are well known kinematical factors

Systematical uncertainties

• Beam and target polarization:	5.3%
• Smearing effects:	<5.5 %
• Uncertainty in F_2:	2%
• for resonance region: assume $A_2=0.06$	

⇒ Ignorance of A_2: Dominant uncertainty in the resonance region of up to 17%

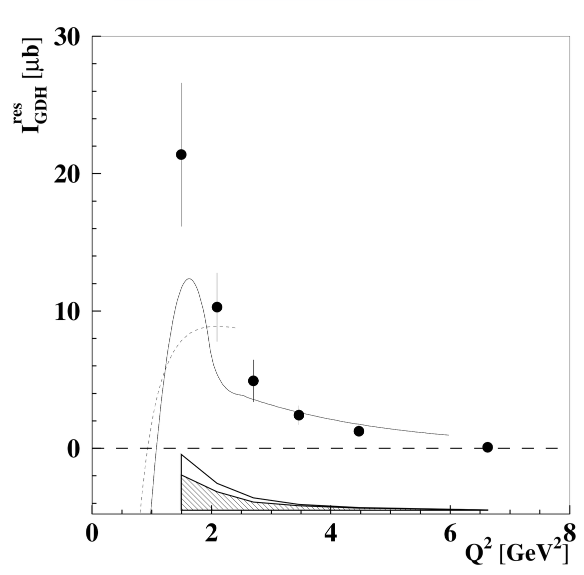
Monte Carlo studies



- Parameterization of F_2 taken from
  A. Bodek et al., Phys. Rev. **D 20** (1979) 1471
- R=0.18 in the resonance region
- Smearing effects were found to be small
- Good agreement between data and Monte Carlo
- Energy resolution ≈240 MeV ⇒ No separation of resonances
- Contamination to the resonance region from elastic and DIS regions amount to 10% – 2% and 7% – 16% resp. depending on Q^2

Experimental Results

Resonance Region

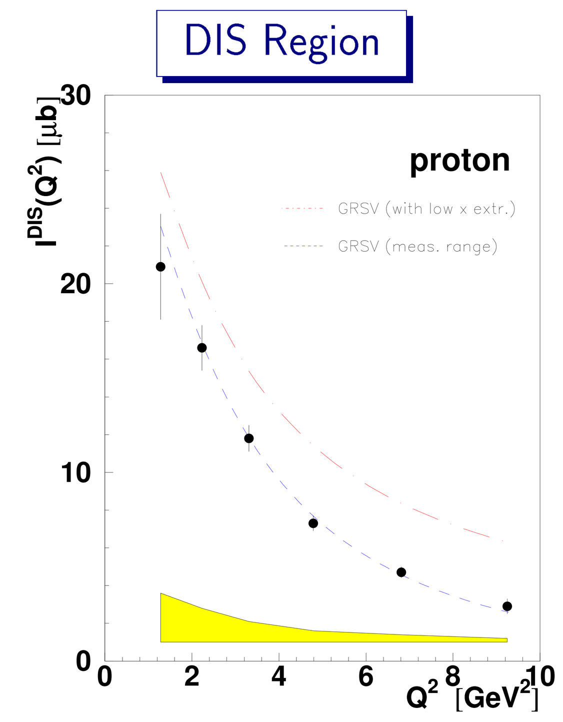


1.0 GeV$^2\leq W^2\leq4.2$ GeV^2

- V. Burkert, Z. J. Li: Phys. Rev. **D 47** (1993) 46
  (based on resonance form factors) (dotted line)
- I. G. Aznauryan: Phys. of At. Nucl **58** (1995) 1014
  (infinitely narrow resonances) (solid line)
- Other models restricted to lower Q^2
- Data: HERMES publication: hep–ex/0008037, PLB in press

Existing models are not sufficient to describe the data

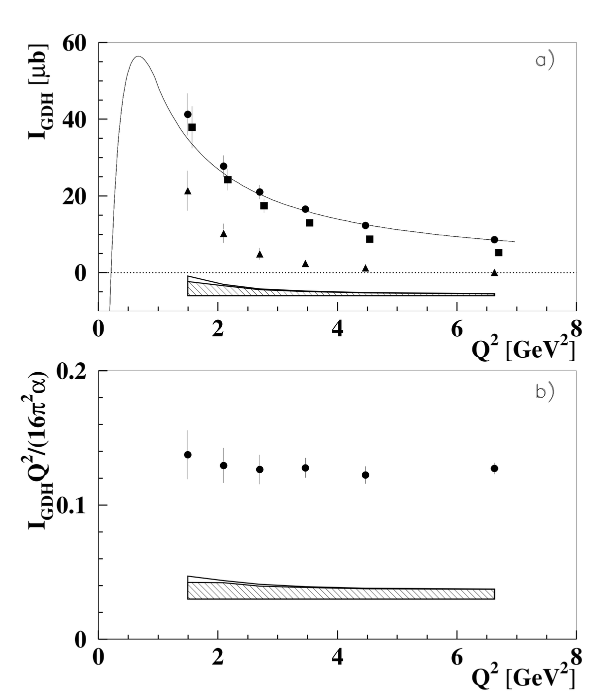
DIS Region



$$I(Q^2)=\int_{\nu_0}^{23.5}\frac{\sigma_{1/2}-\sigma_{3/2}}{\nu}d\nu \qquad I(Q^2)=\int_{\nu_0}^{\infty}\frac{\sigma_{1/2}-\sigma_{3/2}}{\nu}d\nu$$

- NLO fit: M. Glück et al. Phys. Rev **D 53** (1996) 6620
- HERMES data: K. Ackerstaff et al. Phys. Lett. **B 444** (1998) 531
- Extrapolation for $W^2>45$ GeV^2: 3.5 μb (nearly independent of Q^2)
  (following: N. Bianchi, E. Thomas, Phys. Lett. **B 450** (1999) 439)

Combined Result



(▲) show the data in the resonance region. (■) show all data measured by HERMES.
(•) show the complete result including the high energy extrapolation
Solid line: J. Soffer, O. V. Teryaev: Phys. Rev. **D 51** (1995) 25

DIS region important for full integral
No significant sign of higher twist