The Q^2–Dependence of the Generalised GDH Integral for the Deuteron, Proton and Neutron

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GDH sum rule for real and virtual photons

Generalisation of the GDH Integral

The Gerasimov–Drell–Hearn (GDH) sum rule connects an energy–weighted integral of the difference of the helicity–dependent real–photon absorption cross sections with the anomalous contribution to the magnetic moment $\kappa = \frac{\mu M_t}{eI} - Z$ of the target nucleus with atomic number Z :

$$\int_{\nu_0}^{\infty} [\sigma^{\rightleftarrows}(\nu) - \sigma^{\rightrightarrows}(\nu)] \frac{d\nu}{\nu} = -\frac{4\pi^2 I\alpha}{M_t^2}\kappa^2, \tag{1}$$

where $\sigma^{\rightleftarrows}$ and $\sigma^{\rightrightarrows}$ are the photoabsorption cross sections for relative orientation of the photon spin anti–parallel and parallel to the nucleus spin I, ν is the photon energy in the target rest frame, M_t is the nucleus mass, μ is its magnetic moment, Z is the charge in units of the elementary charge e, α the electromagnetic fine–structure constant and ν_0 is the photoabsorption threshold. This sum rule provides an interesting link between the helicity–dependent dynamics and a static ground state property of the target nucleus.
The GDH integral can be generalised to non–zero photon virtuality Q^2 in terms of the helicity–dependent virtual–photon absorption cross sections $\sigma^{\rightleftarrows}$ and $\sigma^{\rightrightarrows}$:

$$I_{GDH}(Q^2) = \int_{\nu_0}^{\infty} [\sigma^{\rightleftarrows}(\nu, Q^2) - \sigma^{\rightrightarrows}(\nu, Q^2)] \frac{d\nu}{\nu}. \tag{2}$$

The cross section difference appearing in the integrand is given by

$$\Delta\sigma = \sigma^{\rightleftarrows} - \sigma^{\rightrightarrows} = \frac{8\pi^2\alpha}{M_t K} A_1 F_1. \tag{3}$$

Proton – Neutron Difference

The generalised GDH integral can be written in terms of the spin structure functions g_1 and g_2 as:

$$I_{GDH} = \frac{8\pi^2\alpha}{M} \int_0^{x_0} \frac{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)}{K} \frac{dx}{x}, \tag{4}$$

where g_1 and g_2 are the polarised structure functions of the nucleon, $\gamma^2 = Q^2/\nu^2$, $x = Q^2/2M\nu$, $x_0 = Q^2/2M\nu_0$. At the relatively large Q^2-values considered here, Eq. (4) simplifies to

$$I_{GDH}(Q^2) = \frac{16\pi^2\alpha}{Q^2}\Gamma_1. \tag{5}$$

In the Bjorken limit the difference $\Gamma_1^p - \Gamma_1^n$ is given by the Bjorken sum rule. It can be derived using only current algebra and isospin symmetry. This sum rule relates the difference of the first moments of g_1^p and g_1^n at fixed Q^2 to the well–measured neutron beta–decay coupling constant $g_a = |g_A/g_V| = 1.2670 \pm 0.0035$ (PDG):

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6} \cdot g_a \cdot C_{ns}(\alpha_s(Q^2)), \tag{6}$$

where C_{ns} is the non–singlet QCD correction calculated thus far up to $\mathcal{O}(\alpha_s^3)$ in the modified minimal subtraction ($\overline{MS}$) scheme.
Experimental verification of the Bjorken sum rule at finite Q^2 provides a fundamental test of QCD. A measurement of $I_{GDH}^p - I_{GDH}^n$ at large enough Q^2 provides such a test.

Data Analysis

- 1.2 GeV$^2 \leq Q^2 \leq 12$ GeV^2 in 6 bins
- 1.0 GeV$^2 \leq W^2 \leq 4.2$ GeV^2 (nucleon resonance region)
  and 4.2 GeV$^2 \leq W^2 \leq 45$ GeV^2 (DIS region)

$$I_{GDH}(Q^2) = \int_{\nu_0}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu} d\nu = \frac{8\pi^2\alpha}{M} \int_0^{x_0} \frac{A_1 F_1}{\nu\sqrt{1+\gamma^2}} \frac{dx}{x}$$

$$A_1 = \frac{A_\parallel}{D} - \eta A_2 \qquad F_1 = F_2 \cdot \frac{1+\gamma^2}{2x(1+R)}$$

$$A_\parallel = \frac{N^- L^+ - N^+ L^-}{N^- L_p^+ + N^+ L_p^-}$$

η, D are well known kinematical factors

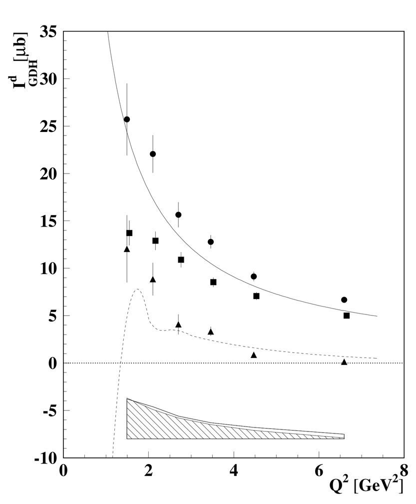
Systematical uncertainties

- Beam and target polarization: 5.5%
- Smearing effects: up to 14 % of partial integrals
- Uncertainty in F_2: 5%
- for resonance region: assume $A_2^{d(p)} = 0(0.06)$

$\Rightarrow$ Ignorance of $A_2^{d(p)}$: Dominant uncertainty in the resonance region of up to 30(17)%

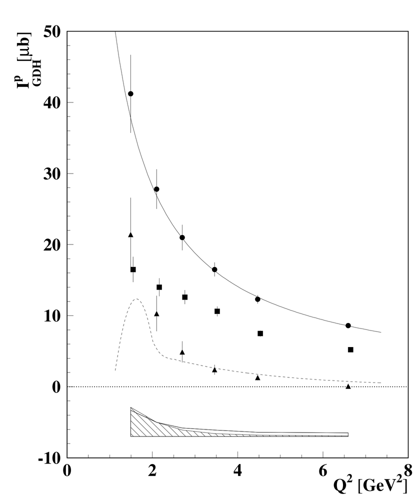
Generalised GDH Integral for the Deuteron, Proton and Neutron

Deuteron Result



The generalised GDH integral I_{GDH}^d for the deuteron nucleus, shown as a function of Q^2 for the three kinematic regions considered: nucleon-resonance region (triangles), DIS region (squares), and full W^2–region (circles) including extrapolation to the unmeasured part. The error bars show the statistical uncertainties. The solid curve is taken from Soffer and Teryaev and represents a prediction for the full I_{GDH}^d. The dashed curve represents a model for the nucleon-resonance region from Aznauryan. The systematic uncertainties of the full integral are given as a band; the hatched area inside represents the systematic uncertainty of the nucleon-resonance region alone. Note that some data points are slightly shifted for better visibility.

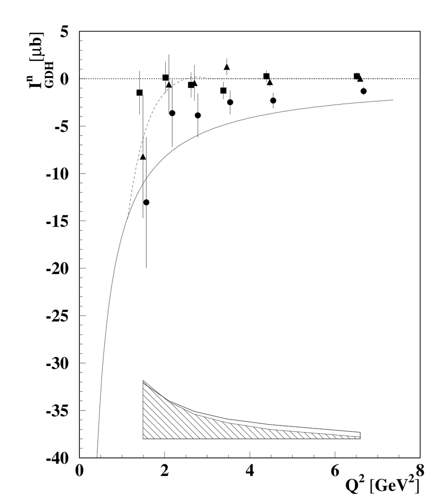
Proton Result



The generalised GDH integral I_{GDH}^p for the proton using the same notation as in the previous figure. The solid curve is taken from Soffer and Teryaev, the dashed curve follows the resonance model of Aznauryan. The data were published earlier by HERMES, but are reanalysed for the present poster using improved knowledge on the target polarisation.

References for HERMES GDH papers:
HERMES Collab., K. Ackerstaff *et al.*, Phys. Lett. **B 444** (1998) 531.
HERMES Collab., A. Airapetian *et al.*, Phys. Lett. **B 494** (2000) 1.
HERMES Collab., A. Airapetian *et al.*, submitted to Eur. Phys. J. **C**.

Neutron result



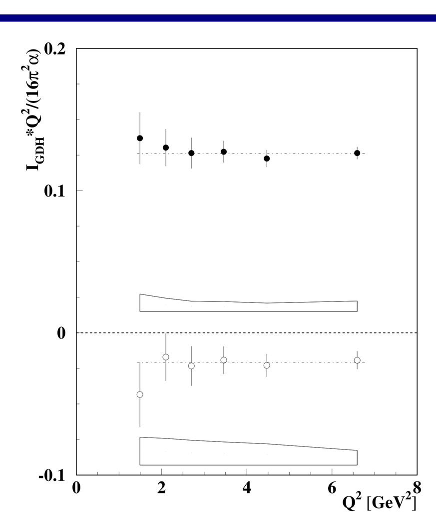
Neutron ($\omega_d = 0.058 \pm 0.010$ is the Deuteron D–state probability):

$$I_{GDH}^n = \frac{I_{GDH}^d}{1 - 1.5\omega_d} - I_{GDH}^p. \tag{7}$$

The generalised GDH integral I_{GDH}^n for the neutron obtained from the deuteron and proton using the same notation as in the previous figures for the symbols and theoretical curves.
References for theoretical curves:
J. Soffer and O.V. Teryaev, Phys. Rev. **D 51** (1995) 25;
J. Soffer and O.V. Teryaev, Phys. Rev. Lett. **70** (1993) 3373.
I. G. Aznauryan, Phys. of At. Nucl. **58** (1995) 1014 and private communication.

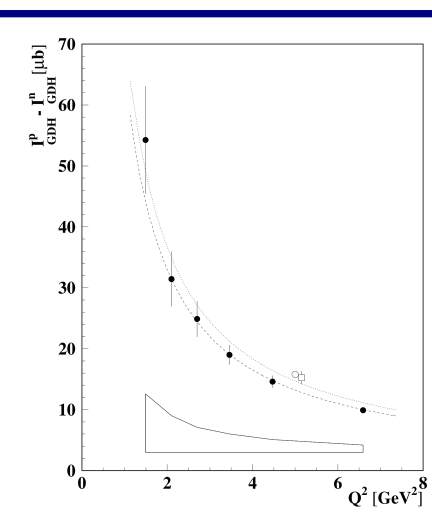
Final Results

Q^2–dependence for the Proton and the Neutron



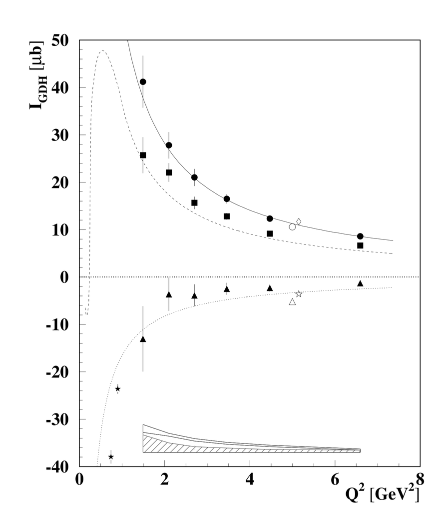
The figure shows Q^2–dependence of the generalised GDH integrals for the proton (filled circles) and neutron(open circles) after the leading–twist dependence, $Q^2/(16\pi^2\alpha)$, has been divided out. The error bars represent the statistical errors. The systematic uncertainties are represented by the respective error bands. The dash–dotted lines are straight line fits to the data. Any contributions from resonance form factors or higher–twist contributions should become visible as a deviation from a flat line. Considering the statistical and systematic uncertainties of the present measurement, no deviation from a leading–twist behaviour can be seen. In other words, the experimental data obtained for the proton and the neutron are consistent with the naive expectation that the $1/Q^2$ expansion is a good approximation down to $Q^2 \simeq 2$ GeV^2.

Proton – Neutron Difference



The Q^2–dependence of the generalised GDH integral for the proton–neutron difference. The dotted curve represent the prediction of Soffer and Teryaev. The dashed curve represents a simple $1/Q^2$ fit to the data leading to $I_{GDH}^{p-n}(Q^2 = 5$ GeV$^2)$ of 14.3 $\pm 0.9 \pm 1.3$ μb. For large Q^2 this difference is expected to obey the Bjorken sum rule. The open symbols represent the measurements of the Bjorken sum rule from E–143 (square) and E–155 (circle). They are slightly shifted in Q^2 for clearer representation. At $Q^2 = 0$ the GDH sum rule gives a value of 29 μb. The error bars represent the combined statistical uncertainties; the band indicates the systematic uncertainty.
References for theoretical curves:
J. Soffer and O.V. Teryaev, Phys.Rev. **D 56** (1997) 7458;
J. Soffer and O.V. Teryaev, hep–ph/0207252.

Combined Results and Summary



The Q^2–dependence of the generalised GDH integrals, calculated over the full W^2–region, for the deuteron nucleus (squares), proton (circles) and neutron (triangles). The curves shown are the predictions for the various targets according to Soffer and Teryaev. The error bars represent the statistical uncertainties. The bands represent the systematic uncertainties (open: neutron, lined: deuteron, cross–hatched: proton). The open symbols at $Q^2 = 5$ GeV^2 represent the measurements from E–143 (shifted to the left) and E–155 (shifted to the right) on proton, deuteron and neutron. The stars represent the two highest Q^2 bins of the neutron measurement from Hall A at Jlab.