Tracking QCD - Instantons

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1. **Setting the Stage**
2. **Small Instantons & Deep Inelastic Scattering**
(in collaboration with A. Ringwald)
3. **Larger-size Instantons**

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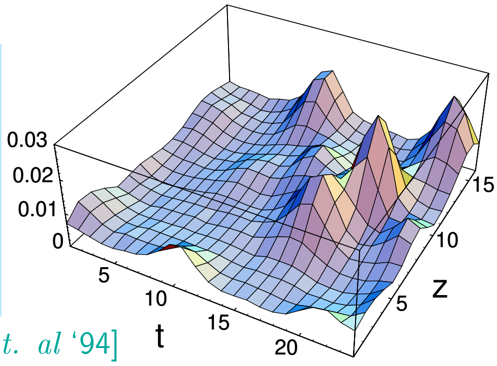
1. Setting the Stage

- **Instantons:** a basic aspect of QCD
  [Belavin *et al* '75, 't Hooft '76]

Non-perturbative fluctuations of gauge fields with typical size $\langle\rho\rangle \approx 0.5$ fm

"instantaneous" in time and space

Lagrange Density



[Chu *et. al* '94]

- Associated with non-trivial topology of QCD vacuum; I's ($\overline{I}$'s) carry conserved, topological charge $+1(-1)$
- **Theoretically:** Play an important rôle in interface region between partonic and hadronic description of strong int's
- Direct experimental evidence for instantons lacking until now!

♣ However,. . . [A. Ringwald & F. Sch. '94 - '01]
Characteristic short distance manifestation of instantons may be exploited for experimental search:

◇ I's induce hard, "chirality"-violating processes, forbidden in usual perturbative QCD ! ($\Leftrightarrow$ABJ-Anomaly) ['t Hooft '76]

◇ Theoretical prediction of rate and characteristic event signature possible in **DIS** (***I***-perturbation theory)

◇ Rate in measurable range at **HERA**!

♣ Unique opportunity in **DIS** at **HERA** for discovering instanton-induced processes!

♣ Key-property for calculability within I-perturbation theory:
Due to hard deep-inelastic scale $\mathcal{Q}$, only instantons of small size $\rho \lesssim \mathcal{Q}^{-1}$ contribute

- On the other hand: Understanding of larger-size instantons & their impact on high-energy scattering (Diffraction ($I\!\!P$), **HERA**-physics?): important theoretical challenge. . .
  [D. Kharzeev,Y.V. Kovchegov & E. Levin '00, V. Shuryak $et.al$ '00]

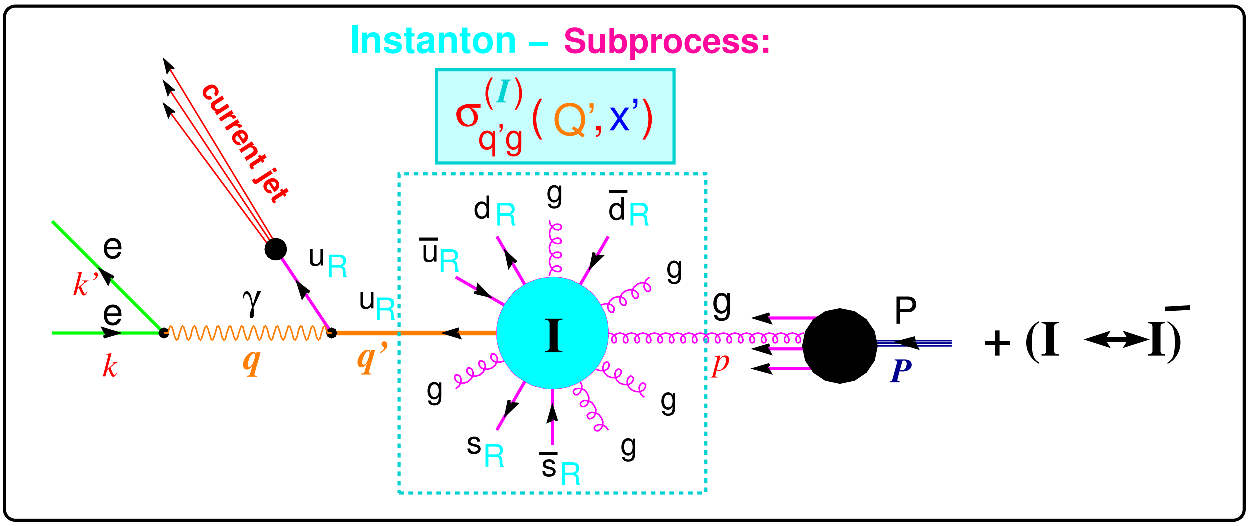
- Report on respective work in progress:
  [F. Sch. '01, F. Sch. & A. Utermann '01]

2. Small Instantons & Deep-Inelastic Scattering

[Ringwald & F. Sch., Phys. Lett **B438** (1998) 217,
Moch, Ringwald & F. Sch., Nucl. Phys. **B507** (1997)]

- I-perturbation theory: dilute gas of I's and $\bar{I}$'s, small α_s, light quarks with $\mathrm{m}_q \cdot \rho_{\mathrm{eff}} < 1$

- Leading, I-induced, chirality-violating process in **deep-inelastic** regime of $e^{\pm}\mathrm{P}$ scattering:



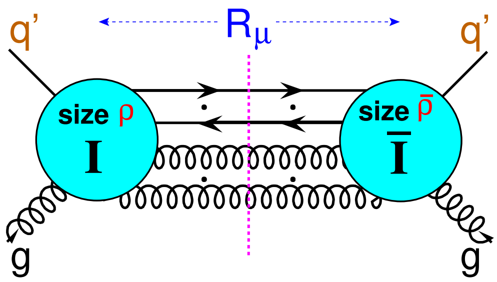
- Factorization in Bjorken limit of I-subprocess variables $\boldsymbol{(Q'^{\,2}, x')}$: large $Q'^{\,2} = -q'^{\,2} > 0$; $x' = \frac{Q'^{\,2}}{2p\cdot q'} \leq 1$ fixed

$$\frac{\mathrm{d}\sigma_{\mathrm{HERA}}{}^{(I)}}{\mathrm{d}x'\mathrm{d}Q'^{\,2}} \simeq \frac{\mathrm{d}\mathcal{L}_{q'g}^{(I)}}{\mathrm{d}x'\mathrm{d}Q'^{\,2}} \cdot \sigma_{q'g}^{(I)}(Q',x')$$

- Differential luminosity, $\mathrm{d}\mathcal{L}_{q'g}^{(I)}$: number of $q'g$ collisions per eP collision $\Leftrightarrow$ integrals over the various known flux factors.

- Essential instanton-dynamics in total cross-section of $\boldsymbol{I}$-subprocess $q'g \overset{I}{\Rightarrow} X$!

$\sigma^{(I)}_{q'g} \sim$



- Being an observable, $\sigma^{(I)}_{q'g}(Q', x')$ involves **integrations** over all $\boldsymbol{I}$ and $\overline{\boldsymbol{I}}$- "collective coordinates",

$\boldsymbol{I}$-size ρ, $\overline{\boldsymbol{I}}$-size $\overline{\rho}$, $I\overline{I}$-distance 4-vector R_μ,
and $U{=}I\overline{I}$ relative color orientation

$$\sigma^{(\boldsymbol{I})}_{q'\,g} = \int d^4R\ \mathrm{e}^{\mathrm{i}\,(p+q')\cdot R} \int_0^\infty d\rho \int_0^\infty d\overline{\rho}\ \mathrm{e}^{-(\rho+\overline{\rho})Q'}$$
$$\times\ D(\rho)\,D(\overline{\rho}) \int dU\ \mathrm{e}^{-\frac{4\pi}{\alpha_s}\Omega\left(U,\frac{R^2}{\rho\overline{\rho}},\frac{\overline{\rho}}{\rho}\right)} \{\ldots\}$$

Two crucial quantities of $\boldsymbol{I}$-calculus:

$D(\rho) =$ (reduced) $\boldsymbol{I}$-size distribution , $\Omega\left(U,\frac{R^2}{\rho\overline{\rho}},\frac{\overline{\rho}}{\rho}\right) = I\overline{I}$-interaction

◇ Both are $\boxed{\textbf{known}}$ in $\boldsymbol{I}$-perturbation theory, formally for $\alpha_s(\mu_r)\ln(\mu_r\rho) \ll 1$ and $\frac{R^2}{\rho\overline{\rho}} \gg 1$, respectively ($\mu_r$ = renormalization scale).

◇ $D(\rho) \propto \rho^{6-2/3n_f+\mathcal{O}(\alpha)}$ generically spoils calculability of $\boldsymbol{I}$-observables, since $(\rho, \overline{\rho})$-integrals diverge for large ρ

♣ **Exception** in **DIS**: "form factor" $\exp(-Q'(\rho+\overline{\rho}))$ insures **small** $(\boldsymbol{I}, \overline{\boldsymbol{I}})$'s with $(\rho, \overline{\rho}) \lesssim 1/Q'$ despite $D(\rho)$

♣ For (large) $Q' \neq 0$, all collective coordinate integrations in $\sigma^{(I)}_{q'g}$ in terms of unique saddle point:

$U^* \Leftrightarrow$ most attractive color orientation

$$\rho^* = \overline{\rho}^* \sim \frac{4\pi}{\alpha_s(\frac{1}{\rho^*})}\frac{1}{Q'}; \qquad \frac{R^{*\,2}}{\rho^{*\,2}} \overset{Q'\ \text{large}}{\sim} 4\frac{x'}{1-x'}$$

◇ Saddle point$\Rightarrow$ indeed, for large Q' and small $(1-x')$: dilute gas $\Leftrightarrow \frac{R}{\rho} \gg 1$ of small $\boldsymbol{I}$'s $\Leftrightarrow \rho \sim \frac{1}{Q'}$!

$D(\rho)$ and $\Omega\left(U, \frac{R^2}{\rho\overline{\rho}}, \frac{\overline{\rho}}{\rho}\right)$: Link between **DIS** and **lattice** observables in QCD vacuum!

◇ **Lattice** $\Rightarrow$ Independent, non-perturbative information on range of validity of their predicted form in (ρ, R)

◇ 1-to-1 saddle-point correspondence of eff. $\boldsymbol{I}$-coordinates

$\Rightarrow$ "fiducial" (Q', x') region for our predictions in **DIS**!

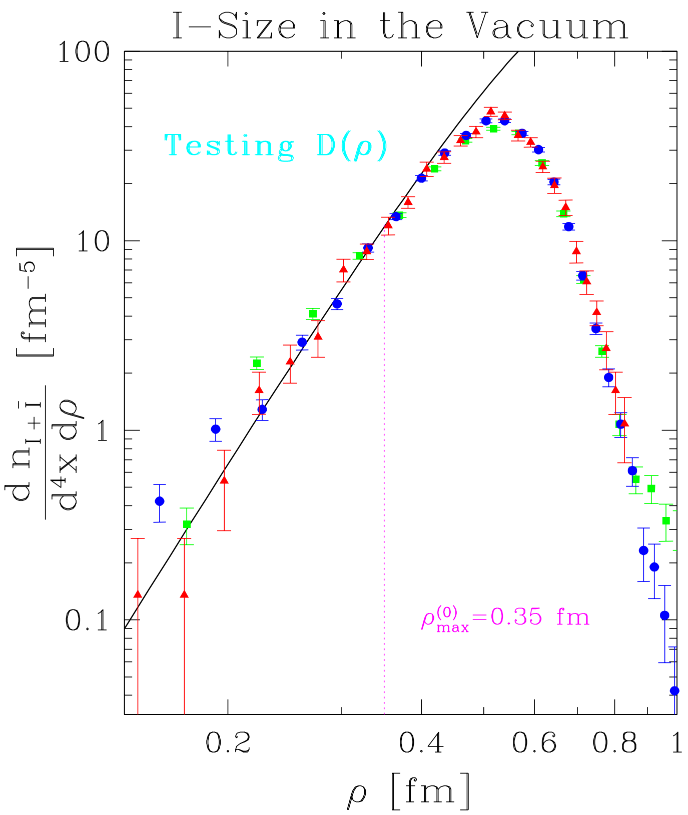
Region of Validity of the Predictions in (Q', x')

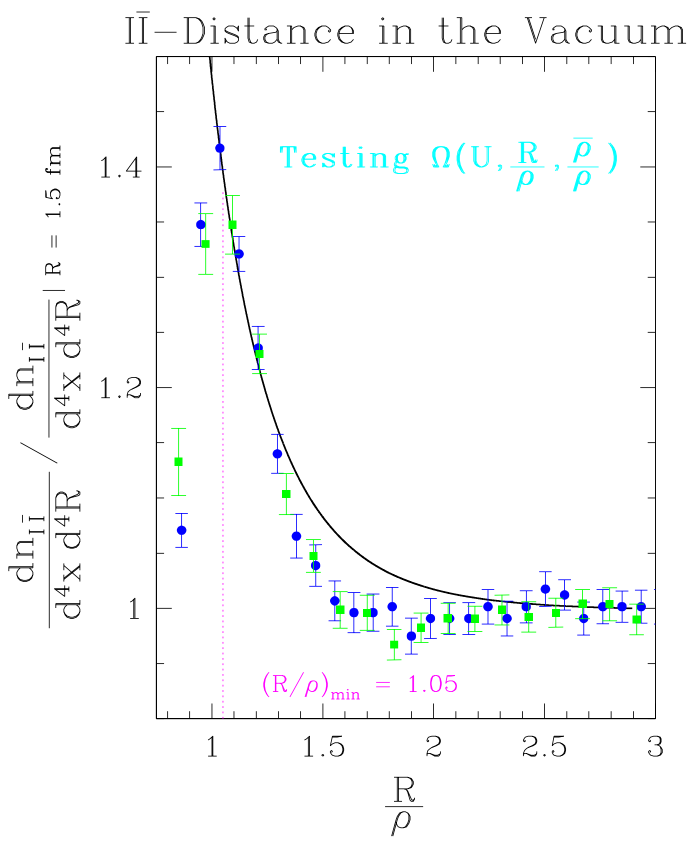
[Ringwald & F. Sch., Phys. Lett. **B 459** (1999) 249; **B 438** (1998) 217; **B 503** (2001) 331]

Lattice

◇ High-quality 'data': QCD $_{n_f=0}$ [UKQCD, Smith & Teper '98]
$\Lambda^{(n_f=0)}_{\overline{\rm MS}} = (238\pm19)$ MeV [ALPHA-Collaboration '99]

♣ ≈ **parameter-free** comparison of I-size and $I\bar{I}$-distance distributions with I-perturbation theory versus (ρ, R):





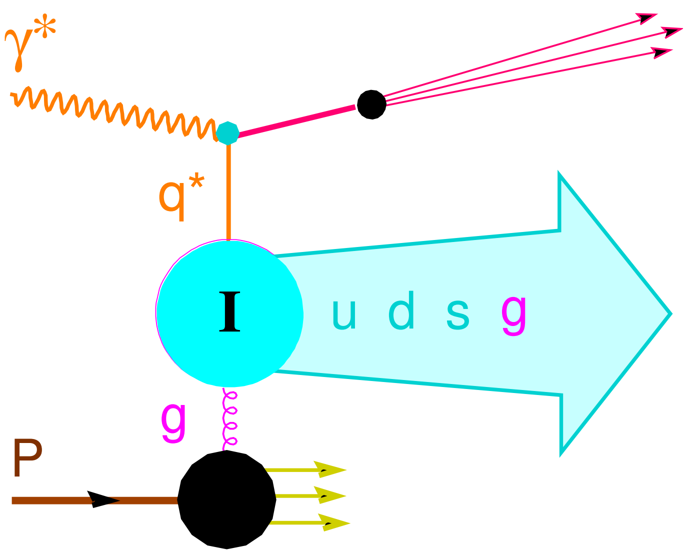
♣ **Results:** I-perturbation theory quantitatively valid for

$$\left.\begin{array}{lcl} \rho\cdot\Lambda^{(n_f=0)}_{\overline{\rm MS}} & \lesssim & 0.42 \\ R/\rho & \gtrsim & 1.05 \end{array}\right\} \overset{\text{saddle pt}}{\Rightarrow} \left\{\begin{array}{lcl} Q'/\Lambda^{(n_f)}_{\overline{\rm MS}} & \gtrsim & 30.8, \\ x' & \gtrsim & 0.35, \end{array}\right.$$

Characteristic Final-State Signature

Indispensable Tool: Monte-Carlo generator package for simulation of instanton events at **HERA**:

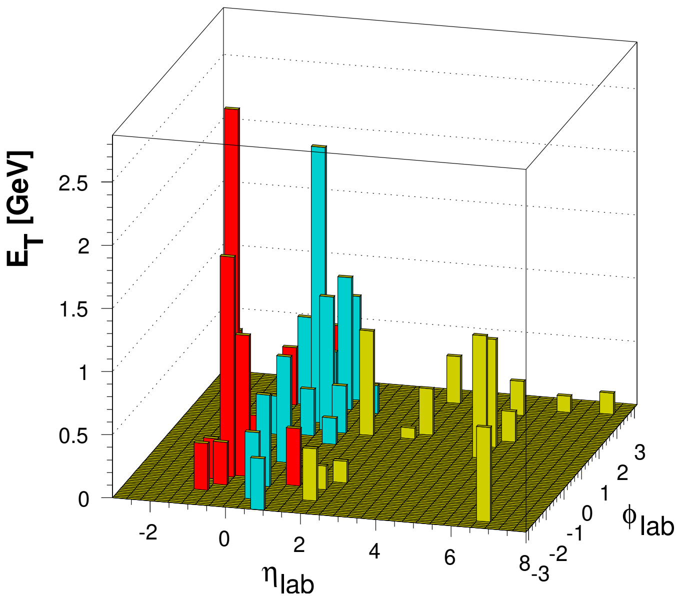
"QCDINS 2.0" [Gibbs, Ringwald & F. Sch. '95, Ringwald & F. Sch, Comp. Phys. Com. **132** (2000) 267 (**long-wite-up**); **WEB**: http://www.desy.de/∼t00fri/qcdins/qcdins.html]



- current jet
- "band"-region: ⇔

In I-rest system: "Fireball" decaying **isotropically** into

$n_f \cdot q + n_f \cdot \overline{q} + \mathcal{O}(\frac{1}{\alpha}) \cdot g = \mathcal{O}(10)$ partons



◇ Large total E_t
◇ Large multiplicity
◇ "u, d, s flavour democracy": **Strangeness** $\Rightarrow K's, \Lambda's$
◇ No further jets

Theory vs. First Dedicated Search Results (H1)

[A. Ringwald & F. Sch., Phys. Lett. **B503** (2001) 331]

- Preliminary results from first dedicated search for I-induced **DIS** events by **H1**-Collaboration

- Status as of *ICHEP2000*, Osaka, July 2000
  c.f. following experimental talk by B. Koblitz (H1).

- **Search Strategy and Observables**: $\Leftarrow$
  [T. Carli, J. Gerigk, A. Ringwald & F. Sch., DESY-PROC-1999-02]

  Isolation of "I-enriched data" by means of cuts to a set of most I-sensitive observables. Place cuts such as to maximize

  $$\boxed{I\text{-separation power} = \frac{\epsilon_I}{\epsilon_{\rm nDIS}}}$$

  in terms of efficiencies $\epsilon_I > 10\%$ (**QCDINS**) and $\epsilon_{\rm nDIS}$ for instanton and normal **DIS** events, respectively.

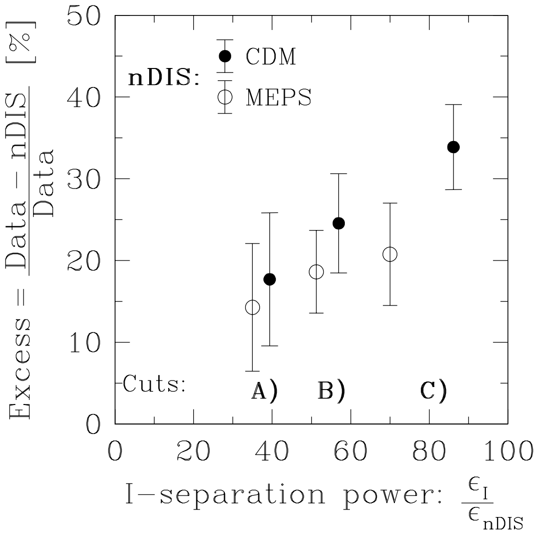
- **nDIS**: Two different Monte-Carlo's (**uncertainties!**)

  $$\left\{ \begin{array}{l} \textbf{CDM} = \text{“Color Dipole Model” } (\Rightarrow \text{ARIADNE}) \text{ and} \\ \textbf{MEPS } (\Rightarrow \text{RAPGAP}) \end{array} \right\}$$

- **3 cut scenarios: A), B), C)**:

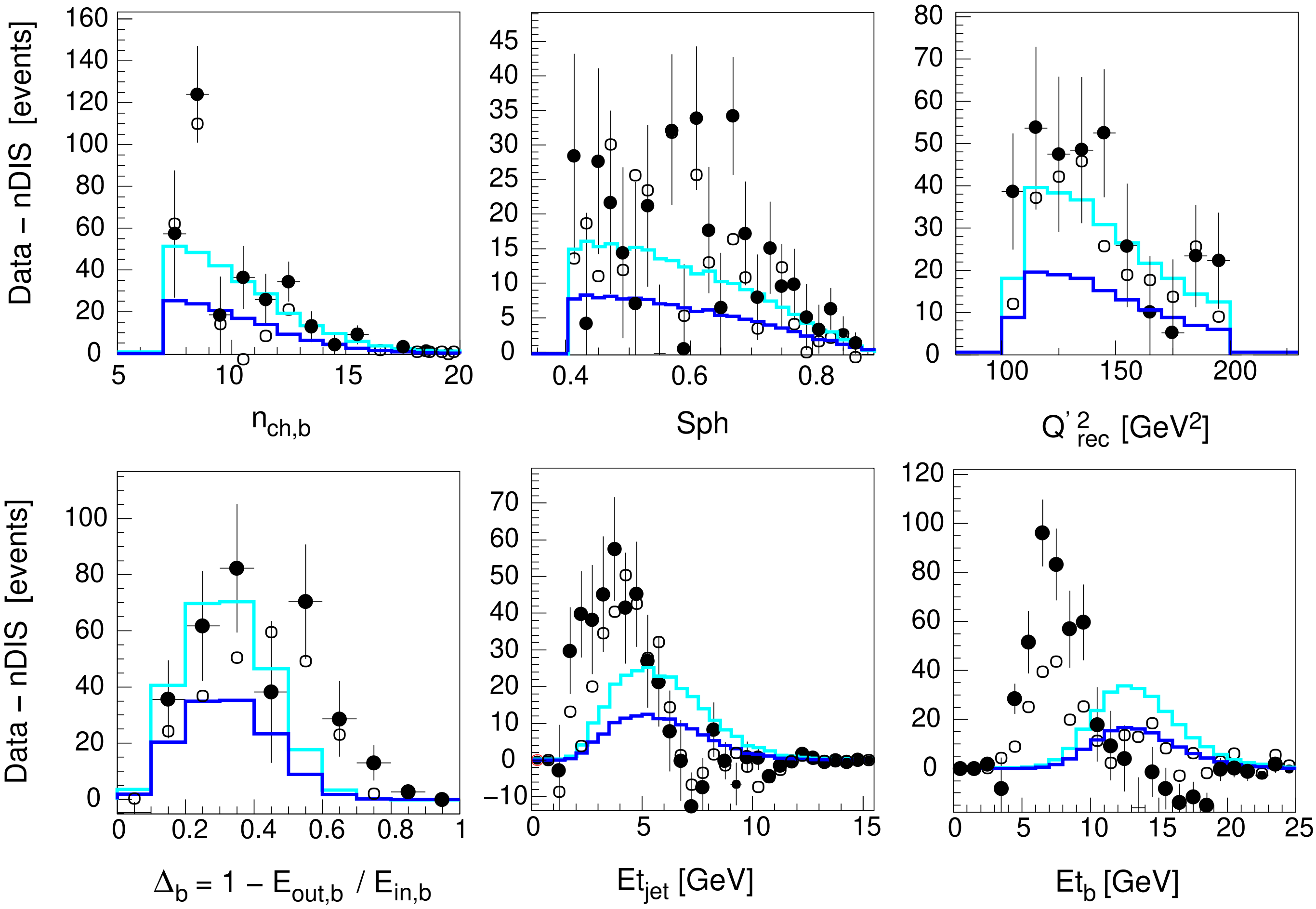
  $\Leftrightarrow$ increasing I-separation power ($\Leftarrow$ **MC** generators), focussing on basic characteristics of I-induced events!

♣ Significant excess, increasing with I-separation power!



[H1 preliminary]

Cuts B): 1332 events; excess: $\left\{ \begin{array}{l} 327^{+82}_{-70} \text{ CDM} \\ 248^{+75}_{-46} \text{ MEPS} \end{array} \right.$



Data − CDM
Data − MEPS } **no x' cut**

QCDINS 2.0 (default); x'>0.35
QCDINS 2.0 ; x'>0.325

♣ In 4 out of 6 observables, **excess** intriguingly similar in size & shape to our predictions from **QCDINS**.

▽ $\boldsymbol{Et}_{\rm jet}$ and $\boldsymbol{Et_b}$ excess: peak offset to the left and too narrow.

▽ **Caveat:** So far, **H1** data incorporate only part of the **theoretically** required cuts: $Q'^{\,2} \gtrsim 113\ {\rm GeV}^2$ (OK), but **no** x'-cut and **no** Q^2-cut.

H1 compared data to our predictions from **QCDINS MC with default** $(Q'^{\,2}, x')$**-cuts**, and **no** Q^2**-cut**

▽ **Missing x'-cut in data:** Not too bad, since

QCDINS with default x'**-cut** models sharp suppression of $\boldsymbol{I}$-effects, apparent in lattice data for $R/\rho \lesssim 1.0 - 1.05$, i.e. $x' \lesssim 0.3 - 0.35$ (c.f. Fig. of $\boldsymbol{I\bar{I}}$-distance distribution)

Yet, substantial unnecessary uncertainty in predicted magnitude of I-signal introduced! (c.f. previous Fig.)

▽ **Missing Q^2-cut:** $Q^2_{\rm min} = \mathcal{O}(100\ {\rm GeV}^2)$ **necessary**, since

– Assures dominance of "planar" (handbag-type) graphs

– Non-planar contributions involve I's of size $\sim 1/Q$ rather than $1/Q'$! These need to be small, too.

– Non-planar graphs hard/slow to calculate ⇒ ignored in **QCDINS**!

- [A. Ringwald & F. Sch., Phys. Lett. **B503** (2001) 331] $\Rightarrow$
  - $\boldsymbol{Et}_{\text{jet}}$ and $\boldsymbol{Et_b}$ distributions for I-events strongly affected by missing Q^2-cut, unlike the 4 other observables .
  - By cutting out PS-region ($\Leftrightarrow$ small Q^2), where **QCDINS** unreliable, ($\boldsymbol{Et}_{\text{jet}}$, $\boldsymbol{Et_b}$)-peaks left-shifted and narrower, in good agreement with shape-normalized **H1**-excess $\Rightarrow$

> ♣ Improved overall agreement and understanding. . .
>
> ♣ Both the Q^2-cut and x'-cut **can** and **should** be implemented experimentally. $\Rightarrow$ Strong reduction of remaining (size-)ambiguities!
>
> - Basic remaining problematics: Excess based on **MC** for **nDIS**. I-signal where **nDIS** not too well known . . .

3. Larger-size Instantons

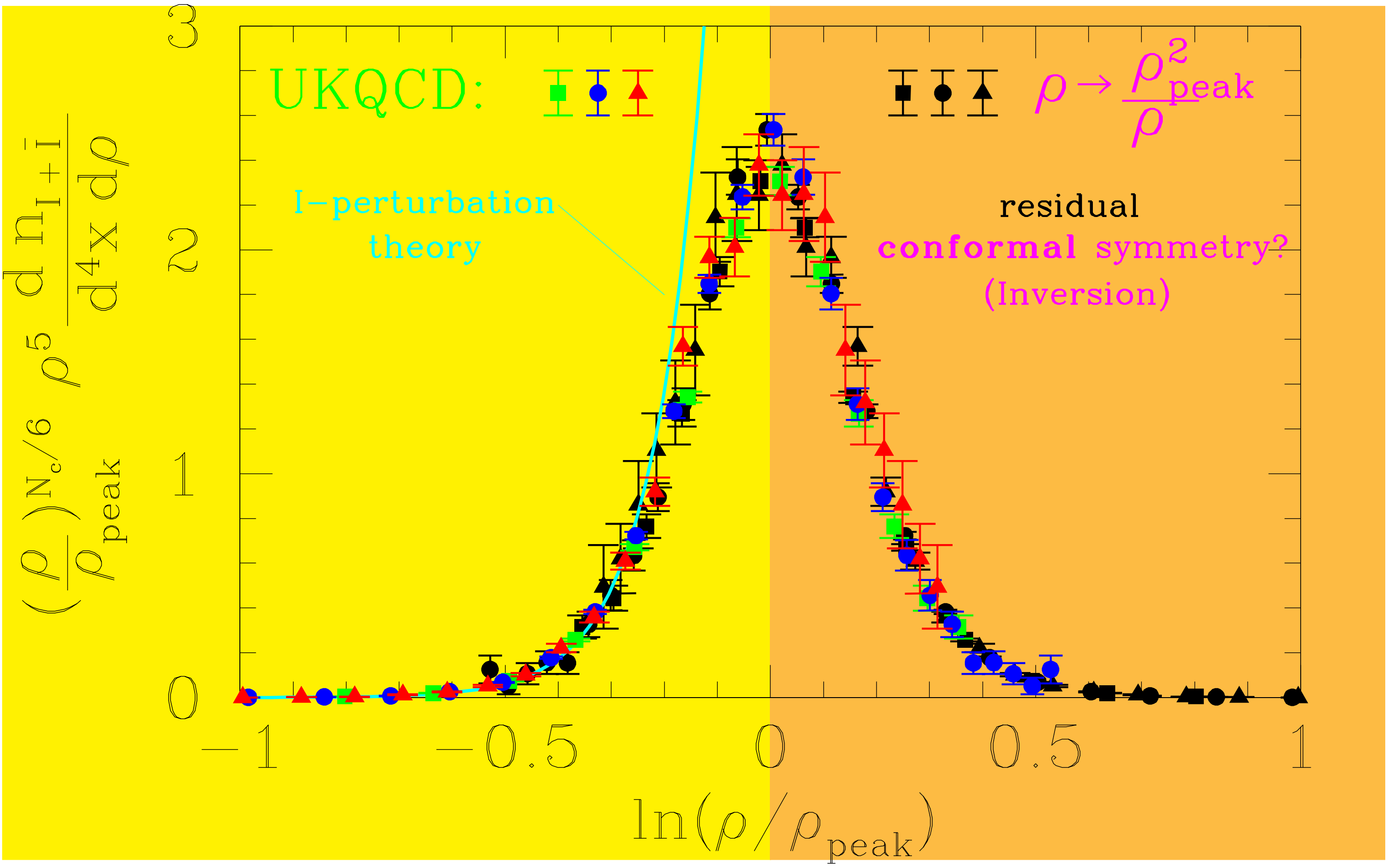
$$\text{Large-size instantons} \begin{cases} \text{non-perturbative long-distance regime} \\ \text{rôle in chiral symm. break., confinement?} \\ \text{piling up for smaller } Q^2 : I\!\!P\text{omeron?} \end{cases}$$

Practical: Larger-size I's $\Leftrightarrow$ smaller $Q'^2_{\min}$, larger I-search regime at **HERA**!

Main theoretical problem: solid framework, small expansion parameter, **symmetries** . . . ?$\Leftarrow$ Crucial guidance from $\boxed{\text{lattice}}$

QCD Vacuum

Lattice data $\Rightarrow$



♣ **Striking** (Fig.): Why are large I's ($\rho \gtrsim \rho_{\rm peak} \approx 0.6$ fm) so much and suddenly suppressed (c.f. I-pert. theory!)

- **Interesting:** [Shuryak '99] $\Rightarrow$ Large I's squeezed by onset of color Meissner effect ((dual) Superconductivity)

♣ **[F. Sch. '01]**: Apparent $\boxed{\textbf{inversion symmetry } \rho \Leftrightarrow \frac{\rho_{\rm peak}^2}{\rho}}$ (Fig.)

– relating non-perturbative regime ($\rho \gtrsim \rho_{\rm peak} \approx 0.6$ fm) to I-pert. theory for small-ρ I's!

– "Relict" from **conformal invariance** of I-sector at classical level?

- Full conformal group $\left\{ \begin{array}{llll} \text{Poincaré} & & \Rightarrow & \text{conserved} \\ \text{Scale trf's}: & x'_\mu & \Rightarrow x_\mu/\lambda & \text{broken} \\ \textbf{Inversion}: & x'_\mu & \Rightarrow \frac{\rho_0^2}{x^2}\, x_\mu & \textbf{?} \end{array} \right.$

- Scale trf's broken by regulariz./renorm., $\langle \theta^\mu_\mu \rangle \sim -\langle G^2 \rangle \neq 0$

- **Note:** Inversion not continuously connected to identity!

- E.g. classical I-gauge field under inversion $x'_\mu \Rightarrow \frac{\rho_0^2}{x^2}\, x_\mu$

$$\boxed{A'^{\,(I)\,a}_{\mu\,\mathrm{sing.\,g.}}(x';\tfrac{\rho_0^2}{\rho}) = A^{(\overline{I})\,a}_{\mu\,\mathrm{reg.\,g.}}(x;\rho)}$$

Note: $\rho \Leftrightarrow \frac{\rho_0^2}{\rho}$!

- Behaviour of 1 I vacuum-to-vacuum (tunnel) amplitude ($\Leftrightarrow$ I-size distribution), under conformal trf's is calculable! [Yoneya '77, Folov & Schwarz '79, Corrigan *et al* '79] $\Rightarrow$

- May isolate non-invariant factor $\propto (\rho/\rho_0)^{-N_c/6}$ (from non-zero mode det's) (c.f. Fig.)

- **Intriguing consequence:**

  Define (non-perturbative) "I-scheme" for $\alpha(\frac{s_I}{\rho})$, by requiring validity of perturbative expression for all ρ $[s = \mu_r\,\rho = \mathcal{O}(1)]$

$$\left(\frac{\rho}{\rho_{\rm peak}}\right)^{Nc/6} \rho^5 \frac{dn_{I+\bar{I}}}{d^4x\,d\rho} \equiv \left(\frac{\rho}{\rho_{\rm peak}}\right)^{Nc/6} f_{\rm known}(\alpha(\tfrac{s_I}{\rho}); s_I)$$

Inversion sym.$\Rightarrow$ $$= \left(\frac{\rho_{\rm peak}}{\rho}\right)^{Nc/6} f_{\rm known}(\alpha(\tfrac{s_I\,\rho}{\rho^2_{\rm peak}}); s_I)$$

♣ Solve for $\alpha(\frac{s_I}{\rho})$ in terms of $\alpha(\frac{s_I\,\rho}{\rho^2_{\rm peak}})$; (Lambert W-function)

♣ With as. freedom form for $\alpha(\frac{s_I\,\rho}{\rho^2_{\rm peak}}) \overset{\rho\ \rm large}{\approx} \frac{2\pi}{\beta_0\ \log(\frac{s_I\,\rho}{\rho^2_{\rm peak}\,\Lambda})} + \ldots,$

$\Rightarrow$ $\boxed{\alpha(\frac{s_I}{\rho}) \overset{\rho\ \rm large}{\approx} \sigma\rho^2 + \ldots \Leftrightarrow \text{linearly rising } q\bar{q}\text{-potential!}}$

Instanton Contribution to Color-Dipole Picture

[F. Sch. & A. Utermann '01, in progress]

- **Project:** Investigate intriguing possibility of larger-size I's piling up at small $x_{\rm Bj}$ $\Rightarrow$ Diffraction ($I\!\!P$) $\Leftarrow$ **Lattice**
  [D. Kharzeev, Y.V. Kovchegov & E. Levin '00, V. Shuryak *et.al* '00]

- **Careful start:** Large Q^2, within validity of I-pert. theory

- Consider "simplest" I-process, $\boxed{\gamma^*\,g \overset{(I)}{\Rightarrow} q_R\,\overline{q}_R}$ $(n_f = 1)$

- Transform our published results [Moch, Ringwald & F. Sch. '97] into color-dipole picture, compare with pQCD expressions

- General structure analogous to pQCD , in terms of wave functions $\mid \Psi^{(I)}(\vec{r}, z)_{\mathrm{L,T}} \mid^2$ and dipole cross section $\sigma^{(I)}_{(q\overline{q})P}$
- **Most interesting difference:** Square of transverse size $\vec{r}^2$ of $q\overline{q}$-pair competes with I-size parameters $\rho\overline{\rho}$, appearing in additional integrations!

  ρ, $\overline{\rho}$ fixed:

  $$\mid \Psi^{(I)}(\vec{r}, z)_{\mathrm{L}} \mid^2 \ \sigma^{(I)}_{(q\overline{q})P} \ \stackrel{\vec{r}^2 \ \text{small}}{\propto} \ \frac{Q^2}{\alpha_s} \rho\overline{\rho}\, g(x_{\mathrm{Bj}}, Q^2) \neq \ 0$$

- Effective I-size matters for saturation, rather than $\vec{r}^2$!

- unlike pQCD : (color transparency, scaling)

  $$\mid \Psi(\vec{r}, z)_{\mathrm{L}} \mid^2 \ \sigma_{(q\overline{q})P} \ \stackrel{\vec{r}^2 \ \text{small}}{\propto} \ \alpha_s \log(\vec{r}^2)^2\, Q^2\, \vec{r}^2\, g(x_{\mathrm{Bj}}, Q^2) \ \Rightarrow 0$$

♣ Larger-size instantons & their impact on high-energy scattering (Diffraction ($I\!P$), **HERA**-physics?): important theoretical challenge. . .

♣ Hope for progress via crucial guidance from high-quality **lattice** 'data'

♣ Large-size and small-size I's related by conformal **inversion** symmetry!?

♣ First results from study of I's in color-dipole picture

No apparent color transparency for vanishing transverse $q\overline{q}$-distance. . .