Confronting Instanton Perturbation Theory with QCD Lattice Results

F. Schrempp, DESY

in collaboration with

A. Ringwald

1. **Instanton-Perturbation Theory in DIS**
2. **Confrontation with Lattice Results (UKQCD)**
3. **Conclusions**

Copy available via WWW:

http://www.desy.de/~t00fri/talks.html

– Oxford, June 2nd 1999 –

1. Instanton-Perturbation Theory in DIS

[Ringwald & F. Sch., Phys. Lett **B438** (1998) 217,
Moch, Ringwald & F. Sch., Nucl. Phys. **B507** (1997) 134]

- **DIS** distinguished, since hard instanton-induced processes
  - ♣ are calculable within instanton-perturbation theory
  - ♣ can possibly be also detected experimentally ⇐ **HERA** .
    [Ringwald & F. Sch. '94 - '99]
  - ♣ Key feature: hard generic DIS scale $\mathcal{Q}$ cuts off instantons with large size $\rho \gtrsim \mathcal{Q}^{-1}$, over which we have no control!

2.1 Exclusive Amplitudes

[Moch, Ringwald & F. Sch., Nucl.Phys. **B**507 (1997) 134]

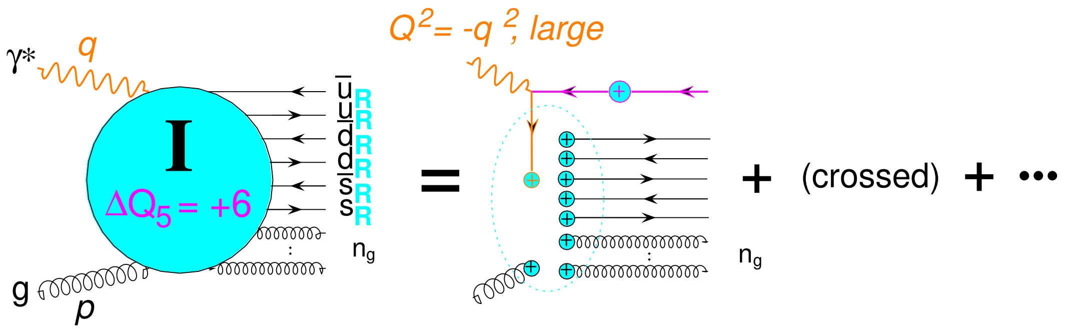
- Consider Instanton (I)-induced amplitude for exclusive DIS processes,

$\triangle Q_5 = 2\, n_f$:

$$\gamma^* + \mathrm{g} \Rightarrow \sum_{\text{flavours}}^{n_f} [\overline{q}_R + q_R] + n_g\, g$$

vanishing to any order of conventional P.T in (massless) QCD:

- I-Perturbation Theory: Expand (Euclidean) path integral for relevant Green's functions about classical instanton solution:



- Fourier-transform (F. T.) with respect to external lines.
- LSZ amputate external legs.
- Analytically continue to Minkowski space.

- Basic building blocks: (singular gauge)
  in (modified) Feynman rules of I-perturbation theory:
  - ◇ Instanton gauge field $A_\mu^{(I)}(x;\rho,U)$ [Belavin *et al.* '75];
  - ◇ Quark zero mode fields $\kappa^{(I)}(x;\rho,U)$, $\overline{\phi}^{(I)}(x;\rho,U)$ (Weyl basis);
  - ◇ (Non-zero mode) quark propagators [Brown *et al.* '78] $S^{(I)}(x,y;\rho,U)$, $\overline{S}^{(I)}(x,y;\rho,U)$ in the I-background

- Collective coordinates: $\boldsymbol{\rho}$ = I-size, $(U)^k_\alpha$ = I-color orientation (color: $k=1,2,3$ and spinor: $\alpha=1,2$ indices).

- Must be integrated over in amplitudes:

$$\mathcal{T}_\mu^{(I)}(q,p,\ldots) = \int dU \int_0^\infty d\rho \;\; D(\rho) \;\; \mathcal{A}_\mu^{(I)}(q,p,\ldots;U,\rho)$$

- (reduced) I-size distribution $D(\rho)$ as generic measure:

$$D(\rho) = d\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left(-\frac{2\pi}{\alpha_s(\mu_r)}\right)\frac{(\rho\,\mu_r)^{b}}{\rho^5}$$

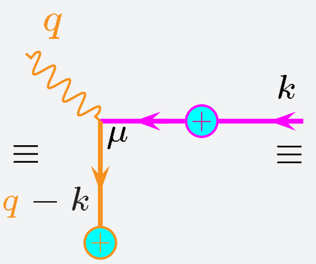
(renormalization scale μ_r)

b	$\frac{1}{D}\frac{\mathrm{d}D}{\mathrm{d}\mu_r}$	Ref.
β_0	$\mathcal{O}(\alpha_s)$	't Hooft '76
$\beta_0 + \frac{\alpha_s(\mu_r)}{4\pi}(\beta_1 - 12\beta_0)$	$\mathcal{O}(\alpha_s{}^2)$	Morris, Ross, & Sachrajda '85

QCD β-function coefficients: $\beta_0 = 11 - \frac{2}{3}\mathrm{n_f}$; $\beta_1 = 102 - \frac{38}{3}\mathrm{n_f}$
d is a known, scheme-dependent constant

- Q^2 only enters in F.T'ed $\gamma^* q\overline{q}$-vertices,

$\mathcal{V}_\mu(q,-k;\rho) \equiv$



$$\equiv \int d^4x\, \mathrm{e}^{-\mathrm{i}q\cdot x}\left[\overline{\phi}^{(I)}(x;\rho)\overline{\sigma}_\mu \boldsymbol{S}^{(I)}(x,-k;\rho)(-\mathrm{i}\overline{k})\right]$$

- After a long and tedious calculation ($v = v_\mu \sigma_\mu,\ \overline{v} = v_\mu \overline{\sigma}_\mu$),

$$\lim_{k^2 \to 0} \mathcal{V}_\mu(q, -k; \rho) = 2\pi \mathrm{i} \rho^{3/2} \left[\epsilon \sigma_\mu \overline{V}(q, q'; \rho)\, U^\dagger\right] ,$$

with $q' = q - k,\ Q' = \sqrt{-q'^2} \equiv \sqrt{-t} > 0,\ Q = \sqrt{-q^2} \geq 0,$

$$\overline{V}(q, q'; \rho) = \left[\frac{\overline{(q')}}{Q'^2} + \frac{\overline{q - q'}}{Q'^2 - Q^2}\right] \rho Q' \, K_1(\rho Q') - \frac{\overline{q - q'}}{Q'^2 - Q^2} \rho Q \, K_1(\rho Q)$$

- Various terms strongly constrained by manifest gauge invariance

$$\boldsymbol{q}^{\boldsymbol{\mu}} \mathcal{T}_\mu^{(I)}(q, p, \ldots) = 0$$

- 2^{nd} term ("contact" term) in $\overline{V}(q, q'; \rho)$ necessary to cancel an unphysical pole in physical region at $Q'^2 = Q^2$!

- In DIS: Exponential decrease of all form factors (Bessel K_1 functions)

$$K_1(\rho Q) \overset{\rho Q \Rightarrow \infty}{\sim} \exp\left[-\rho Q\right].$$

$\Rightarrow$ Integration over the instanton size ρ in amplitudes $\mathcal{T}_\mu^{(I)}(q, p, \ldots)$ is finite for large $Q > 0$

[as long as the internal quarks are not moving collinear to the incoming gluon (like in pQCD!)].

♣ **Hard**, partonic scattering processes (fixed-angle) in **DIS** are reliably calculable in $\boldsymbol{I}$-perturbation theory.

♣ Only small size instantons, $\boldsymbol{\rho} < 1/\mathcal{Q}$, are probed in **DIS**.

♣ Amplitudes manifestly gauge invariant.

♣ Important Question: Is the hard photoproduction limit $(Q^2 \Rightarrow 0)$ with a high E_t of the current quark jet within the reach of $\boldsymbol{I}$-perturbation theory?

- **True** in ordinary QCD perturbation theory...

♣ Motivation: "Monster" fireball events (ZEUS) & lots of further interesting data. . .

- For $\boldsymbol{Q^2 \Rightarrow 0}$: $\rho Q K_1(\rho Q) \stackrel{Q \Rightarrow 0}{\Rightarrow} 1,$,

- $\Rightarrow$ $2^{\rm nd}$ term in $\overline{V}(q,q';\rho)$ $\boldsymbol{\rho}$-independent and $\boldsymbol{\rho}$-integration IR-divergent within $\boldsymbol{I}$-perturbation theory, since there $D(\rho) \sim \rho^6$.

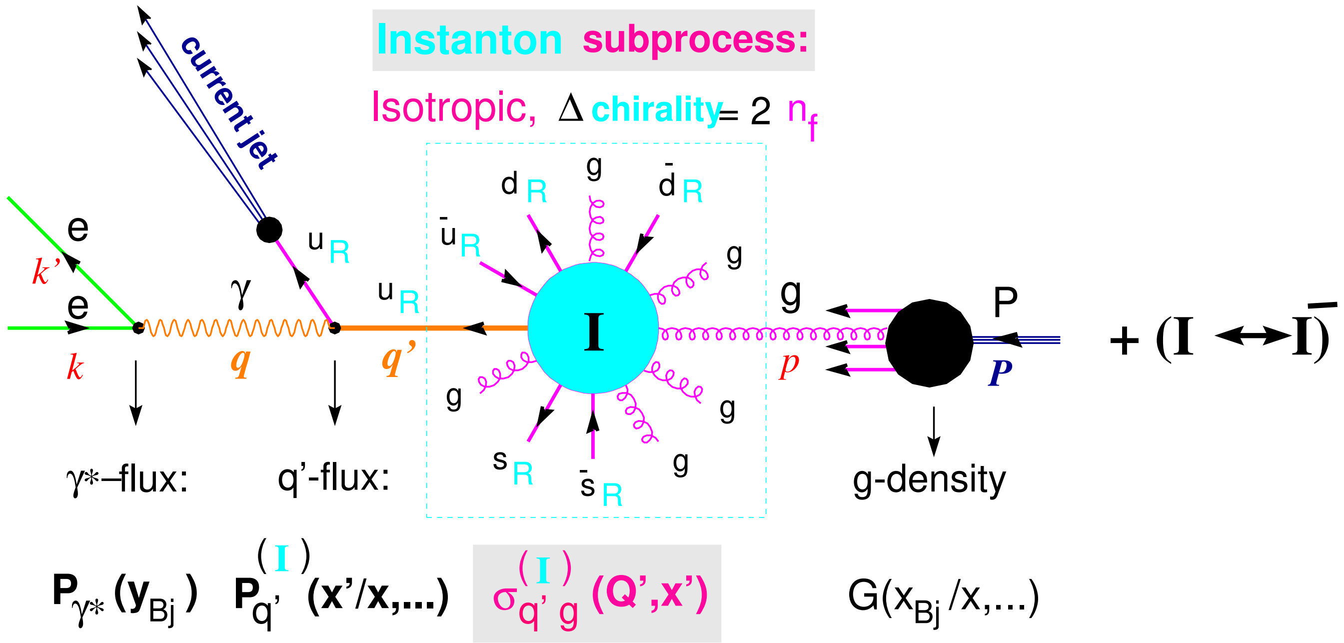
- This divergence is independent of current-quark E_t:

$$Q'^{\,2} = Q^2 + \frac{E_t}{\sqrt{s}} \left(s \exp(-\eta) - Q^2 \exp(\eta)\right) \overset{Q \Rightarrow 0}{\Rightarrow} E_t \sqrt{s} \exp(-\eta)$$

1.2 Inclusive DIS Processes

[Ringwald & F. Sch., Phys. Lett **B438** (1998) 217;
Moch, Ringwald & F. Sch., to appear]

- Leading instanton-induced, chirality violating process in DIS regime of $e^{\pm}P$ scattering for large $Q^2 = -q^2$, $Q'^{\,2} = -q'^{\,2}$:



- Instanton-subprocess with own **Bjorken**-variables, $(\boldsymbol{x'}, \boldsymbol{Q'^2})$

$$Q'^2 \equiv -q'^2 > 0; \qquad 0 \le x_{Bj} \le x \le x' = \frac{Q'^2}{2pq'} \le 1$$

- Show within $\boldsymbol{I}$-perturbation theory in Bjorken limit:

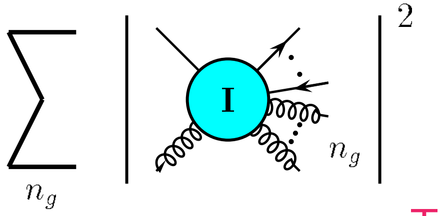
$$\frac{d\sigma_{\mathrm{HERA}}{}^{(I)}}{dx' dQ'^2} \simeq \frac{d\mathcal{L}_{q'g}^{(I)}}{dx' dQ'^2} \cdot \boxed{\sigma_{q'g}^{(I)}(Q', x')}$$

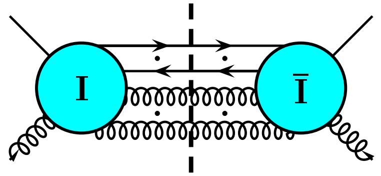
- Calculable differential $q'g$ luminosities $d\mathcal{L}_{q'g}^{(I)}$:
  - Involve integrations over γ^*-flux P_{γ^*}, q'-flux $P_{q'}^{(I)}$ and g-density G.
  - Account for number of $q'g$ collisions per eP collision.

- Crucial $\boldsymbol{I}$-dynamics in total cross section $\boxed{\sigma_{q'g}^{(I)}(Q', x')}$ of $\boldsymbol{I}$-induced subprocess $q'g \overset{\boldsymbol{I}}{\Rightarrow} X$!

- Two alternative ways to obtain the latter:

Summation of Exclusive Cross-Sections	Optical Theorem and $I\overline{I}$-Valley Method





Total cross-section obtained in the form

$$\sigma^{(I)}_{q'\,p} = \int d^4R\; e^{i(q'+p)\cdot R} \int_0^\infty d\rho \int_0^\infty d\overline{\rho}\; D(\rho)D(\overline{\rho}) \int dU\, e^{-\frac{4\pi}{\alpha_s}\Omega\left(U,\frac{R^2}{\rho\overline{\rho}},\frac{\overline{\rho}}{\rho}\right)} \{\ldots\}\, e^{-Q'(\rho+\overline{\rho})}$$

R_μ corresponds to

energy-momentum conservation,

$$(2\pi)^4\delta^{(4)}(p+q'-\textstyle\sum_i k_i) = \int d^4R\, e^{i(p+q'-\sum_i k_i)\cdot R}$$

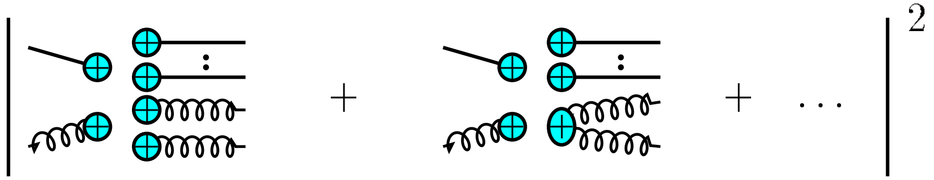
$$\int \frac{d^4k_i}{(2\,\pi)^3}\,\delta^{(+)}(k_i^2)\;\exp\left[-i\,k_i\cdot R\right] = \frac{1}{(2\pi)^2}\,\frac{1}{-R^2+i\epsilon R_0}\;,$$

$I\overline{I}$-distance 4-vector

$\Omega\left(U, \frac{R^2}{\rho\overline{\rho}}, \frac{\overline{\rho}}{\rho}\right)$ corresponds to

summation and exponentiation of final-state gluons,

$$\frac{\Omega}{f(U)} = -6\left(\frac{\rho\overline{\rho}}{-R^2+i\epsilon R_0}\right)^2 + 12\left(\frac{\rho\overline{\rho}}{-R^2+i\epsilon R_0}\right)^3\left(\frac{\rho}{\overline{\rho}}+\frac{\overline{\rho}}{\rho}\right) + \ldots$$



interaction between I and $\overline{I}$,

$$\Omega \equiv S^{(I\overline{I})} - 1,$$

with $I\overline{I}$ valley action

$$S^{(I\overline{I})} \equiv \frac{\alpha_s}{4\pi}\,S\left[A^{(I\overline{I})}_\mu\right].$$

Summation of Exclusive Cross-Sections

I-perturbation theory $\Leftrightarrow$

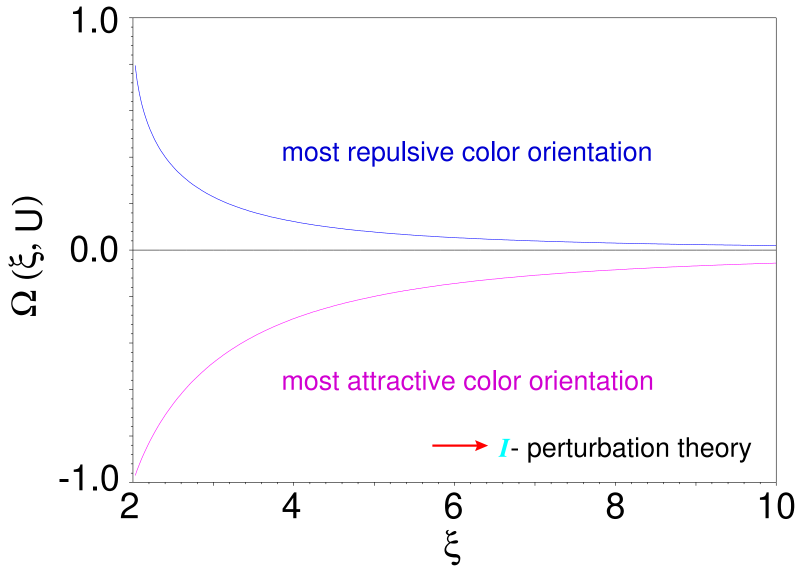
Ω known as series for large $\frac{R^2}{\rho\overline{\rho}}$.

Optical Theorem and $I\overline{I}$-Valley Method

$\Omega(\xi, U)$ analytically known for

all conformal separations $\xi \equiv \frac{R^2}{\rho\overline{\rho}} + \frac{\rho}{\overline{\rho}} + \frac{\overline{\rho}}{\rho}$

[Yung '88, V.V. Khoze, Ringwald '91; Verbaarschot '91]



Saddle-point evaluation of collective coordinate integrations,

$U^* \Leftrightarrow$ most attractive $I\overline{I}$ color orientation

$$\rho^* = \overline{\rho}^* \sim \frac{4\pi}{\alpha_s Q'} \qquad \frac{R^*}{\rho^*} \sim 2\sqrt{\frac{x'}{1-x'}}$$

(x', Q') related in a **one-to-one** correspondence to collective coordinates (R^*, ρ^*)

- In summary:

I-induced rate in **DIS** in terms of two crucial quantities of I-calculus:

$D(\rho) =$ I-size distribution, $\Omega\left(U, \frac{R^2}{\rho\overline{\rho}}, \frac{\overline{\rho}}{\rho}\right) =$ $I\overline{I}$-interaction

◇ **known** in I-perturbation theory, for $\alpha(\mu_r)\ln(\mu_r\rho) \ll 1$ and $R^2/\rho\overline{\rho} \gg 1$, respectively.

◇ $\Omega\left(U, \frac{R^2}{\rho\overline{\rho}}, \frac{\overline{\rho}}{\rho}\right)$ analytically known within "$I\overline{I}$-valley" approximation

◇ Powerlaw behaviour $D(\rho) \propto \rho^{6-2/3n_f+\mathcal{O}(\alpha)}$ generically spoils applicability of I-perturbation theory, since I-size integrals diverge, except in **DIS**, where "form factors" $F(Q'\rho)\,F(Q'\overline{\rho}) \propto \exp(-Q'(\rho+\overline{\rho}))$ insure $\rho \lesssim 1/Q'$

♣ $D(\rho)$ and $\Omega\left(U, \frac{R^2}{\rho\overline{\rho}}, \frac{\overline{\rho}}{\rho}\right)$ provide the link between I-observables in **DIS** and in the QCD vacuum ($\Leftarrow$ lattice)

2. Confrontation with Lattice Results (UKQCD)

[Ringwald & F. Sch., hep-lat/9903039, to appear in Phys. Lett. **B**]

- **Lattice**: "Snapshots" of the QCD -vacuum in (Euclidean) space-time, including all non-perturbative, topological features like instantons
- Recent high-quality lattice simulation of quenched QCD ($n_f = 0$) [UKQCD, Smith & Teper, '98,. . .]

$\beta \equiv 6/g_0^2(a)$:	$=$	6.4	6.2	6.0	
i. e. lattice spacing a :	$\simeq$	0.055	0.072	0.10	fm
volume $V = (n_s^3\, n_t)\, a^4$:	$=$	$32^3\, 64$	$24^3\, 48$	$16^3\, 48$	a^4
	$\simeq$	18.6	18.0	18.2	fm^4

topological structure over wide range of sizes $a < \rho < V^{1/4}$ accessible.

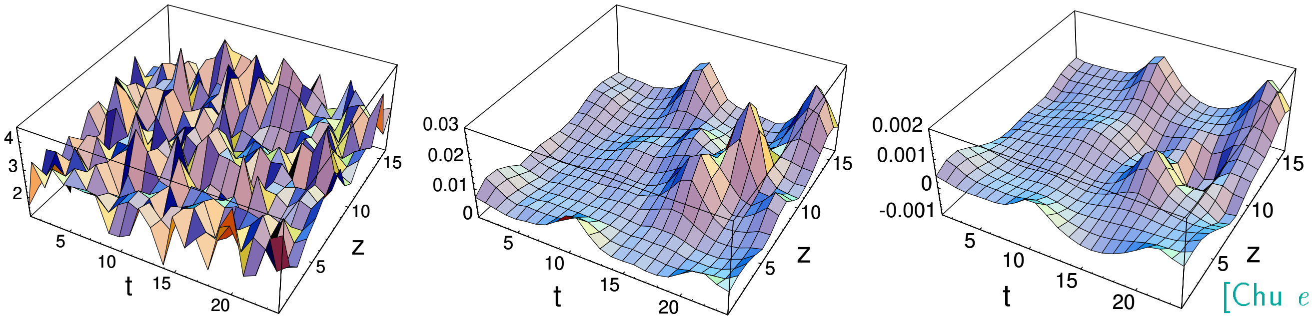
♣ $\boxed{\text{Range of validity of } I\text{-perturbation theory?}}$

- Crucial: $\Rightarrow$ "fiducial" (Q', x') region for **DIS** predictions!
- Interesting question by itself. . .

2.1 Cooling

- Designed to filter out (dominating) fluctuations of short wavelength $\mathcal{O}(a)$, while affecting topological fluctuations of size $\rho \gg a$ comparatively little.

Lagrange Density **Topological Charge Density**



[Chu *et. al* '94]

Before Cooling **After** Cooling

- **In practice**: n_{cool} sweeps trough the lattice links replacing existing link matrices $\Rightarrow$ new ones, locally minimizing (or reducing) the action

- $\Rightarrow$ Sequence of increasingly smooth or 'cold' lattice fields. May then study local topological structures ($\boldsymbol{I}$, $\overline{\boldsymbol{I}}$) via pattern recognition algorithms (c. f. Fig.)

- Cooling also removes $\boldsymbol{I}$'s and $\overline{\boldsymbol{I}}$'s, e.g. via $I\overline{I}$ annihilation $\Rightarrow$ **UKQCD**: "under-relaxed" (i. e. slow) cooling

- No objective criterium available yet, on quantitative amount of "vacuum damage" introduced by cooling. . .

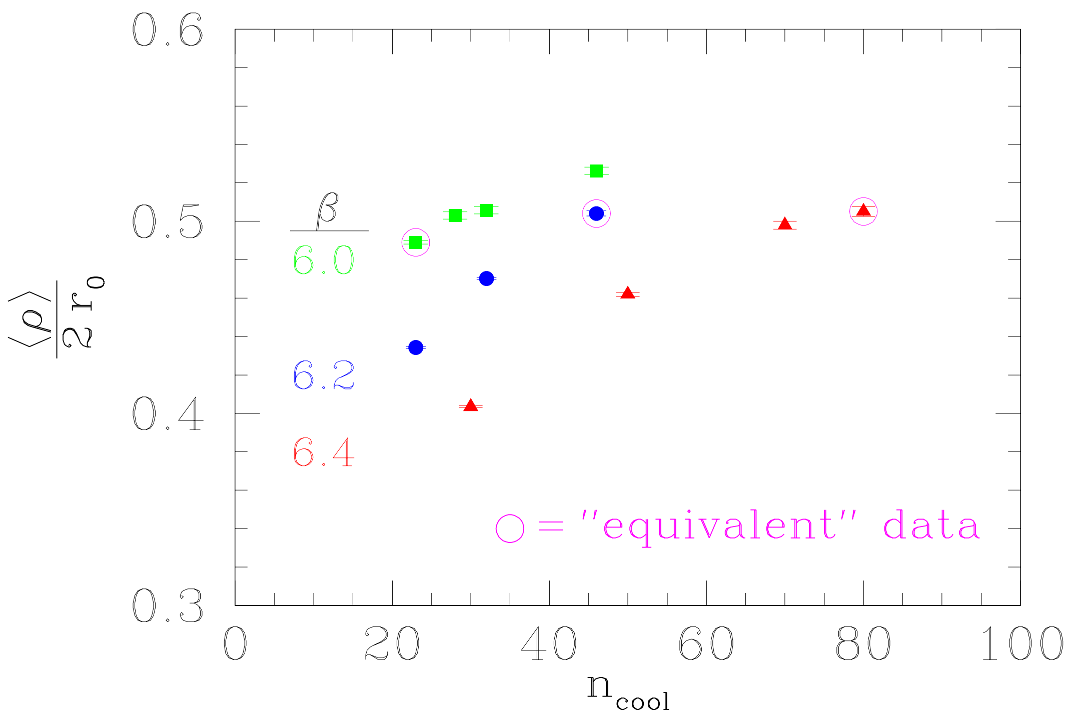
- Two proposals to approach physics of uncooled vacuum:

  ◇ Try extrapolating observables to $n_{\text{cool}} \Rightarrow 0$ [J. Negele '98]
  $\Leftarrow$ unrealistic: $\boldsymbol{I}$, $\overline{\boldsymbol{I}}$-density gets far too large. . .

  $\Rightarrow$ **UKQCD**: Starting from smallest reliable $n_{\text{cool}} \simeq 23$ at $\beta = 6.0$, say, find further "equivalent" pairs (β, n_{cool}) giving $\sim$ same shape & normalization of distributions.

◇ **Result:** pairs $(\beta, n_{\rm cool}) = (6.0, 23)$, $(6.2, 46)$, $(6.4, 80)$ used for continuum limit, $a \to 0$!

- **Example**: average I-size $\langle\rho\rangle$
- strong dependence on $n_{\rm cool}$ for fixed β, unlike "equivalent" data.



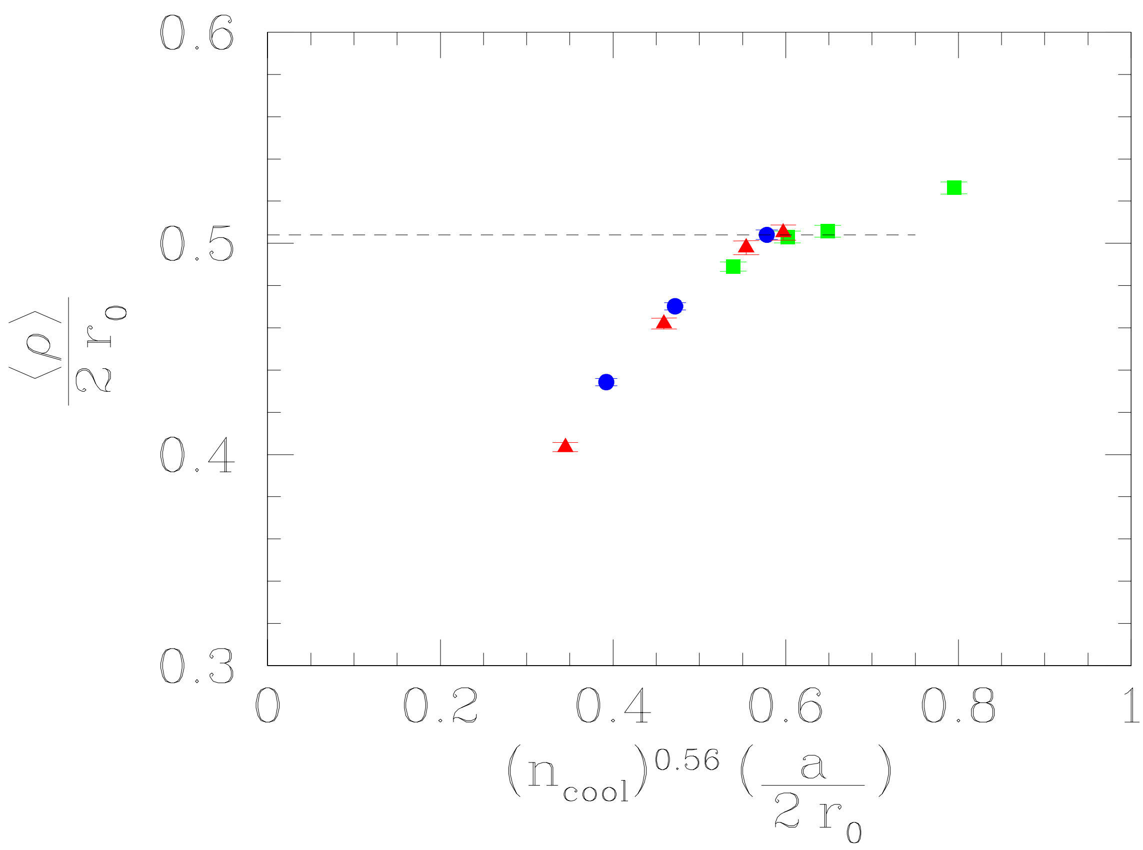
- **Throughout**: use Sommer scale $2\,r_0 \simeq 1$ fm ($\Leftrightarrow$ force $F_{\overline{q}q}$ between static quarks: $r_0^2 F_{\overline{q}q}(r_0) = 1.65$) rather than string tension σ to convert to physical units.
- Crucial question remains: Why $(\beta, n_{\rm cool}) = (6.0, 23)$ as "starting pair" distinguished?
- Striking observation: [Ringwald & F. Sch., hep-lat/9903039]

Cooling is kind of "diffusion process": length scale up to which it smoothes configurations grows with $n_{\rm cool}$.

- $\Rightarrow$ random walk argument:

"cooling radius": $\boxed{\frac{r_{\rm cool}}{2\,r_0} = {\rm const.}(\sqrt{n_{\rm cool}})^{1+\delta}\frac{a}{2\,r_0}}$; $\quad \delta$ small

- **Fig.:** $\boxed{\frac{\langle\rho\rangle}{2 r_0} \text{ vs. } \frac{r_{\rm cool}}{2\,r_0}}$ for all $(\beta, n_{\rm cool})$ on a single smooth curve, exhibiting a clear plateau $\Leftrightarrow$ "equivalent" data!



- $\boxed{\text{In summary:}}$ plateau region = correct regime for extracting physical information about instantons!

- Scaling in $r_{\rm cool}$ also for all other observables extracted by **UKQCD**.

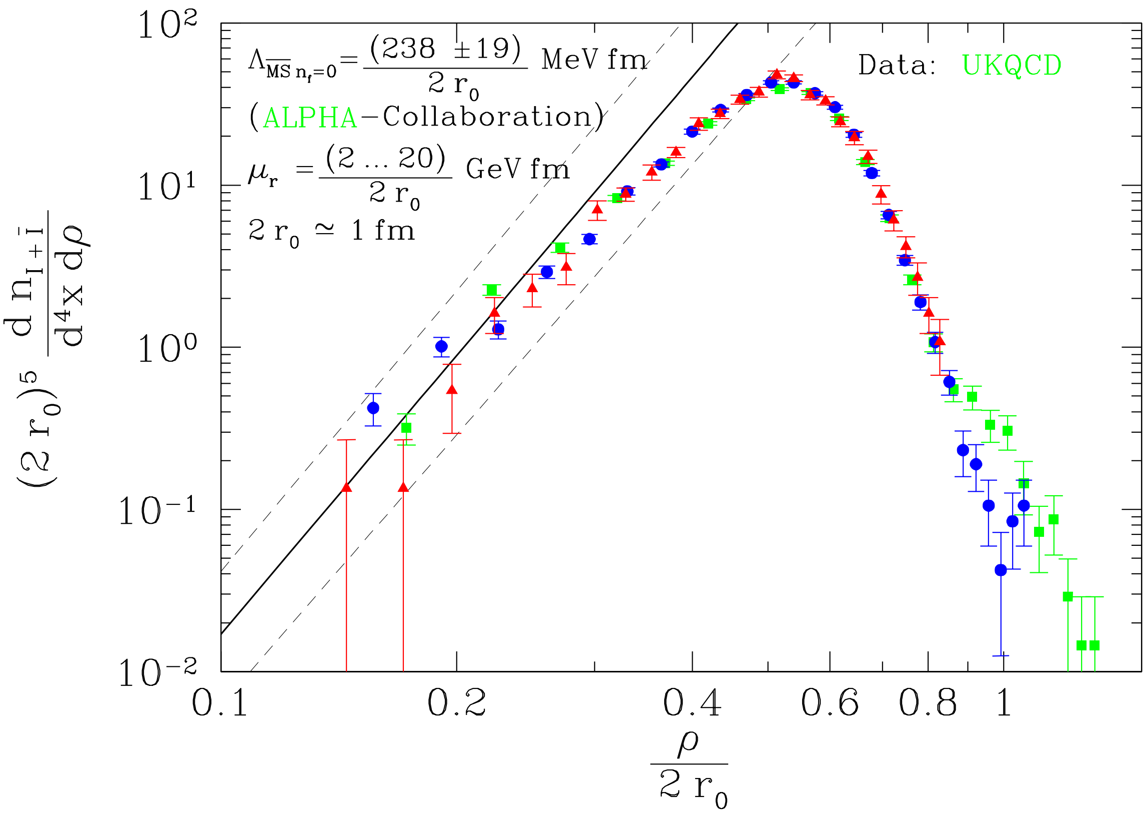
2.2 I-Size Distribution

- UKQCD measured: $\frac{d\, n_{I+\bar{I}}}{d^4x\, d\rho}$ = number of I's and $\bar{I}$'s per unit volume per unit size (pattern recognition)

- Perform continuum limit of "equivalent" data via ansatz

$$(2r_0)^5 \frac{d\, n_{I+\bar{I}}}{d^4x\, d\rho} = \text{function}\left(\frac{\rho}{\langle \rho(a)\rangle}\right),$$

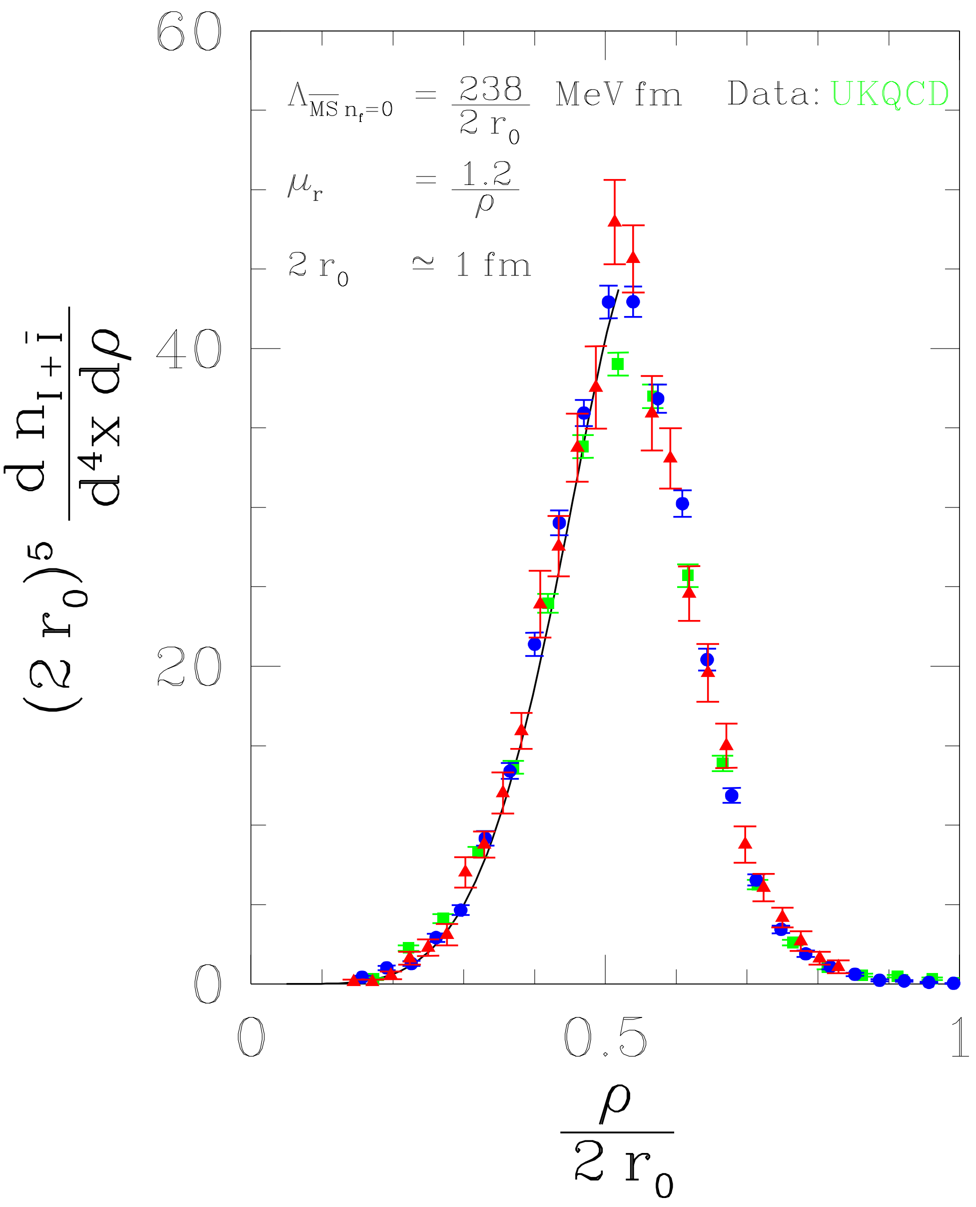
and rescaling by $\rho \Rightarrow \langle\rho(0)\rangle/\langle\rho(a)\rangle \cdot \rho$,
with $\langle\rho(0)\rangle/2r_0$ from linear extrapolation in $(a/2r_0)^2$.



- Parameter-free **agreement** in shape and normalization,

$$(2r_0)^5 \frac{d(n_{I+\bar{I}})}{d^4x\, d\rho} \simeq 2\, D(\rho)_{|I-\text{ pert. theory}}, \quad \text{for} \quad \rho \lesssim 0.3 \text{ fm}$$

- Error band $\Leftrightarrow$ error of $2r_0\Lambda_{\overline{\mathrm{MS}}\,n_f=0} = (238\pm19)$ MeV fm [ALPHA-Collaboration, Nucl. Phys **B 544** (1999) 669]
- Almost independent of μ_r for large range $2 \lesssim \mu_r \lesssim 20$ GeV.
- Improved choice: $\mu_r\rho = \mathcal{O}(1)$, to minimize $\ln(\mu_r\rho)$-corrections $\Rightarrow$ **agreement** up to the peak, $\rho \lesssim 0.5$ fm !



- **Note:** As function of α, D has a peak structure like lattice data! Maximum around $\alpha_{\mathrm{peak}} \approx \pi/N_c \simeq 1$ for $N_c = 3$

- Define non-perturbative "instanton-scheme" for α, by requiring exact validity of I-perturbative form for $D(\rho)$,

$$\rho^5 \frac{d\, n_{I+\overline{I}}}{d^4x\, d\rho} \equiv 2\, d_{\overline{\mathrm{MS}}} \left(\frac{2\pi}{\alpha_{\mathrm{I}}(\frac{1}{\rho})}\right)^{2\, N_c} \mathrm{e}^{-\frac{2\pi}{\alpha_{\mathrm{I}}(\frac{1}{\rho})}}\, s_{\mathrm{I}}{}^{\beta_0+\frac{\alpha_{\mathrm{I}}(\frac{1}{\rho})}{4\pi}(\beta_1-4\, N_c\beta_0)}$$

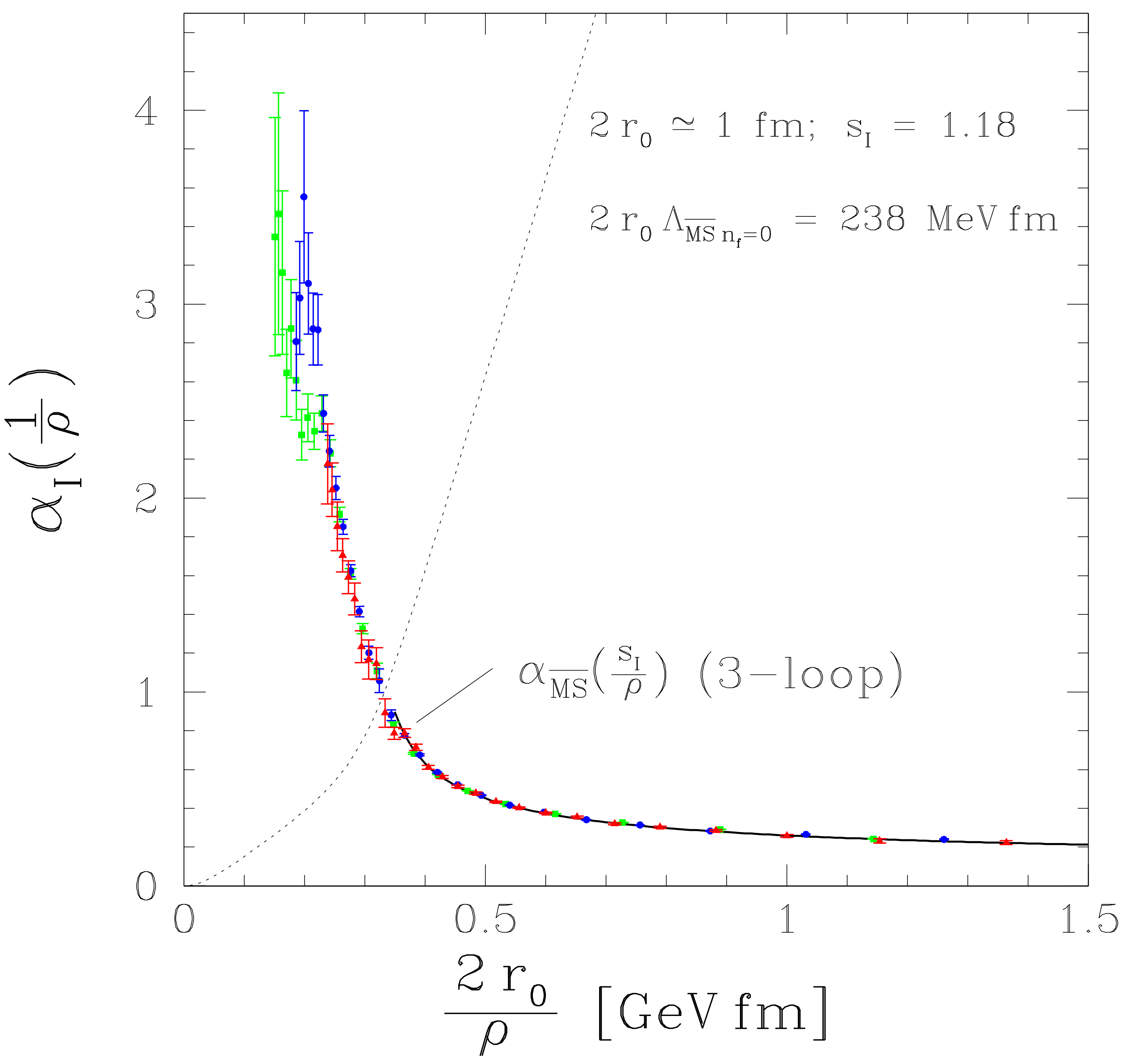
- standard 2-loop scheme conversion formula at short distances

  with $s_I = \Lambda_{\overline{\mathrm{MS}}}/\Lambda_I$ and $\alpha_{\overline{\mathrm{MS}}}(s_I/\rho) \simeq \alpha_I(1/\rho)$

- find from peak-region: $s_I = 1.18$, $\alpha_{I\,\mathrm{peak}} = 0.98 \Rightarrow$

  in perturbative regime, I-scheme close to $\overline{\mathrm{MS}}$-scheme.

- extract α_I for each data point of $(I + \overline{I})$-size distribution (c. f. Fig.)



- second solution $\alpha_{\boldsymbol{I}}^{(2)}$, vanishing in IR-limit (dotted line), with monopole-like relation in peak-region,

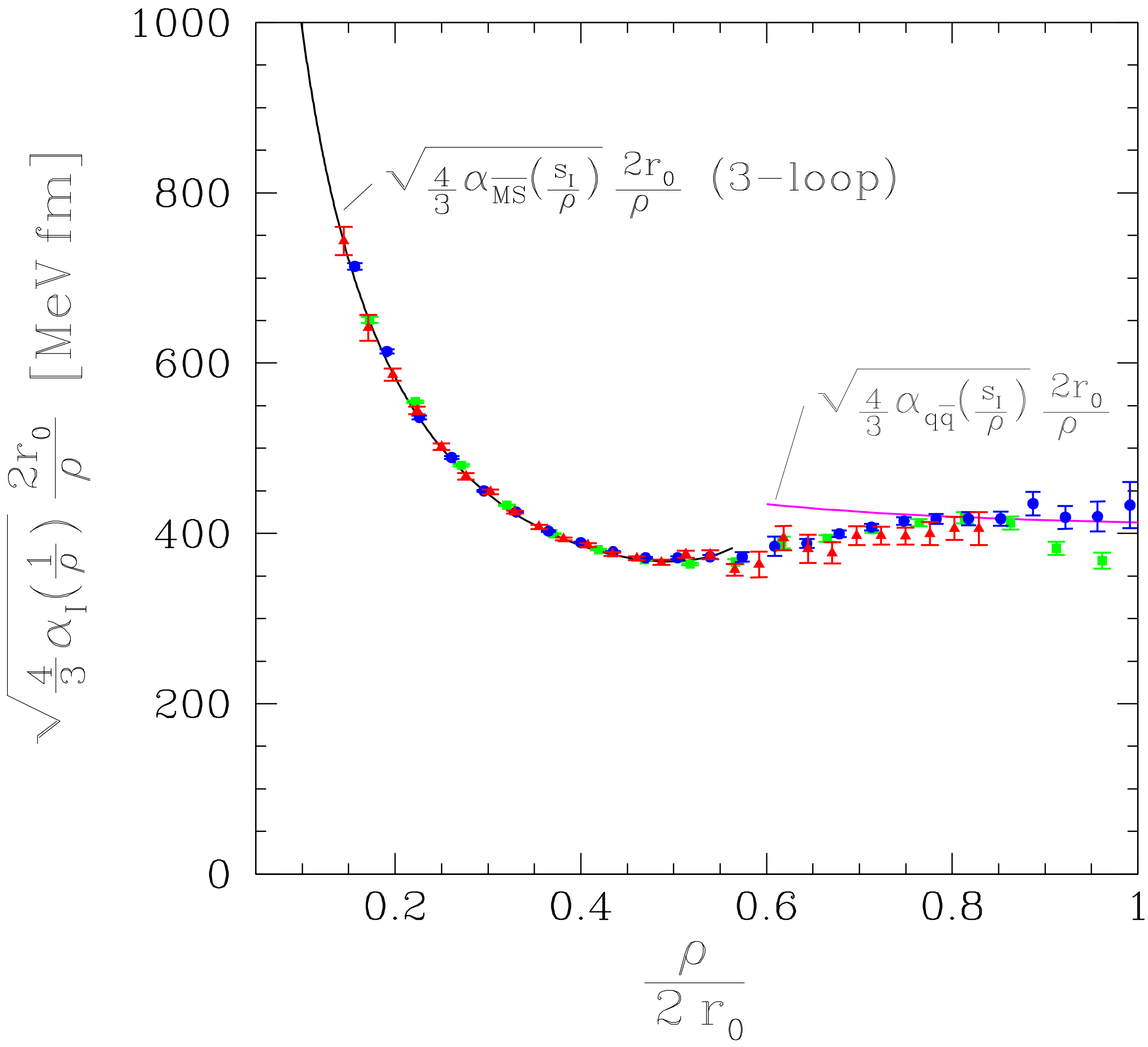
$$\boxed{\alpha_{\boldsymbol{I}}^{(2)} \cdot \alpha_{\boldsymbol{I}} \approx 1}$$ and $\alpha_{\boldsymbol{I}}^{(2)}(\mu) \propto (\mu/\Lambda_{\boldsymbol{I}})^{\beta_0/2N_c}$ in UV limit.

Analoguous to [I. Kogan & M. Shifman '95]: two (dual) phases in SUSY Yang Mills (exact NSVZ β-function) ?

- **Remarkable:** In non-perturbative long-distance regime, $\alpha_I(\frac{1}{\rho})$ approaches $\alpha_{q\bar{q}}(\frac{s_I}{\rho})$ $\Leftrightarrow$ static $q\bar{q}$-potential $V_{q\bar{q}}$

$$\frac{4}{3}\frac{\alpha_{q\bar{q}}(1/r)}{r^2} \equiv \frac{dV_{q\bar{q}}(r)}{dr} \simeq \sigma + \frac{\pi}{12}\frac{1}{r^2} \overset{r\to\infty}{\Rightarrow} \sigma,$$

with string tension σ and $2r_0\sqrt{\sigma} = (472.4 \pm 4.3)$ MeVfm [M. Guagnelli *et al.* '98]



- Transition between perturbative and confinement sensitive physics seems rather sharp, located around peak of $\frac{d\,n_{I+\bar{I}}}{d^4x\,d\rho}$

2.3 $I\bar{I}$-Distance Distribution

- UKQCD measured: $\frac{d(n_{I\bar{I}})}{d^4x\, d^4R}$ = $I\bar{I}$-distance distribution = number of unlike charges in the spherical shell of width δR a distance R away from each charge, normalized by shell volume

- First direct test of $I\bar{I}$-valley form for $I\bar{I}$-interaction Ω!

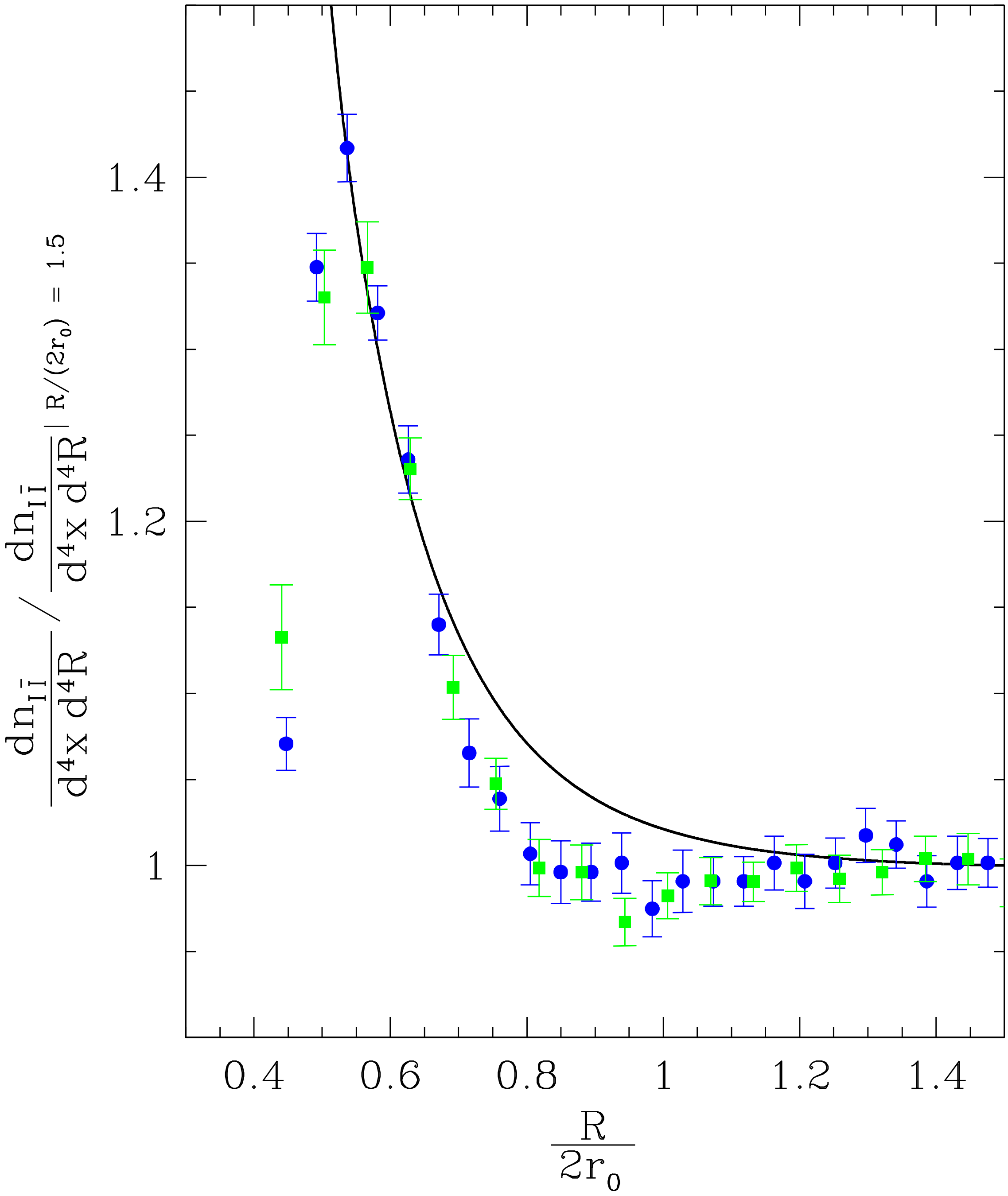
$$\frac{d(n_{I\bar{I}})}{d^4x\, d^4R} \stackrel{?}{\simeq} N \int_0^\infty d\rho \int_0^\infty d\bar{\rho}\; D(\rho)\, D(\bar{\rho}) \int dU\, \mathrm{e}^{-\frac{4\pi}{\alpha}\, \Omega\left(U, \frac{R^2}{\rho\bar{\rho}}, \frac{\bar{\rho}}{\rho}\right)}$$

- Substitute $D(\rho)$ and $\alpha = \alpha_I(\frac{s_{I\bar{I}}}{\sqrt{\rho\bar{\rho}}})$ by lattice results, perform 4 integrations numerically. 1 parameter: $s_{I\bar{I}} \simeq 0.7 = \mathcal{O}(1)$

- Continuum limit of "equivalent" data analogously to I-size distribution ($\beta = 6.4$ data missing!):

$$\frac{dn_{I\bar{I}}}{d^4x\, d^4R} \Big/ \frac{dn_{I\bar{I}}}{d^4x\, d^4R}_{|\, R \text{ large}} = \text{function}\left(\frac{R}{r(a)}\right)$$

- Rescale data by $R \Rightarrow r(0)/r(a) \cdot R$ with linear extrapolation of scaling factor $r(a)/r_{|\beta=6.2}$ in $(a/r_0)^2$ to $a = 0$.



- Good **agreement** for $R \gtrsim 0.5$ fm, i. e. $R/\rho \gtrsim 1$

3. Conclusions

- $\boldsymbol{I}$-perturbation theory for **DIS** well under control.
- $D(\rho)$ and $\Omega\left(U, \frac{R^2}{\rho\overline{\rho}}, \frac{\overline{\rho}}{\rho}\right)$ provide the link between $\boldsymbol{I}$-observables in **DIS** and in the QCD vacuum ($\Leftarrow$ lattice)
- $(\beta, n_{\rm cool})$ dependence of UKQCD data scales as function of a "cooling radius" $r_{\rm cool}/2r_0$
- Found a "plateau" of $\langle\rho\rangle/2r_0$ vs. $r_{\rm cool}/2r_0$, where "equivalent" UKQCD data are located $\Rightarrow$ appropriate for continuum limit!
- Parameter-free agreement of measured $\boldsymbol{I}$-size distribution with $\boldsymbol{I}$-perturbation theory for $\rho/2r_0 \leq 0.3$
- Extract α in "$\boldsymbol{I}$-scheme" from measured $\boldsymbol{I}$-size distribution
- First direct support for $I\overline{I}$-valley form for $I\overline{I}$-interaction Ω!

◇ Lattice constraints on (R, ρ) from high-quality UKQCD data translate into "fiducial" kinem. region in **DIS**

◇ Surprising **agreement** of lattice results with $\boldsymbol{I}$-perturbation theory very interesting by itself.

◇ Promising: Try replacing $D(\rho)$ and $I\overline{I}$-interaction Ω by the actual form measured on the lattice! $\Rightarrow$ **DIS**-cuts may be much relaxed, high-E_T fireballs in photoproduction. . .