Instanton-Induced Processes in Deep-Inelastic Scattering

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in collaboration with

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1. **Introduction**
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– Bad Honnef, March 1997 –

1. Introduction

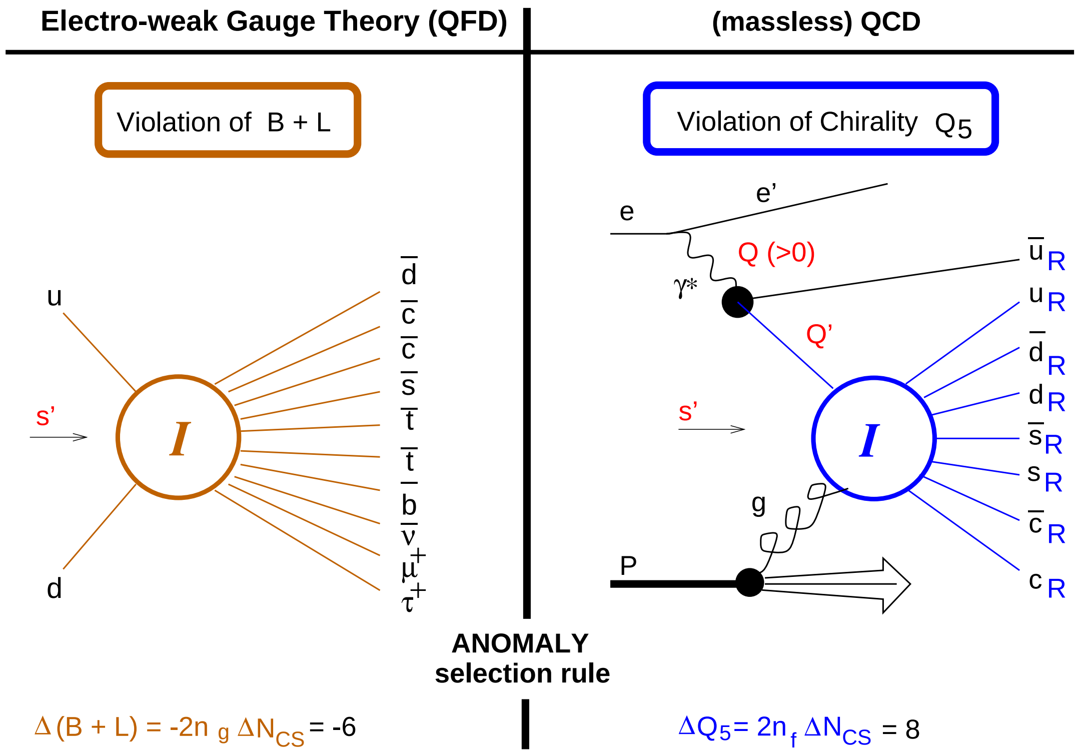
- Perturbative formulation of the Standard Modell theoretically consistent and agrees with present experiments.

- However, even for small coupling, there are processes that cannot be described by conventional perturbation theory:

  Certain fermionic quantum numbers, which are conserved to any order in perturbation theory, are violated due to Adler-Bell-Jackiw anomalies:

$$B+L \text{ in QFD}; \quad \text{Chirality } (Q_5) \text{ in (massless) QCD}.$$

- Corresponding "anomalous" processes are induced ['t Hooft '76] by instantons [Belavin *et al.* '75] :



- Instantons:
  - Represent tunnelling transitions in YM theories between classically degenerate, topologically inequivalent vacua.
  - Tunnelling rate exponentially suppressed at low energy, $\sim \exp(-4\pi/\alpha)$.
  - Strong growth with increasing energy, via production of additional gauge bosons $A_\mu^{(I)} \sim 1/g$ [Ringwald '90; Espinosa '90] ,

- Motivation for deep-inelastic scattering (DIS):
  - Theoretically clean:

    Possible to isolate well-defined, calculable instanton contributions in the regime of small QCD-gauge coupling at hard momentum scale [Balitsky, Braun '93; Moch, Ringwald, F. Sch. '96] .
  - Experimentally accessible:
    * DIS at HERA offers a unique window to detect QCD-instanton induced processes through their characteristic multi-particle final-state signature [Ringwald, F. Sch. '94, '96, Gibbs, Ringwald, F. Sch. '95, Gibbs, Greenshaw, Milstead, Ringwald, F. Sch. '96] .

♣ An experimental discovery of such a novel, non-perturbative manifestation of non-abelian gauge theories would clearly be of basic significance.

2. Instanton-Induced Processes in Leading Semi-Classical Approximation

- Instanton calculations in QCD are generically plagued by infrared divergencies associated with the integration over the instanton size ρ.

- Illustrate, that, to leading-order in the semi-classical approximation, instanton-induced amplitudes in DIS:

  – do not suffer from such divergencies: the typical hard momentum scale $\mathcal{Q}$ in deep-inelastic scattering provides a dynamical IR cutoff for the instanton size,

$$\rho \lesssim \mathcal{O}(1/\mathcal{Q}),$$

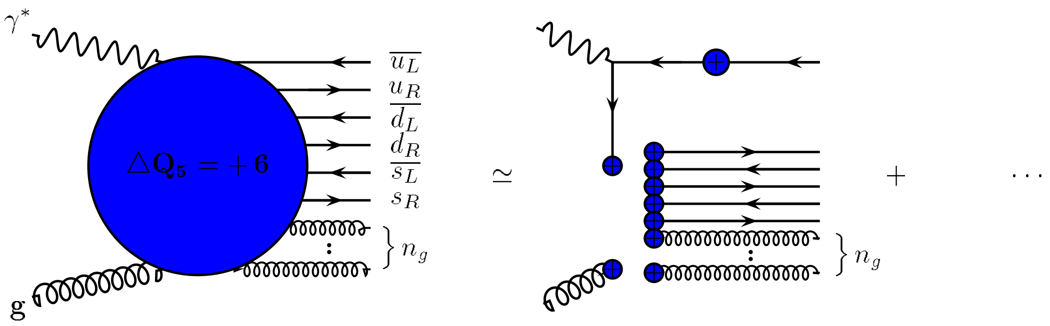
  such that the integration over the size parameter is finite;

  – are well-defined in the fixed-angle regime, in the same manner as amplitudes in perturbative QCD;

  – bear a remarkable resemblence to electro-weak $B+L$ violation.

- Consider amplitude for generic $\triangle Q_5 = 2\,n_f$ process,

$$\gamma^* + \mathrm{g} \Rightarrow \sum_{\text{flavours}}^{n_f} [\overline{\mathrm{q}_L} + \mathrm{q}_R] + n_\mathrm{g}\,\mathrm{g} \; :$$



- Expand (Euclidean) path integral for relevant Green's functions about classical instanton solution.
- Fourier-transform with respect to external lines.
- LSZ amputate external legs.
- Analytically continue to Minkowski space.

- Basic building blocks:

  – Classical instanton gauge field (singular gauge) [Belavin *et al.* '75] :

$$A^{(I)}_{\mu'}(x) \;=\; -\mathrm{i}\,\frac{2\,\pi^2}{g_s}\,\rho^2\,U\left(\frac{\sigma_{\mu'}\,\overline{x} - x_{\mu'}}{2\,\pi^2\,x^4}\right)U^\dagger\,\frac{1}{\Pi_x}\,,$$

$$\Pi_x \;\equiv\; 1 + \frac{\rho^2}{x^2}\,,$$

depending on collective coordinates:

$\rho = I$-size, $\qquad (U)^k_\alpha = I$-color orientation

(color: $k = 1,2,3$ and spinor: $\alpha = 1,2$ indices).

– Fermion zero modes in the instanton background (Weyl basis)[1] ['t Hooft '76] :

$$\kappa^{(I)m}{}_{\dot\beta}(x) = 2\,\pi\,\rho^{3/2}\,\epsilon^{\gamma\delta}\,(U)^m_\delta\,\frac{\overline{x}_{\dot\beta\gamma}}{2\,\pi^2\,x^4}\,\frac{1}{\Pi_x^{3/2}},$$

$$\overline{\phi}^{(I)\dot\alpha}{}_l(x) = 2\,\pi\,\rho^{3/2}\,\epsilon_{\gamma\delta}\,\left(U^\dagger\right)^\gamma_l\,\frac{x^{\delta\dot\alpha}}{2\,\pi^2\,x^4}\,\frac{1}{\Pi_x^{3/2}},$$

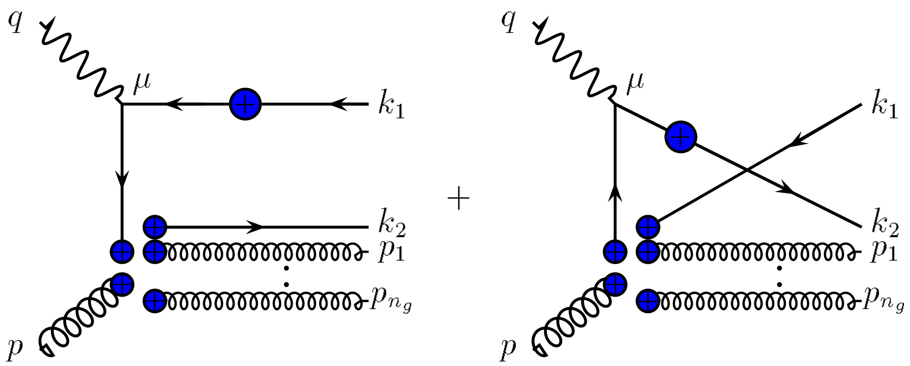
– Fermion propagators in the instanton background [Brown *et al.* '78] :

$$S^{(I)}(x,y) = \frac{1}{\sqrt{\Pi_x\Pi_y}}\left[\frac{x-y}{2\pi^2(x-y)^4}\right.$$
$$\left.\left(1+\rho^2\frac{\left[U\,x\,\overline{y}\,U^\dagger\right]}{x^2y^2}\right) + \frac{\rho^2\,\sigma_\mu}{4\pi^2}\frac{\left[U\,x\,(\overline{x}-\overline{y})\,\sigma_\mu\,\overline{y}\,U^\dagger\right]}{x^2(x-y)^2y^4\Pi_y}\right],$$

$$\overline{S}^{(I)}(x,y) = \cdots$$

[1] We use the standard notations, in Euclidean space: $\sigma_\mu = (-\mathrm{i}\,\vec\sigma, 1)$, $\overline{\sigma}_\mu = (\mathrm{i}\,\vec\sigma, 1)$, and in Minkowski space: $\sigma_\mu = (1, \vec\sigma)$, $\overline{\sigma}_\mu = (1, -\vec\sigma)$, where $\vec\sigma$ are the Pauli matrices. Furthermore, for any four-vector v_μ, we use the shorthand $v \equiv v\cdot\sigma$, $\overline{v} \equiv v\cdot\overline{\sigma}$.

- Leading-order amplitude ($n_f = 1$) takes the following form in Euclidean space [Moch, Ringwald, F. Sch. '96] :



$$\mathcal{T}_\mu^{(I)\,a\,a_1\ldots a_{n_g}} = -\mathrm{i}\,e_q\,d\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right]$$

$$\int dU \int_0^\infty \frac{d\rho}{\rho^5}\,(\rho\,\mu_r)^{\beta_0}\,\lim_{p^2\to 0} p^2\,\mathrm{tr}\left[\lambda^a\,\epsilon_g(p)\cdot A^{(I)}(p)\right]$$

$$\prod_{i=1}^{n_g}\lim_{p_i^2\to 0} p_i^2\,\mathrm{tr}\left[\lambda^{a_i}\,\epsilon_g^*(p_i)\cdot A^{(I)}(-p_i)\right]\,\times$$

$$\chi_R^\dagger(k_2)\left[\lim_{k_2^2\to 0}(\mathrm{i}k\!\!\!/_2)\,\kappa^{(I)}(-k_2)\,\lim_{k_1^2\to 0}\mathcal{V}_\mu^{[\overline{\phi}^{(I)}]}(q,-k_1)\right.$$

$$\left.+\lim_{k_2^2\to 0}\mathcal{V}_\mu^{[\kappa^{(I)}]}(q,-k_2)\,\lim_{k_1^2\to 0}\overline{\phi}^{(I)}(-k_1)\,(-\mathrm{i}\,\overline{k\!\!\!/}_1)\right]\chi_L(k_1)\,,$$

with generic notation for Fourier transforms,

$$f(\ldots,k,\ldots) = \int d^4x\,\mathrm{e}^{-\mathrm{i}\,k\cdot x}\,f(\ldots,x,\ldots)\,.$$

- Instanton-density with renormalization scale μ_r,
  ['t Hooft '76; Bernard '79] ,

$$d\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right]\,(\rho\,\mu_r)^{\beta_0}\ ,$$

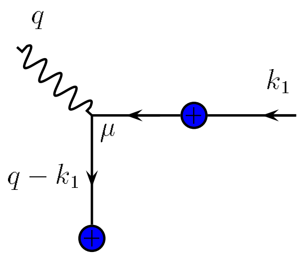
  - is 1-loop RG-invariant.
  - strongly IR-divergent if integrated over ρ
- F.T. and LSZ-amputation of the classical instanton gauge field $A^{(I)}_{\mu'}$ and the quark zero modes κ and $\overline{\phi}$, respectively, is straightforward [SVZ '80; Ringwald '90; Espinosa '90] ,

$$\begin{aligned}
\lim_{p^2\to 0} p^2 \mathrm{tr}\left[\lambda^a A^{(I)}_{\mu'}(p)\right] &= \frac{2\pi^2}{g_s}\rho^2 \mathrm{tr}\left[\lambda^a U\,[p_{\mu'} - \sigma_{\mu'}\overline{p}]\,U^\dagger\right],\\
\lim_{k_2^2\to 0} (\mathrm{i}k_2)^{\alpha\dot\alpha}\kappa^{(I)}{}^{i}_{\dot\alpha}(-k_2) &= 2\pi\rho^{3/2}(U)^i_\beta\epsilon^{\beta\alpha},\\
\lim_{k_1^2\to 0} \overline{\phi}^{(I)}{}^{\dot\gamma}_{j}(-k_1)(-\mathrm{i}\overline{k}_1)_{\dot\gamma\delta} &= 2\pi\rho^{3/2}\epsilon_{\beta\delta}\left(U^\dagger\right)^\beta_j .
\end{aligned}$$

- F.T. and LSZ-amputation of the photon-fermion vertices,

$$\begin{aligned}
\mathcal{V}^{[\overline{\phi}^{(I)}]}_\mu(q,-k_1) &\equiv \int d^4x\,\mathrm{e}^{-\mathrm{i}q\cdot x}\left[\overline{\phi}^{(I)}(x)\overline{\sigma}_\mu S^{(I)}(x,-k_1)(-\mathrm{i}\overline{k}_1)\right],\\
\mathcal{V}^{[\kappa^{(I)}]}_\mu(q,-k_2) &\equiv \int d^4x\,\mathrm{e}^{-\mathrm{i}q\cdot x}\left[(\mathrm{i}k_2)\overline{S}^{(I)}(-k_2,x)\sigma_\mu\kappa^{(I)}(x)\right],
\end{aligned}$$

  e.g.



$$\equiv \int d^4x\, \mathrm{e}^{-\mathrm{i}q\cdot x} \left[\overline{\phi}^{(I)}(x)\overline{\sigma}_\mu S^{(I)}(x,-k_1)(-\mathrm{i}\overline{k}_1)\right]$$

quite non-trivial and has important physical consequences.

- After a long and tedious calculation . . .

$$\lim_{k_1^2\to 0} \mathcal{V}_\mu^{[\overline{\phi}^{(I)}]}(q,-k_1) = 2\pi\mathrm{i}\rho^{3/2}\left[\epsilon\sigma_\mu \overline{V}(q,k_1;\rho)U^\dagger\right] ,$$

$$\lim_{k_2^2\to 0} \mathcal{V}_\mu^{[\kappa^{(I)}]}(q,-k_2) = 2\pi\mathrm{i}\rho^{3/2}\left[U V(q,k_2;\rho)\overline{\sigma}_\mu\epsilon\right] ,$$

where

$$V(q,k;\rho) = \frac{k}{2q\cdot k}\rho\sqrt{q^2}\,K_1\left(\rho\sqrt{q^2}\right) + \left[\frac{(q-k)}{(q-k)^2} - \frac{k}{2q\cdot k}\right]\rho\sqrt{(q-k)^2}\,K_1\left(\rho\sqrt{(q-k)^2}\right) .$$

- The integration over the instanton size ρ in $\mathcal{T}_\mu^{(I)}$ is finite due to the exponential decrease of the form factors in $V(q,k;\rho)$,

$$\mathcal{Q}\rho K_1(\mathcal{Q}\rho) \overset{\mathcal{Q}\rho\to\infty}{\to} \sqrt{\frac{\pi}{2}}\sqrt{\mathcal{Q}\rho}\,\exp\left[-\mathcal{Q}\rho\right] .$$

- After continuation to Minkowski space:

Typical hard scale in deep-inelastic scattering,

$$\mathcal{Q} \equiv \min\left\{Q \equiv \sqrt{-q^2}\,,\ \sqrt{-(q-k_1)^2}\,,\ \sqrt{-(q-k_2)^2}\right\} \geq 0\,,$$

provides a dynamical IR cutoff for the instanton size,

$$\rho \lesssim \mathcal{O}(1/\mathcal{Q}).$$

Therefore, deep-inelastic scattering very well suited for studying manifestations of QCD-instantons.

- Next steps:
  - Insert all LSZ-amputated F.T.s. into $\mathcal{T}_\mu^{(I)}$.
  - Continue to Minkowski space.
  - Perform integration over $\rho \Rightarrow$
- Result:

$$\mathcal{T}_\mu^{(I)\,a\,a_1\ldots a_{n_g}} = \mathrm{i}\, e_q\, 4\,\pi^2\, d\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right]$$
$$\times 2^{\beta_0+2n_g}\,\Gamma\left(\frac{\beta_0+2n_g+1}{2}\right)\Gamma\left(\frac{\beta_0+2n_g+3}{2}\right)$$
$$\times\left(\frac{\pi^3}{\alpha_s}\right)^{\frac{1+n_g}{2}}\int dU\,\mathrm{tr}\left[\lambda^a\,U\,\left[\epsilon_g(p)\cdot p-\epsilon_g(p)\,\overline{p}\right]\,U^\dagger\right]$$
$$\times\prod_{i=1}^{n_g}\mathrm{tr}\left[\lambda^{a_i}\,U\,\left[\epsilon_g^*(p_i)\,\overline{p_i}-\epsilon_g^*(p_i)\cdot p_i\right]\,U^\dagger\right]$$

$$\times \left\{ \left[U\, \chi_R^\dagger(k_2)\, \epsilon \right] \left[\epsilon\, v(q, k_1; \rho) \overline{\sigma}_\mu\, \chi_L(k_1)\, U^\dagger \right] - \left[U\, \chi_R^\dagger(k_2)\, \sigma_\mu\, \overline{v}(q, k_2; \rho)\, \epsilon \right] \left[\epsilon\, \chi_L(k_1)\, U^\dagger \right] \right\} ,$$

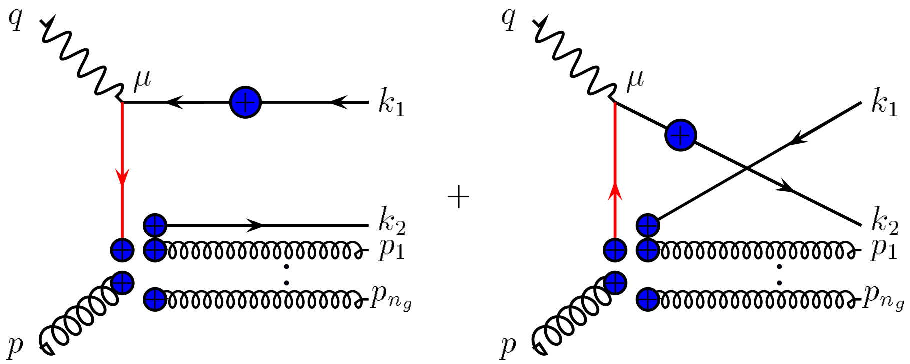
with

$$v(q, k; \mu_r) \equiv \frac{1}{\mu_r^{1+2n_g}} \left\{ -\frac{k}{2q \cdot k} \left(\frac{\mu_r}{\sqrt{-q^2}} \right)^{\beta_0 + 2n_g + 1} + \left[\frac{(q-k)}{-(q-k)^2} + \frac{k}{2q \cdot k} \right] \left(\frac{\mu_r}{\sqrt{-(q-k)^2}} \right)^{\beta_0 + 2n_g + 1} \right\} .$$

- Highly non-trivial check:
  Electromagnetic current conservation,

$$q^\mu \mathcal{T}_\mu^{(I)\, a\, a_1 \ldots a_{n_g}} = 0.$$

- Like in perturbative QCD:
  Leading-order I-induced amplitudes well-behaved as long as we avoid the collinear singularities, arising when the internal quark virtualities,



vanish,

$$-(q-k_1)^2 \text{ and/or } -(q-k_2)^2 \to 0.$$

Fixed-angle scattering processes at high Q^2 and moderate multiplicity are reliably calculable in (instanton) perturbation theory.

- Validity of approximation:

  Average instanton size $\langle\rho\rangle$ contributing for a given virtuality $\mathcal{Q}$,

$$\langle\rho\rangle \equiv \frac{\int\limits_0^\infty d\rho\, \rho\, \rho^{\beta_0+1+2n_g}\, K_1(\rho\, \mathcal{Q})}{\int\limits_0^\infty d\rho\, \rho^{\beta_0+1+2n_g}\, K_1(\rho\, \mathcal{Q})} \simeq \frac{\beta_0 + 3/2 + 2n_g}{\mathcal{Q}}\,.$$

  Should be small, in order that higher-order corrections can be neglected; i.e. we have to require

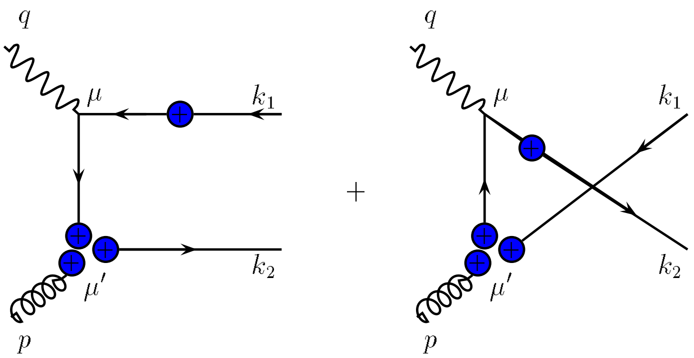
$$\langle\rho\rangle \ll 1/\Lambda \quad \Rightarrow \quad \mathcal{Q} \gtrsim 5 \text{ GeV}.$$

Example: The Simplest Process ($n_f = 1, n_g = 0$)

[Moch, Ringwald, F. Sch. '96]

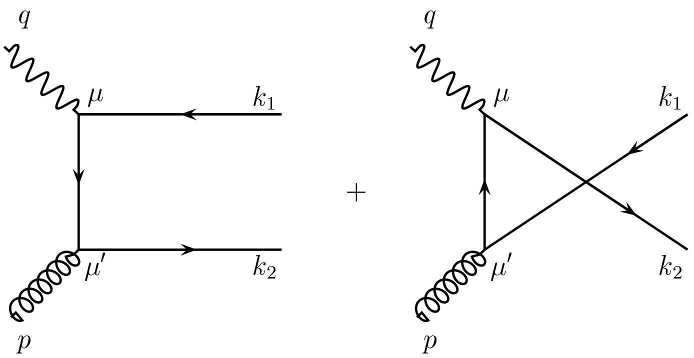
- Compare simplest I-induced process ($n_f = 1, n_g = 0$),

$$\gamma^* + \mathrm{g} \to \overline{\mathrm{q}_L} + \mathrm{q}_R \; ; \quad (\triangle Q_5 = 2) \; ,$$



with the appropriate leading-order perturbative QCD, chirality-conserving process,

$$\gamma^* + \mathrm{g} \to \overline{\mathrm{q}_L} + \mathrm{q}_L \; ; \quad (\triangle Q_5 = 0) \; .$$



- Simplest process can be considered as lower bound on total instanton-induced contribution.

  Contains all essential features of dominant multi-gluon process.

- Compare differential cross-sections,

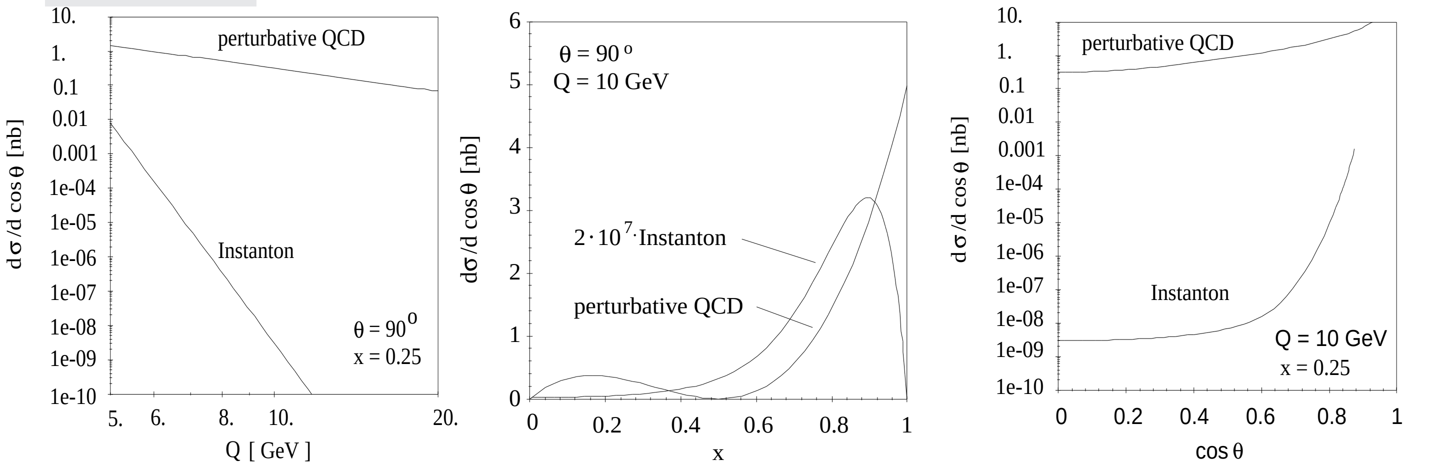
$$\frac{d\sigma}{d\cos\theta}(Q, x, \cos\theta)\,, \quad \text{with} \quad x = \frac{Q^2}{2p\cdot q}\,.$$

  θ: $\gamma^* g$-c.m. scattering angle

  Choice of parameters: $e_q = 2/3$, $\Lambda = 234$ MeV, $\mu_r = Q$.

  Used 2-loop renormalization improvement of instanton-density [Morris, Ross, Sachrajda '85] .

- Result:

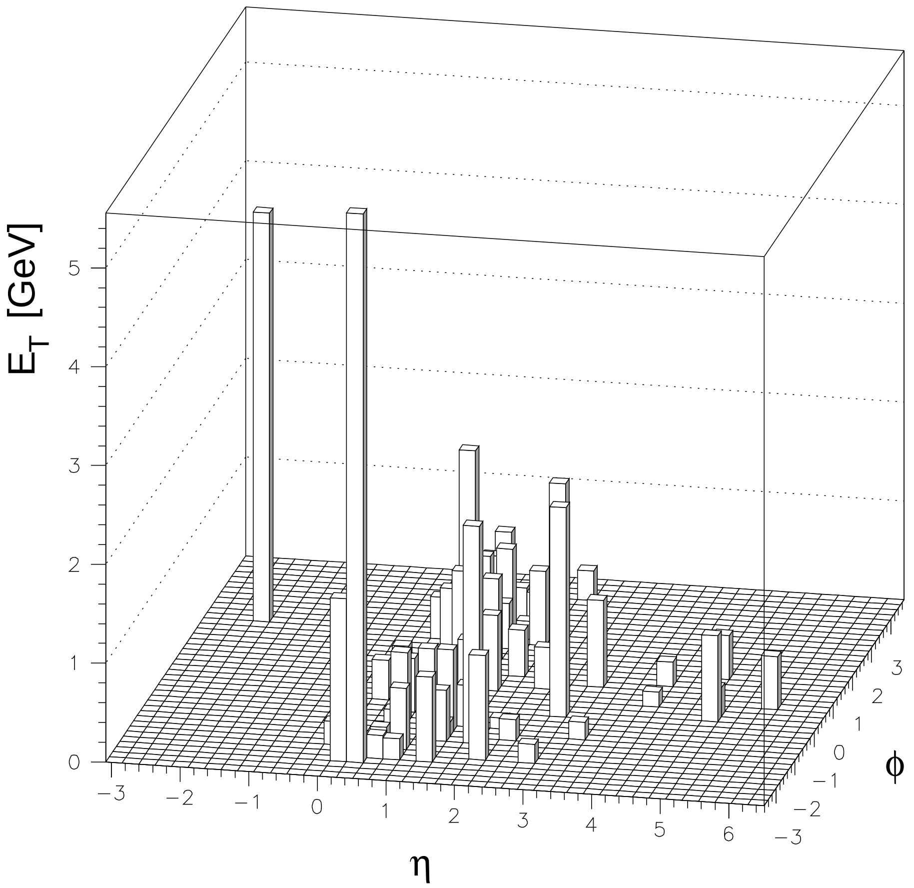


- Fixed-angle scattering at large enough Q^2 well under contrôl (left and middle Fig.).

- Collinear singularity ($\cos\theta \to 1$) for instanton case much stronger than perturbative one (right Fig.).

3. Ongoing Searches

- Monte-Carlo Simulation for HERA: "**QCDINS 1.3**", with hadronization ("HERWIG 5.8") [Gibbs, Ringwald, F. Sch. '95 & in prep.]
- Lego-Plot of Typical Event:



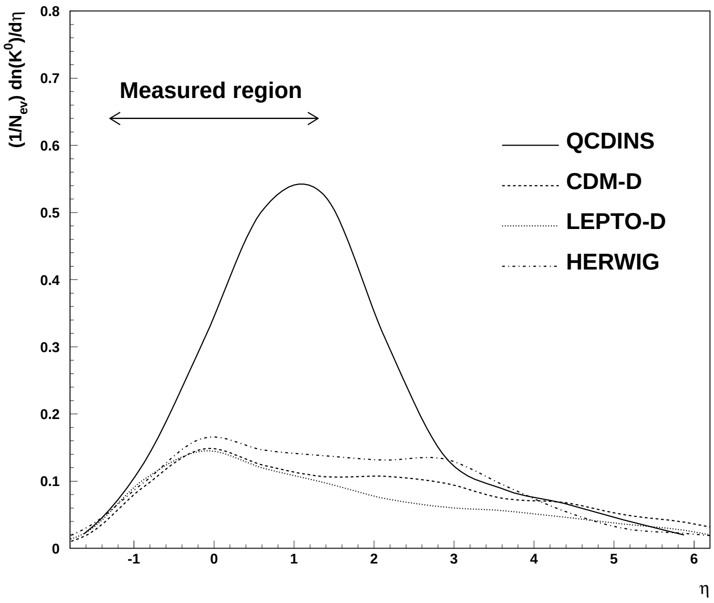
  – Current jet
  – Hadronic "band": ⇔ **isotropy** in $\boldsymbol{I}$-rest system
    * Large total $E_T = \mathcal{O}(20)$ GeV
    * Large multiplicity $N_{\rm band} = \mathcal{O}(25)$
    * No jets in "band" !
    * Characteristic flavor flow:

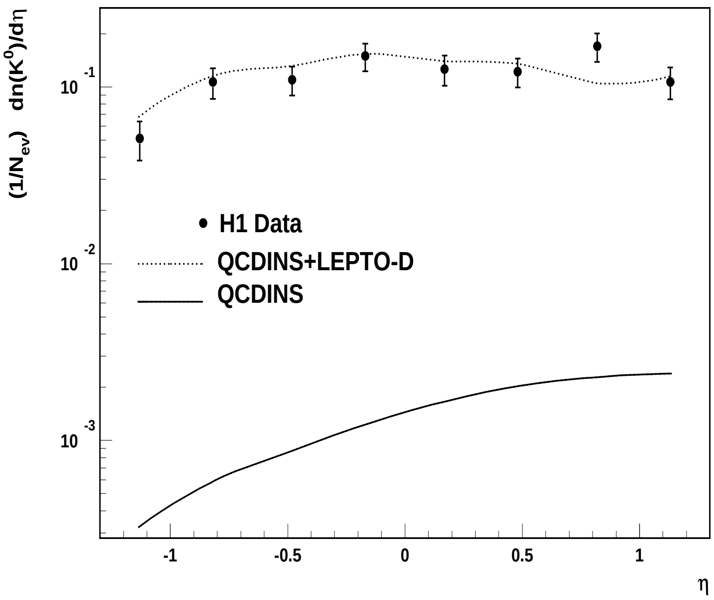
      ⋆ **Strangeness** ⇒ K_S^0's   ⋆ **Charm** ⇒ μ's

First Search for I-induced Events at HERA

[H1 Collaboration, S. Aid et.al., '96]

- Expect strong enhancement of K^0-flow in "band" for I-induced events! (left Fig.)





- H1 Collaboration:

  First experimental upper bound for I -induced cross-section at HERA, based on observed K_S^0's (right Fig.),

$$\sigma_{e\,P}{}^{(I)} < 0.9\ \mathrm{nb}\ (95\%\ \mathrm{CL});$$

$$10^{-3} < x_{\rm Bj} < 10^{-2},\ 0.1 < y_{\rm Bj} < 0.6,\ 10 < Q^2 < 70\ \mathrm{GeV}^2.$$