Instantons in the QCD Vacuum and High-Energy Scattering

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1. **Setting the Stage**
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3. **Instantons and the Color-Dipole Picture of High-Energy Scattering** (with A. Utermann)
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1. Setting the Stage

- **High-energy scattering dominantly at small p_T:**
  - vast phenomenology, characterized by strikingly simple, semi-classical, **geometrical** features:

    c.f. [B. & F. Schrempp '73 - '79]

    - universal hadronic **"radius"** $R_T \sim \mathcal{O}(1\mathrm{fm})\ \ [1 + \epsilon\, ln(s)]$
    - $\sim$ "black disk" **diffraction**: $\begin{cases} \sigma_T \overset{\text{s large}}{\sim} 2\pi\, R_T^2 \\ \sigma_{\rm el}/\sigma_T \Rightarrow 1/2 \end{cases}$
    - $\frac{d\,\sigma}{d\,p_T^2} \sim R_T^4\, \exp(-R_T\, p_T)$, or more generally
    - "geometrical scaling" for impact parameter profile: $T(b^2, s) = T(\frac{b^2}{R_T(s)^2})$ [Buras & Dias de Deus '74]

- Persisting theoretical challenge ($\gtrsim 25$ years) to understand non-perturbative "geometrization" within QCD :

  - important rôle of **HERA** physics regime (**DIS**)
  - allows to approach the non-perturbative small $(x_{\rm bj},\ Q^2)$ limit from "perturbative side". However. . .
  - Resummation attempts (**BFKL '77**) of perturbative QCD very involved & poorly convergent [**F**adin & **L**ipatov '98]. . .

♣ **More promising:** approaches via truely non-perturbative (field) configurations, associated with distinct intrinsic scale

- in addition, e.g. from "Heidelberg school": $\left\{ \right.$ Dosch, Simonov *et al* '87-; Nachtmann '91- ;. . .

close connection:

$\underbrace{\textbf{(Euclidean) vacuum structure}}_{\text{lattice data!}} \Leftrightarrow$ **scattering processes**

Prominent example:

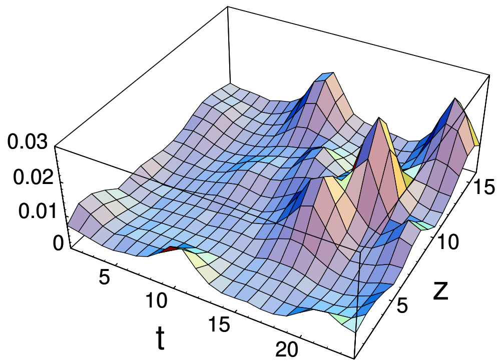
Instantons: [Belavin *et al* '75, 't Hooft '76]

Non-perturbative fluctuations of gluon fields with **marked** size $\langle\rho\rangle \approx 0.5$ fm

"instantaneous" in time and space

Associated with non-trivial topology of QCD vacuum;

Lagrange Density



♣ **Small** Instantons $\Leftrightarrow$ **DIS** [Ringwald & F. Sch. '94 - '01]

◇ induce hard, "chirality"-violating processes, forbidden in usual perturbative QCD ! ['t Hooft '76]

◇ "$\boldsymbol{I}$-perturbation theory" in **DIS** (hard scale $\mathcal{Q}$)): $\rho_{\rm eff} \propto \frac{1}{\mathcal{Q}} < \langle\rho\rangle$ and $\alpha_s(\mathcal{Q}^2)$ small; I-scale $\langle\rho\rangle$ unimportant!

◇ at **HERA**: rate calculable, in measurable range and experimental searches under way (**H1**)!. . .

In this talk: concentrate on an intriguing **conjecture**:

♣ **Larger-size** Instantons: $\rho \approx \langle\rho\rangle \approx 0.5$ fm $\Leftrightarrow$
semi-classical, geometrical high-energy scattering: $R_T \Leftrightarrow \langle\rho\rangle$
[Kharzeev, Levin *et al* '99 - '00; Shuryak, Zahed *et al.* '00 -; F. Sch '01; F. Sch. & Utermann '02]

Requires careful strategy [F. Sch '01; F. Sch. & Utermann '02]:

(1) Exploit lattice results on I-size distribution in the (Euclidean) vacuum $\Rightarrow$ Sect.2

(2) Transform our published **DIS**-results (I-pert. theory) for high Q^2 into popular "color-dipole" scattering picture.

(3) With the help of (1), carefully increase "$q\bar{q}$-dipole" size $|\vec{r}| \sim 1/Q$ towards hadronic dimensions $\mathcal{O}$(fm)

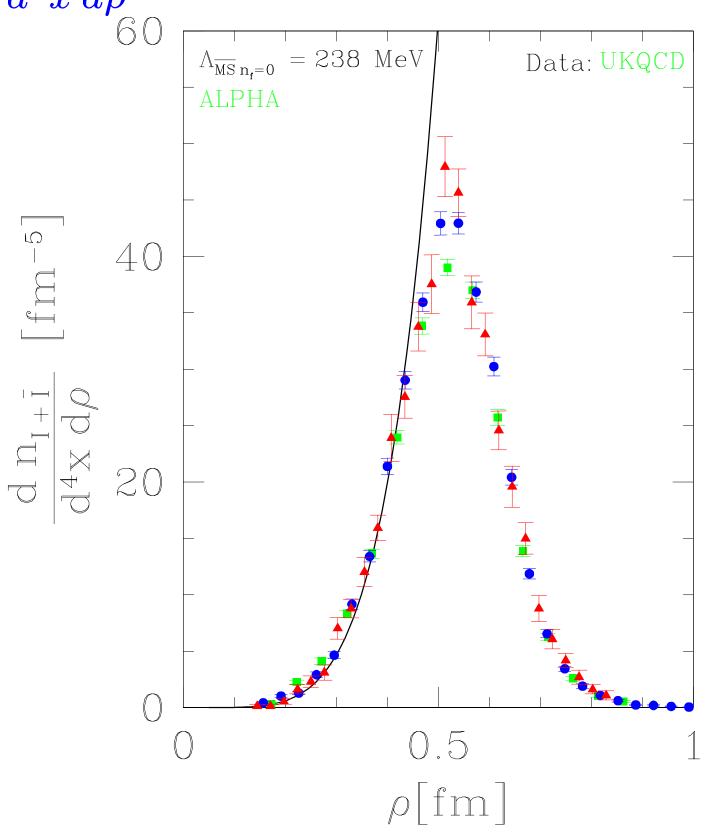
"Saturation" of ($q\bar{q}$ P)- dipole cross section?? $\sigma_{\rm DP}(r) \overset{?}{\propto} R_T^2$
What is scale R_T?

Instantons in the QCD Vacuum

- I-**gauge field** $A^{(I)}_\mu(x;\rho,x_0)$, explicitly known self-dual solution of 4d (Euclidean) Yang-Mills field Eqs., depending on "collective coordinates" like I-size ρ, I-position x_0,

- to be integrated out in all observables: $O = \int_0^\infty d\rho D(\rho)[\ldots]$ with weight $D(\rho)$

- "I-size distribution" $D(\rho) \equiv \frac{d\,n_I}{d^4x\,d\rho}$ crucial in I-calculus
  - **known** in I-pert. theory, formally for $\alpha_s(\mu_r)\ln(\mu_r\rho) \ll 1$:
  - Strong power-law $D(\rho) \propto \rho^{6-2/3n_f}$, a main reason for breakdown of I-pert. theory for larger-size I's (Fig)!
  - **However**, $\frac{d\,n_I}{d^4x\,d\rho}$ also "measured" on the lattice (Fig)



- **Lessons from the lattice**:
  - Instantons represent non-perturbative vacuum structures of marked size $\langle\rho\rangle \approx 0.5$ fm
  - Parameter-free agreement with I-pert. theory for $\rho < \langle\rho\rangle$, [Ringwald & F. Sch. Phys. Lett. B '99], followed by dramatic disagreement above $\langle\rho\rangle$!

- **Working hypothesis:** Enlarge region of validity of our I-pert. theory results by replacing throughout

$$\boxed{D_{\mathrm{I-pert.th}}(\rho) \Rightarrow D_{\mathrm{lattice}}(\rho)}$$

3. Instantons and the Color-Dipole Picture of High-Energy Scattering

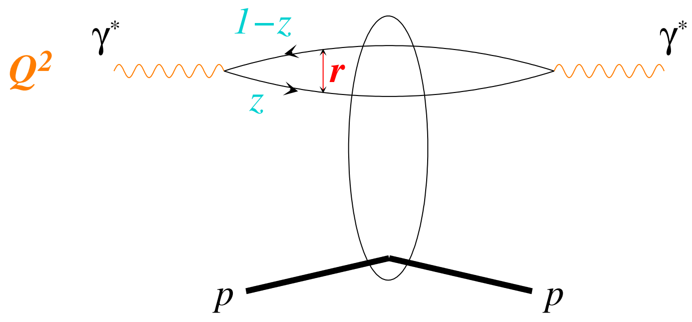
[F. Sch '01; F. Sch. & Utermann '02]

Reminder: "Color-Dipole" Scattering Picture in DIS

[Bjorken & Kogut '73; Frankfurt & Strikman '88; Mueller '90; Nikolaev & Zakharov '91,'94; Buchmüller, McDermott & Hebecker '97 -]

- Convenient & intuitive framework for studying transition at small x_{bj} from hard **DIS** (Q^2 large) to soft diffractive scattering ($Q^2, x_{\mathrm{bj}} \Rightarrow 0, \frac{Q^2}{x_{\mathrm{bj}}} \approx W^2$ large, fixed)

- **DIS** $\Leftrightarrow$ virtual Compton scattering on Nucleon (Fig)



- For small $x_{\rm bj}$: lifetime of γ^* $\Rightarrow$ $q\bar{q}$-fluctuation $\sim \frac{1}{x_{\rm bj}\, m_{\rm P}} \gg$ $(q\bar{q})\,{\rm P}$ interaction time in P-rest system $\Rightarrow$ 2-step process:

- γ^* fluctuates into $(q\bar{q})$-dipole [(light-cone) wave function $\Psi_{\rm L,\,T}^{\gamma^*\to q\bar{q}}$] which subsequently interacts with Proton target [dipole cross section $\sigma_{\rm DP}$], before recombining again into γ^*

$$\sigma_{\rm L,T}(x_{\rm bj},Q^2) = \int_0^1 dz \int d^2\boldsymbol{r}\; |\Psi_{\rm L,T}^{\gamma^*\to q\bar{q}}(z,\boldsymbol{r})|^2\, \sigma_{\rm DP}(\boldsymbol{r},\ldots)$$

- z: long. momentum fraction of γ^* carried by q (Fig)
- $\boldsymbol{r}$: transverse $(q\bar{q})$-dipole size (Fig)
- $\left\{\begin{array}{l}\text{longitud.}\\ \text{transverse}\end{array}\right\}$ photon-wave function perturbatively calculable:

$$\Psi_{\rm L}^{\gamma^*\to q\bar{q}} = e_q \frac{2\sqrt{6\alpha}}{2\pi} z(1-z) Q\, K_0(\hat{Q}\,r);\;\; \hat{Q} = \sqrt{z(1-z)}\,Q$$

$$\Psi_{\rm T}^{\gamma^*\to q\bar{q}} = e_q \frac{\sqrt{6\alpha}}{2\pi} \sqrt{z^2+(1-z)^2}\hat{Q}\, K_1(\hat{Q}\,r)$$

- $\sigma_{\rm DP}(\boldsymbol{r},\ldots)$ includes non-perturbative part of interaction

- Simplest pQCD model: $(q\bar{q})$ P interaction via the exchange of 2 gluons: $\Rightarrow$ with $G(x_{\rm bj}, Q^2)$ = gluon density in P,

$$\text{small } r^2 \lesssim \frac{1}{Q^2}: \quad \sigma_{\rm DP}(\boldsymbol{r}, x_{\rm bj}) \approx \frac{\pi^2}{3}\alpha_s\, \boldsymbol{r^2}\, [x_{\rm bj}\, G(x_{\rm bj}, \overline{Q^2})]$$

- Intuitive, geometrical picture for $\sigma_{\rm DP}(\boldsymbol{r})$, describing transition from hard to soft physics as function of $\boldsymbol{r}$:

  - small r $\lesssim \frac{1}{Q^2}$: $\sigma_{\rm DP}(\boldsymbol{r}) \propto$ dipole area $= \pi \boldsymbol{r^2} \Rightarrow 0$

    "color transparency"

  - large r $\gtrsim$ hadronic size $\boldsymbol{R_T}$: $\sigma_{\rm DP}(\boldsymbol{r}) \propto \pi\, R_T^2$

    "saturation"

- We are now ready to ask: What is R_T?; could it be related to instanton size $\langle\rho\rangle \approx 0.5$ fm?

"Color-Dipole" Scattering with an Instanton in the Background

[F. Sch. & Utermann '02]

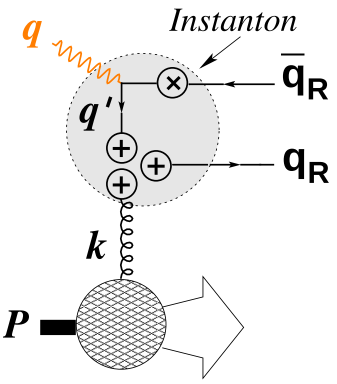
- **Conservative start:** Large Q^2, small x_{bj} & cuts such that I-perturbation theory remains valid

- **In this talk:** Consider "simplest" I-induced hard process,

$$\boxed{\gamma^*\, g \overset{(I)}{\Rightarrow} q_R\, \overline{q}_R} \quad (n_f = 1, \text{ no final-state gluons}),$$

worked out carefully in [Moch, Ringwald & F. Sch., NP **B** '97]

- Transform our published results into color-dipole picture



+ (t => u)

$$\begin{aligned} Q^2 &= -q^2 \\ x_{\rm bj} &= \frac{Q^2}{2P\cdot q} \\ x &= \frac{Q^2}{2k\cdot q} \\ -t &= -q'^2 \\ -u &= t + \frac{Q^2}{x} \end{aligned}$$

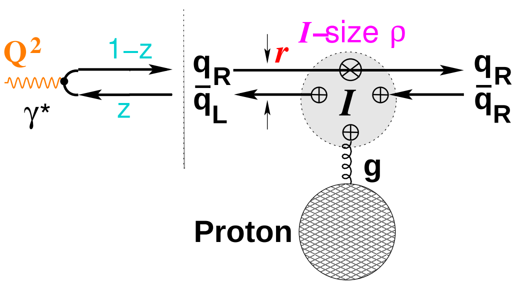
$$\sigma_{L,T}(x_{\rm bj}, Q^2) = \int_{x_{\rm bj}}^1 \frac{dx}{x}\left(\frac{x_{\rm bj}}{x}\right)\, G\left(\frac{x_{\rm bj}}{x}, Q^2\right) \int d\,t \frac{d\hat\sigma_{L,T}{}^{\gamma^* g}(x,t,Q^2)}{dt}$$

E.g.,

$$\frac{d\hat\sigma_L{}^{\gamma^* g}}{dt} = \frac{\pi^7}{2}\frac{e_q^2}{Q^2}\frac{\alpha}{\alpha_s}\left(x(1-x)\sqrt{tu}\left[\frac{I(-t)-I(Q^2)}{Q^2+t} - (t\leftrightarrow u)\right]\right)^2$$

in terms of the master integral,

$$I(\omega^2) = \int_0^\infty d\rho\, D(\rho)\rho^5(\omega\rho)\mathrm{K}_1(\omega\rho) \overset{\omega \text{ large}}{\propto} \frac{1}{\omega}\left(\frac{\mu_r}{\omega}\right)^{11-\frac{2}{3}nf} \Rightarrow 0$$



+ (z => 1−z)

$$\begin{aligned} z &= \frac{-t}{Q^2}x \\ 1-z &= \frac{-u}{Q^2}x \\ \hat{Q}^2 &= z(1-z)Q^2 \\ \boldsymbol{l}^2 &= \hat{Q}^2\left(\frac{1}{x}-1\right) \\ \boldsymbol{r} &\overset{2d\ FT}{\Leftrightarrow} \boldsymbol{l} \end{aligned}$$

with

$$2\pi\tilde{F}(z,\boldsymbol{r}^2;\rho) = \int \frac{d^2\boldsymbol{l}}{2\pi}\, e^{i\,\boldsymbol{l}\cdot\boldsymbol{r}}\, F(z,l^2;\rho) = \int_0^\infty d\,l\,l\,J_0(l r)\,F(z,l^2;\rho);$$

and

$$\int_0^\infty d\,l\,l\,F(z,\boldsymbol{l}^2;\rho)^2 = (2\pi)\int d^2\boldsymbol{r}\,\tilde{F}(z,\boldsymbol{r}^2;\rho)^2,$$

one obtains, after variable and 2d-Fourier transformation,

$$\sigma^{(I)}{}_{L,T} = \int_{x_{\rm bj}}^1 \frac{d\,x}{x}\int dt\,[\ldots] \Rightarrow \int dz\int d^2\boldsymbol{r}\left(|\,\Psi_{L,T}\,|^2\,\sigma_{\rm DP}\right)^{(I)}.$$

- **Note:**
- We make the same leading $\log\left(\frac{1}{x_{\rm bj}}\right)$ approximation as in analogous pQCD calculation,

$$\left(\frac{x_{\rm bj}}{x}\right)\,G\left(\frac{x_{\rm bj}}{x},Q^2\right) \approx x_{\rm bj}\,G(x_{\rm bj},Q^2),$$

- The asymmetric $(q\bar{q})$-configuration: $z \gg (1-z)$ or $1-z \gg z$ dominates, as usual
- Validity of strict I-pert. theory, i.e $D(\rho) \equiv D_{\rm I-pert.theory}(\rho)$, requires Q, $\sqrt{-t}$, $\sqrt{-u} \gtrsim \mu_f \approx 6-8$ GeV. Maps into

$$z_{\rm min} \equiv \left(\frac{\mu_f}{Q}\right)^2 \lesssim z \lesssim 1 - z_{\rm min}$$

- Relax after $D(\rho) \Rightarrow D_{\rm lattice}(\rho)$ (at least everything is finite!)
- **Discussion of Results:** $\Rightarrow$

- **Most interesting:** Instanton induces additional length-scale, I-size $\rho \approx \langle\rho\rangle$ besides $(q\bar{q})$-dipole size r,

- As expected intuitively, the 2 scales "compete"! :

$$\left(|\,\Psi_L\,|^2\,\sigma_{\rm DP}\right)^{(I)} \approx \left[e_q\frac{\sqrt{6\alpha}}{2\pi}z(1-z)Q\right]^2 \frac{x_{\rm bj}\,G(x_{\rm bj},Q^2)}{12\alpha_s}\pi^8$$
$$\left(\int_0^\infty d\rho D(\rho)\rho^5\left\{-\frac{d}{dr^2}\left(2r^2\frac{K_1\left(\hat{Q}\sqrt{r^2+\rho^2/z}\right)}{\hat{Q}\sqrt{r^2+\rho^2/z}}\right)-(z\Rightarrow 1-z)\right\}\right)^2$$

- **Note :** $-\frac{d}{dr^2}\left(2r^2\frac{K_1\left(\hat{Q}\sqrt{r^2+\rho^2/z}\right)}{\hat{Q}\sqrt{r^2+\rho^2/z}}\right) \approx \begin{cases} -\frac{K_1\left(Q\rho\sqrt{1-z}\right)}{Q\rho\sqrt{1-z}} & \text{for} \quad r^2 z/\rho^2 \Rightarrow 0 \\ K_0\left(\hat{Q}r\right) & \text{for} \quad r^2 z/\rho^2 \text{ **large** } \end{cases}$

- Hence, in the two limiting cases (pick $z \gg 1-z$)

$r^2 \Rightarrow 0,\ Q\sqrt{1-z}$ large : $\left(|\,\Psi_L\,|^2\,\sigma_{\rm DP}\right)^{(I)}$ **very small** (c.f. pQCD $:\propto r^2 K_0(\hat{Q}r)^2 \Rightarrow 0$)

$r^2 \gtrsim \langle\rho\rangle^2$: $$\boxed{\left(|\,\Psi_{L,T}\,|^2\,\sigma_{\rm DP}\right)^{(I)} \approx |\,\Psi_{L,T}\,|^2_{\rm pQCD}\ \underbrace{\frac{x_{\rm bj}\,G(x_{\rm bj},Q^2)}{12\alpha_s}\pi^8\left(\int_0^\infty d\rho D_{\rm lattice}(\rho)\rho^5\right)^2}_{\sigma^{(I)}{}_{\rm DP}\ \text{“saturates”}}}$$

Conclusions

- Fresh look at striking semi-classical, **geometrical** features of **soft** high-energy scattering

- Why instantons?
  - towards soft scattering regime, instanton contributions tend to get strongly enhanced (decreasing Q^2, $x_{\rm bj}$)
  - $\oplus$ **lattice**: I's indeed associated with quite a sharp intrinsic scale, that may naturally be responsible for semi-classical "geometrization" of QCD ,

$$R_T \propto \int_0^\infty d\rho D_{\rm lattice}(\rho)\rho^5,$$

- Born out via explicit transcription of our previous I-results (**DIS**) into intuitive "color-dipole" scattering picture:
  - Unlike pQCD , competition between two length-scales: $(q\bar{q})$-dipole size r and I-size $\rho \approx \langle\rho\rangle \approx 0.5$ fm
  - If $r^2 \gtrsim \langle\rho\rangle^2$, $\Rightarrow$ geometrical limit, i.e. $\sigma^{(I)}{}_{\rm DP}$ "saturates",

$$\sigma^{(I)}{}_{\rm DP} \propto \pi \left(\int_0^\infty d\rho D_{\rm lattice}(\rho)\rho^5\right)^2$$

- Need to work out final-state gluons and realistic $n_f > 1$ (in progress), before ready for phenomenology. . .