QCD Instantons and Small-x Diffraction

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in collaboration with

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1. Setting the Stage
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4. Conclusions

[F. Sch., J. Phys. **G28** (2002) 915;
F. Sch. & Utermann, Proc. DIS2002; Phys. Lett. **B543** (2002) 197]

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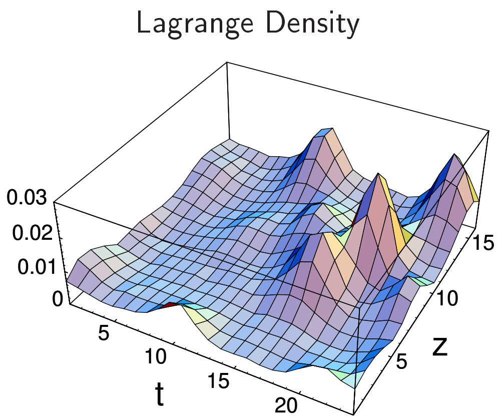
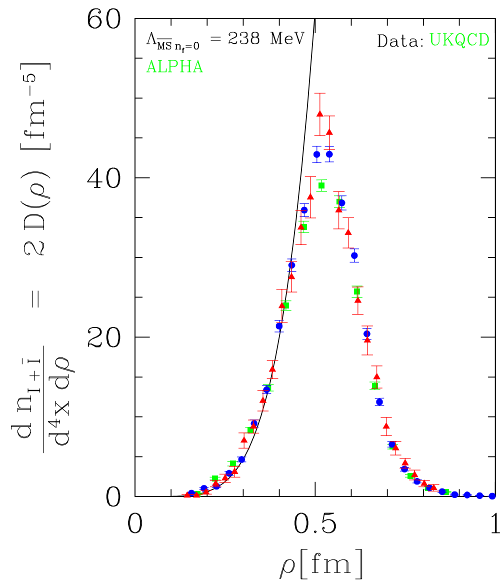
–SEWM02, Heidelberg, 2. Oct 2002 –

1. Setting the Stage

- Instantons (I) [Belavin *et al*'75]:



- Non-perturbative fluctuations of gauge fields, "instantaneous" in (euclidean) space and time (Fig.).



- Distribution of I-size ρ, sharply localized around $\langle\rho\rangle \approx 0.5$ fm ($\Leftarrow$ **UKQCD** lattice data, Fig.)

[Ringwald & F.Sch. '94-01]:

Small QCD instantons $\Leftrightarrow$ Deep Inelastic Scattering (DIS)

– Induce hard, chirality-violating processes, forbidden in pQCD ['t Hooft '76]

– Unique opportunity in **DIS** at **HERA**: Calculable, sizable rate and characteristic event signature

– "I-perturbation theory" in **DIS** warranted by hard scale $\mathcal{Q}$: $\alpha_s(\mathcal{Q}^2)$ small and $\rho_{\rm eff} \sim \frac{1}{\mathcal{Q}} \lesssim \langle\rho\rangle$; I-scale $\langle\rho\rangle$ unimportant $\Rightarrow$ Fig.

– At **HERA**: 1st dedicated search for **DIS** processes induced by small I's $\Rightarrow$ Intriguing (but yet inconclusive) excess of events in an "I-sensitive data sample" **H1**-Collab., hep-ex/0205078

♣ $\boxed{\text{In this talk}}$: Rôle of larger-size I's and associated intrinsic scale $\langle\rho\rangle \approx 0.5$ fm for decreasing $(Q^2,\ x)$ towards soft regime?

- $\boxed{\text{Focus on intriguing possibility:}}$ Kharzeev, Kovchegov & Levin '01; Shuryak *et al* '00 - '02, F. Sch. '01; F. Sch. & Utermann '02
  - Larger-size I's: building blocks for **diffractive** processes
  - I-scale $\langle\rho\rangle$ reflected in **geometrization** of soft QCD

- 2 qualitative arguments:

1. I's $\Leftrightarrow$ truely non-perturbative gluons, naturally introducing an intrinsic size scale $\langle\rho\rangle \approx 0.5$ fm.

   Matching "diffractive size scale": $R_{I\!P} = R\sqrt{\frac{\alpha'_{I\!P}}{\alpha'}} \approx 0.5$ fm, $R \approx 1$ fm, small $I\!P$omeron slope $\alpha'_{I\!P} \approx \frac{1}{4}\,\alpha'$

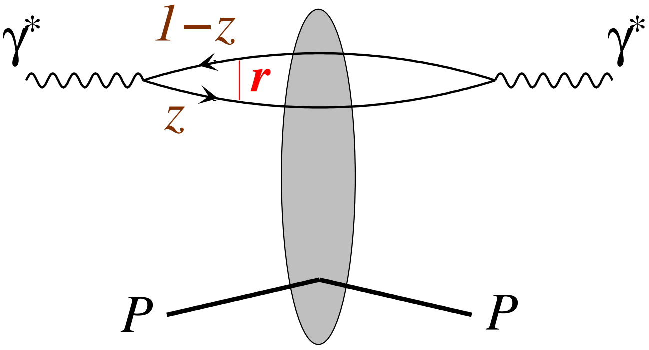
2. I-suppression known to decrease strongly towards **IR** regime/high energies:

   ◦ Trend from I-perturbation theory [Ringwald & F. Sch. '94, '98].

   ◦ Tunnelling between vacua $(E \approx 0) \overset{\text{large energies}}{\Rightarrow}$ via sphaleron configuration $(E \approx M_{\rm sph} \propto \frac{1}{\alpha_s\,\rho_{\rm eff}})$ on top of barrier [Aoyama & Goldberg '87; Ringwald '90,..., Shuryak *et al* '02].

- A convenient framework for studying the transition from hard to soft physics (diffraction):

Colour-dipole picture at small x in DIS: [Nikolaev & Zakharov '90, Mueller '94]



z: longitudinal photon momentum fraction carried by the quark

$\mathbf{r}$: transverse $(q\bar{q})$-size

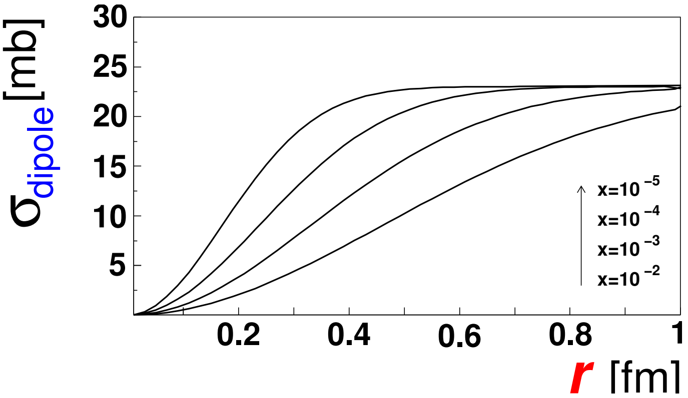
- **Intuitively:** For small x, in P rest frame, γ^* fluctuates mainly into $q\bar{q}$-dipole far upstream with

$$\tau_{q\bar{q}-\mathrm{formation}} \gg \tau_{(q\bar{q})\,P-\mathrm{interaction}} \quad \Rightarrow \quad \text{factorization}$$

$$\boxed{\sigma_{L,T}(x,Q^2) = \int_0^1 dz \int d^2\mathbf{r}\; |\Psi_{L,T}^{\gamma^*\to q\bar{q}}(z,r)|^2\, \sigma_{\mathrm{dipole}}(r,\ldots)}$$

- (light-cone) **wave function**, calculable in pQCD :

$$\Psi_{L,T}^{pQCD}\;(z,r) \sim \mathrm{K}_{0,1}(\sqrt{z(1-z)}\,rQ)$$



"saturation": $\boxed{\sigma_{\mathrm{dipole}} \overset{r \gtrsim 0.5\mathrm{fm}}{\Rightarrow} \text{constant}}$

"color transparency": $\boxed{\sigma_{\mathrm{dipole}} \overset{r\to 0}{\propto} \pi\, r^2}$

pQCD : [Low, '75; Nikolaev & Zakharov, '90]

2. Instantons and the Colour-Dipole Picture at Small x

[F. Sch.,'01; F. Sch. & Utermann '02]

- $\sigma_{\rm dipole}(r, ..)$ in general includes non-perturbative effects
- Do instantons of size $\sim \langle\rho\rangle$ in background, give rise to "saturating", geometrical form,

$$\sigma^{(I)}{}_{\rm dipole}(r, \ldots) \overset{\rm r \gtrsim \langle\rho\rangle}{\sim} \pi\langle\rho\rangle^2 \ ?$$

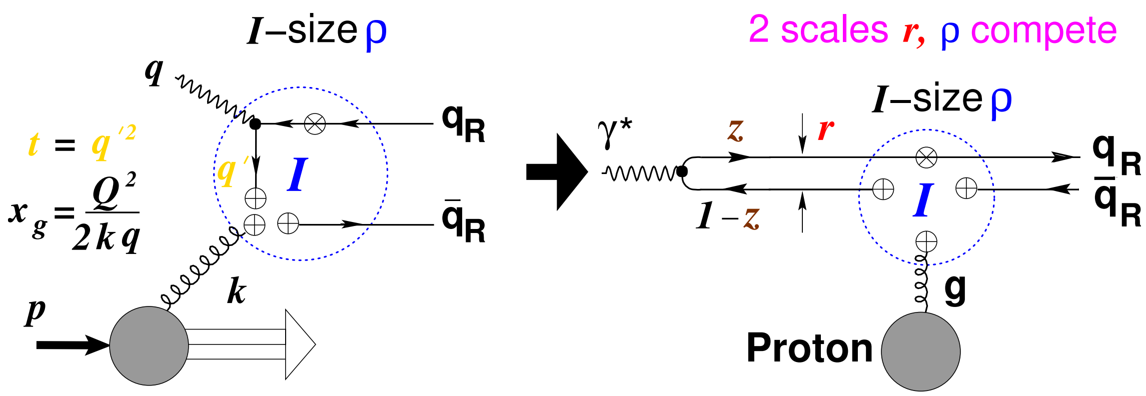
♣ $\boxed{\text{Strategy}}$:

- Conservative start: Large Q^2 and cuts $\Rightarrow$ I-pert. theory
- Transform published results on I-induced **DIS** processes, [Moch, Ringwald & F. Sch., Nucl. Phys. **B** '97; Ringwald & F. Sch., Phys. Lett. **B** '98], into colour-dipole picture
- Carefully increase $q\bar{q}$-dipole size r towards hadronic dimensions with help of lattice results for 2 crucial building blocks of I-calculus:

  $\boxed{I\text{-size distribution } D(\rho)}$ and $\boxed{I\bar{I}\text{- interaction } \Omega^{I\bar{I}}}$

- Then, the above restrictions are now longer necessary (i.e. at least everything remains finite)

- Simplest I-induced (sub)process, $\gamma^* + g \overset{(I)}{\Rightarrow} q_{\rm R} + \overline{q}_{\rm R}$,
  i.e. $n_f = 1$, no final-state gluons



- Starting point: [Moch, Ringwald & F. Sch., Nucl. Phys. **B507** (1997) 134]

$$\sigma^{\gamma^* P}_{L,T}(x, Q^2) = \int_x^1 \frac{d\,x_g}{x_g} \left(\frac{x}{x_g}\right) G\left(\frac{x}{x_g}, Q^2\right) \int d\,t\, \frac{d\hat{\sigma}^{\gamma^* g}{}_{L,T}}{d\,t}(x_g, t, Q^2)$$

e.g.: $\frac{d\,\hat{\sigma}^{\gamma^* g}{}_L}{d\,t} \overset{(I)}{=} \frac{\pi^7}{2} \frac{e_q^2}{Q^2} \frac{\alpha_{\rm em}}{\alpha_s} \left[x_g(1-x_g)\sqrt{tu}\, \frac{\mathcal{R}(\sqrt{-t}) - \mathcal{R}(Q)}{Q^2 + t} - (t \leftrightarrow u)\right]^2$

with the 'master' integral, $\mathcal{R}(\mathcal{Q}) = \int_0^\infty d\rho\; D(\rho)\rho^5\; (\mathcal{Q}\rho) {\rm K}_1(\mathcal{Q}\rho)$

Note: $\mathcal{R}(0) = \int_0^\infty d\rho\; D_{\rm lattice}(\rho)\rho^5 \approx 0.6\ {\rm fm}[\sim \langle\rho\rangle]$ **finite**

- With variable transformation and 2d-Fourier transformation $\boxed{(x_g, t)} \Leftrightarrow \boxed{(z, \mathbf{r})}$,

- indeed obtain colour-dipole structure,

$$\int_x^1 \frac{d\,x_g}{x_g}\{\ldots\} \int dt\, \frac{d\hat{\sigma}^{\gamma^* g}{}_{L,T}}{dt} \Rightarrow \int dz \int d^2\mathbf{r}\; \left(|\Psi_{L,T}|^2\, \sigma_{\rm dipole}\right)^{(I)}.$$

- **Most interesting:** Background instanton induces additional length-scale, I-size $\rho \approx \langle\rho\rangle$ besides $(q\bar{q})$-dipole size r,

- As expected intuitively, the 2 scales $r,\ \rho$ "compete"! : $(\hat{Q} = \sqrt{z(1-z)}\,Q)$

e.g.
$$\left(|\Psi_L|^2\,\sigma_{\rm dipole}\right)^{(I)} \approx |\,\Psi_L{}^{\rm pQCD}\,(z,r)\,|^2\,\frac{1}{\alpha_{\rm s}}\,x\,G(x,\mu^2)\,\frac{\pi^8}{12}$$
$$\times\left\{\int_0^\infty d\rho D_{\rm lattice}(\rho)\,\rho^5\left(\frac{-\frac{d}{dr^2}\left(2r^2\frac{{\rm K}_1(\hat{Q}\sqrt{r^2+\rho^2/z})}{\hat{Q}\sqrt{r^2+\rho^2/z}}\right)}{{\rm K}_0(\hat{Q}r)} - (z\leftrightarrow 1-z)\right)\right\}^2,$$

- **Note:** $-\frac{d}{dr^2}\left(2r^2\frac{K_1(\hat{Q}\sqrt{r^2+\rho^2/z})}{\hat{Q}\sqrt{r^2+\rho^2/z}}\right) \approx \begin{cases} -\frac{K_1(Q\rho\sqrt{1-z})}{Q\rho\sqrt{1-z}} & \text{for} \quad r^2 z/\rho^2 \Rightarrow 0 \\ K_0\left(\hat{Q}r\right) & \text{for} \quad r^2 z/\rho^2 \ \textbf{large} \end{cases}$

- Hence, in the two limiting cases (pick $z \gg 1-z$)

$r^2 \Rightarrow 0$: $\left(|\,\Psi_{\frac{L}{T}}\,|^2\,\sigma_{\rm dipole}\right)^{(I)} = \mathcal{O}(1)$ but **tiny** (c.f. pQCD $:\propto r^2 K_0(\hat{Q}r)^2 \Rightarrow 0$)

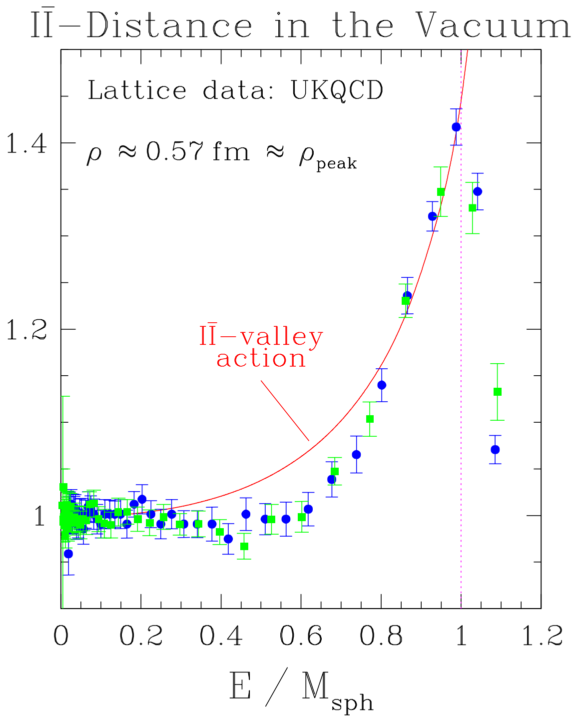
$r^2 \gtrsim \langle\rho\rangle^2$: $\left(|\,\Psi_{\frac{L}{T}}\,|^2\,\sigma_{\rm dipole}\right)^{(I)} \approx |\,\Psi_{\frac{L}{T}}^{pQCD}\,|^2\,\underbrace{\frac{x\,G(x,Q^2)}{12\alpha_s}\pi^8\left(\int_0^\infty d\rho D_{\rm lattice}(\rho)\,\rho^5\right)^2}_{\sigma^{(I)}_{\rm dipole}\ \textbf{"saturates"}}$

More realistic: $\gamma^* + g \overset{(I)}{\Rightarrow} n_f\,(q_{\rm R} + \bar{q}_{\rm R}) + \text{gluons}$

- **Start from**: I-perturbation theory results: [Ringwald & F. Sch. '98]; final-state gluons $\leftrightarrow$ $I\bar{I}$-"valley" interaction ${\Omega^{I\bar{I}}}_{\rm valley}$ known for all $I\bar{I}$-distances R/ρ [Khoze & Ringwald '91; Verbaarschot'91]
- Crucial building block entering $\sigma^{(I)\,\rm gluons}_{\rm dipole}$ in semi-hard regime,

$$\boxed{\left\langle e^{-\frac{4\pi}{\alpha_s}\Omega^{I\bar{I}}}\right\rangle_{U,\rho,\bar{\rho}}, \quad \text{with} \quad \left\{ \begin{array}{l} \rho, \bar{\rho} \approx \rho_{\rm peak} \approx 0.59\ {\rm fm}[\approx \ \langle\rho\rangle], \\ \rho_{\rm peak} = \text{peak of } \rho^5\, D_{\rm lattice}(\rho) \end{array} \right.}$$

- Directly measured on the lattice (UKQCD) through R-distribution $dn_{I\bar{I}}/d^4z\,d^4R$ [Ringwald & F. Sch. '99]
- For $\rho, \bar{\rho} \approx \rho_{\rm peak}$: new saddle point $R^*/\rho \Rightarrow 1 \Leftrightarrow E/M_{\rm sph} \Rightarrow 1$ in $\sigma^{(I)\,\rm gluons}_{\rm dipole} \approx \int dE \int d^4R\,\{\ldots\}$ [F. Sch. & Utermann '02]



- $\sigma_{\rm dipole}$ dominated by QCD - sphaleron, as expected! Barrier height: $M_{\rm sph} \approx \frac{3\pi}{4}\frac{1}{\alpha_s\,\rho}$ [Ringwald & F. Sch. '94; Diakonov & Petrov '94]
- $\Omega^{I\bar{I}} \approx {\Omega^{I\bar{I}}}_{\rm valley}$ right up to QCD - sphaleron peak,
- $\sigma_{\rm dipole}$ "saturates" $\propto$ I-area $= \pi \left(\int_0^\infty d\rho D_{\rm lattice}(\rho)\rho^5\right)^2$

3. Wegner-Wilson Loop Scattering in an Instanton Background

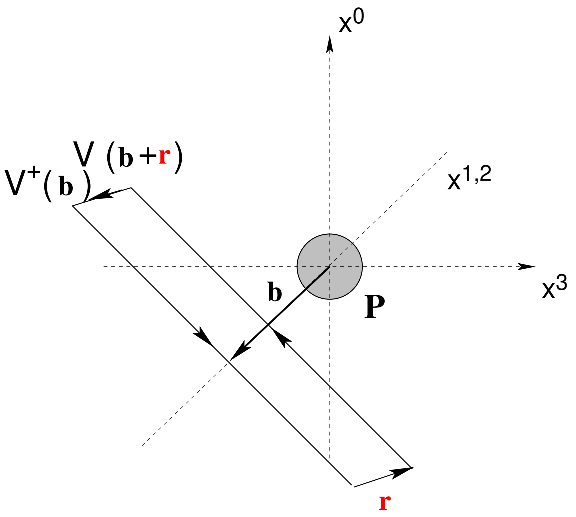
[F. Sch. & Utermann '02]

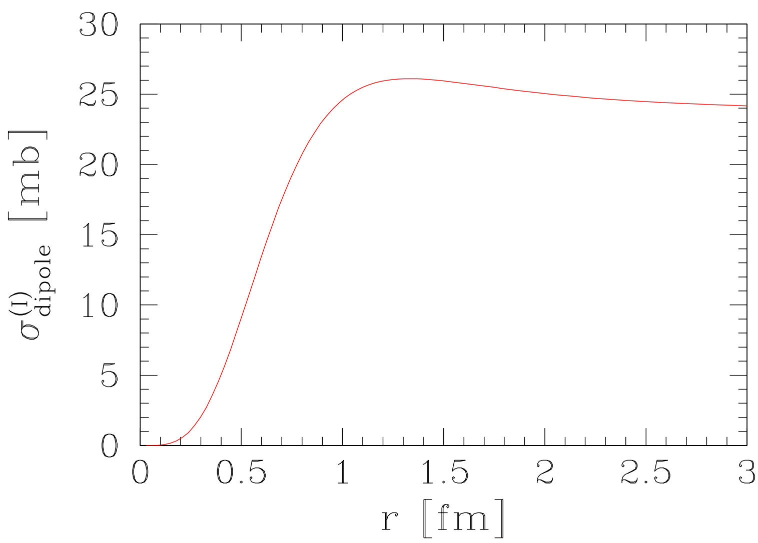
[Nachtmann '91, '97, Dosch *et al.* '90 -'02, Buchmüller, Hebecker *et al.* '96 -. . . , Shuryak *et al.* '00 -'02]

- $s \gg -t$: Eikonal regime, scattering of (light-like) Wegner-Wilson (W-W) loops $\Leftrightarrow$ $q\bar{q}$-color dipoles (c.f. talk by Pirner)
- **Consider:** Instanton background $G_\mu(x) = G_\mu^{(I)}(x)$,
- $\sigma_{\rm dipole}$ in "Semiclassical approach" [Buchmüller, Hebecker *et al.* '96 -]
  W-W loop in color field of proton (Fig. left)

$$\begin{aligned}
\sigma_{\rm dipole}(r) &= \frac{1}{N_c}\int d^2\mathbf{b}\,\mathrm{tr}\left[T(\mathbf{r},\mathbf{b})T(\mathbf{r},\mathbf{b})^\dagger\right];\\
T(\mathbf{r},\mathbf{b}) &= \left\langle V^\dagger(\mathbf{b})\,V(\mathbf{b}+\mathbf{r}) - 1\right\rangle_{G^{(I)}};\\
\langle\ldots\rangle_{G^{(I)}} &: \quad \mathcal{D}G^a_\mu \Rightarrow d^4z\,d\rho\,D_{\rm lattice}(\rho)\,dU.
\end{aligned}$$

- Analytic solution! "black-disk" saturation: $\sigma_{\rm dipole} \approx 2\pi\rho_{\rm peak}{}^2$
- Fast raise of $\sigma_{\rm dipole}$ around $r \approx \rho_{\rm peak} \approx 0.59$ fm (Fig. right)





4. Conclusions

- Larger-size I's: building blocks for **diffractive** processes?
- I-scale $\langle\rho\rangle$ reflected in **geometrization** of soft QCD ?

- Two different approaches for $\sigma_{\rm dipole}$ as function of dipole size r:
  - I-perturbation theory $\oplus$ lattice data
  - Wegner-Wilson loop scattering in I-background
- Similar and $\sim$ realistic results, with $\sigma_{\rm dipole}$ reflecting I-scale $\langle\rho\rangle$!
  - From Color transparency at small r,
  - strong raise around $r \approx \langle\rho\rangle \Rightarrow$ Saturation $\propto \pi\langle\rho\rangle^2$
- Interesting: Comparison with stochastic vacuum approach...