# Instanton-Events in Hard Photoproduction - Can they be Predicted - ?

F. Schrempp, DESY

in collaboration with

A. Ringwald

1. **The Problem**
2. **Going Beyond Instanton Perturbation Theory**
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## 1. The Problem

- **DIS** distinguished, since hard instanton-induced processes

- are calculable within instanton-perturbation theory
  [Balitsky & Braun '93; Ringwald & F. Sch. '94 - '98;
  Moch, Ringwald & F. Sch. '97]

- Key feature: hard generic DIS scale $\mathcal{Q}$ cuts off instantons with large size $\boldsymbol{\rho} \gtrsim \boldsymbol{\mathcal{Q}}^{-1}$, over which we have no control!
  [Moch, Ringwald & F. Sch. '97]

♣ The Question: Is the hard photoproduction limit $(Q^2 \Rightarrow 0)$ with a high $E_T$ of the current quark jet within the reach of $\boldsymbol{I}$-perturbation theory?

- **True** in ordinary QCD perturbation theory...

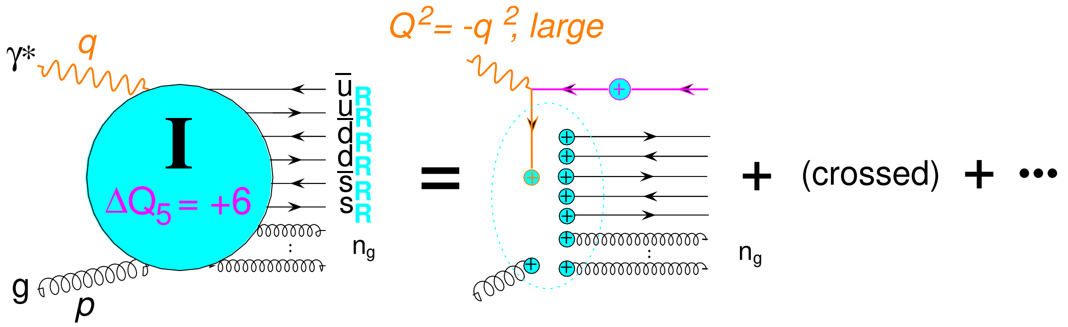
♣ The Motivation: "Monster" Fireball events & lots of further interesting data. . .

- Recall main arguments in case of Instanton $(\boldsymbol{I})$-induced amplitude for exclusive DIS processes,

$\triangle Q_5 = 2\, n_f$ :   $\gamma^* + \mathrm{g} \Rightarrow \sum_{\text{flavours}}^{n_f} [\overline{q}_R + q_R] + n_g\, g$

vanishing to any order of conventional P.T in (massless) QCD

- $I$-Perturbation Theory: Expand (Euclidean) path integral for relevant Green's functions about classical instanton solution:




- Fourier-transform (F. T.) with respect to external lines.
- LSZ amputate external legs.
- Analytically continue to Minkowski space.
- Basic building blocks: (singular gauge) in (modified) Feynman rules of $I$-perturbation theory:
  - Classical instanton gauge field $A_\mu^{(I)}(x;\rho,U)$ [Belavin *et al.* '75] ;
  - Quark zero mode fields $\kappa^{(I)}(x;\rho,U)$, $\overline{\phi}^{(I)}(x;\rho,U)$ (Weyl basis);
  - (Non-zero mode) quark propagators $S^{(I)}(x,y;\rho,U)$, $\overline{S}^{(I)}(x,y;\rho,U)$ in the $I$-background [Brown *et al.* '78]

- Collective coordinates: $\boldsymbol{\rho} = I$-size, $(U)^k_\alpha = I$-color orientation (color: $k = 1, 2, 3$ and spinor: $\alpha = 1, 2$ indices).

- Must be integrated over in Amplitudes:

$$\mathcal{T}^{(I)}_\mu(q, p, \ldots) = \int dU \int_0^\infty d\rho\, D(\rho)\, \mathcal{A}^{(I)}_\mu(q, p, \ldots; U, \rho)$$

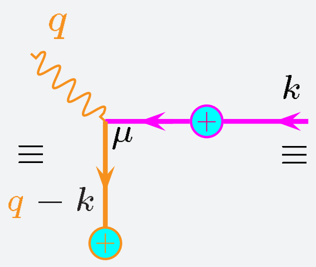
- $\boldsymbol{I}$-density (renormalization scale $\mu_r$) as generic measure:

$$D(\rho) = d\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left(-\frac{2\pi}{\alpha_s(\mu_r)}\right)\frac{(\boldsymbol{\rho}\,\mu_r)^{\boldsymbol{b}}}{\boldsymbol{\rho}^5}$$

| **b** | $\frac{1}{D}\frac{\mathrm{d}D}{\mathrm{d}\mu_r}$ | Ref. |
|---|---|---|
| $\beta_0$ | $\mathcal{O}(\alpha_s)$ | 't Hooft '76 |
| $\beta_0 + \frac{\alpha_s(\mu_r)}{4\pi}(\beta_1 - 12\beta_0)$ | $\mathcal{O}(\alpha_s{}^{\mathbf{2}})$ | Morris, Ross, & Sachrajda '85 |

QCD $\beta$-function coefficients: $\beta_0 = 11 - \frac{2}{3}\mathrm{n_f}$; $\beta_1 = 102 - \frac{38}{3}\mathrm{n_f}$

- $Q^2$ enters only in F.T'ed $\gamma^* q\overline{q}$-vertices,

$\mathcal{V}_\mu(q, -k; \rho) \equiv$ 

$\equiv \int d^4x\, \mathrm{e}^{-\mathrm{i}q\cdot x}\left[\overline{\phi}^{(I)}(x;\rho)\overline{\sigma}_\mu \boldsymbol{S}^{(I)}(x, -k; \rho)(-\mathrm{i}\overline{k})\right]$

- After a long and tedious calculation ($v = v_\mu \sigma_\mu,\ \overline{v} = v_\mu \overline{\sigma}_\mu$),

$$\lim_{k^2\to 0} \mathcal{V}_\mu(q,-k;\rho) = 2\pi \mathrm{i} \rho^{3/2} \left[\epsilon \sigma_\mu \overline{V}(q,q';\rho)\, U^\dagger\right] ,$$

with $q' = q-k,\ Q' = \sqrt{-q'^{\,2}} \equiv \sqrt{-t} > 0,\ Q = \sqrt{-q^2} \geq 0$,

$$\begin{aligned}\overline{V}(q,q';\rho) &= \left[\frac{\overline{(q')}}{Q'^{\,2}} + \frac{\overline{q-q'}}{Q'^{\,2}-Q^2}\right] \rho Q'\, K_1(\rho Q') \\ &\quad -\frac{\overline{q-q'}}{Q'^{\,2}-Q^2}\, \rho Q\, K_1(\rho Q)\end{aligned}$$

- Various terms strongly constrained by manifest gauge invariance $q^\mu \mathcal{T}_\mu^{(I)}(q,p,\ldots) = 0$ !

- $2^{\rm nd}$ term ("contact" term) in $\overline{V}(q,q';\rho)$ necessary to cancel an unphysical pole in physical region at $Q'^{\,2} = Q^2$!

- In DIS: Exponential decrease of all form factors (Bessel $K_1$ functions) $K_1(\rho Q) \overset{\rho Q \Rightarrow \infty}{\sim} \exp\left[-\rho Q\right]$.

- $\Rightarrow$ Integration over the instanton size $\rho$ in amplitudes $\mathcal{T}_\mu^{(I)}(q,p,\ldots)$ is finite for large $Q > 0$ [as long as the internal quarks are not moving collinear to the incoming gluon (like in pQCD!)].

- For $Q^2 \Rightarrow 0$: $\rho Q K_1(\rho Q) \overset{Q\Rightarrow 0}{\Rightarrow} 1,$ ,

- $\Rightarrow$ $2^{\rm nd}$ term in $\overline{V}(q, q'; \rho)$ $\rho$-independent and $\rho$-integration IR-divergent within $I$-perturbation theory, since there $D(\rho) \sim \rho^6$.

- This divergence is independent of current-quark $E_T$:

$$Q'^2 = Q^2 + \frac{E_T}{\sqrt{s}} \left(s \exp(-\eta) - Q^2 \exp(\eta)\right) \overset{Q\Rightarrow 0}{\Rightarrow} E_T \sqrt{s} \exp(-\eta)$$

## 2. Going Beyond Instanton Perturbation Theory

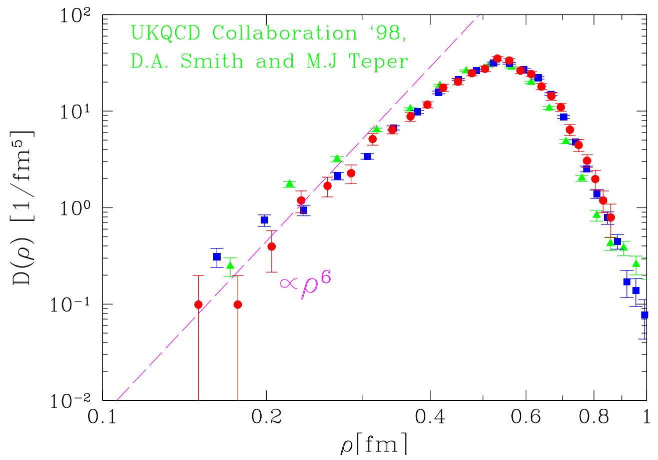
♣ Proposal: Attempt to extend our predictions beyond proper regime of $I$-perturbation theory, by taking the two most strongly varying entries directly from lattice simulations:

$$I - \text{density } D(\rho) \text{ and } I\overline{I} - \text{interaction } \Omega(R, \rho, \overline{\rho}, U)$$

- $\Rightarrow$ Photoproduction limit finite and calculable and kinematical range of predictions in DIS much extended!

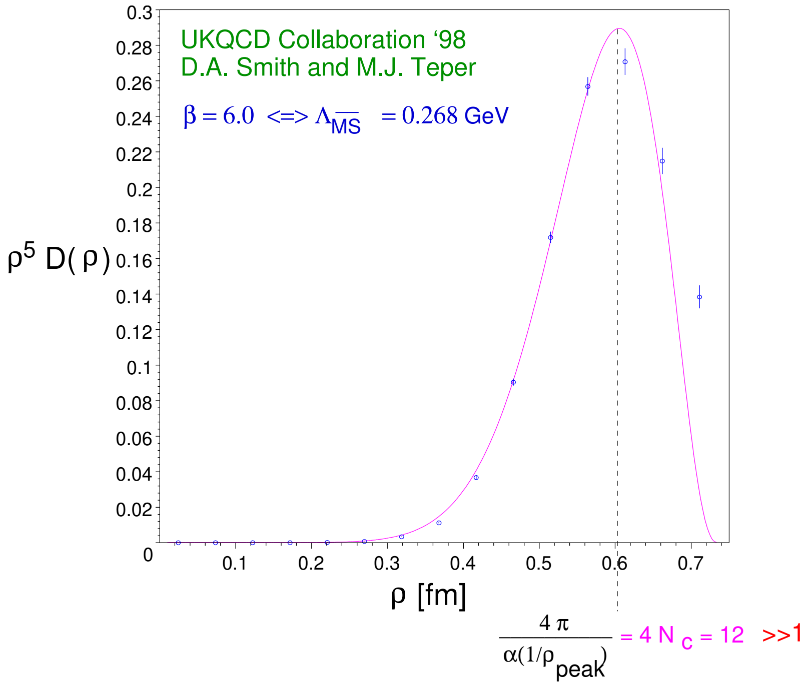
- Prize to pay:
  Some phenomenological input$\Rightarrow$ less rigor. . .

- $I$-density: for small-size instantons, $\rho \lesssim \rho_{\rm max} \simeq 0.3$ fm , well described by $I$-perturbation theory, $D(\rho)_{|n_f=0} \propto \rho^6$:




- Plausible choice of renormalization scale $\mu_r$ in perturbative form of $D(\rho, \mu_r)$: $\mu_r = 1/\rho$, extends good description beyond the peak region around $\rho_{\text{peak}} \simeq 0.5$ fm!

- $\rho^5 D(\rho)$ depends on $\rho$ only through $\alpha_s(1/\rho)$:




- $D(\rho) \overset{\rho\to\infty}{\Rightarrow} 0$, hence $\rho$-integration IR-convergent, also for photoproduction!

- $I\overline{I}$ – interaction $\Omega(R,\rho,\overline{\rho},U)$:

- At **HERA** need:

$$\sigma^{(I)}_{q'p} = \int d^4R\, \mathrm{e}^{\mathrm{i}\,(p+q')\cdot R} \int_0^\infty d\rho\, D(\rho)\mathrm{e}^{-Q'\rho} \int_0^\infty d\overline{\rho}\, D(\overline{\rho})\mathrm{e}^{-Q'\overline{\rho}}$$

$$\int dU\, \{\ldots\}\, \mathrm{e}^{-\frac{4\pi}{\alpha_S(1/\sqrt{\rho\overline{\rho}})}\,\Omega(R,\rho,\overline{\rho},U)}$$

- On the **lattice** measure:

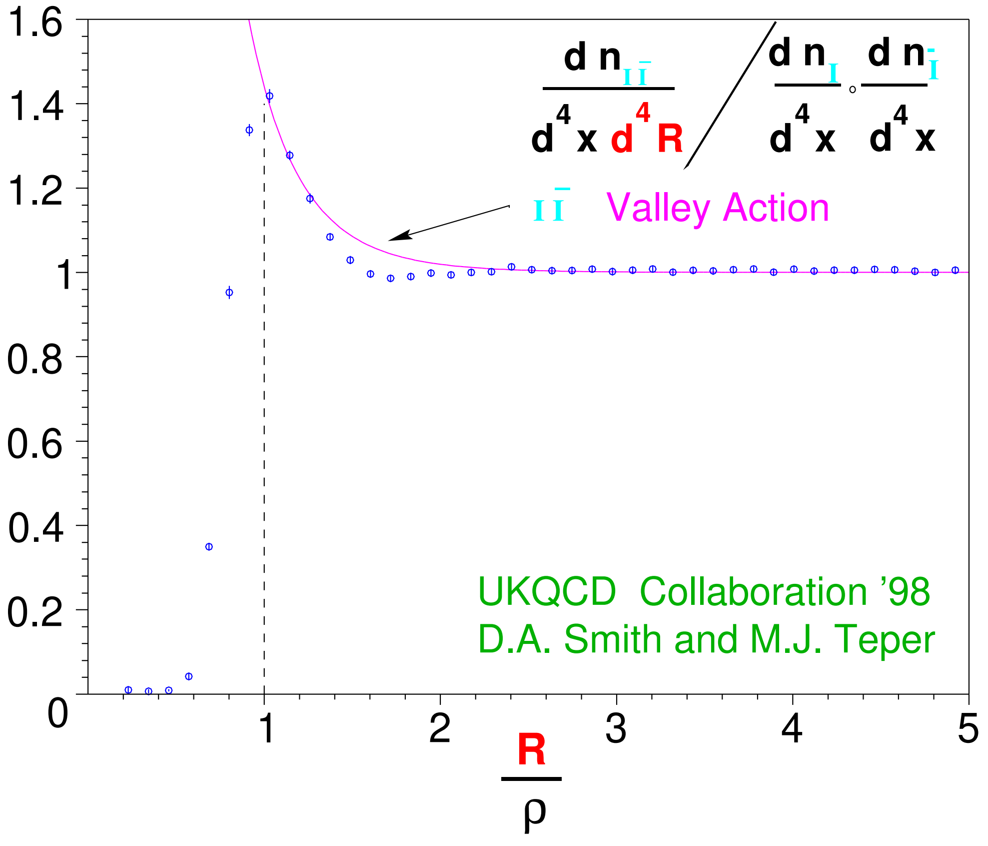
$$\frac{\mathrm{d}n_{I\overline{I}}}{\mathrm{d}^4x\,\mathrm{d}^4R} \Big/ \frac{\mathrm{d}n_I}{\mathrm{d}^4x}\cdot\frac{\mathrm{d}n_{\overline{I}}}{\mathrm{d}^4x} = \frac{1}{\int_0^\infty d\rho\, D(\rho) \int_0^\infty d\overline{\rho}\, D(\overline{\rho})}$$

$$\times \int_0^\infty d\rho\, D(\rho) \int_0^\infty d\overline{\rho}\, D(\overline{\rho}) \int dU\, \mathrm{e}^{-\frac{4\pi}{\alpha_S(1/\sqrt{\rho\overline{\rho}})}\,\Omega(R,\rho,\overline{\rho},U)}$$

$R^2$ large
$\Rightarrow 1$

- Good description for $R/\rho \gtrsim 1$ in terms of known $I\overline{I}$-valley action $(\xi = \frac{R^2}{\rho\overline{\rho}} + \frac{\rho}{\overline{\rho}} + \frac{\overline{\rho}}{\rho} \gtrsim 2)$,

$$\Omega\,(R,\rho,\overline{\rho},U) \equiv S^{I\overline{I}}(\xi;U) - 1 = \Sigma_1(\xi)\,f(U) + \Sigma_2(\xi)\,g(U);$$

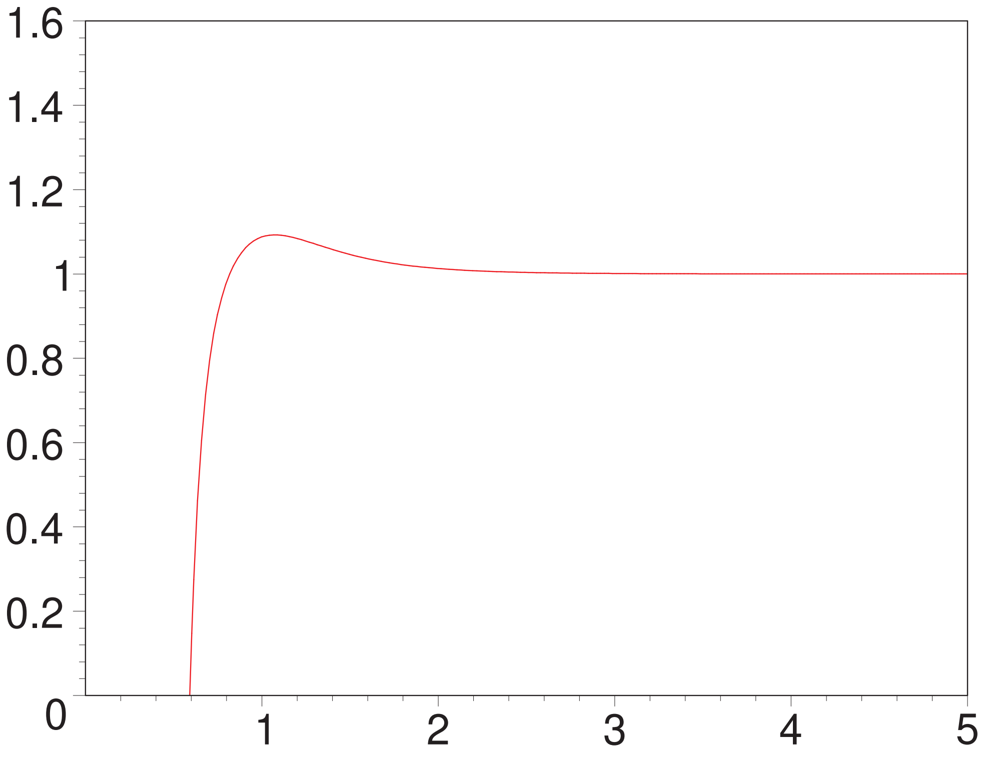



- Slight modification of subdominant

$$\Sigma_2(\xi) \Rightarrow \frac{\xi+3/2}{\xi-3/2}\,\Sigma_2(\xi) \overset{R/\rho\ \mathrm{large}}{\Rightarrow} \Sigma_2(\xi)$$

gives needed strong repulsion at small $R/\rho$!

- $\Rightarrow$ **HERA**-cross section does not "explode" anymore for small $x' \lesssim 0.35$!
- May considerably relax $(x', Q')$ cuts. . .


1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5

## 3. Outlook

- Very promising interface between lattice results and $I$-induced processes at **HERA**;
- may help to extend significantly the range of instanton predictions, including "monster" fireball events in photoproduction!
- Some crucial further calculations may be done during this workshop! Interested volunteers are most welcome. . .