QCD-Instantons at HERA

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– DIS '98, Brussels, April 1998 –

1. Introduction

- Gauge fields $A_\mu^a(x)$ in SU(N) Yang-Mills theories (in 4d Euclidean space-time) are known to possess an integer **topological** charge Q,

$$Q = \int \mathrm{d}^4 x q(x); \qquad q(x) = \frac{g^2}{8\pi^2}\frac{1}{4} G_{\mu\nu}^a(x)\tilde{G}_{\mu\nu}^a(x);$$

in terms of the field strength tensor $G_{\mu\nu}^a(x)$ and its dual $\tilde{G}_{\mu\nu}^a(x) = 1/2\epsilon_{\mu\nu\rho\sigma}G_{\rho\sigma}^a(x)$, $a = 1 \ldots N^2 - 1$.

- Action S and Lagrange density $\mathcal{L}(x)$ (without fermions):

$$\begin{aligned} S &= \int \mathrm{d}^4 x \mathcal{L}(x) = \int \mathrm{d}^4 x \frac{1}{4} G_{\mu\nu}^a(x)\, G_{\mu\nu}^a(x); \\ &= \frac{1}{4}\int \mathrm{d}^4 x \left[\pm G_{\mu\nu}^a(x)\tilde{G}_{\mu\nu}^a(x) + \frac{1}{2}\underbrace{\left(G_{\mu\nu}^a(x) \mp \tilde{G}_{\mu\nu}^a(x)\right)^2}_{\geq 0}\right] \\ &\geq \frac{8\pi^2}{g^2} \mid Q \mid . \end{aligned}$$

- Ordinary perturbation theory resides in the sector: $\boldsymbol{Q = 0}$. Expansion about trivial field configuration $\boldsymbol{A_\mu^a(x) \equiv 0, S = 0}$.

- **(Anti-) Instantons:** non-perturbative fluctuations of the gauge-fields, representing topology changing tunnelling processes: $\boldsymbol{Q = \pm 1}$.
  - Solutions of classical Euclidean field equations with $G_{\mu\nu}^a(x) = \pm\tilde{G}_{\mu\nu}^a(x)$:

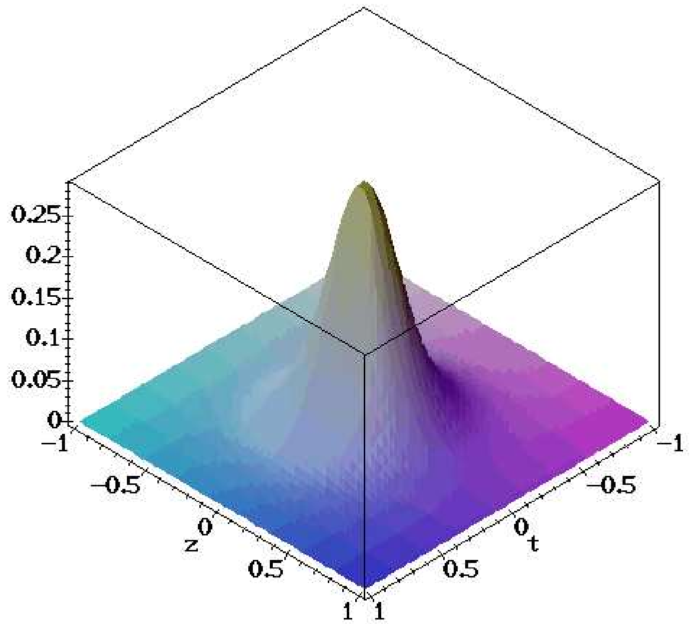
$$\boldsymbol{A_\mu^{a\,(I)}(x;\rho) = \frac{2}{g}\cdot\frac{\eta_{a\mu\nu}\,x_\nu}{x^2+\rho^2},}$$

Instanton **size parameter** $\boldsymbol{\rho > 0}$ and t'Hooft coefficients $\eta_{a\mu\nu}$

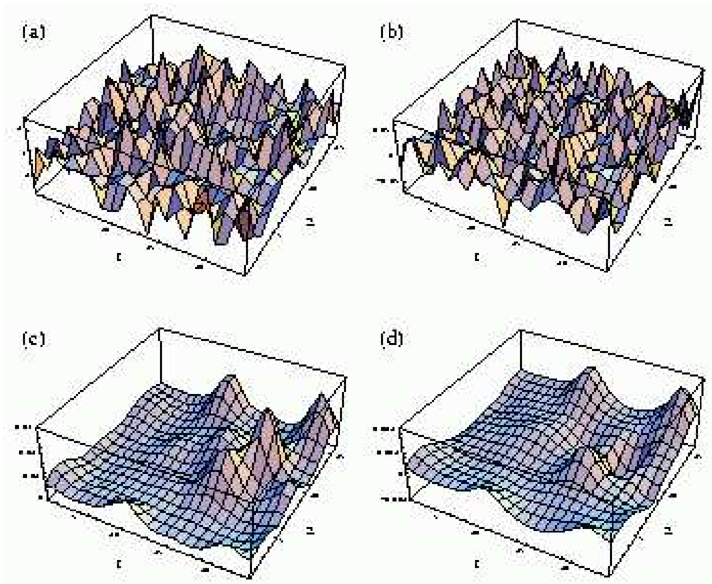
– Minimize the action for $Q = 1$,

$$\mathcal{L}^{(I)}(x) = \frac{48}{g^2} \cdot \frac{\rho^4}{(x^2+\rho^2)^4} \quad \Rightarrow \quad \boxed{S^{(I)} = \frac{8\pi^2}{g^2}}$$

.



– can be seen and studied in **lattice simulations** (⇒ Sect.2):



Before "cooling"

After cooling

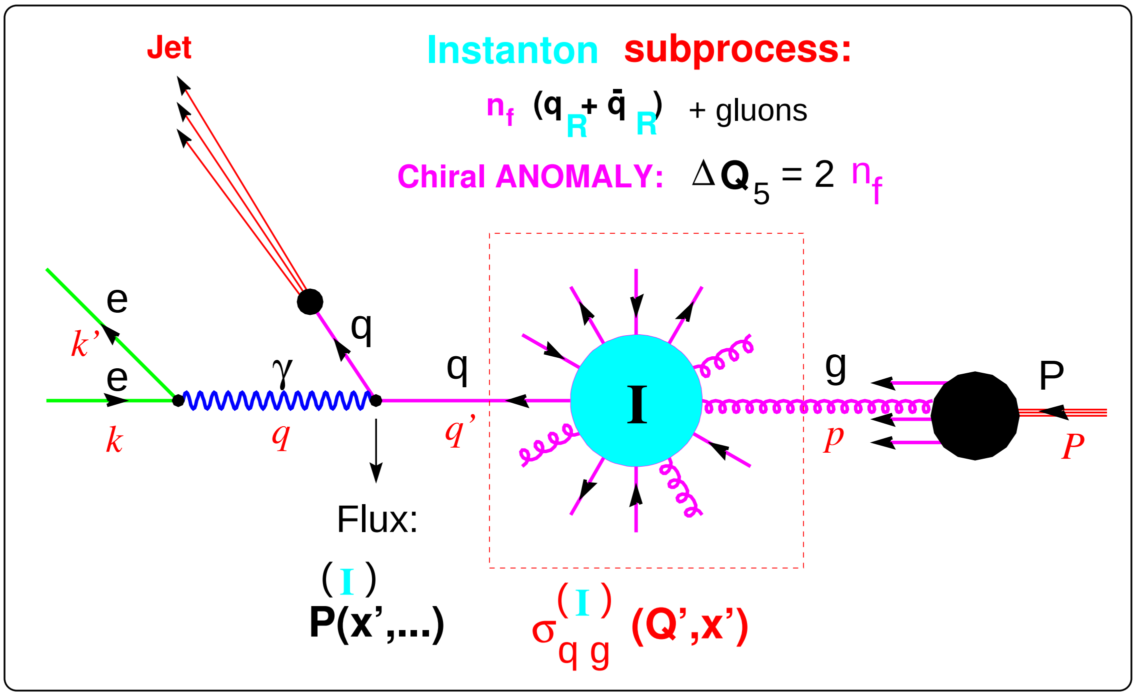
[Chu et al. '94]

Lagrange density *L(x)*   topological charge density q(x)

– See several Instantons($q(x) > 0$) and 1 Anti-Instanton ($q(x) < 0$)!

2. Which is the "Fiducial" Kinematical Region?
– Restrictions from Lattice-QCD Simulations

- Leading Instanton(I)-induced, chirality violating process in DIS regime of $e^{\pm}P$ scattering:



- Instanton-subprocess with own Bjorken-variables:

$$Q'^2 = -q'^2 > 0; \quad 0 \le x' = \frac{Q'^2}{2p\cdot q'} \le 1$$

- State of the art:
  - **Our Framework:** Perturbation theory about the QCD **Instanton**,

$$\mid Q \mid = 1 : A^a_\mu(x) = A^{a\,(I)}_\mu(x;\rho,\ldots), \; S^{(I)} = \frac{8\pi^2}{g_s^2} = \frac{2\pi}{\alpha_s},$$

    with α_s being the small parameter.
  - Integration over Instanton-size **finite!**; usual infrared divergencies (large ρ) **absent**, since

♣ inverse, large momentum scale $1/Q'$ provides a dynamical infrared cutoff, $\rho \lesssim 1/Q'$! [Moch, Ringwald, F. Sch., Nucl. Phys. B507 (1997) 134].

$\Rightarrow$ Hence for the first time well-defined, calculable contribution from small Instantons in DIS!

- Validity of Instanton-perturbation theory requires large $Q' \gtrsim Q'_{\rm min}$; $x' \gtrsim x'_{\rm min} > 0$!

- Final state properties and $\sigma^{(I)}_{\rm HERA}(Q' > Q'_{\rm min}, x' > x'_{\rm min})$ from Monte Carlo generator **QCDINS**. New upgrade:**1.5.2**.
  [Gibbs, Ringwald, F. Sch., '95-'98]

- Prediction of I-induced rates requires knowledge of "fiducial" $(\boldsymbol{Q'}, \boldsymbol{x'})$ region, where I-perturbation theory may be trusted, i.e.

- How big are $\boldsymbol{Q'_{\rm min}}, \boldsymbol{x'_{\rm min}}$?? $\Leftarrow$ Crucial input into QCDINS MC!

- Important information from recent Lattice-QCD simulations :
  4d Euclidean space-time made discrete, typically,

  lattice spacing: a $=$ 0.055 - 0.1 fm
  lattice volume: V $= [16^3 \cdot 48 - 32^3 \cdot 64]a^4$
  $= [0.88^3 \cdot 2.64 - 3.2^3 \cdot 6.4]\mathrm{fm}^4$

  [UKQCD, Smith & Teper '98]

- Up to now: only gluons $(n_f = 0)$

♣ Snapshots of the QCD vacuum including **all** possible non-perturbative features like Instantons!

- Study properties of Instanton ensemble with sizes $a \lesssim \rho \lesssim V^{1/4}$, like I-density $D(\rho)$, average distance $\langle R \rangle$ to nearest neighbours $(I\overline{I})$, validity of dilute gas picture,. . .

- Lattice "data" **independent** of I-perturbation theory !

- Compare with I-perturbative predictions; see where deviations start $\Rightarrow \boldsymbol{Q'}_{\text{min}}, \boldsymbol{x'}_{\text{min}}$!

- **Caveats:** Lattice artefacts $\Leftrightarrow$ continuum limit; dependence on "cooling",. . .

♣ I-density $D(\rho)$ (tunnelling probability!):

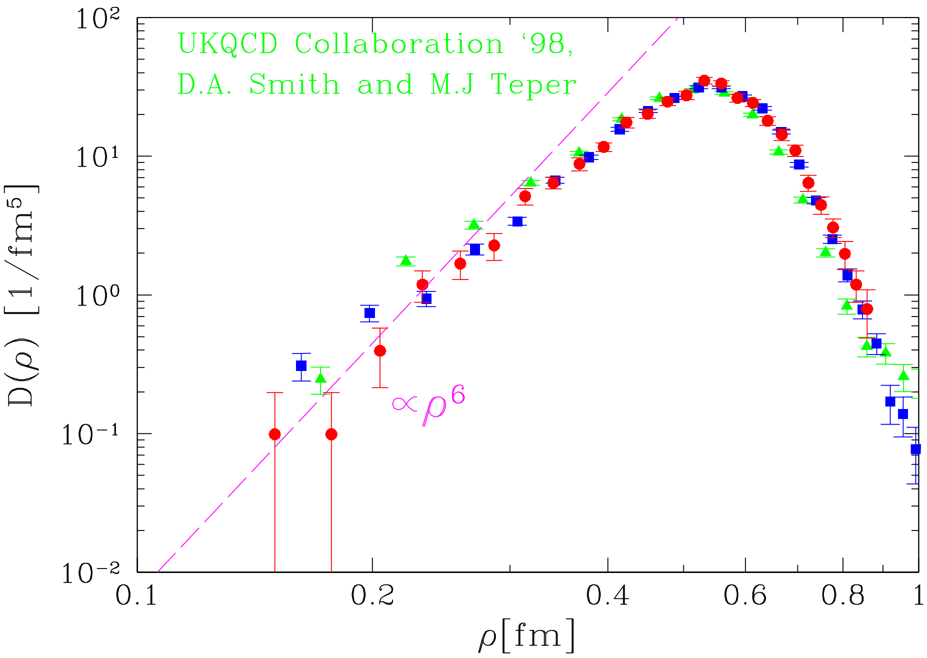
– I-perturbation theory: (renormalization scale μ_r)

$$D(\rho) \equiv \frac{\mathrm{d}n_I}{\mathrm{d}^4x\,\mathrm{d}\rho} = d\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left(-\frac{2\pi}{\alpha_s(\mu_r)}\right)\frac{(\boldsymbol{\rho}\mu_r)^b}{\boldsymbol{\rho}^5}$$

b	$\frac{1}{D}\frac{\mathrm{d}D}{\mathrm{d}\mu_r}$	Ref.
β_0	$\mathcal{O}(\alpha_s)$	't Hooft '76
$\beta_0 + \frac{\alpha_s(\mu_r)}{4\pi}(\beta_1 - 12\beta_0)$	$\mathcal{O}(\alpha_s{}^2)$	Morris, Ross, & Sachrajda '85

QCD β-function coefficients: $\beta_0 = 11 - \frac{2}{3}\mathrm{n_f}$; $\beta_1 = 102 - \frac{38}{3}\mathrm{n_f}$

– Lattice "data" $\Leftrightarrow$ I-perturbation theory: $D(\rho)_{|n_f=0} \propto \boldsymbol{\rho}^6$ (Fig.)



$\Rightarrow$ $\boxed{I\text{-perturbation theory for } \rho \lesssim \rho_{\max} = 0.3 \text{ fm}}$

♣ Average $I\overline{I}$ distance $\langle R \rangle$ [UKQCD, Smith & Teper '98]

i) $\langle R_{I\overline{I}} \rangle \simeq 0.45$ fm $< \langle R_{II+\overline{II}} \rangle$ ($a \to 0$ extrapolation)

ii) Ratio has better stability against "cooling": $\frac{\langle R_{I\overline{I}} \rangle}{\langle \rho \rangle} \simeq 0.83$

i)+ii) $\Rightarrow$ **entire** I-ensemble **not** like dilute gas!

♣ What about **smaller** Instantons?

– Consider total topological Charge[2] and total number of charges:

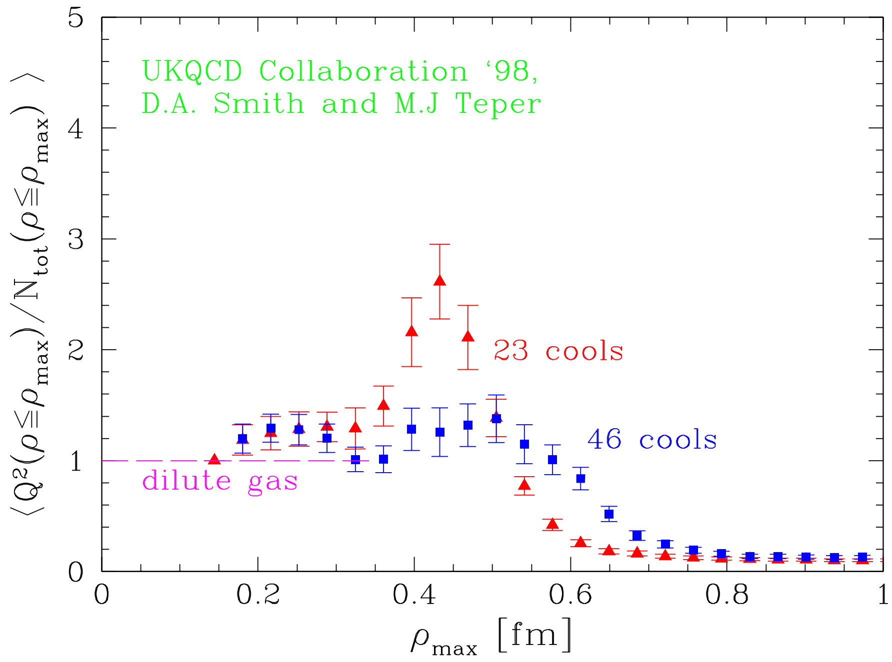
$$Q^2 = (n \cdot (+1) + \bar{n} \cdot (-1))^2; \quad N_{\mathrm{tot}} = n + \bar{n}.$$

– For a dilute gas, number fluctuations are poissonian and correlations among n and $\bar{n}$ distributions absent:

$$\langle (n-\bar{n})^2 \rangle = \langle n+\bar{n} \rangle \text{ or } \boxed{\langle \frac{Q^2}{N_{\rm tot}} \rangle = 1}$$

– Check whether this holds for Instantons **smaller than** $\boldsymbol{\rho}_{\rm max}$, i.e. whether,

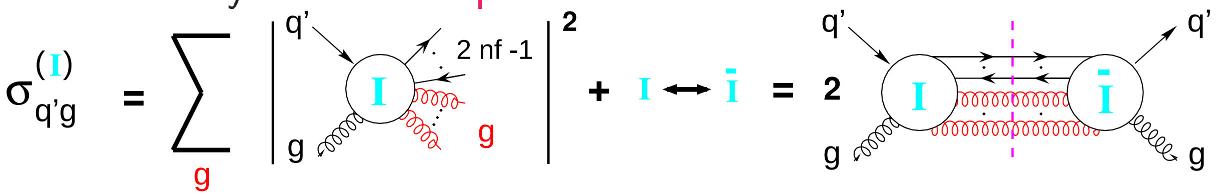
$$\boxed{\left\langle \frac{Q^2(\rho \lesssim \rho\text{max})}{N_{\rm tot}(\rho \lesssim \rho\text{max})} \right\rangle \overset{?}{\simeq} 1} \text{ ; (Fig.)}$$



– $\Rightarrow$ $\boxed{\sim \text{dilute gas for } \rho \lesssim \rho_{\rm max} \simeq 0.3 \text{ fm}}$

♣ How to relate this to **DIS?**

– Crucial Instanton dynamics in I-subprocess total cross section, evaluated by means of optical theorem:



– involving $q'g \Rightarrow q'g$ forward elastic amplitude in $\boldsymbol{I\bar{I}}$**-background**.

– New "collective coordinate" besides $I(\overline{I})$ size $\rho(\bar{\rho})$:

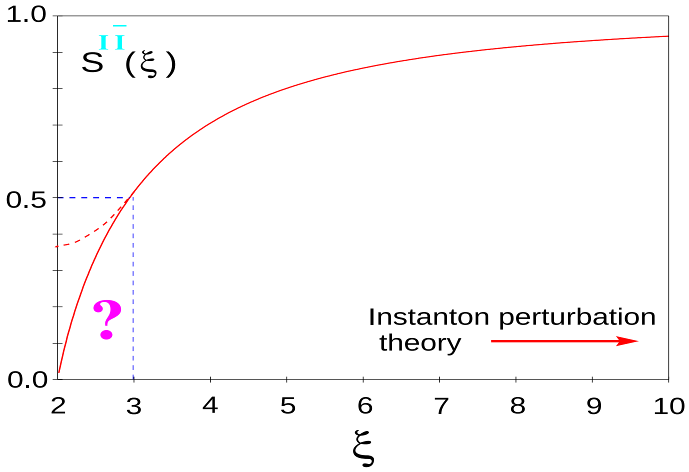
$$\boxed{R = I\overline{I}\text{-separation}} \Leftarrow \text{Lattice constraints!}$$

– To extrapolate somewhat beyond I-perturbation theory, use $\boxed{I\overline{I}\text{-valley method}}$, an effective resummation of final-state gluons in terms of **known** $I\overline{I}$-valley action $\boxed{0 < S^{I\overline{I}}(\xi) < 1}$, depending only on "conformal parameter"

$$\boxed{\xi = \frac{R^2}{\rho\bar{\rho}} + \frac{\rho}{\bar{\rho}} + \frac{\bar{\rho}}{\rho}}$$

Previously assumed:

$$\boxed{\xi \geq 3, \text{ i.e. } S^{I\overline{I}} \gtrsim \tfrac{1}{2}}$$



– Every observable $(\sigma^{(I)}_{q'g}, \ldots)$ involves **integration** over all "collective coordinates", $\rho, \overline{\rho}, R, \ldots$:

$$\sigma^{(I)}_{q'g}(Q', x') = \int_0^\infty d\rho \int_0^\infty d\overline{\rho} \underbrace{D(\rho)D(\overline{\rho})}_{I,\overline{I}-\text{densities}\Leftarrow\text{Lattice!}} e^{-(\rho+\overline{\rho})Q'}$$

$$\times \int^\infty d\xi \mathcal{M}(\xi, x', Q', \ldots) e^{-\frac{4\pi}{\alpha_s}(S^{I\overline{I}}(\xi)-1)}$$

– Integrations by means of **saddle-point method**;

– unique **saddle point** $(\rho^* = \overline{\rho}^*, \xi^*) \Leftrightarrow (\rho, \overline{\rho}, \xi)$-values by which the **integrals** are **dominated**!

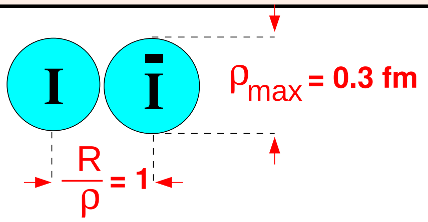
– Saddle point provides one-to-one **mapping** of Instanton variables $(\rho, R/\rho) \Longleftrightarrow$ HERA kinematics (Q', x'):

$$\overline{\rho}^* = \rho^* = \frac{1}{Q'} v(\alpha(Q'), \alpha(\mu_r), x');$$

$$\sqrt{\xi^* - 2} \equiv \frac{R^*}{\rho^*} = w(\alpha(Q'), \alpha(\mu_r), x');$$

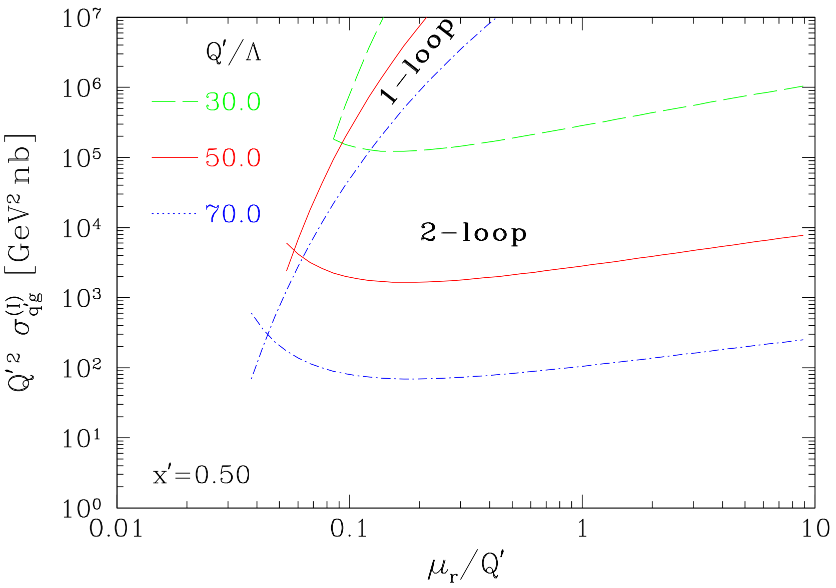
– "Fiducial" kinematical region from lattice constraints:

$$\left.\begin{array}{lll} \rho & \lesssim & 0.3\ \mathrm{fm}; \\ \frac{R}{\rho} & \gtrsim & 1 \end{array}\right\} \Rightarrow \left\{\begin{array}{lllll} Q' & \gtrsim & Q'_{\mathrm{min}} & \simeq & 8\ \mathrm{GeV}; \\ x' & \gtrsim & x'(Q'_{\mathrm{min}}) & \simeq & 0.35. \end{array}\right.$$

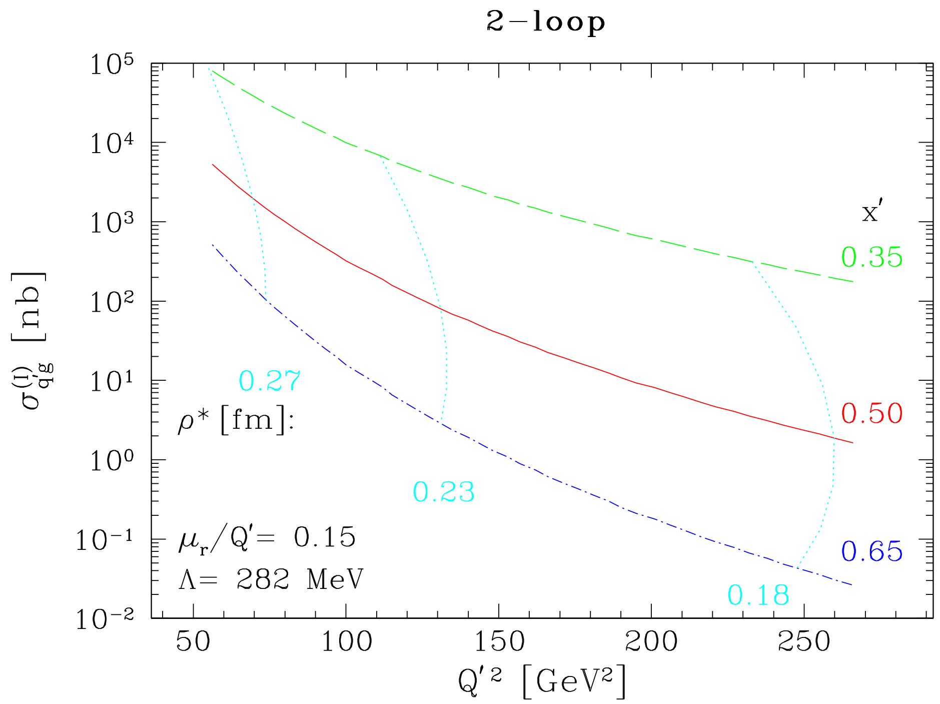


3. Controlling the Residual Renormalization Scale Dependence in $\sigma^{(I)}_{\mathrm{HERA}}$

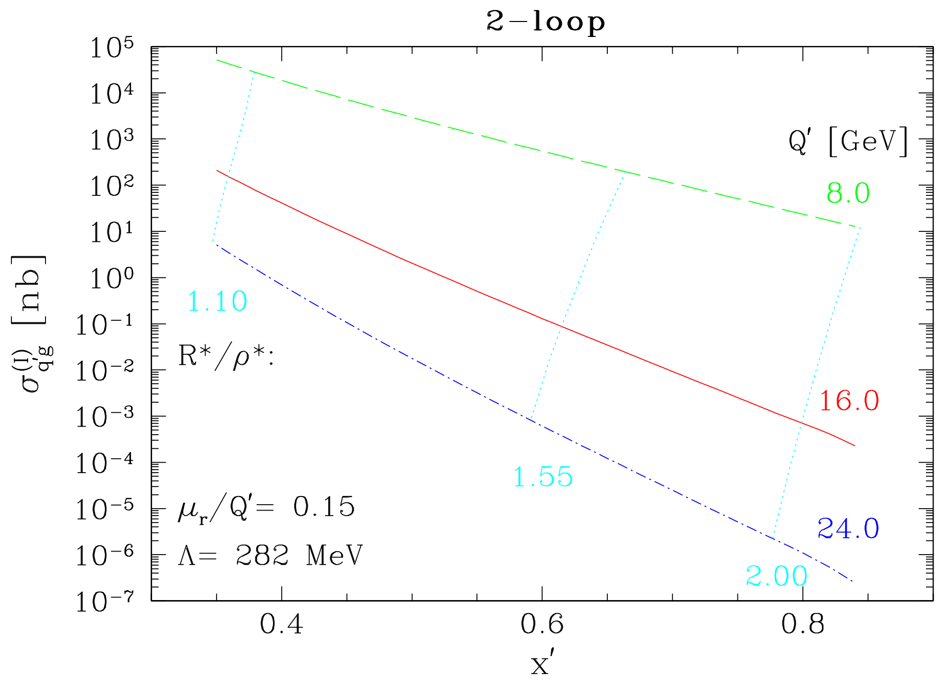
- Great progress by using **2-loop RG invariant form** of I-density $D(\rho)$ in calculation of I-subprocess cross sections: Residual dependence on renormalization scale $\boldsymbol{\mu_r}$ **strongly reduced**! (Fig.)



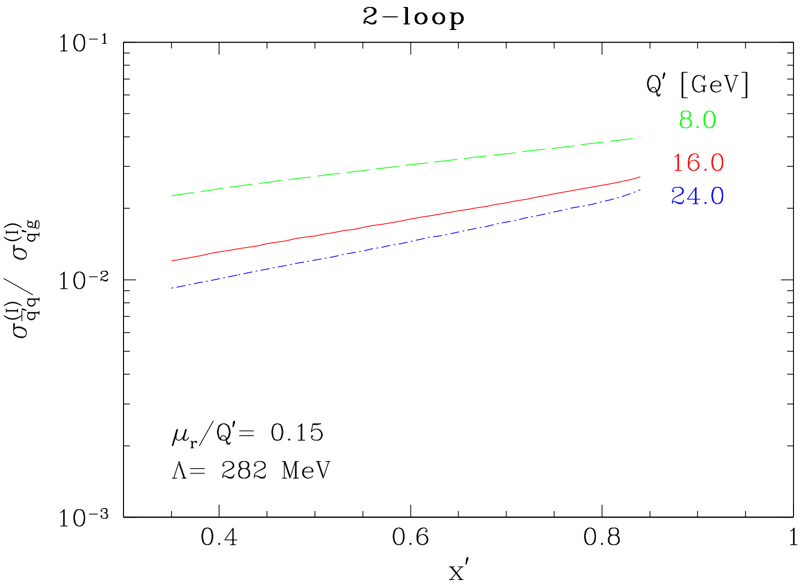
- Use as "best scheme": $\mu_r = 0.15\ Q'$ throughout, for which $\partial\sigma^{(I)}_{q'g}/\partial\mu_r \simeq 0$.



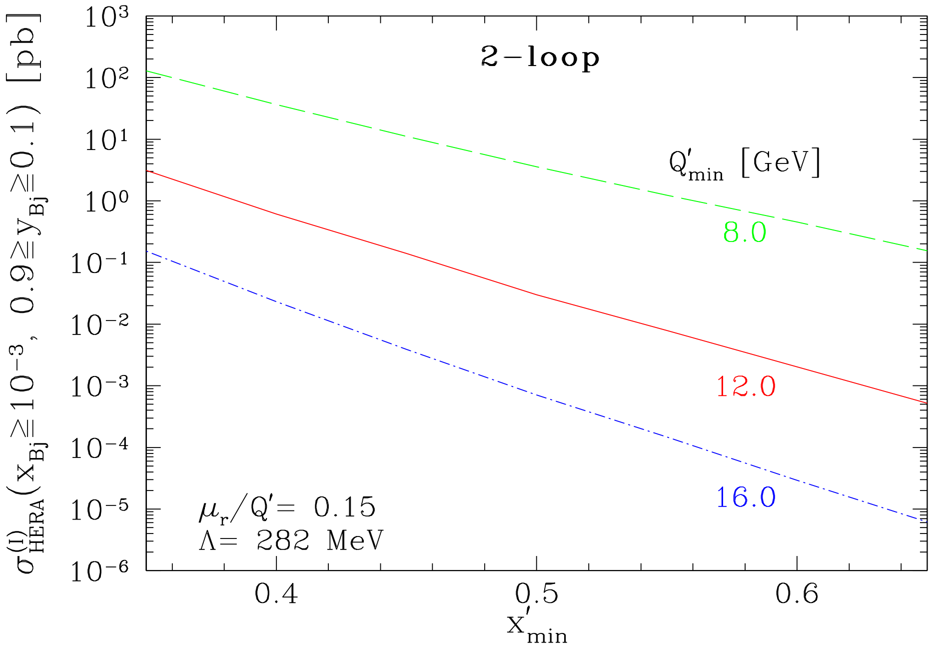
- Q' dependence of $\sigma^{(I)}_{q'g}$ **probes Instanton size ρ.**



- x' dependence of $\sigma^{(I)}_{q'g}$ **probes $I\bar{I}$ distance R in units of the size ρ.**



- At small $x_{\rm Bj}$, the I-induced contribution from quarks in the proton may be neglected as compared to the gluons.



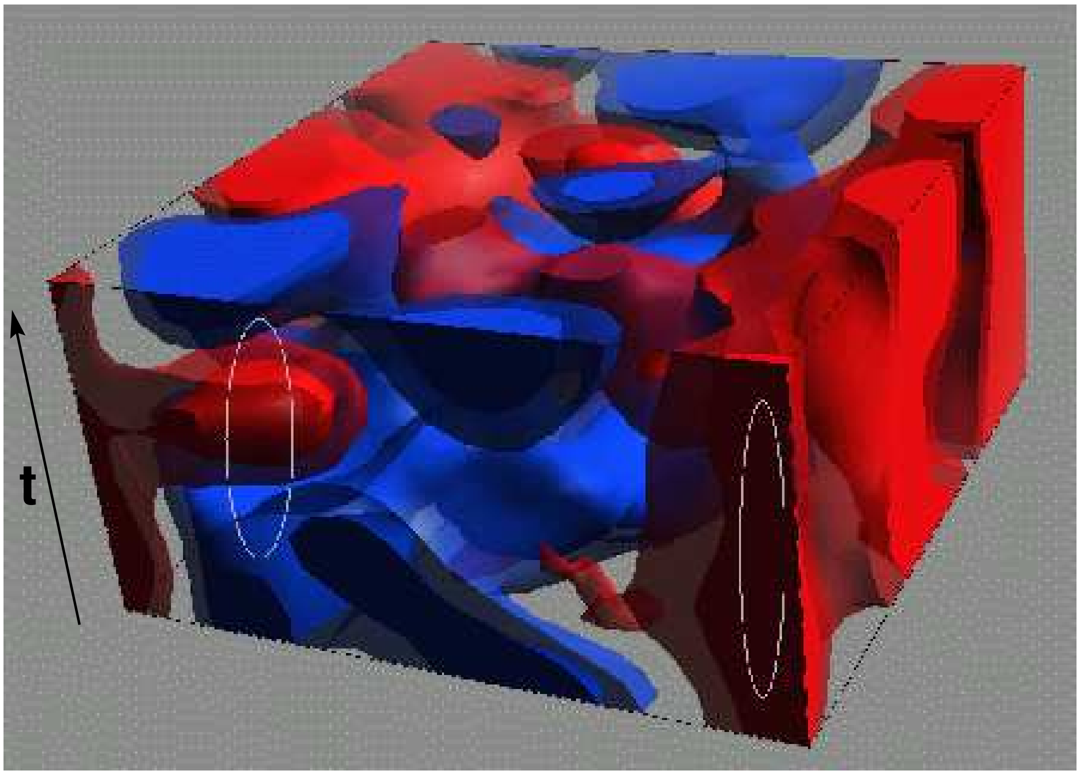
- The finalized I-induced cross-section at **HERA**, as a function of $x'_{\rm min}$ and $Q'_{\rm min}$ in leading semi-classical approximation as obtained with the new release **QCDINS 1.5.2**!

4. Conclusions

- Topology, an important aspect of non-abelian gauge theories like QCD, with Instantons being the simplest non-trivial "building blocks"!

  Effects on hadron spectroscopy ($\eta - \eta'$ mass difference...), presumably on chiral-symmetry breaking, etc...

Isosurfaces of topological charge density q(x)



Forcrand et al. '97

- Much recent activity in the "lattice-community" to study the topological features i.e. Instantons of the QCD ground state ("vacuum").
  [UKQCD, Smith & Teper '98, Forcrand *et al.* '97, DeGrand *et al.* '97, Michael & Spencer '95, Chu *et al.* '94. . .]
- Tried to link the lattice results with I-induced processes at **HERA** $\Rightarrow$

  "Fiducial" region in (Q', x')!

- At **HERA**: Filter out Instantons too big or too close to each other for I-perturbation theory to apply, by restricting $\boxed{Q' \gtrsim 8\ \mathrm{GeV}, x' \gtrsim 0.35}$.

- Much improved new calculation of I-induced $\sigma^{(I)}_{\mathrm{HERA}}$, with only weak residual dependence on renormalization scale μ_r remaining!

- New release of **QCDINS MC: 1.5.2**, ready to be used for further experimental searches for **Instantons** at **HERA**!