QCD-Instantons in $e^{\pm}P$ Scattering

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in collaboration with

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1. Introduction

- Instantons [Belavin *et al.* '75]:

  – Tunnelling transitions in YM theories, connecting degenerate vacua of different topology;

  – induce processes forbidden in perturbation theory, yet bound to exist due to Adler-Bell-Jackiw anomalies of certain fermionic quantum numbers ['t Hooft '76]:

  $B + L$ in QFD; Chirality (Q_5) in (massless) QCD.

  – suppressed at low energies, $\sim \exp\{-4\pi/\alpha\}$,

♣ may become **unsuppressed, i. e. observable** at high energies via production of additional gauge bosons $A_\mu^I \sim 1/g$ [Ringwald '90; Espinosa '90]

♣ An experimental discovery of such a novel, non-perturbative manifestation of non-abelian gauge theories would clearly be of basic significance.

Distinguished rôle of $e^{\pm}P$ scattering involving hard momentum scale $\mathcal{Q}$ $\left\{ \begin{array}{l} \text{deep} - \text{inelastic (DIS)} \\ \text{hard photoproduction?} \end{array} \right.$

♣ Experimentally accessible: $\Rightarrow$ **HERA**

- QCD-instantons less suppressed than QFD-instantons ($\alpha_s \gg \alpha_W$).
- Unique window to detect QCD-instanton induced processes through their characteristic multi-particle final-state signature [Ringwald & F. Sch. '94 '96; Gibbs, Ringwald & F. Sch. '95; Gibbs, Greenshaw, Milstead, Ringwald & F. Sch. '96] .
- c.f. instanton talks in WG III $\left\{ \begin{array}{ll} \text{A. Ringwald} & \text{(phenomenology)} \\ \text{T. Carli (H1)} & \text{(exp. searches at HERA)} \end{array} \right.$

♣ Theoretically:

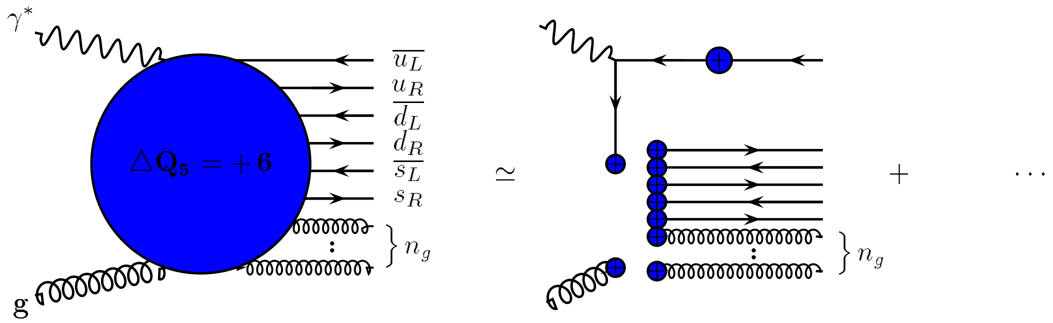
- Possible to make contact with experiment! [Balitsky & Braun '93; Ringwald & F. Sch '94 '96; Moch, Ringwald & F. Sch. '96 and in prog.]
- Absence of generic IR divergencies from integration over I-size ρ, since hard scale $\mathcal{Q}$ provides dynamical cutoff, $\rho \lesssim \mathcal{O}(1/\mathcal{Q})$, .
- Instanton-induced amplitudes well-defined and calculable for small $\alpha_s(\mathcal{Q})$ and fixed scattering angles.
- Inclusive framework to systematically calculate properties of I-induced multi-parton final state; accounts for exponentiation of produced gluons including final-state tree-graph corrections

2. Instanton-Induced Processes in Leading Semi-Classical Approximation

- Consider Instanton-induced amplitude for $\triangle Q_5 = 2\,n_f$ process,

$$\gamma^* + \mathrm{g} \Rightarrow \sum_{\text{flavours}}^{n_f} [\overline{\mathrm{q}_L} + \mathrm{q}_R] + n_{\mathrm{g}}\,\mathrm{g} \;:$$

vanishing to any order of conventional P.T in (massless) QCD:



- Expand (Euclidean) path integral for relevant Green's functions about classical instanton solution.
- Fourier-transform (F. T.) with respect to external lines.
- LSZ amputate external legs.
- Analytically continue to Minkowski space.

- Basic building blocks: $\left(\Pi_x \equiv 1 + \frac{\rho^2}{x^2}\right)$,

  – Classical instanton gauge field (singular gauge) [Belavin *et al.* '75] :

$$A^{(I)}_{\mu'}(x) \quad = \quad -\mathrm{i}\,\frac{2\,\pi^2}{g_s}\,\rho^2\,U\left(\frac{\sigma_{\mu'}\,\overline{x} - x_{\mu'}}{2\,\pi^2\,x^4}\right)U^\dagger\,\frac{1}{\Pi_x}\,,$$

collective coordinates: $\rho = I$-size, $\quad (U)^k_\alpha = I$-color orientation

(color: $k = 1, 2, 3$ and spinor: $\alpha = 1, 2$ indices).

– Quark zero modes in the instanton background
(Weyl basis)[1] ['t Hooft '76] :

$$\kappa^{(I)m}{}_{\dot\beta}\,(x) \;=\; 2\,\pi\,\rho^{3/2}\,\epsilon^{\gamma\delta}\,(U)^{m}{}_{\delta}\,\frac{\overline{x}_{\dot\beta\gamma}}{2\,\pi^2\,x^4}\,\frac{1}{\Pi_x^{3/2}}\,,$$

$$\overline{\phi}^{(I)\dot\alpha}{}_{l}\,(x) \;=\; 2\,\pi\,\rho^{3/2}\,\epsilon_{\gamma\delta}\,\left(U^\dagger\right)^{\gamma}{}_{l}\,\frac{x^{\delta\dot\alpha}}{2\,\pi^2\,x^4}\,\frac{1}{\Pi_x^{3/2}}\,,$$

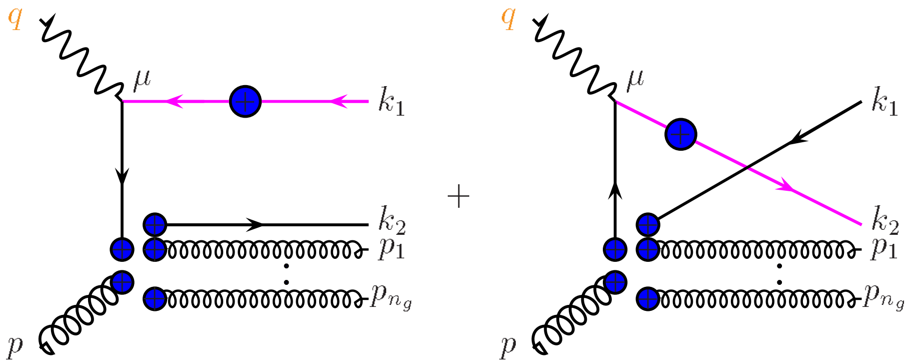
– (Non-zero mode) quark propagators in the instanton background
[Brown *et al.* '78] :

$$S^{(I)}(x,y) = \frac{1}{\sqrt{\Pi_x\Pi_y}}\left[\frac{x-y}{2\pi^2(x-y)^4}\left(1+\rho^2\frac{\left[U\,x\,\overline{y}\,U^\dagger\right]}{x^2y^2}\right)+\frac{\rho^2\,\sigma_\mu}{4\pi^2}\frac{\left[U\,x\,(\overline{x}-\overline{y})\,\sigma_\mu\,\overline{y}\,U^\dagger\right]}{x^2(x-y)^2y^4\Pi_y}\right],$$

$$\overline{S}^{(I)}(x,y) = \cdots$$

- Leading-order amplitude $(n_f = 1)$ in Euclidean space after F. T. with respect to external lines [Moch, Ringwald & F. Sch. '96] :

[1] Standard notations: in Euclidean space, $\sigma_\mu = (-\mathrm{i}\,\vec\sigma, 1)$, $\overline{\sigma}_\mu = (\mathrm{i}\,\vec\sigma, 1)$, and in Minkowski space, $\sigma_\mu = (1, \vec\sigma)$, $\overline{\sigma}_\mu = (1, -\vec\sigma)$, where $\vec\sigma$ are the Pauli matrices. Furthermore, for any four-vector v_μ, we use the shorthand $v \equiv v\cdot\sigma$, $\overline{v} \equiv v\cdot\overline{\sigma}$.



$$\mathcal{T}_{\mu}^{(I)\,a\,a_1\ldots a_{n_g}} = -\mathrm{i}\,e_q\,d\,\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right]$$

$$\int dU \int_0^\infty \frac{d\rho}{\rho^5}\,(\rho\,\mu_r)^{\beta_0} \lim_{p^2\to 0} p^2\,\mathrm{tr}\left[\lambda^a\,\epsilon_g(p)\cdot A^{(I)}(p)\right]$$

$$\prod_{i=1}^{n_g} \lim_{p_i^2\to 0} p_i^2\,\mathrm{tr}\left[\lambda^{a_i}\,\epsilon_g^*(p_i)\cdot A^{(I)}(-p_i)\right]\;\times$$

$$\chi_R^\dagger(k_2)\left[\lim_{k_2^2\to 0}\,(\mathrm{i}k\!\!\!/_2)\,\kappa^{(I)}(-k_2)\lim_{k_1^2\to 0}\,\mathcal{V}_\mu(q,-k_1)\right.$$

$$\left.+\lim_{k_2^2\to 0}\,\mathcal{V}_\mu^c(q,-k_2)\lim_{k_1^2\to 0}\,\overline{\phi}^{(I)}(-k_1)\,(-\mathrm{i}\,\overline{k\!\!\!/}_1)\right]\chi_L(k_1)\,,$$

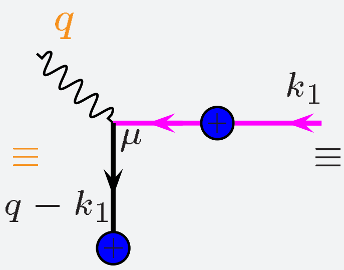
- Instanton-density with renormalization scale μ_r ['t Hooft '76; Bernard '79],

$$d\,\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right]\,(\rho\,\mu_r)^{\beta_0}\;;\;\text{d known constant}$$

  - is 1-loop RG-invariant; in practice:$\Rightarrow$ 2-loop RG-invariance [Morris, Ross, Sachrajda '85] .
  - strongly IR-divergent if integrated over ρ

- F.T. and LSZ-amputation of instanton gauge field $A_{\mu'}^{(I)}$ and quark zero modes κ and $\overline{\phi}$ is straightforward [SVZ '80; Ringwald '90; Espinosa '90],

- LSZ-amputation of current quark in F.T'ed photon-fermion vertices, $\mathcal{V}_\mu(q,-k_1)$, $\mathcal{V}^c_\mu(q,-k_2)$, e.g.

$\mathcal{V}_\mu(q,-k_1) \equiv$ 

$$\equiv \int d^4x\, \mathrm{e}^{-\mathrm{i}q\cdot x}\left[\overline{\phi}^{(I)}(x)\overline{\sigma}_\mu S^{(I)}(x,-k_1)(-\mathrm{i}\overline{k}_1)\right]$$

quite non-trivial and has important physical consequences.

- After a long and tedious calculation . . .

$$\lim_{k_1^2\to 0} \mathcal{V}_\mu(q,-k_1) = 2\pi\mathrm{i}\rho^{3/2}\left[\epsilon\sigma_\mu\overline{V}(q,k_1;\rho)\,U^\dagger\right],$$

$$\lim_{k_2^2\to 0} \mathcal{V}^c_\mu(q,-k_2) = 2\pi\mathrm{i}\rho^{3/2}\left[U\,V(q,k_2;\rho)\overline{\sigma}_\mu\epsilon\right],$$

where

$$V(q,k;\rho) = \frac{k}{2q\cdot k}\rho\sqrt{q^2}\,K_1\left(\rho\sqrt{q^2}\right) + \left[\frac{(q-k)}{(q-k)^2} - \frac{k}{2q\cdot k}\right]\rho\sqrt{(q-k)^2}\,K_1\left(\rho\sqrt{(q-k)^2}\right).$$

- The integration over the instanton size ρ in $\mathcal{T}^{(I)}_\mu$ is finite due to the exponential decrease of the form factors in $V(q,k;\rho)$,

$$\mathcal{Q}\rho K_1(\mathcal{Q}\rho) \overset{\mathcal{Q}\rho\to\infty}{\to} \sqrt{\frac{\pi}{2}}\sqrt{\mathcal{Q}\rho}\,\exp\left[-\mathcal{Q}\rho\right].$$

- After continuation to DIS-regime of Minkowski space: Hard scale,

$$\mathcal{Q} \equiv \min\left\{Q \equiv \sqrt{-q^2},\, \sqrt{-(q-k_1)^2},\, \sqrt{-(q-k_2)^2}\right\} \geq 0,$$

provides a dynamical IR cutoff for the instanton size,

$$\rho \lesssim \mathcal{O}(1/\mathcal{Q}).$$

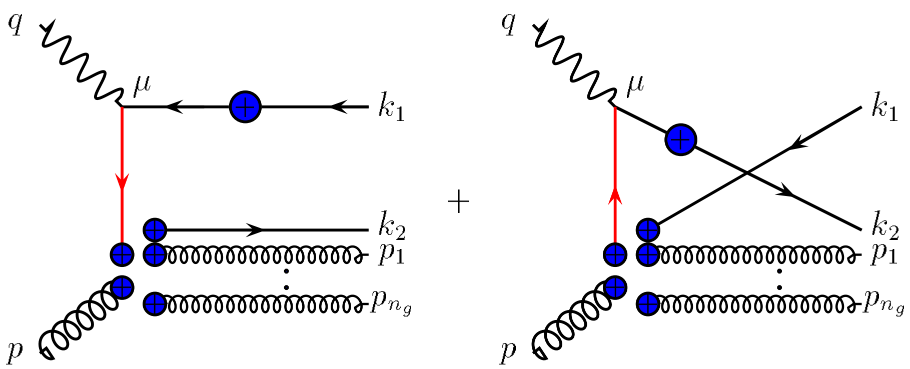
Therefore, deep-inelastic scattering very well suited for studying manifestations of QCD-instantons.

- Next steps:
  - Insert all LSZ-amputated F.T.s. into $\mathcal{T}_\mu^{(I)}$.
  - Continue to Minkowski space.
  - Perform integration over ρ analytically $\Rightarrow$ finite result
- Highly non-trivial check: Electromagnetic current conservation,

$$q^\mu \mathcal{T}_\mu^{(I)\,a\,a_1\ldots a_{n_g}} = 0.$$

- Like in perturbative QCD:

  Leading-order I-induced amplitudes well-behaved as long as we avoid the collinear singularities, arising when the internal quark virtualities vanish, $t \equiv -(q-k_1)^2$ and/or $u \equiv -(q-k_2)^2 \to 0$.



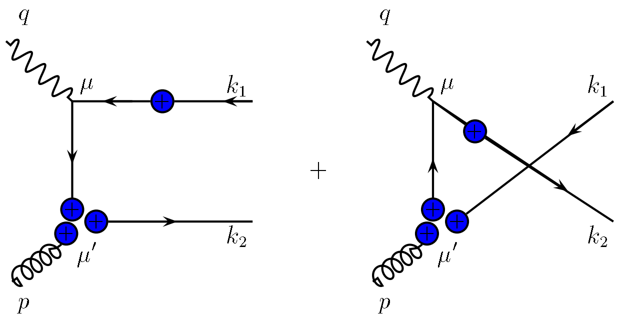
Fixed-angle scattering processes at high Q^2 and moderate multiplicity are reliably calculable in (instanton) perturbation theory.

Example: The Simplest Process ($n_f = 1, n_g = 0$)

[Moch, Ringwald & F. Sch. '96]

$$\gamma^* + \mathrm{g} \to \overline{\mathrm{q}_L} + \mathrm{q}_R\ ; \qquad (\triangle Q_5 = 2)$$

,



May be considered as lower bound on total instanton-induced contribution. Contains all essential features of dominant multi-gluon process.

$$\frac{d\sigma^{(I)}}{dt} = \pi^2\ \frac{x(1-x)}{Q^4}\frac{\alpha e_q^2}{2}\mathcal{N}^2\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^{13}\exp\left[-\frac{4\,\pi}{\alpha_s(\mu_r)}\right]\left(\frac{\mu_r^2}{Q^2}\right)^b$$

$$\left[\left(\frac{Q^2}{-t}\right)^{b+1} + \left(\frac{Q^2}{-u}\right)^{b+1} + 2tu\frac{\left(\left(\frac{Q^2}{-t}\right)^{\frac{b+1}{2}} - 1\right)\left(\left(\frac{Q^2}{-u}\right)^{\frac{b+1}{2}} - 1\right)}{(t+Q^2)(u+Q^2)}\right],$$

$$\mathcal{N} \equiv \sqrt{\frac{2}{3}}\,\pi^2\,d\,2^b\,\Gamma\left(\frac{b+1}{2}\right)\Gamma\left(\frac{b+3}{2}\right)\ ; \qquad x = \frac{Q^2}{2p\cdot q}$$

$$b \equiv \beta_0 + \frac{\alpha_s(\mu_r)}{4\,\pi}\,(\beta_1 - 12\,\beta_0)$$ $\Leftrightarrow$ 2-loop RG improvement of I-density.

- Dependence on renormalization scale μ_r very weak!

- Validity of approximation:

  Average instanton size $\langle\rho\rangle$ contributing for a given virtuality $\mathcal{Q}$,

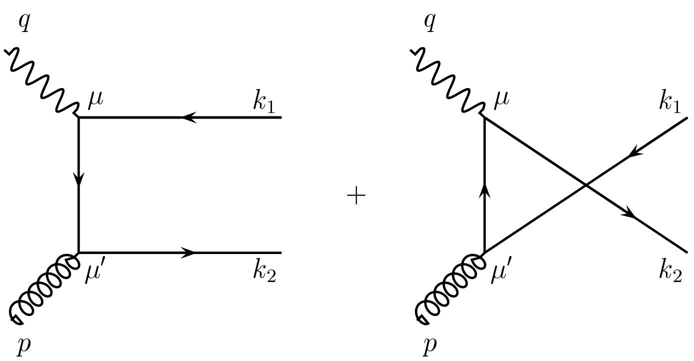
$$\langle\rho\rangle \equiv \frac{\int\limits_0^\infty d\rho\, \rho\, \rho^{\beta_0+1}\, K_1(\rho\, \mathcal{Q})}{\int\limits_0^\infty d\rho\, \rho^{\beta_0+1}\, K_1(\rho\, \mathcal{Q})} \simeq \frac{\beta_0 + 3/2}{\mathcal{Q}}\,.$$

  Should be small, in order that higher-order corrections of I-pert. theory can be neglected;
  [Callan et.al '78, Andrei & Gross '78, Appelquist & Shankar '78, SVZ '80]

$$\langle\rho\rangle \lesssim \frac{1}{500\ \mathrm{MeV}} < \frac{1}{\Lambda} \quad \Rightarrow \quad \mathcal{Q} \gtrsim 5\ \mathrm{GeV}.$$

- Compare with the appropriate leading-order perturbative QCD, chirality-conserving process,

$$\gamma^* + \mathrm{g} \to \overline{\mathrm{q}_L} + \mathrm{q}_L\ ; \qquad (\triangle Q_5 = 0)\ .$$

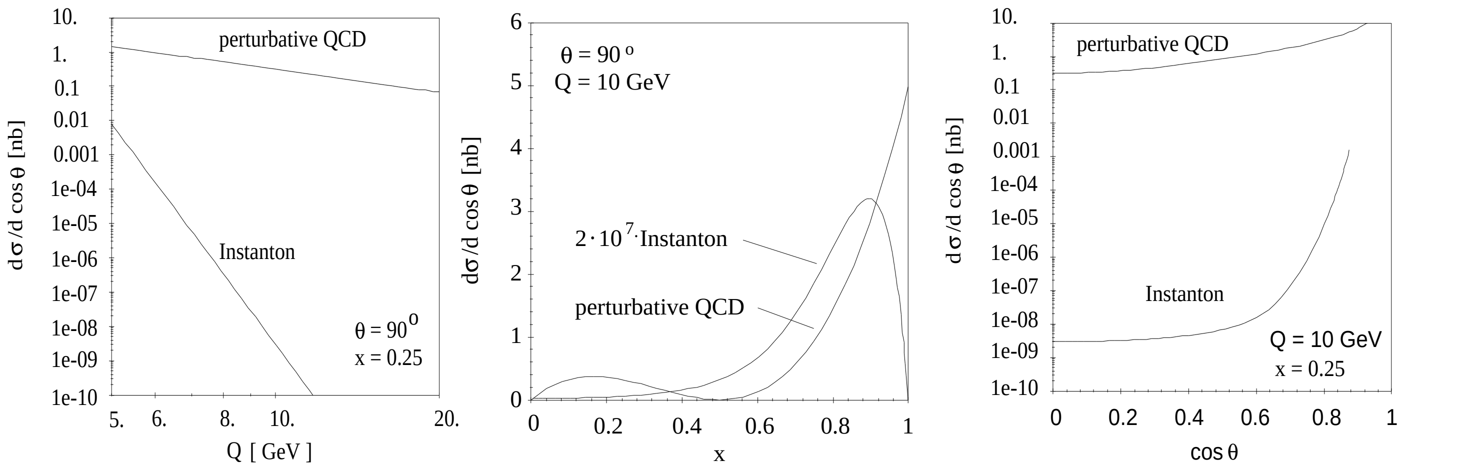


- Compare fixed-angle differential cross-sections,

$$\frac{d\sigma}{d\cos\theta}(Q, x, \cos\theta)\,, \qquad \text{with scattering angle } \theta \text{ in } \gamma^* g \text{ c.m.s.}$$

Parameters: $e_q = 2/3$, $\Lambda = 234$ MeV, $\mu_r = Q$.

- Result:



- Fixed-angle scattering at large enough Q^2 well under control (left and middle Fig.).

- Collinear singularity ($\cos\theta \to 1$) for instanton case much stronger than perturbative one (right Fig.).

- Further important application: [Moch, Ringwald & F. Sch., in prog.]

  Study hard photoproduction limit, with large γ^* virtuality Q^2 substituted by large k_T of produced q-jet,

$$Q^2 \to 0, x \to 0, \frac{Q^2}{x} \to s \ (\text{large}), k_T = \frac{\sqrt{s}}{2} \sin\theta \text{ large}$$

3. Inclusive Approach to Multi-Particle Final State

- Main task: Best theoretical framework for making contact with Experiment (HERA) without upsetting validity of I-P.T.?

- Best bet experimentally: hunting for I- "footprints" in multi-particle final state [Ringwald & F. Sch. '94 '96].

- Momentum space picture $\Rightarrow$ Control over I-approximations also at small $\{x_{Bj}, Q^2\}$ through kinematical cuts on (reconstructed) final-state momentum variables!

I-induced cross sections for multi-parton final state?

a) Exclusive, "brute-force" method: Multi-parton cross-sections by squaring $\gamma^* + \mathrm{g} \Rightarrow \sum_{\text{flavours}}^{n_f} [\overline{\mathrm{q}_L} + \mathrm{q}_R] + n_\mathrm{g}\, \mathrm{g}$ amplitudes from Sect.2 and summing over unobserved partons.

b) Prefer inclusive approach: Based on [Ringwald & F. Sch. '94 '96, Moch, Ringwald & F. Sch. in prog.]

Systematic characterization of final state in terms of $\sigma_{\text{tot}}^{(I)}$ and 1,2,... parton inclusive cross sections.

- Elegant, implicit summation over exponentiating leading order gluon emission and gluonic final-state tree-graph corrections.

optical theorem: $\sigma^{(I)}_{\rm tot} \quad \Leftrightarrow \quad {\rm Disc}\{T^{(I)}_{2\to 2}\}$

Mueller- optical "theorems" [A.H. Mueller '70]:

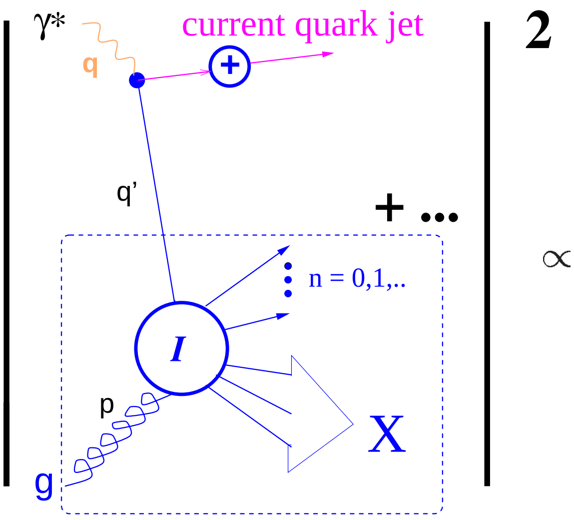
n=1,2. . . particle inclusive cross sections $\quad \Leftrightarrow \quad {\rm Disc}\{T^{(I)}_{2+n\to 2+n}\}$

⊕ $I\bar{I}$-valley method [Yung '88, Khoze & Ringwald '91, . . .]

⊕ saddle-point integration over collective $I\bar{I}$ coordinates.

- Reduction to I-induced subprocess via

$d\sigma^{\rm incl.\ (I)}_{\gamma^* g \to \{n+1\} + X} \sim$



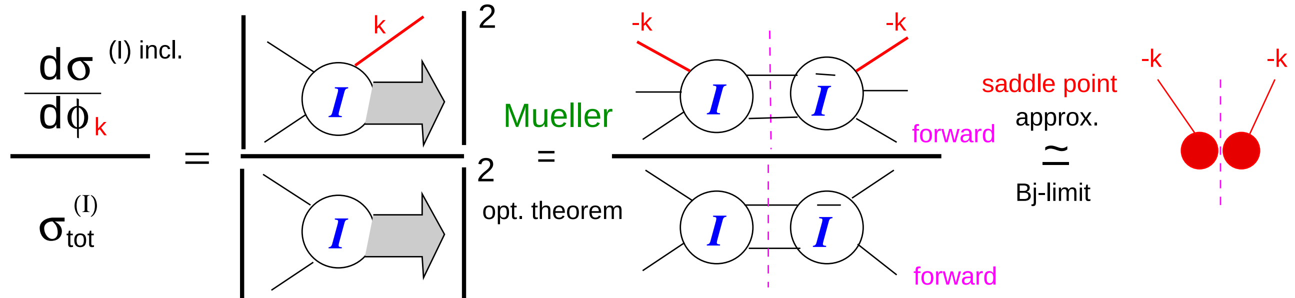
$$\propto \frac{\alpha}{Q^2}\int_x^1 \frac{dx'}{x'}\int^{Q^2\frac{x'}{x}} dQ'^2 P_{q^*/\gamma^*}\, d\sigma^{{\rm incl.}(I)}(q^* g \to \{n\} + X)$$

Instanton-induced subprocess

Bjorken variables: x', $Q'^2 = -q'^2$

- Examples:

  – 1-parton inclusive $(n = 1)$:

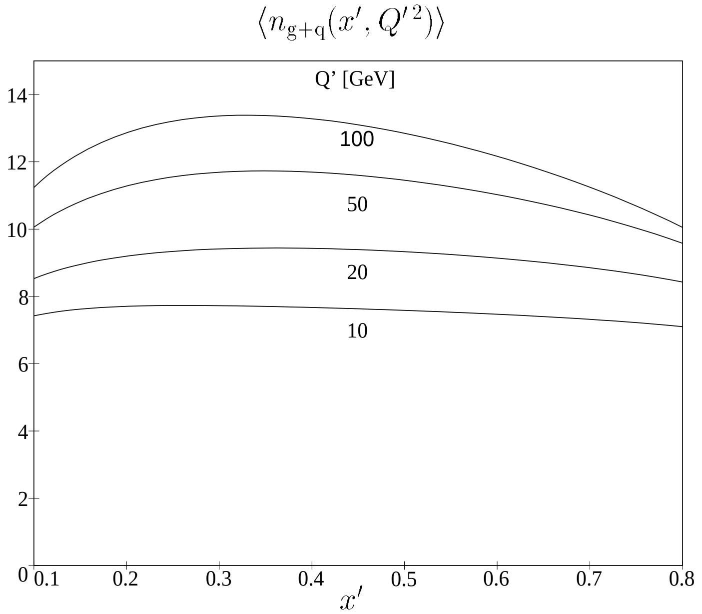


$I\bar{I}$ valley method

Product of amp'ed, FT'ed fields in bg. of $I\bar{I}$ valley configuration

Output:

Isotropy $\qquad \frac{d\langle E_T\rangle}{d\eta_I} = \frac{\sqrt{s'}}{\langle n\rangle}\frac{1}{\cosh\,\eta_I} \qquad \boldsymbol{\langle n_{\mathrm{q+G}}(x', Q'^2)\rangle}$



– 2-parton inclusive $(n = 2)$:
  * $\langle n^2\rangle - \langle n\rangle^2 \Rightarrow \lim_{Q'^2\to\infty} \sigma_n{}^{(I)\ \mathrm{excl.}} =$ **Poisson**
  * momentum correlations, ...

- Highly non-trivial consistency conditions
  [De Tar, Freedman & Veneziano '71; L. Brown '71]

energy-momentum/charge/... sum rules

e.g.

$$\sum_{\mathrm{q,G}} \int d\phi_k\ k_\mu\ \frac{1}{\sigma^{(I)}_{\mathrm{tot}}}\frac{d\sigma^{(I)}}{d\phi_k} = (p+q')_\mu$$

– $\sim$ o.k. in Bjorken limit ($Q'^2 \to \infty$, x' fixed)

Quite stable/reliable results on relative rates $(1/\sigma^{(I)}_{\mathrm{tot}})d\sigma^{\mathrm{incl.(I)}}$, as functions of η, ϕ, E_T, since poorly known factors cancel in ratios!

4. Conclusions

- Distinguished rôle of $e^{\pm}P$ scattering involving hard momentum scale $\mathcal{Q}$.

- Experimentally:

  HERA offers unique window in DIS (and hard photoproduction?) to detect I-induced events through their characteristic multi-particle final-state signature.

- Theoretically:

  - Absence of IR divergencies associated with integration over I-size ρ.
  - Chirality-violating Amplitudes well-defined and calculable for small $\alpha_s(\mathcal{Q})$ and fixed scattering angles.
  - Inclusive framework to systematically calculate properties of I-induced multi-parton final state;

    accounts for exponentiation of produced gluons including final-state tree-graph corrections:

    * Optical theorem and Mueller-optical theorems, to express various I-induced inclusive cross sections as discontinuities of elastic n $\rightarrow$ n forward amplitudes $T^{(I)}_{n\rightarrow n}$
    * Uniform evaluation by means of $I\bar{I}$ valley method and saddle-point integration over collective coordinates.
    * Stable results, consistent with stringent energy-momentum sum rules.