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# QCD instantons at HERA—an introduction

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**Abstract.** We review our ongoing theoretical and phenomenological study of the discovery potential for instanton-induced deep-inelastic scattering (DIS) events at HERA. Constraints from recent lattice simulations will be exploited and translated into a 'fiducial' kinematical region for our predictions of the instanton-induced DIS cross section.

## 1. Instantons

Non-Abelian gauge theories like QCD are known to exhibit a rich vacuum structure. The latter includes *topologically* non-trivial fluctuations of the gauge fields, carrying an integer topological charge $Q$. The simplest building blocks of topological structure are instantons ($Q = +1$) and anti-instantons ($Q = -1$) which are well known explicit solutions of the Euclidean field equations in four dimensions [1].

Instantons (I) are widely believed to play an important role in various *long-distance* aspects [2] of QCD.

First of all, they may provide a solution of the famous $U_A(1)$ problem [3] ($m_{\eta'} \gg m_\eta$), with the corresponding pseudoscalar mass splitting related to the topological susceptibility in the pure gauge theory by the well known Witten–Veneziano formula [4]. Moreover, a number of authors have attributed a strong connection of instantons with chiral symmetry breaking [5, 2] as well as the hadron and glueball spectrum.

However, there are also very important *short-distance* implications [6–10] of QCD instantons to which this paper is devoted.

instantons are known to induce certain processes which violate *chirality* in accordance with the general axial-anomaly relation [3] and which are forbidden in conventional perturbation theory. Of particular interest in this context is the *deep inelastic scattering* (DIS) regime. Here, hard I-induced processes may both be *calculated* [8–10] within *I-perturbation theory* and possibly be *detected experimentally* [7, 11–13]. As a key feature it has recently been shown [8], that in DIS the generic hard scale $\mathcal{Q}$ cuts off instantons with *large size* $\rho \gg \mathcal{Q}^{-1}$, over which one has no control theoretically.

Our finalized results [9, 10] for inclusive I-induced DIS cross sections are summarized in sections 2 and 4. Their weak residual renormalization-scale dependence is quite remarkable.

As a second main point of this review (section 3), constraints from recent lattice simulations will be exploited [9, 14] and translated into a 'fiducial' kinematical region for our predictions of the I-induced DIS cross section based on I-perturbation theory. In section 5 we discuss the expected event signature and search strategies based on our Monte Carlo generator [11] QCDINS 1.60. Finally (section 6), we briefly address an interesting class of 'fireball' events, observed in photoproduction, in the context of instantons and put forward a promising

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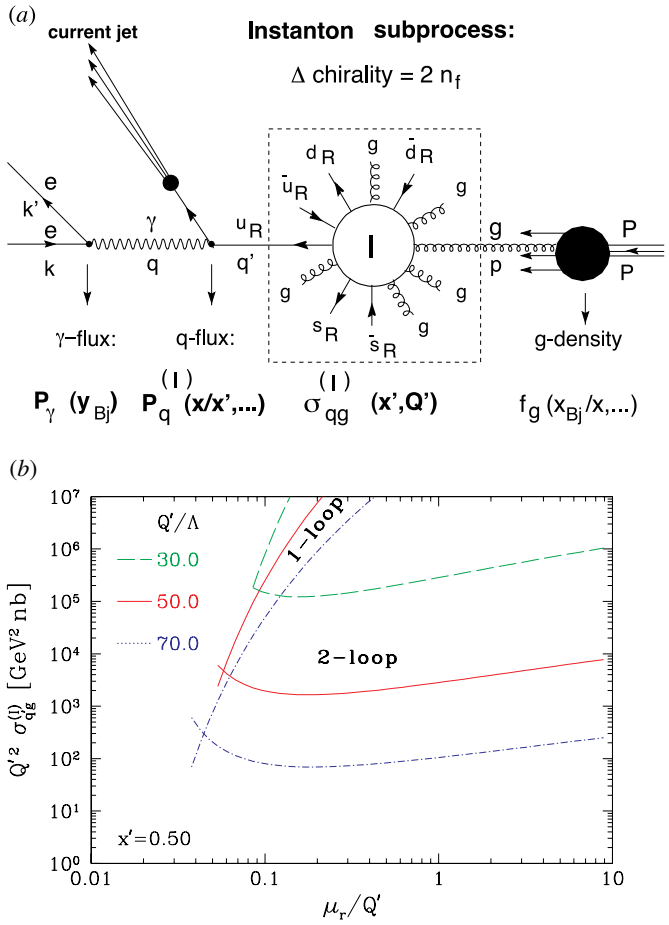


**Figure 1.** (*a*) The leading I-induced process in the DIS regime of $e^{\pm}P$ scattering ($n_f = 3$). (*b*) Illustration of the weak residual renormalization-scale ($\mu_r$) dependence of the resulting I-subprocess cross section $\sigma_{q\,g}^{(I)}(Q', x')$.

proposal [14] on extending our theoretical predictions beyond the regime of strict I-perturbation theory.

## 2. DIS cross sections in I-perturbation theory

In I-perturbation theory one expands the relevant Green functions about the known, classical instanton solution $A_\mu = A_\mu^{(I)} + \cdots$ instead of the usual (trivial) field configuration $A_\mu^{(0)} = 0$ and obtains a corresponding set of modified Feynman rules. Like in conventional pQCD, the gauge coupling $\alpha_s$ has to be small.

The leading I-induced process in the DIS regime of $e^{\pm}P$ scattering for large photon virtuality $Q^2$ is illustrated in figure 1(*a*). The dashed box emphasizes the so-called I-*subprocess* with its own Bjorken variables,

$$Q'^2 = -q'^2 \geqslant 0; \qquad x' = \frac{Q'^2}{2p \cdot q'} \leqslant 1. \tag{1}$$

It induces a total chirality violation $\Delta$ chirality $= 2n_f$, in accord with the corresponding axial anomaly [3]. In the Bjorken limit of I-perturbation theory, the dominant I-induced contribution to the inclusive HERA cross section may be shown to take the form [9, 10]

$$\frac{\mathrm{d}\sigma^{(I)}_{\mathrm{HERA}}}{\mathrm{d}x'\,\mathrm{d}Q'^2} \simeq \frac{\mathrm{d}\mathcal{L}^{(I)}_{q\,g}}{\mathrm{d}x'\,\mathrm{d}Q'^2} \cdot \sigma^{(I)}_{q\,g}(Q', x'). \tag{2}$$

The differential luminosity, $\mathrm{d}\mathcal{L}^{(I)}_{q\,g}$, accounting for the number of qg collisions per eP collision, has a convolution-like structure. It involves integrations over the gluon density, the $\gamma$-flux $P_\gamma$ and the known q-flux $P^{(I)}_q$ in the I-background (cf figure 1($a$)). The crucial I-dynamics resides in the I-subprocess total cross section $\sigma^{(I)}_{q\,g}(Q', x')$, on which we focus our attention next [9, 10].

Being an observable, $\sigma^{(I)}_{q\,g}(Q', x')$ involves integrations over all I($\bar{\mathrm{I}}$)-'collective coordinates', including the I ($\bar{\mathrm{I}}$)-sizes $\rho(\bar\rho)$ and the I$\bar{\mathrm{I}}$-distance† four-vector $R_\mu$,

$$\sigma^{(I)}_{q\,g} = \int_0^\infty \mathrm{d}\rho\, D(\rho) \int_0^\infty \mathrm{d}\bar\rho\, D(\bar\rho) \int \mathrm{d}^4R\, \{\ldots\} \mathrm{e}^{-Q'(\rho+\bar\rho)} \mathrm{e}^{\mathrm{i}\,(p+q')\cdot R} \mathrm{e}^{-\frac{4\pi}{\alpha_s}\Omega\left(\frac{R^2}{\rho\bar\rho}, \frac{\bar\rho}{\rho}\right)}. \tag{3}$$

The $\rho(\bar\rho)$-integrals in (3) involve as generic weight the I($\bar{\mathrm{I}}$)-density $D(\rho(\bar\rho))$ [3, 15, 16],

$$\begin{aligned} D(\rho) &= \frac{d}{\rho^5}\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{2N_c} \exp\left(-\frac{2\pi}{\alpha_s(\mu_r)}\right)(\rho\mu_r)^{\beta_0+\frac{\alpha_s(\mu_r)}{4\pi}(\beta_1-4N_c\beta_0)} \\ d &= \frac{2\mathrm{e}^{5/6}}{\pi^2(N_c-1)!(N_c-2)!}\mathrm{e}^{-1.511\,37N_c+0.291\,75n_f} \quad (\overline{\mathrm{MS}}\text{ scheme}); \\ \beta_0 &= \frac{11}{3}N_c - \frac{2}{3}n_f; \qquad \beta_1 = \frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{N_c}\right)n_f, \end{aligned} \tag{4}$$

with renormalization scale $\mu_r$ and $N_c = 3$.

The function $\Omega(R^2/(\rho\bar\rho), \ldots)$ in equation (3), appearing in the exponent with a large numerical coefficient $4\pi/\alpha_s$, incorporates the effects of final-state gluons. Within strict I-perturbation theory, it is given in form of a perturbative expansion [17], while in the so-called I$\bar{\mathrm{I}}$-valley approximation [18, 19] $\Omega$ is associated with an analytically known closed expression [19, 20] for the interaction between I and $\bar{\mathrm{I}}$, $\Omega \simeq \alpha_s/(4\pi)S[A^{\mathrm{I}\bar{\mathrm{I}}}_\mu] - 1$. With both methods agreeing for larger values of $R^2/(\rho\bar\rho)$, we have actually used the valley method in our quantitative evaluation.

Due to the nonvanishing virtuality $Q'^2$ in DIS, the 'form factor' $\exp[-Q'(\rho+\bar\rho)]$ in (3), being associated with the off-shell quark (zero mode) q, suppresses large-size instantons [8–10]. Hence, the integrals in (3) are *finite*. In fact, they are dominated by a unique *saddle-point* [9,10],

$$\rho^* = \bar\rho^* \sim 1/Q'; \qquad R^{*2} \sim 1/(p+q')^2 \Rightarrow \frac{R^*}{\rho^*} \sim \sqrt{\frac{x'}{1-x'}}, \tag{5}$$

from which it becomes apparent that the virtuality $Q'$ controls the effective I-size, while $x'$ determines the effective I$\bar{\mathrm{I}}$-distance (in units of the size $\rho$).

In figure 1($b$), the resulting I-subprocess cross sections (3) are displayed [9] over a *large* range of $\mu_r/Q'$ for fixed $x' = 0.5$ and $Q'/\Lambda = 30, 50, 70$. Apparently, we have achieved great progress in stability and hence predictivity by using the improved expression (4) of the I-density $D(\rho)$, which is renormalization-group (RG) invariant at the two-loop level, i.e. $D^{-1}\,\mathrm{d}D/\mathrm{d}\ln(\mu_r) = \mathcal{O}(\alpha_s^2)$. The residual dependence on the renormalization scale $\mu_r$ is remarkably flat and turns out to be strongly reduced as compared with the one-loop case! Throughout, we choose as the 'best scale', $\mu_r = 0.15\ Q'$, for which $\partial\sigma^{(I)}_{q\,g}/\partial\mu_r \simeq 0$ (cf

† Both an I and an $\bar{\mathrm{I}}$ enter here, since cross sections result from taking the *modulus squared* of an amplitude in the single I background.

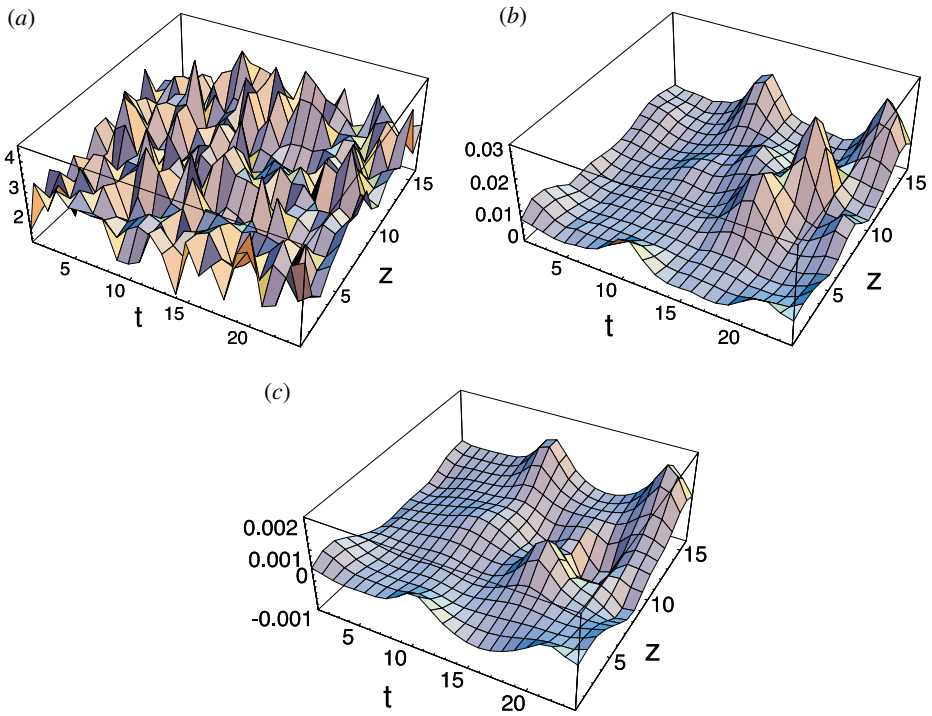


**Figure 2.** Instanton content of a typical slice of a gluon configuration at fixed $x$, $y$ as a function of $z$ and $t$ [22]. (*a*) Lagrange density before 'cooling', with fluctuations of *short* wavelength $\mathcal{O}(a)$ dominating. After 'cooling' by 25 steps, three I and two $\bar{\text{I}}$ may be clearly identified in the Lagrange density (*b*) and the topological charge density (*c*).

figure 1(*b*)). This choice agrees well with the intuitive expectation [8, 6] $\mu_r \sim 1/\langle\rho\rangle \sim Q'/\beta_0 = \mathcal{O}(0.1)\, Q'$.

## 3. 'Fiducial' region from lattice simulations

There has been much recent activity in the lattice community to 'measure' topological fluctuations in lattice simulations [21] of QCD. Being independent of perturbation theory, such simulations provide 'snapshots' of the QCD vacuum including all possible non-perturbative features like instantons (figure 2). Let us discuss next, how these lattice results may be exploited to provide crucial support for the theoretical basis of our calculations in DIS.

To this end, we first perform a *quantitative* confrontation [9, 14] of the predictions from I-perturbation theory with a recent high-quality lattice simulation [23] of QCD (without fermions, $n_f = 0$). The striking agreement which we shall find over a range of I-collective coordinates is a very interesting result by itself.

Next, we recall (cf (3) and (5)) that the collective coordinate integrals in our DIS cross section $\sigma_{qg}^{(I)}(Q', x')$ are dominated by a unique, calculable saddle-point $(\rho^*, R^*/\rho^*)$, in one-to-one correspondence to the conjugate momentum variables $(Q', x')$. This fact then allows us to *translate* the extracted range of validity of I-perturbation theory and the dilute I-gas approximation, $(\rho \leqslant \rho_{\max}, R/\rho \geqslant (R/\rho)_{\min})$, directly into a 'fiducial' kinematical region $(Q' \geqslant Q'_{\min}, x' \geqslant x'_{\min})$ in momentum space!

In lattice simulations 4D-Euclidean space-time is made discrete; specifically, the recent 'data' from the UKQCD collaboration [23], which we shall use here, involve a lattice spacing $a = 0.055-0.1$ fm and a volume $V = l^3_{\text{space}} \cdot l_{\text{time}} = [16^3 \times 48 - 32^3 \times 64]\, a^4$. In principle, such a lattice allows one to study the properties of an ensemble of I and $\bar{\text{I}}$ with sizes $a < \rho < V^{1/4}$. However, in order to make I-effects visible, a certain 'cooling' procedure has to be applied first. It is designed to filter out (dominating) fluctuations of *short* wavelength $\mathcal{O}(a)$ (cf figure 2(*a*)), while affecting the topological fluctuations of much longer wavelength $\rho \gg a$ comparatively little. After 'cooling', I and $\bar{\text{I}}$ can clearly be seen (and studied) as bumps in the lagrange density and the topological charge density (figures 2(*b*) and (*c*)). For a more detailed discussion of lattice-specific caveats, like possible lattice artefacts and the dependence of results on 'cooling' etc., see [21, 23].

Of course, one has to extrapolate the lattice observables to the continuum ($a \Rightarrow 0$), before a meaningful comparison with I-perturbation theory can be made. This is complicated by a strong dependence of the various distributions on the number $n_{\text{cools}}$ of cooling sweeps for *fixed* $\beta = 6/g^2_{\text{lat}}$. In [23], however, *equivalent* pairs $(\beta, n_{\text{cools}})$ were found, for which *shape* and *normalization* of the distributions essentially remain *invariant*. For instance, the continuum extrapolation of the data for the $(\text{I}+\bar{\text{I}})$-density $D_{\text{I}+\bar{\text{I}}}$ at $(\beta, n_{\text{cools}}) = (6.0, 23),\ (6.2, 46),\ (6.4, 80)$, may thus be performed quite reliably [14], by simply rescaling the arguments $\rho \Rightarrow \overline{\rho(0)}/\overline{\rho(a)} \cdot \rho$. Here, $\overline{\rho(0)}$ denotes the continuum limit of the *weakly varying* average $\rho$ values, $\overline{\rho(a)}$, of $D_{\text{I}+\bar{\text{I}}}(\rho, a)$. A *linear* extrapolation in $(a/r_0)^2$ was employed. For consistency and minimization of uncertainties, one should use only a single dimensionful quantity to relate lattice units and physical units. Throughout our analysis, all dimensions are therefore expressed by the so-called Sommer scale [24, 25] $r_0$, with $2r_0 \simeq 1$ fm, which we prefer over the string tension [23]. The resulting 'continuum data' for $D_{\text{I}+\bar{\text{I}}}(\rho)$ are displayed in figure 3. They scale nicely. We are now ready to perform a quantitative comparison with the predictions of I-perturbation theory [14]. For reasons of space, let us concentrate here on the $(\text{I}+\bar{\text{I}})$-density $D_{\text{I}+\bar{\text{I}}}(\rho)$. The prediction (4) of I-perturbation theory is a power law for *small* $\rho$, i.e. approximately $D \sim \rho^6$ for $n_f = 0$. Due to its two-loop RG-invariance the normalization of $D_{\text{I}+\bar{\text{I}}}(\rho)$ is practically independent of the renormalization scale $\mu_r$ over a wide range. It is strongly and exclusively dependent on $r_0 \Lambda_{\overline{\text{MS}} n_f=0}$, for which we take the most recent, accurate result by the ALPHA collaboration [25], $2r_0 \Lambda_{\overline{\text{MS}} n_f=0} = (238 \pm 19)$ MeV fm. In figure 3(*b*) we display both this parameter-free prediction from (4) of I-perturbation theory and the continuum limit of the UKQCD data in a log–log plot, to clearly exhibit the expected power law in $\rho$. The agreement in shape *and* normalization for $\rho \lesssim 0.3(2r_0) \simeq 0.3$ fm is striking, indeed, notably in view of the often criticized 'cooling' procedure and the strong sensitivity to $\Lambda_{\overline{\text{MS}} n_f=0}$.

By a similar analysis [14], we were able to infer from the 'equivalent' UKQCD lattice data a range of validity $R/\rho \gtrsim 1$ of the valley expression for the $\text{I}\bar{\text{I}}$-interaction $\Omega(R^2/(\rho\bar{\rho}), \ldots)$ in (3). Finally, we have confirmed [23, 14] the approximate validity of the dilute-gas picture for sufficiently *small* instantons† with $\rho \lesssim (0.3\text{–}0.5)$ fm. The latter results are based on the 'packing fraction' [23] being $<1$ and a test of the dilute-gas identity: $\langle Q^2 \rangle = N_{\text{tot}}$. Here Q is the topological charge and $N_{\text{tot}}$ the total number of charges. These results strongly support the reliability of our calculations in DIS.

By means of the discussed saddle-point correspondence (5), these lattice constraints may be converted into a 'fiducial' region for our cross section predictions in DIS [9],

$$\left.\begin{array}{lcccl} \rho^* & \leqslant & \rho^*_{\text{max}} & \simeq & 0.3 \text{ fm}; \\ \frac{R^*}{\rho^*} & \geqslant & \left(\frac{R^*}{\rho^*}\right)_{\text{min}} & \simeq & 1 \end{array}\right\} \Rightarrow \left\{\begin{array}{lcccl} Q' & \geqslant & Q'_{\text{min}} & \simeq & 8 \text{ GeV}; \\ x' & \geqslant & x'_{\text{min}} & \simeq & 0.35. \end{array}\right. \tag{6}$$

† Note that the full $(\text{I}+\bar{\text{I}})$-ensemble without the size restriction is known *not* to be a dilute gas [23, 21].

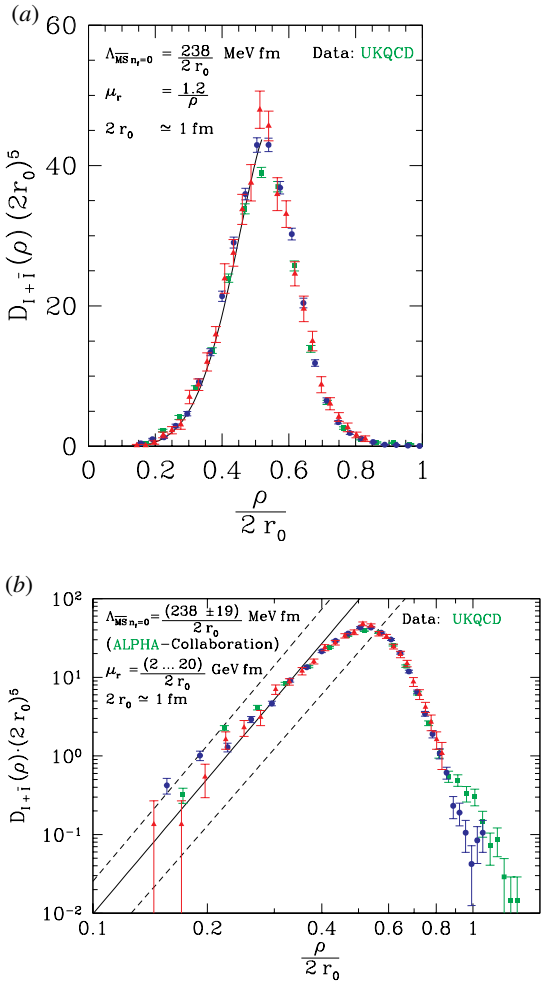


**Figure 3.** Continuum limit [14] of 'equivalent' UKQCD data [23] for the (I + $\bar{\text{I}}$)-density at $(\beta, n_{\text{cools}}) = (6.0, 23)$ (□), $(6.2, 46)$ (○), $(6.4, 80)$ (△). The striking agreement with $2\,D(\rho)$ of I-perturbation theory from (4) is apparent [14]. The three-loop form of $\alpha_s$ with $\Lambda_{\overline{\text{MS}}\,n_f=0}$ from ALPHA [25] was used. (*a*) For $\mu_r = 1.2/\rho$, the agreement extends up to the peak; (*b*) log–log plot to exhibit the expected power law $\sim\rho^6$ and the agreement in magnitude for small $\rho$ over a wide range of $\mu_r$. The dashed error band results from varying $\Lambda_{\overline{\text{MS}}\,n_f=0}$ and $\mu_r$ within its error and given range, respectively.

## 4. HERA cross section

Figure 4 displays our finalized I-induced cross section at HERA [9, 10], as function of the cuts $x'_{\text{min}}$ and $Q'_{\text{min}}$. For the minimal cuts (6) extracted from the UKQCD lattice simulation, we obtain a surprisingly large cross section,

$$\sigma^{(I)}_{\text{HERA}}(x' \geqslant 0.35,\, Q' \geqslant 8\ \text{GeV}) \simeq 126\ \text{pb}; \qquad x_{\text{Bj}} \geqslant 10^{-3}; \qquad 0.9 \geqslant y_{\text{Bj}} \geqslant 0.1. \tag{7}$$

Hence, with the total luminosity accumulated by experiments at HERA, $\mathcal{L} = \mathcal{O}(80)\ \text{pb}^{-1}$, one already expects $\mathcal{O}(10^4)$ I-induced events on tape from this kinematical region. Note also that the cross section quoted in equation (7) corresponds to a fraction of I-induced to normal DIS

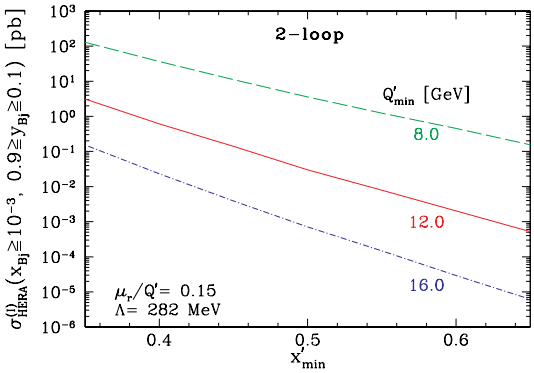


**Figure 4.** I-induced cross section at HERA as function of the cuts in $(x', Q')$.

(nDIS) events of

$$f^{(I)} = \frac{\sigma^{(I)}_{\rm HERA}}{\sigma^{\rm (nDIS)}_{\rm HERA}} = \mathcal{O}(1)\%; \qquad \text{for} \quad x_{\rm Bj} \geqslant 10^{-3}; \quad 0.9 \geqslant y_{\rm Bj} \geqslant 0.1. \tag{8}$$

This is remarkably close to the published upper limits on the fraction of I-induced events [26], which are also on the one percent level.

There are still a number of significant uncertainties in our result for the cross section. For *fixed* $Q'$ and $x'$ cuts, one of the dominant uncertainties arises from the experimental uncertainty in the QCD scale $\Lambda$. In the two-loop expression for $\alpha_s$ with $n_f = 3$ (massless) flavours we used the value $\Lambda_{\overline{\rm MS}_{n_f=3}} = 282$ MeV, corresponding to the central value of the DIS average for $n_f = 4$, $\Lambda_{\overline{\rm MS}_{n_f=4}} = 234$ MeV [27]. If we change $\Lambda_{\overline{\rm MS}_{n_f=3}}$ within the allowed range, $\approx \pm 65$ MeV, the cross section (7) varies between 26 pb and 426 pb. Minor uncertainties are associated with the residual renormalization-scale dependence (cf figure 1(*b*)) and the choice of the factorization scale. Upon varying the latter by an order of magnitude, the changes are in the $\mathcal{O}(20)\%$ range only.

By far the dominant uncertainty in $\sigma^{(I)}_{\rm HERA}$ arises, however, from the uncertainty in placing the $(x', Q')$ cuts (cf figure 4). Hence, the constraints (6) from lattice simulations are extremely valuable for making concrete and reliable predictions of the I-induced rate at HERA.

## 5. Signatures and searches

An indispensable tool for investigating the structure of the I-induced final state and for developing optimized search strategies is our Monte Carlo generator for I-induced DIS-events, QCDINS 1.60. Besides the matrix element for the I-induced hard subprocess, it provides leading-log parton showers and hadronization via its interface to HERWIG 5.9.

The characteristic features of the I-induced final state are illustrated in figure 5(*a*) displaying the lego plot of a typical event from QCDINS 1.60 (cf also figure 1(*a*)): Besides a single (not very hard) current-quark jet, one expects an accompanying densely populated *'hadronic band'*. For $x_{\rm Bj\,min} \simeq 10^{-3}$, say, it is centred around $\bar{\eta} \simeq 2$ and has a width of $\Delta\eta \simeq \pm 1$. The band directly reflects the *isotropic* production of an I-induced 'fireball' of $\mathcal{O}(10)$ partons in the I-rest system. Both the total transverse energy $\langle E_T \rangle \simeq 15$ GeV and the charged particle multiplicity $\langle n_c \rangle \simeq 13$ in the band are far higher than in normal DIS events. Finally, each I-induced event has to contain strangeness (and possibly also charm) such that the number of $\rm K^0$ amounts to $\simeq 2.2$/event.

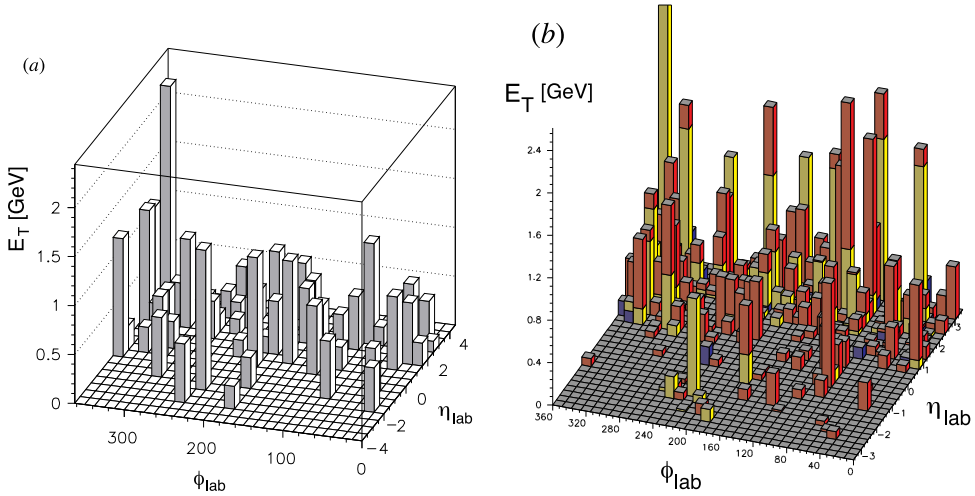


**Figure 5.** (*a*) Lego plot of a typical I-induced event from QCDINS 1.60. (*b*) An interesting *real* 'fireball' event in photoproduction from ZEUS [30] with very large total $E_T$ and multiplicity.

Despite the high expected rate (7) of I-induced events at HERA, no *single* observable is known (yet) with sufficient nDIS rejection. Hence, a dedicated multi-observable analysis seems to be required. Neural network filters are being tried and exhibit a very good analysing power if applied to $\gtrsim \mathcal{O}(5)$ observables [28]. Strategies to produce 'I-enriched' data samples and to reconstruct $(Q'^2, x')$ are under study and look quite promising [28]. Clearly, in all cases, a good understanding of the perturbative QCD background in the *tails* of the considered distributions is required.

## 6. Going beyond I-perturbation theory

A class of striking 'fireball' events in photoproduction, with large total $E_T$ and large multiplicity, has been reported [29] at this meeting (see e.g. figure 5(*b*)). While the quantitative analysis is still in an early stage, these events seem to exhibit all characteristics of I-induced events (cf figure 5(*a*)). However, it appears that—unlike ordinary QCD perturbation theory—the hard photoproduction limit, $Q^2 \Rightarrow 0$, $(E_T)_{\text{jet}}$ large, is not within the reach of strict I-perturbation theory. The reason is that in the $Q^2 \Rightarrow 0$ limit, we encounter a contribution to the photoproduction cross section, which tends to *diverge*, if integrated over the I-size $\rho$. This IR divergence at large $\rho$ is independent of the $E_T$ of the (current quark) jet and directly associated with the 'bad' large-$\rho$ behaviour of the *perturbative* expression (4) for the I-density, $D(\rho) \sim \rho^{6-2/3n_f}$. In contrast, the *actual* form of $D(\rho)$ (cf figure 3) is strongly peaked around $\rho \simeq 0.5$ fm, and appears to *vanish* exponentially fast for larger $\rho$. The above 'fireball' events and more generally, the ongoing I-searches at HERA, provide plenty of motivation for trying to extend our calculational framework beyond strict I-perturbation theory. We are thus led to make the following promising proposal in this direction.

One may try and replace the most strongly varying entries in the perturbative calculations, the I-density $D(\rho)$ and the I$\bar{\text{I}}$-interaction $\Omega(R^2/(\rho\bar{\rho}), \ldots)$, in (3), by their *actual* form as extracted from the recent non-perturbative lattice results [14].

The I-rates in photoproduction and the I-contributions to further interesting observables may then be calculated, and the $(Q'^2, x')$ cuts in I-searches be considerably relaxed! Due

to the strong peaking of $D(\rho)$, only the region around $\rho \simeq 0.5$ fm enters and the dilute-gas approximation may well continue to hold up to the peak (cf section 3).

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