The TPOL interaction point: how to implement it in a Monte Carlo

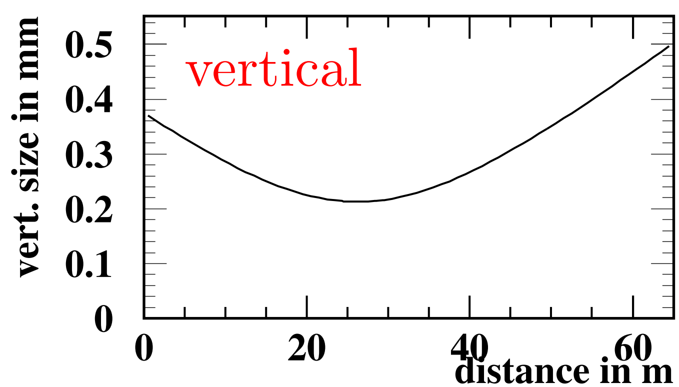
- The electron beam
- Folding in the laser beam
- Results

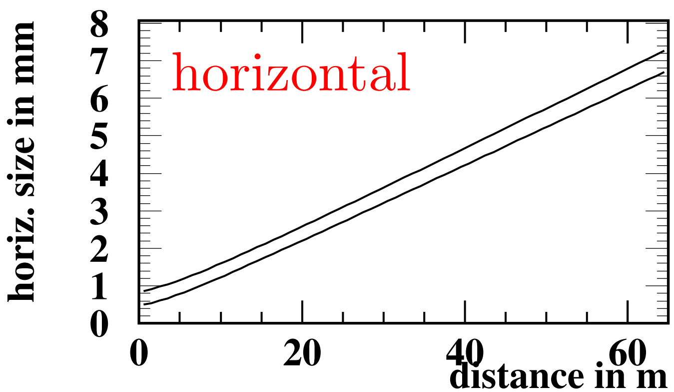
The electron beam

- Basic machine parameters at HERA II are:
  vertical: $\alpha_y = 1.45$, $\beta_y = 56.21$, m, $\epsilon_y \approx 2.5\,\text{mm}\,\mu\text{rad}$
  Horizontal: $\alpha_x = -0.41$, $\beta_x = 8.48\,\text{m}$, $\epsilon_x \approx \epsilon_y/0.03$
  Note: α, β from Eliana, ϵ_y from HERA II measurements, ϵ_x from NIM paper (might be out-of-date)

- $\alpha_y > 0$ and $\alpha_x < 0$ means beam is focussed in y and defocussed in x

- Formula for beam envelope along D is given in NIM paper:
  $\sigma(D) = \sqrt{\epsilon(\beta - 2\alpha * D + \gamma * D^2)}$
  Note: $\gamma = \frac{1+\alpha^2}{\beta}$ is the beam divergence at $D = 0$





The electron beam (continued)

- Probability density to decribe the electron beam size y and slope s_y:

  $f_y(y, s_y) = \frac{1}{2\pi\epsilon_y} \exp[-\frac{1}{2\epsilon_y}(\beta_y s_y^2 + 2\alpha_y y\, s_y + \gamma_y y^2)]$

  Test: beam size at distance D:

  $\sigma(D)^2 = \langle (y + D * s_y)^2 \rangle = \int dy \int ds_y (y + D * s_y)^2\, f_y(y, s_y) = \epsilon(\beta - \alpha D + \gamma D^2)$

  beam size y and beam slope s_y are anti-correlated

- To generate correlated random-numbers, find orthogonal linear combination of y and s_y in argument of exponential:

  $\frac{1}{\epsilon_y}(\beta_y s_y^2 + 2\alpha_y y\, s_y + \gamma_y y^2) = \frac{1}{\epsilon_y}(\beta_y s_y^2 + 2\alpha_y y\, s_y + \frac{(\alpha_y^2+1)}{\beta_y} y^2) = \frac{\beta_y}{\epsilon_y}(s_y + \frac{\alpha_y}{\beta_y} y)^2 + \frac{1}{\epsilon_y \beta}$

  Generate independent Gaussian random numbers r_1 and r_2.

  Assign $y = \sqrt{\epsilon_y \beta_y} r_1$ and $(s_y + \frac{\alpha_y}{\beta_y} y) = \sqrt{\frac{\epsilon_y}{\beta_y}} r_2$.

  Calculate $s_y = \sqrt{\frac{\epsilon_y}{\beta_y}} r_2 - \alpha_y \sqrt{\frac{\epsilon_y}{\beta_y}} r_1 = \sqrt{\epsilon_y \gamma_y}((1 - C_y^2) r_2 + C_y\, r_1)$ where $C_y = \frac{\alpha_y}{1+\alpha_y^2}$.

Folding in the laser beam

- laser is a round beam with crossing-angle ϕ_y

  Probability density function for laser beam (fixed y):

  $f_l(x,z) = \frac{N}{2\pi\sigma_l} \exp[-\frac{1}{2\sigma_L^2}(x^2 + (y\cos\phi_y - z\sin\phi_y)^2)]$

- Full probability density (contains LASER and electron beam parameters):

  $f(x,y,z,s_x,s_y) = N \times f_x(x,s_x) \times f_y(y,s_y) \times f_l(x,y,z)$

- Need to find independent linear combinations and generate five random numbers

  Argument of exponential for the horizontal direction:

  $\frac{\beta_x}{\epsilon_x}(s_x + \frac{\alpha_x}{\beta_x}x)^2 + (\frac{1}{\epsilon_x\beta_x} + \frac{1}{\sigma_L^2})x^2$

  Introduce size of overlap-region in x at IP: $\sigma_x = \frac{1}{\frac{1}{\epsilon_x\beta_x}+\frac{1}{\sigma_L^2}}$

- With random numbers r_1 and r_2:

  Calculate $x = \sigma_x r_1$

  and $s_x = \sqrt{\epsilon_x\gamma_x}((1 - C_x^2)r_2 + C_x\frac{\sigma_x}{\sqrt{\epsilon_x\beta_x}}r_1)$

Folding in the laser beam (y and z, results)

- Argument of exponential: $\frac{\beta_y}{\epsilon_y}(s_y + \frac{\alpha_y}{\beta_y} y)^2 + \frac{1}{\epsilon_y \beta_y} y^2 + \frac{1}{\sigma_L^2}(y - z\phi_y)^2$
- Use independent Gaussian random numbers $(r_1, r_2, r_3, r_4, r_5)$

  Horizontal vertex at IP: $x = \sigma_x r_1$

  Horizontal vertex slope at IP: $s_x = \sqrt{\epsilon_x \gamma_x}((1 - C_x^2) r_2 + C_x \frac{\sigma_x}{\sqrt{\epsilon_x \beta_x}} r_1)$

  Vertical vertex at IP: $y = \sqrt{\epsilon_y \beta_y} r_3$

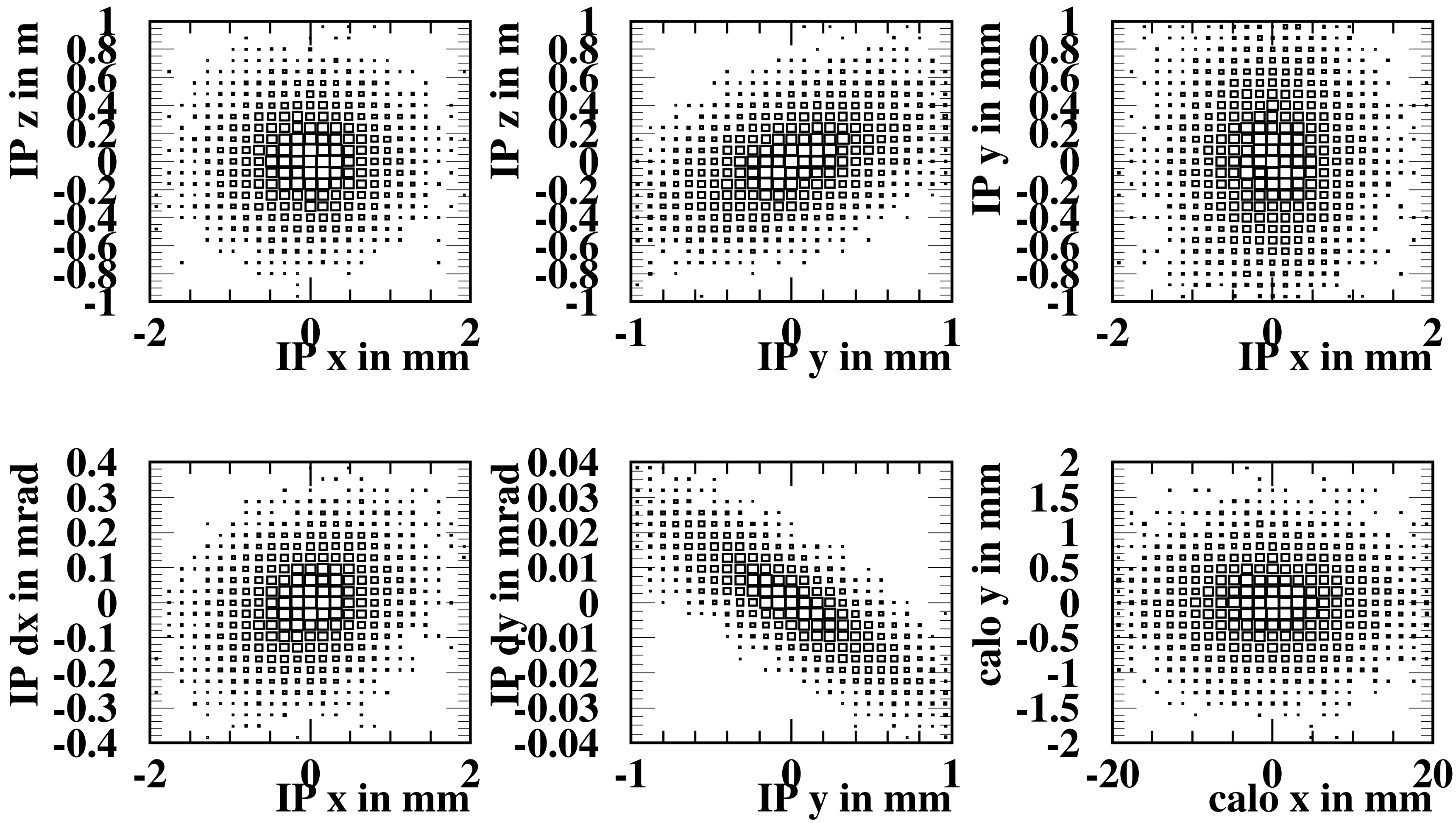
  Vertical vertex slope at IP: $s_y = \sqrt{\epsilon_y \gamma_y}((1 - C_y^2) r_4 + C_y r_3)$

  Longitudinal vertex: $z = \frac{1}{\phi_y}(r_5 \sigma_L + y)$
- Result: (x, s_x) and (y, s_y, z) are correlated
- Check beam size at distance D (Mathematica):

  $\langle (y + s_y D)^2 \rangle \approx \epsilon_y (\beta_y - 2\alpha_y D + \gamma_y D^2)$

  $\langle (x + s_x D)^2 \rangle \approx \frac{\sigma_L^2}{\epsilon_x \beta_x + \sigma_L^2} \epsilon_x (\beta_x - 2\alpha_x D + (\gamma_x + \frac{\epsilon_x}{\sigma_L^2}) D^2)$

Results



IP z in m
IP x in mm
IP z in m
IP y in mm
IP y in mm
IP x in mm
IP dx in mrad
IP x in mm
IP dy in mrad
IP y in mm
calo y in mm
calo x in mm