Limits in High Energy Physics

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Tutorial/lecture for the
Terascale Statistics School

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Exercises

Outline

- Part I: basic concepts, Bayes and Frequentist, simple example
- Part II: Poisson with background, expected limit, CL_s method
- Part III: systematic uncertainties and many channels, hybrid method, profile likelihood
- Exercises: Lecture is interleaved by exercises ~10-15 minutes each. Solutions are discussed in the lecture
- ROOT macros for exercises:

  `www.desy.de/~sschmitt/LimitStatSchool2013/macros`
- If available on our computer, use wget:

```
wget -N -nd www.desy.de/~sschmitt/LimitStatSchool2013/macros.list
wget -N -nd -i macros.list
```

Exercise 1 (Bayes' law)

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

- Disease and a test for the disease
- 0.1% of the population have the disease (prior)
- If one has the disease, the test is positive with 99% probability (likelihood)
- If one does not have the disease, the test is positive with 1% probability
- What is the posterior probability to have the disease, given a positive test?

Exercise 2 (Neyman construction)

- Poisson experiment, determine limits on the parameter μ, given N_{obs}
  - a) determine the range $N_{obs} \leq N \leq \infty$ for CL=0.95 and μ=2,3,5,10. What is the probability to find the measurement in these ranges
  - b) determine the limit on μ for N_{obs}=0,2,10,100
- Hint: the probability to find N in the interval $N_{obs} \leq N \leq \infty$ is given by:

$$\text{Probability: } \sum_{N \geq N_{obs}}^{\infty} \frac{e^{-\mu}(\mu)^N}{N!} = 1-\alpha = 1 - \text{TMath::Prob}(2*\mu, 2*N_{obs})$$

$$\text{Inverse function: } 2*\mu = \text{TMath::ChisquareQuantile}(1-\alpha, 2*N_{obs})$$

(a)

μ	N_{obs}	1-α
2		
3		
5		
10		

(b)

N_{obs}	μ_{limit}
0	
2	
10	
100	

Exercise 3 (Bayesian limit)

- Exercise 3a: Bayesian limit for N_{obs}=0,2,10,100 (flat prior) (use Root macro)
- Exercise 3b: use a prior $P(\mu)=\mu$, N_{obs}={0,2,10,100}
- Exercise 3c: use a flat prior up to μ_{max}=90, set prior to zero above μ_{max}
- Compare to exercise 2

- Bayesian limit with arbitrary prior → numerical integration
- `GetPosterior.C(muLimit,nObs)`

$$\text{Posterior} \sim \int_0^{\mu_0} d\mu \, \text{Prior}(\mu) \frac{\exp[-\mu]\mu^{N_{obs}}}{N_{obs}!}$$

- Vary muLimit until Posterior=0.95

	frequentist	Bayes flat	Bayes $P(\mu)=\mu$	Bayes flat μ_{max}=90
N_{obs}	μ_{limit}	μ_{limit}	μ_{limit}	μ_{limit}
0				
2				
10				
100				

Exercise 4 (limit with background)

- Calculate Frequentist and Bayesian limits for N_{obs}={0,2} and b={0.5,2.0,3.5}

Poisson parameter: $\mu = s + b$

	b=0.5		b=2.0		b=3.5	
	N_{obs}=0	N_{obs}=2	N_{obs}=0	N_{obs}=2	N_{obs}=0	N_{obs}=2
Bayesian						
Frequentist						

- Frequentist: use methods from exercise 2
- Bayes: try to modify exercise 3 macro, or use macro GetPosteriorWithBackground.C

Expected limit (exercise 5)

- Expected limit: limit weighted by background probability

$$\langle s_{\text{limit}} \rangle = \sum_{n=0}^{\infty} \frac{e^{-b} b^n}{n!} \text{LimitOnSignal}(b, n)$$

	b=0.5		b=2.0		b=3.5	
	N_{obs}=0	N_{obs}=2	N_{obs}=0	N_{obs}=2	N_{obs}=0	N_{obs}=2
Bayesian	3.0	5.8	3.0	4.8	3.0	4.3
Frequentist	2.5	5.8	1.0	4.3	-0.5	2.8
Expected						

- Calculate expected limits for b={0.5,2.0,3.5}
- Macro GetExpectedLimit.C

Exercise 6 (CL_s method)

- Frequentist limit: $1 - CL \geq \alpha = \mathrm{CL}_{SB} = P(N \leq N_{\mathrm{obs}}; \mu = s + b)$
- CL_s limit: $1 - CL \geq \mathrm{CL}_S = \frac{\mathrm{CL}_{SB}}{\mathrm{CL}_B} = \frac{P(N \leq N_{\mathrm{obs}}; \mu = s + b)}{P(N \leq N_{\mathrm{obs}}; \mu = b)}$

	b=0.5		b=2.0		b=3.5	
	N_{obs}=0	N_{obs}=2	N_{obs}=0	N_{obs}=2	N_{obs}=0	N_{obs}=2
Bayesian	3.0	5.8	3.0	4.8	3.0	4.3
Frequentist	2.5	5.8	1.0	4.3	-0.5	2.8
CL_s						
Expected						

- Use macro GetCLsLimit.C to calculate CL_s, iterate to get limit

Exercise 7 (limits from hybrid method)

- CL_S limit, systematic error treated with hybrid method $\mu = l(s+b)$
- Background error: zero or σ_b=50% [b_{obs}={0.5, 3.5}]
- Luminosity error: zero or σ_l=10% [l_{obs}=1.0]

CL_S limits	b=0.5		bgr=3.5	
	N_{obs}=0	N_{obs}=2	N_{obs}=0	N_{obs}=2
No syst				
σ_b/b=50%				
σ_l/l=10%				
Both syst.				

Use root macro GetClsSys.C