A NEW METHOD FOR THE HIGH-PRECISION ALIGNMENT OF TRACK DETECTORS

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Abstract
Track detectors in high energy physics experiments require an accurate determination of a large number of alignment parameters. A general method has been developed, which allows the determination of up to several thousand alignment parameters in a simultaneous linear least squares fit of an arbitrary number of tracks. The sensitivity of the method is demonstrated in an example of the simultaneous alignment of a 56-plane drift chamber and a 2-plane silicon tracker. About 1400 alignment parameters are determined in a fit of about fifty thousand tracks.

1. ALIGNMENT PROBLEMS OF TRACK DETECTORS

Alignment problems for large detectors, especially track detectors, in particle physics often require the determination of a large number of alignment parameters, typically of the order of 100 to 1000. Alignment parameters for example define the accurate space coordinates and orientation of detector components. In addition driftchambers require the determination of the drift velocity v_{drift} and of local variations of the drift velocity, and of the Lorentz angle. Figure 1 shows an example of residuals in a silicon vertex track detector before and after an alignment.

In the alignment usually special alignment measurements are combined with data of particle reaction, typically tracks from physics interactions and from cosmics. In this paper the alignment parameters are called *global* parameters; they contribute to all the data. Usually these parameters are *corrections* to default values and are thus relatively small, and the value zero is the initial approximation. Parameter sets valid for a single track are called *local* parameters, examples are track slopes and curvatures.

An approximate alignment method is to perform least squares fits on the data *ignoring* initially the alignment parameters. The *deviations* between the fitted and measured data, the *residuals*, are then used to determine the alignment parameters afterwards. Single fits depend only on the small number of local parameters like slope or curvature of the track, and are easy to perform. This approximate method however is not a statistically correct method, because the (local) fits depend on a wrong model (namely: ignoring the global parameters), and the result are biased. In addition the results may be unstable due to detector inefficiencies. The adjustment of parameters based on the observed (biased) residuals will then result in biased alignment parameters. If these alignment parameters are applied as corrections in repeated fits, the remaining residuals will be reduced, as desired. However, the fitted parameters will still be biased, even more, if this approximate procedure is applied iteratively. The simple residual method is also unable to determine correlated parameters, for example a geometrical shift of a detector position and a shift in drift velocity in a drift chamber.

A better method is an overall fit, with all the (global) alignment parameters and local parameters, perhaps from thousands or millions of events, determined simultaneously. The most important general method for the determination of parameters from measured data is the linear least squares model. It is usually stable, fast and accurate, requires no initial values of parameters and is able to take into account all the correlations between different parameters, even if there are many parameters. There is of course a practical limit in the number of parameters in simultaneous fits, because of space and time limitations. In an alignment fit the size of the vectors and matrices in the least squares solution will be large, of the order of 10^4 or 10^6. With standard methods such a solution is impossible.

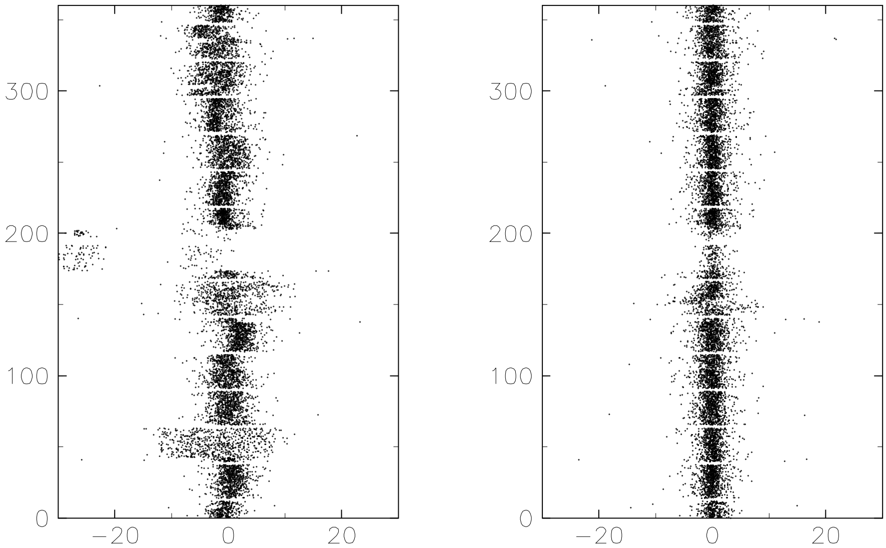

Fig. 1: Residuals between tracks and hits of a silicon tracker are shown as a function of the azimuthal angle (vertical axis, in degrees) before and after an alignment. The residual on the horizontal axis is given in units of 10 μm.

In this paper a special method of solution for this kind of problems is derived. Due to the special structure of the matrices with one set of global parameters and many sets of local parameters the problem can in fact be reduced to a solvable size, without making any approximations. The rather general method is realized in a program called **Millepede** [2].

2. STANDARD LEAST SQUARES

Linear models. In a linear model the measured quantity z_k has an expectation value which is a *linear* function of the parameters $\boldsymbol{a}$. The difference between the measured value and the expectation value is the residuum r_k:

$$z_k = \boldsymbol{a}^T \cdot \boldsymbol{d}_k + r_k \; . \tag{1}$$

The measured data are assumed to be *independent*; the result of a measurement z_k does not dependent on any other measurement. Thus the covariance matrix of the measured data is a diagonal matrix, which simplifies the treatment. The accuracy of the measurement z_k is given by the variance σ_k^2 or the standard deviation σ_k. It is assumed, that the precision is at least approximately known.

The solution of the method of least squares is determined by the minimum of the weighted sum of the square of the residuals

$$S(\boldsymbol{a}) = \sum_k \frac{\left(z_k - \boldsymbol{a}^T \cdot \boldsymbol{d}_k\right)^2}{\sigma_k^2} \; , \tag{2}$$

where the sum (index k) is over all measurements. This sum is a quadratic function of the parameters $\boldsymbol{a}$, and the solution is obtained from a set of linear equations (so-called normal equations of least squares), which can be written in matrix form

$$\boldsymbol{C}\boldsymbol{a} = \boldsymbol{b} \qquad \text{with} \quad \boldsymbol{C} = \sum_k w_k \left(\boldsymbol{d}_k \cdot \boldsymbol{d}_k^T\right) \qquad \text{and} \quad \boldsymbol{b} = \sum_k w_k z_k \boldsymbol{d}_k \tag{3}$$

with a symmetric n-by-n matrix $\boldsymbol{C}$ and a right-hand-side n-vector $\boldsymbol{b}$; the elements of $\boldsymbol{C}$ and $\boldsymbol{b}$ are sums with *independent* contributions from each measurement (because the measured data are independent).

There is no limitation on the number of measured data; it is not necessary to keep all input data in memory, because the complete information from a single measurement is contained in the contributions to $\boldsymbol{C}$ and $\boldsymbol{b}$, which have a fixed size. The solution vector is $\boldsymbol{a} = \boldsymbol{C}^{-1}\boldsymbol{b}$ with the inverse of the matrix $\boldsymbol{C}$, which, by error propagation, is equal to the covariance matrix of the parameters $\boldsymbol{a}$.

What are the limits for the number n of parameters, if the matrix equation is solved by standard methods? The symmetry property of the matrix $\boldsymbol{C}$ may be used to store only the $(n^2+n)/2$ elements on and above the diagonal, and also to reduce the computer time to almost half the time for a non-symmetric matrix. Matrix inversion is an explicit process and requires a CPU time proportional to n^3 for a n-by-n matrix. For a 1000-by-1000 symmetric matrix in double precision (about 4 Mbyte) the CPU time is a few minutes on a standard PC. The time ($\propto n^3$) and space ($\propto n^2$) requirements imply at present a practical limit of the size of the matrix (and number of parameters) between $n = 1\,000$ and $n = 10\,000$.

The accuracy of the computation of the inverse matrix is a critical point, and often computation in double precision is required, especially for large matrices. The accuracy is generally low for a large condition number of the matrix (the condition number is defined as the ratio of the largest to the smallest eigenvalue of the matrix). From the interpretation of the inverse matrix as covariance matrix it is clear, that the condition number will be large if the correlation between parameters is high. The matrix is singular (one or more eigenvalues are zero) in case of a complete correlation between parameters. In this case and also in cases of missing data (e.g. dead channels in an alignment fit) a standard matrix inversion program will fail. A singular value decomposition (SVD) would allow to recognize the cases of zero or small eigenvalues, which would *destroy* the full solution. The standard method in case of zero or very small eigenvalues is to set their inverse to zero; this means essentially to remove parameters or linear combinations of parameters from the solution (i.e. the resulting parameter values are zero); if the parameters are in fact *corrections* this is clearly acceptable. In the program **Millepede** [2] a similar, but simpler and faster strategy is used to avoid the effects of a bad condition number. This strategy also works in cases, where due to missing input data[1] one or more parameters are undefined; the parameters will become zero in this case. The strategy allows in addition to remove certain parameters from the least squares fit without the necessity to reorganize the numbering.

Solution by partitioning. The special structure of the matrix $\boldsymbol{C}$ to be inverted may allow to move the limit of solvable problems further (see also [1]). Below the matrix $\boldsymbol{C}$ is partitioned into submatrices, and the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are partitioned into two subvectors; then the matrix equation becomes

$$\left(\begin{array}{c|c} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} \\ \hline \boldsymbol{C}_{21} & \boldsymbol{C}_{22} \end{array}\right) \left(\begin{array}{c} \boldsymbol{a}_1 \\ \hline \boldsymbol{a}_2 \end{array}\right) = \left(\begin{array}{c} \boldsymbol{b}_1 \\ \hline \boldsymbol{b}_2 \end{array}\right) , \tag{4}$$

where the submatrix $\boldsymbol{C}_{11}$ is a p-by-p square matrix and the submatrix $\boldsymbol{C}_{22}$ is a q-by-q square matrix, with $p+q = n$. If the sub-vector $\boldsymbol{a}_1$ would not exist, the solution for the sub-vector $\boldsymbol{a}_2$ would be defined by the matrix equation

$$\boldsymbol{C}_{22}\ \boldsymbol{a}_2^* = \boldsymbol{b}_2 \qquad \text{with solution} \quad \boldsymbol{a}_2^* = \boldsymbol{C}_{22}^{-1}\ \boldsymbol{b}_2 \ , \tag{5}$$

where the star indicates the special character of this solution. Only the q-by-q sub-matrix $\boldsymbol{C}_{22}$ has to be inverted. The solution $\boldsymbol{a}_2^*$ of course differs from the final solution $\boldsymbol{a}_2$ and is called the *local* solution.

Now, having inverted the sub-matrix $\boldsymbol{C}_{22}$, the submatrix of the complete inverse matrix $\boldsymbol{C}$ corre-ponding to the upper left part $\boldsymbol{C}_{11}$, called $\boldsymbol{B}$, is obtained by the formula

$$\boldsymbol{B} = \left(\boldsymbol{C}_{11} - \boldsymbol{C}_{12}\boldsymbol{C}_{22}^{-1}\boldsymbol{C}_{12}^T\right)^{-1} \ , \tag{6}$$

[1]Large detectors may have dead channels, which means missing data.

which requires in addition to some matrix products the calculation of the inverse of a p-by-p sub-matrix. With this matrix $\boldsymbol{B}$ the solution of the whole matrix equation can written in the form

$$\left(\begin{array}{c} \boldsymbol{a}_1 \\ \hline \boldsymbol{a}_2 \end{array}\right) = \left(\begin{array}{c|c} \boldsymbol{B} & -\boldsymbol{B}\boldsymbol{C}_{12}\boldsymbol{C}_{22}^{-1} \\ \hline -\boldsymbol{C}_{22}^{-1}\boldsymbol{C}_{12}^T\boldsymbol{B} & \boldsymbol{C}_{22}^{-1} - \boldsymbol{C}_{22}^{-1}\boldsymbol{C}_{12}^T\boldsymbol{B}\boldsymbol{C}_{12}\boldsymbol{C}_{22}^{-1} \end{array}\right) \left(\begin{array}{c} \boldsymbol{b}_1 \\ \hline \boldsymbol{b}_2 \end{array}\right) , \tag{7}$$

which can be used to obtain the complete result ($\boldsymbol{a}_1$ and $\boldsymbol{a}_2$). Note that to solve the matrix equation two smaller matrices, a symmetric p-by-p and a symmetric q-by-q matrix, have to be inverted. The sub-vector $\boldsymbol{a}_1$ is obtained by the product

$$\boldsymbol{a}_1 = \boldsymbol{B}\boldsymbol{b}_1 - \boldsymbol{B}\boldsymbol{C}_{12}\boldsymbol{C}_{22}^{-1}\boldsymbol{b}_2 = \boldsymbol{B}\left(\boldsymbol{b}_1 - \boldsymbol{C}_{12}\boldsymbol{a}_2^*\right) , \tag{8}$$

which is simplified by using the special local solution $\boldsymbol{a}_2^* = \boldsymbol{C}_{22}^{-1}\boldsymbol{b}_2$. If the interest is the determination of this sub-vector $\boldsymbol{a}_1$ only, while the sub-vector $\boldsymbol{a}_2$ is not needed, then only the equation (8) has to be solved. Some computer time can be saved by this method, especially if the matrix $\boldsymbol{C}_{22}^{-1}$ is easily calculated or already known before. In special cases the method can be applied repeatedly, and therefore problems with a huge number of parameters become solvable. The method is not an approximation, but is exact and it takes into account all the correlations. This type of application is discussed in more detail in the next chapter.

It seems that this possibility of removing unnecessary parameters, but still getting the correct solution for the remaining parameters in least squares problems was already known and has been used in the nineteenth century; one example is [3].

Constraints. If there are explicit relations between the parameters of a least squares fit, then they should be explicitly taken into account in the solution. The case of a single linear constraint between parameters of the form $\boldsymbol{f}^T \cdot \boldsymbol{a} = \boldsymbol{f}_0$ is assumed here. The Lagrange multiplier method is the standard method to include constraints in a least squares solution; one additional vector $\boldsymbol{\lambda}$ is introduced, with one element per constraint. The equation (3) is modified by the additional parameters to the form

$$\left(\begin{array}{c|c} \boldsymbol{C} & \boldsymbol{f} \\ \hline \boldsymbol{f}^T & \boldsymbol{0} \end{array}\right) \left(\begin{array}{c} \boldsymbol{a} \\ \hline \boldsymbol{\lambda} \end{array}\right) = \left(\begin{array}{c} \boldsymbol{b} \\ \hline \boldsymbol{f}_0 \end{array}\right) , \tag{9}$$

where the matrix on the left hand side is still symmetric. *Linear* least squares problems with *linear* constraints can be solved direcly, without iterations and without the need for initial values of the parameters.

3. LEAST SQUARES WITH GLOBAL AND LOCAL PARAMETERS

Local parameters. Local parameters are denoted by $\alpha_j,\ j = 1, 2, \ldots \nu$. A set of measurements z is considered. A single value z from the set is ideally given by a linear expression $z = \alpha_1 \cdot \delta_1 + \alpha_2 \cdot \delta_2 + \ldots \alpha_\nu \cdot \delta_\nu$, depending on the parameters α_j and on (known) constant factors δ_j. More complete, the dependence can be written as

$$z_k = \alpha_1 \cdot \delta_{1k} + \alpha_2 \cdot \delta_{2k} + \ldots \alpha_\nu \cdot \delta_{\nu k} = \sum_{j=1}^{\nu} \alpha_j \cdot \delta_{jk} , \tag{10}$$

with an additional index k indicating the k-th measured value.

As an example the measurement of a straight particle track is considered, where each z_k is a coordinate measurement at plane k. The local parameters are intercept (α_1) and slope (α_2) of the particle track. If the distance of the plane is denoted by S_k, the linear track model can be written as

$$z_k = \alpha_1 \cdot 1 + \alpha_2 \cdot S_k .$$

From a least squares fit of the model to the set of measured z-values the local parameters intercept and slope can be determined. The formalism of least squares allows to take into account the different precision of the different measurements, by assigning a weight to each measurement of $w_k = 1/\sigma_k^2$. The least squares solution, minimizing the weighted sum of squared residuals, is defined by the so-called normal equations, which can be written in terms of matrices

$$\boldsymbol{\Gamma}\,\boldsymbol{\alpha} = \boldsymbol{\beta}\,, \qquad \text{solved by} \quad \boldsymbol{\alpha} = \boldsymbol{\Gamma}^{-1}\boldsymbol{\beta}\;. \tag{11}$$

The elements of the matrix $\boldsymbol{\Gamma}$ and the vector $\boldsymbol{\beta}$ are formed by sums

$$\Gamma_{ij} = \sum_k w_k \cdot \delta_{ik}\delta_{jk} \qquad\qquad \beta_j = \sum_k w_k \cdot z_k \delta_{jk}\;. \tag{12}$$

These formulas are valid in the case of *uncorrelated* z-measurements (uncorrelated measurements are assumed thoughout this paper). The inverse matrix $\boldsymbol{\Gamma}^{-1}$ is the covariance matrix of the parameter vector $\boldsymbol{\alpha}$.

Global parameters. Now global parameters are considered, which contribute to *all* the measurements. They are assumed to be *alignment parameters* here, although the formalism is general and can also be applied to other problems. Aligment parameters are defined to represent *corrections* to ideal (design) values; this parametrization adds further terms to the equations (10)

$$z = \underbrace{a_1 \cdot d_1 + a_2 \cdot d_2 + \ldots a_n \cdot d_n}_{\text{global parameters}} + \underbrace{\alpha_1 \cdot \delta_1 + \alpha_2 \cdot \delta_2 + \ldots \alpha_\nu \cdot \delta_\nu}_{\text{local parameters}} = \sum_{i=1}^{n} a_j \cdot d_j + \sum_{j=1}^{\nu} \alpha_j \cdot \delta_j\;. \tag{13}$$

Usually only few terms containing global parameters are nonzero for a single z measurement. In the following it is assumed that, as written in equation (13), the dependence on the global parameters is *linear*. A non-linear relationship for the global parameters would require iterations, starting with reasonable initial values for the parameters, and assuming a linearized expression in each iteration.

The simultaneous fit of global and local parameters. In the following it is assumed that there is a set of N partial measurements. Each partial measurement, with index i, depends on local parameters $\boldsymbol{\alpha}_i$, and all of them depend on the global parameters. In a simultaneous fit of all global parameters plus local parameters from N subsets of the data there are in total $(n + N \cdot \nu)$ parameters, and the standard solution requires the solution of $(n + N \cdot \nu)$ equations with a computation proportional to $(n + N \cdot \nu)^3$. Below it is shown, that the problem can be reduced to n equations with a computation proportional n^3.

Generalizing the formalism of equations (10) to the complete measurement, starting from equations of the type of (13), one obtains a system of normal equations with large dimensions, as is shown in equation (14). The matrix on the left side of equation (14) has, from each partial measurement, three types of contributions. The first part is a contribution of a symmetric matrix $\boldsymbol{C}_i$, of dimension n (number of global parameters). All the matrices $\boldsymbol{C}_i$ are added up in the upper left corner of the big matrix of the normal equations. The second contribution is the symmetric matrix $\boldsymbol{\Gamma}_i$ (compare equation (12)), which gives a contribution to the diagonal of the big matrix and is depending only on the i-th partial measurement. The third contribution is a rectangular matrix $\boldsymbol{G}_i$, with a row number of n (global) and a column number of ν (local).

There are two contributions to the vector of the normal equations, $\boldsymbol{b}_i$ for the global and $\boldsymbol{\beta}_i$ for the local parameters. The complete matrix equation is given by

$$\left(\begin{array}{c||c|c|c}
\sum \boldsymbol{C}_i & \cdots & \boldsymbol{G}_i & \cdots \\
\hline\hline
\vdots & \ddots & 0 & 0 \\
\hline
\boldsymbol{G}_i^T & 0 & \boldsymbol{\Gamma}_i & 0 \\
\hline
\vdots & 0 & 0 & \ddots
\end{array}\right) \cdot \left(\begin{array}{c}
\boldsymbol{a} \\
\hline\hline
\vdots \\
\hline
\boldsymbol{\alpha}_i \\
\hline
\vdots
\end{array}\right) = \cdot \left(\begin{array}{c}
\sum \boldsymbol{b}_i \\
\hline\hline
\vdots \\
\hline
\boldsymbol{\beta}_i \\
\hline
\vdots
\end{array}\right) \tag{14}$$

In this matrix equation the matrices $\boldsymbol{C}_i, \boldsymbol{\Gamma}_i, \boldsymbol{G}_i$ and the vectors $\boldsymbol{b}_i$ and $\boldsymbol{\beta}_i$ contain contributions from the i-th partial measurement. Ignoring the global parameters one could solve the normal equations $\boldsymbol{\Gamma}_i \boldsymbol{\alpha}_i^* = \boldsymbol{\beta}_i$ for each partial measurement separately by

$$\boldsymbol{\alpha}_i^* = \boldsymbol{\Gamma}_i^{-1} \boldsymbol{\beta}_i \, . \tag{15}$$

The complete system of normal equations has a special structure, with many vanishing sub-matrices. The only connection between the local parameters of different partial measurements is given by the sub-matrices $\boldsymbol{G}_i$ und $\boldsymbol{C}_i$,

The aim of the fit is solely to determine the global parameters; final best parameters of the local parameters are not needed. The matrix of equation (14) is written in a partitioned form. The general solution can also be written in partitioned form. Many of the sub-matrices of the huge matrix in equation (14) are zero and this has the effect, that the formulas for the sub-matrices of the inverse matrix are very simple. By this procedure the normal equations

$$\left(\quad \boldsymbol{C'} \quad \right) \left(\boldsymbol{a} \right) = \left(\boldsymbol{b'} \right) , \tag{16}$$

are obtained, which only contain the global parameters, with a modified matrix $\boldsymbol{C'}$ and a vector $\boldsymbol{b'}$,

$$\boldsymbol{C'} = \sum_i \boldsymbol{C}_i - \sum_i \boldsymbol{G}_i \boldsymbol{\Gamma}_i^{-1} \boldsymbol{G}_i^T \qquad \boldsymbol{b'} = \sum_i \boldsymbol{b}_i - \sum_i \boldsymbol{G}_i \left(\boldsymbol{\Gamma}_i^{-1} \boldsymbol{\beta}_i \right) \, . \tag{17}$$

This set of normal equations contains explicitly only the global parameters; implicitly it contains, through the correction matrices, the complete information from the local parameters, influencing the fit of the global parameters. The parentheses in equation (17) represents the solution for the local parameters, ignoring the global parameters. The solution $\boldsymbol{a} = \boldsymbol{C'}^{-1} \boldsymbol{b'}$ represents the solution vector $\boldsymbol{a}$ with covariance matrix $\boldsymbol{C'}^{-1}$. The solution is direct, no iterations are required. Iterations may be necessary for other reasons, namely

- the equations depend *non-linearly* on the global parameters; the equations have to be linearized;
- the data contain outlier, which have to be removed in a sequence of cuts, becoming narrower during the iteration;
- the accuracy of the data is not known before, and has to be determined from the data (after the alignment).

Application. A very important point in problems discussed above is the definition of the set of global parameters. It is essential to define them in a way, which is sensitive to the data used to determine them. Computing the inverse of a huge (symmetric) matrix can be a delicate numerical problem. The matrix will be singular or close to a singular matrix, if one or more of the global parameters are not really determined (and constrained) by the data, and this could destroy the whole determination. This will happen in case of a strong correlation between different global parameters.

It is wise to start from few global parameters and to observe the correlations between them, before adding more global parameters. For example if too many global parameters are used in a detector alignment, the whole detector may *shift* and *rotate* freely in the fit. In those cases one could either fix certain global parameters or one has to introduce constraints (e.g. average displacement and rotation equal to zero).

Experience with the method shows that it is essential to make a simultaneous alignment of all track detector components, which are also used for the measurement of tracks, while independent internal alignment of the single track detector components may not lead to a good overall result. It is also important to use, if possible, different data sets simultaneously. For example in a track detector alignment high-momentum cosmic muons may be useful, which traverse the whole detector and thus relate (and constrain) different parts of the detector. But this should be done simultaneously with tracks from physics events, because these events finally require a good alignment.

The method described here has been realized in the program **Millepede** [2], written in Fortran77. It provides a set of subroutines for the mathematical methods and allows to adapt the method to a particular problem. The program allows to adjust the dimension of vectors and matrices via `PARAMETER`-statements and to introduce linear constraints. Throughout the program use is made of the sparse character of the vectors and matrices in order to reduce the execution time. The program includes a method for outlier rejection, which in practice may be essential in alignment problems especially if some detector components have a bad initial alignment. This feature requires to iterate, realized with an intermediate data file, which is written during the first iteration and read during the other iterations.

Sometimes the model underlying the alignment is *nonlinear* and also constraints may be nonlinear. The standard method to treat these problems is linearization: the nonlinear equation is replaced by a linear equation for the correction of a parameter (Taylor expansion); this requires a good approximate value of the parameter.

4. ALIGNMENT OF THE CENTRAL TRACK DETECTORS IN THE H1 EXPERIMENT

The example described is the alignment in the $r\varphi$-plane perpendicular to the beam line of a 56-plane drift chamber and a 2-plane silicon vertex detector in the H1 detector at HERA [4]. Both are cylindrical detectors; the drift chamber has a length (z) of about 2 m, and extending from 20.3 cm to 84.4 cm in radius r. The silicon vertex detector [5] has two planes around the beampipe made of 0.15 mm Aluminium and 0.9 mm carbon fiber. The drift chamber and the silicon vertex detector have a $r\varphi$ resolution of about 150 μm and 15 μm, respectively. These detectors are interspersed by additional chambers for the z-measurement.

The central jet chambers CJC1 and CJC2 ([4]) with an active length of 2200 mm and an outer radius of 844 mm have in total 2640 anode sense wires parallel to the beam with two adjacent cathode planes (wires) shaping the drift field. A jet chamber cell extends azimuthally from the sense wire plane to both adjacent cathode wire planes, and radially extends over the full radial span of CJC1 and CJC2 with no further subdivision. The jet cells are tilted by about 30° such that in the presence of the magnetic field (1.1 Tesla) the ionization electrons drift approximately perpendicular to high momentum vertex tracks.

The silicon vertex detector CST consists of two cylindrical layers of double sided, double metal silicon sensors read out by a custom designed analog pipeline chip. The two layers of the CST are formed from 12 and 20 faces at radii of 5.75 cm and 9.75 cm, respectively. One face or 'ladder' consists of six

row.	number	parameter	σ	unit
1	2	Δx	1	μm
2	2	$\Delta x/\Delta z_r$	2	μm
3	2	Δy	1	μm
4	2	$\Delta y/\Delta z_r$	2	μm
5	2	$\Delta\varphi$	10	μrad
6	2	$\Delta\varphi/\Delta z_r$	10	μrad
7	2	$\Delta\alpha_{\text{Lor}}$	100	μrad
8	2	$\Delta v_{\text{drift}-}/v_{\text{drift}}$	10^{-5}	
9	2	$\Delta v_{\text{drift}+}/v_{\text{drift}}$	10^{-5}	
10	2	$\Delta T_0 \times v_{\text{drift}}$	< 1	μm
11	2	wire staggering in wire plane	few	μm
12	2	wire staggering perp wire plane	few	μm
13	2	sagging in wire plane	few	μm
14	2	sagging perp. wire plane	few	μm
15	180	$\Delta v_{\text{drift}}/v_{\text{drift}}$ per cell half	few	10^{-4}
16	112	$\Delta v_{\text{drift}}/v_{\text{drift}}$ per layer half	few	10^{-4}
17	330	$\Delta T_0 \times v_{\text{drift}}$ per group	10	μm
18	56	wire position in driftdir. per layer	10	μm
19	56	$\Delta T_0 \times v_{\text{drift}}$ per layer	10	μm
20	56	Δx_W per layer	few 10	μm
21	112	$\Delta v_{\text{drift}}/v_{\text{drift}}$ for $I_e/50$ mA	few 10^{-4}	
22	90	$\Delta v_{\text{drift}}/v_{\text{drift}}$ per layer	few 10^{-4}	
23	90	Δy_W per layer	few 10	μm
24	90	$(\Delta y_W)^2$ per layer^2	few 10	μm
25	64	Δ in ladder	few	μm
26	64	Δ perp. ladder	few	μm
27	64	rel. Δ in ladder (z_r)	few	μm
28	64	rel. Δ perp. ladder (z_r)	10	μm
29	64	rel. Δ perp. ladder (φ)	few	μm

Table 1: Alignment parameters determined in the **Millepede** fit. The **Millepede** accuracy is given by one standard deviation σ. The different parameters are discussed in the text.

silicon sensors and aluminium nitride hybrids at each end. A double layer of carbon fiber strips with a total thickness of 700 μm and a height of 4.4 mm is glued to the edges. The position of the ladders in a layer are shifted tangentially to ensure an overlap in $r\varphi$ of adjacent active areas. The active length in z is 35.6 cm for both layers.

Data taken with the H1 detector are reconstructed online and the reconstruction modules also determine important parameters like average drift velocities, Lorentz-angle and T_0 of the CJC1 and CJC2. In addition beam parameters like the $r\varphi$ position of the primary interaction point are recorded.

In the alignment procedure using the program **Millepede** a large number of parameters is determined; these parameter are in general corrections to detector parameters and represent *small* corrections. It turned out that a *common* alignment of the drift chamber and the silicon detector is essential, after an internal detector alignment, which already determines reasonable starting values for alignment parameters. For each of the drift chambers CJC1 and CJC2 14 global parameters representing an overall shift or tilt are introduced. Local variations of the drift velocity v_{drift} for cells halfs and layers halfs are observed, which are parametrized by 180 + 112 corrections, which change with the HV configuration. For each wire group (8 wires) corrections to T_0 are introduced (330 corrections). For the silicon vertex detector CST an internal alignment has already been done, using the same techniques, with in total 384 local parameters. In the common alignment fit 5 parameters per half ladder representing shifts are introduced (320 parameters). Table 1 contains all parameters. A far as possible the parameters are defined with the dimension *length*. For example the time-zero parameters are multiplied by the mean drift time. Angles are parametrized as shifts relative to a normalized length parameter. For example z_r is defined as z/z_{max} and has a range –1 to +1. The shift Δx with z is parametrized as $\Delta x/z_r$ and has the dimension length.

In addition to physics events (ep interactions) also cosmic tracks with and without B-field were simultaneously used as input to the alignment. With combined fits to both halfs of cosmic tracks parts

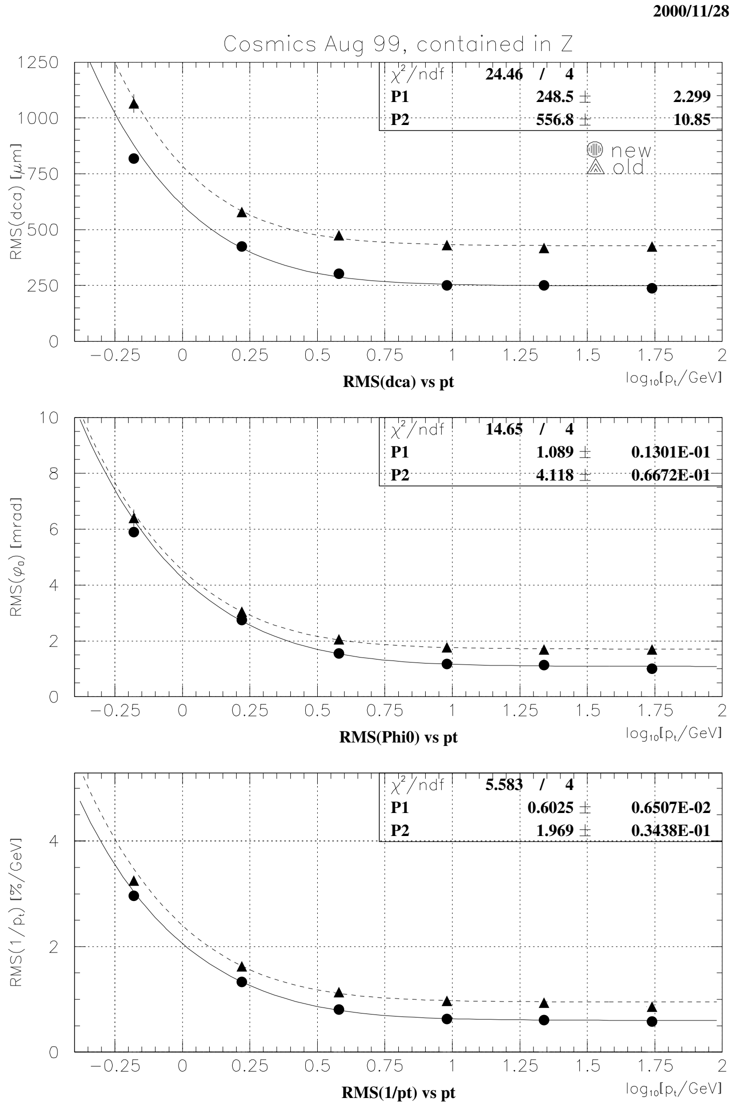

Fig. 2: The standard deviations for the three track parameters d_{ca} (distance of closest aproach to the beam axis), the azimuthal angle φ and of the inverse transverse momentum $1/p_t$ are shown as a function of $\log_{10} p_t$/GeV, before (old) and after (new) the improved alignment. The standard deviations are determined from the difference between the two parts of cosmic tracks.

of the detectors opposite in φ angle are strongly correlated. This allows an accurate interpolation from the drift chamber to the silicon tracker with an accuracy $\sigma = 30$ to 40 μm, which is of the same order of magnitude as the silicon tracker accuracy. In total about 50 000 events are used as input. Constraints require e.g. a zero overall shift of the detector. The CJC2 is fixed (first five parameters) vs. the cryostat on the basis of external survey data as reference.

An overview about the corrections determined in the alignment fit is given in Table 1, which has 29 rows of different corrections. In detail these corrections are as follows.

Rows 1 – 6 These rows contain geometrical parameters; in an external survey the positions are measured with an accuracy of a few 100 μm and angles with an accuracy of a few 10 μrad. This accuracy is increased by a large factor in the alignment fit.

Row 6 This parameter is determined from $B = 0$ cosmics (straight line track model).

Rows 11 – 14 The parameters are determined only as a check. Their actual values are determined from hit triplets as a function of angle β and z_r (β is the angle between the track and the normal to the drift direction).

Row 15,16 The parameters change with the HV configuration.

Row 18,20 Corrections for the wire position in and perpendicular to the drift direction. These corrections are determined in a cross check with $B = 0$ cosmic tracks, otherwise the data from a survey of flanges are used ($< 10\mu$m).

Row 21 These correction are used to describe the changes of drift velocity v_{drift} by variations of the electron beam current I_e due to space charge effects; they are determined once.

Row 22 – 24 These parameters change with repair/breakdown of HV cards. They are introduced in order to correct for bad cathode HV.

Row 25 – 29 Corrections for the silicon vertex detector, five per half ladder, determined in and perpendicular to ladder, on average and as function of z and of φ.

The alignment is done for run ranges and repeated after e.g. a change of the HV configuration. Run-to-run variations of parameters (e.g. the average drift velocity v_{drift}) are taken into account by the data from the online reconstruction as mentioned above.

track fit	$\sigma(d_{\text{ca}})$ [cm]	$\sigma(1/p_t)$ [(GeV/c)$^{-1}$]	remark
only 1 track with Si hits	0.0209	$5.58 \cdot 10^{-3}$	drift chamber resolution
both tracks with at least 1 Si hit	0.0032	$2.26 \cdot 10^{-3}$	Si tracker resolution
both tracks with at least 2 Si hits	0.0028	$2.17 \cdot 10^{-3}$	Si tracker resolution

Table 2: Standard deviations in the parameters d_{ca} and $1/p_t$, determined from the difference between the two parts of cosmic tracks.

The alignment procedure allows to reach the intrinsic resolution not only locally but also for complete tracks. For the drift chambers CJC1 and CJC2 the local hit resolution can be determined from local hit triplets as a function of drift distance; the minimum is about 120 μm for 1 cm drift distance. The minimum value from track residuals is about 125 μm. For the CST the intrinsic point resolution of 15 μm from hits overlaps is also reached globally. Figure 2 shows the improvement for track parameters, determined from data in the comparison of parameters between the two parts of cosmic tracks. A significant improvement is visible for larger track momenta. In Table 2 the standard deviations in the parameters d_{ca} and $1/p_t$, determined from the difference between the two parts of cosmic tracks, are given for three different track classes.

ACKNOWLEDGEMENTS

We would like to thank the organizers of the conference on Advanced Statistical Techniques in Particle Physics for their hospitality and the stimulating atmosphere in Durham.

References

[1] V. Blobel and E. Lohrmann, Statistische und numerische Methoden der Datenanalyse, Teubner, Stuttgart und Leipzig (1998)

[2] A program description for **Millepede** and the code is available via `http://www.desy.de/~blobel/`.

[3] Schreiber, O. (1877): Rechnungsvorschriften für die trigonometrische Abteilung der Landesaufnahme, Ausgleichung und Berechnung der Triangulation zweiter Ordnung. Handwritten notes. Mentioned in W. Jordan (1910): Handbuch der Vermessungskunde, Sechste erw. Auflage, Band I, §III: 429-433. J.B.Metzler, Stuttgart.

[4] H1 Collaboration, The H1-detector at HERA, Nucl. Instrum. Meth., **A386**, 310 – 396 (1997)

[5] D. Pitzl et al., The H1 silicon vertex detector, Nucl. Instrum. Meth., **A454**, 334 – 349 (2000)