Introduction

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Statistical (and numerical) methods of data analysis

- Current physics experiments take data at a high rate: many Mbytes per second over a long period of time
- The essential result may be a single number, or a limit
- Statistical and numerical methods are needed in order to extract meaningful information from experimental data.

Examples for new data from HEP

Amplitude of the CP-violating asymmetry:

$\sin 2\beta$	$=$	$0.34 \pm 0.20(\text{stat}) \pm 0.05(\text{syst})$	BABAR
$\sin 2\beta$	$=$	$0.58^{+0.32\ +0.09}_{-0.34\ -0.10}$	BELLE

Measurement of the anomalous g value, $a_\mu = (g-2)/2$, for the positive muon:

$$a_{\mu^+} = 11\ 659\ 202(14)(6) \times 10^{-10}\ (1.3\ \text{ppm}) \qquad \text{BNL}$$

Difference between the weighted experimental result and the theoretical value from the standard model corresponds to 2.6 times the error.

Example: neutrino mass

Data (1996) on the electron-neutrino mass m_ν from tritium β decay

Experiment	m_ν^2	(eV^2)	m_ν 95 % C.L. limit
Tokyo	-65	$\pm 85 \pm 65$	< 13.0 eV
Los Alamos	-147	$\pm 68 \pm 41$	< 9.3 eV
Zürich	-24	$\pm 48 \pm 61$	< 11.7 eV
Mainz	-39	$\pm 34 \pm 15$	< 7.2 eV
Livermore (LLNL)	-130	$\pm 20 \pm 15$	< 7.0 eV
Moskau (INR)	-22	± 4.8	< 4.4 eV

First error of m_ν^2 is statistical, second systematic. Upper limits for the neutrino mass are calculated using a certain method for 95 % confidence level.

PDG 2000: $m_\nu < 3$eV Interpretation of tritium β decay is complicated by anomalies near the endpoint ("unknown effects cause an accumulation of events near the endpoint of the electron spectrum"), and the limits are not without ambiguity.

Example: Number of light neutrinos

Mark II collaboration at SLAC (1989): analysis which assumed Standard Model couplings but which fit for the Z mass and the number of neutrinos N_ν with result:

$$N_\nu = 2.8 \pm 0.6$$

95 % C.L. limit $N_\nu < 3.9$ excludes fourth massless neutrino

- Existence of at least three types on neutrinos was well established
- Ability to exclude $N_\nu = 4$ in this experiment due to downward fluctuation of measured N_ν from 3 to 2.8

PDG 2000: $N_\nu = 2.994 \pm 0.012$ from Standard Model fits to LEP data.

Literature

R. Barlow Statistics: A Guide to the Use of Statistical Methods in the Physical Sciences, Wiley, 1989

V. Blobel and E. Lohrmann Statistische und numerische Methoden der Datenanalyse, Teubner, 1998

S. Brandt Datenanalyse, Spektrum, 1999

G. Cowan Statistical Data Analysis, Oxford, 1998

L. Lyons Statistics for nuclear and particle physicists, Cambridge, 1989

W.H. Press, S.A. Teukolsky, W.T. Vetterling, B.P. Flannery, Numerical Recipes in [...C, Fortran] (various progr. lang.), The Art of Scientific Computing

Literature on algorithms

Robert Sedgewick Algorithms, Addison-Wesley (various editions)

Sara Base Computer Algorithms, Addison-Wesley

Algorithms: Complex problems are divided into smaller problems, which hopefully can be solved by standard algorithms.

Important standard problems in data organisation

- sorting of items according to a key (ascending, descending)
- search in a list

Algorithms usually apply to a set of objects of a certain size n. The dependence $f(n)$ of the computer time or the number of operations on n for larger values of n (leading dependence) is often an essential property of an algorithm.

Typical leading dependences

Sometimes important:

$A(n) =$ average behaviour $\qquad$ $W(n) =$ worst case behaviour

Typical leading dependences on the number n of objects in algorithms with examples of algorithms for sorting and searching.

function	sorting	searching
$f(n) = \text{const.}$		hash search
$f(n) = \text{const.} \cdot \log n$		binary search
$f(n) = \text{const.} \cdot n$		sequential search
$f(n) = \text{const.} \cdot n \log n$	quicksort, heapsort	
$f(n) = \text{const.} \cdot n^2$	quicksort with sorted input	
$f(n) = \text{const.} \cdot 2^n$		

Topics

- Probability and statistics
- Monte Carlo methods
- Parameter estimation and information
- Least squares methods
- Maximum likelihood method and function minimization
- Confidence intervals and limits
- Unfolding