# Vorlesung 1: Reelle Funktionen

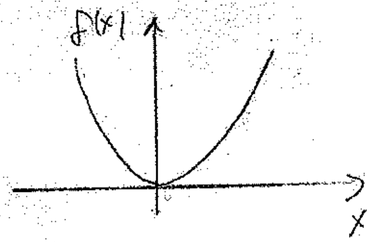
Seien $D, W \subseteq \mathbb{R}$ Teilmengen von $\mathbb{R}$

Betrachte die Funktion oder Abbildung

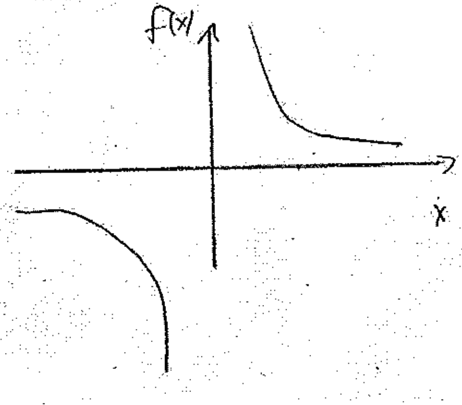
$$f: \underbrace{D}_{\text{Definitionsbereich}} \longrightarrow \underbrace{W}_{\text{Wertebereich}}$$

$$\forall\ x \in D \longmapsto \underbrace{f(x)}_{\text{eindeutig!}} \in W \quad (\text{Menge aller Funktionswerte})$$

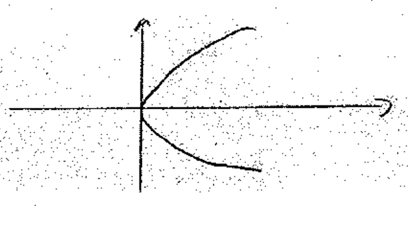
**Beispiel** i) $f(x) = x^2, \quad D = \mathbb{R}, \quad W = \mathbb{R}^+$




ii) $f(x) = \frac{1}{x}, \quad D = \mathbb{R} \setminus \{0\}, \quad W = \mathbb{R} \setminus \{0\}$



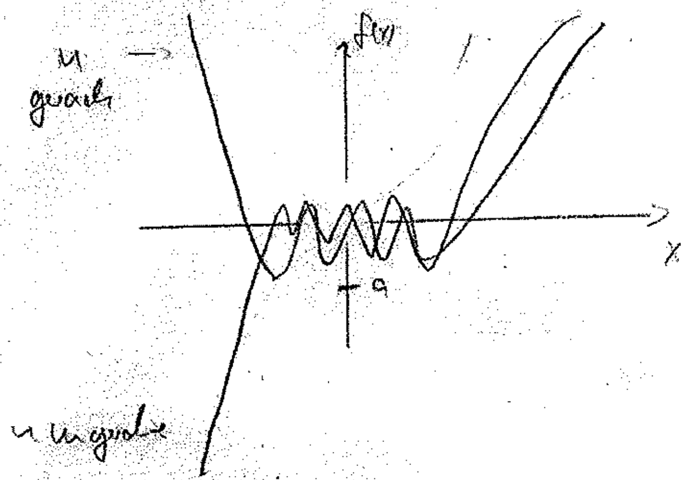

iii)



kein Fkt!

spezielle Fkt, die oft in der Physik auftreten:

- Polynome $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n = \sum_{i=0}^{n} a_i x^i$

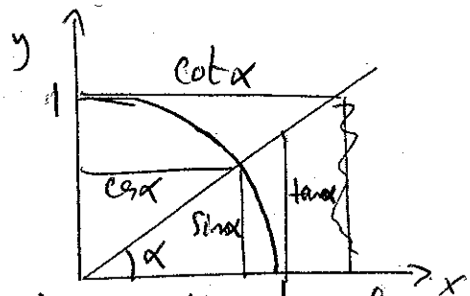



$D = \mathbb{R}$

$W = \mathbb{R}$, $[a, \infty]$

n ungerade, n gerade

- rationale Fkt: $f(x) = \dfrac{P_1(x)}{P_2(x)}$, $P_{1,2}$ Polynome

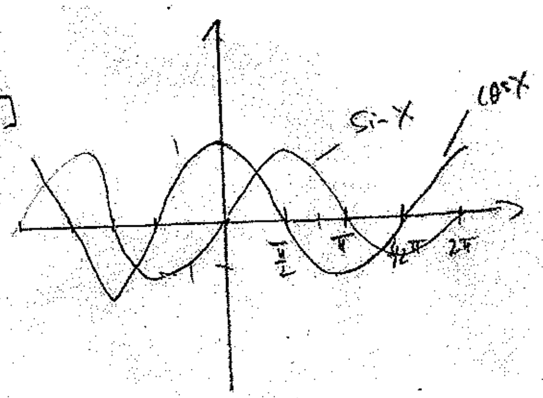


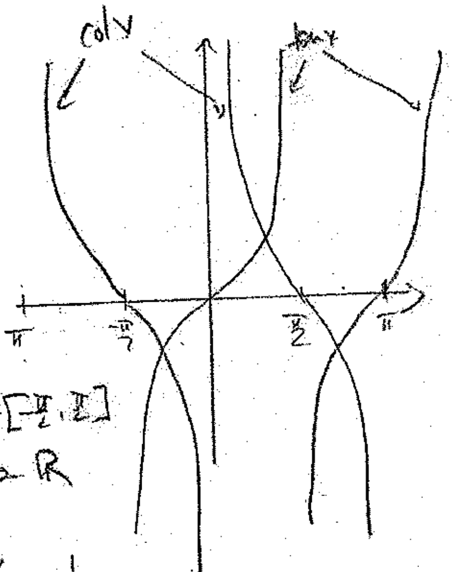

- trigonometrische Fkt:

$f(x) = \sin x$, $f(x) = \cos x$, $f(x) = \tan x$, $f(x) = \cot x$

$D = \mathbb{R}$

$W = [-1, 1]$




$D = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$W = \mathbb{R}$

Es gilt: $\tan x = \dfrac{\sin x}{\cos x}$, $\cot x = \dfrac{\cos x}{\sin x} = \dfrac{1}{\tan x}$, $\sin^2 x + \cos^2 x = 1$

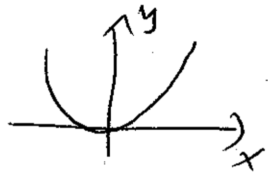
Eine Funktion $f(x)$ heißt 1.3

[gerade oder (Achsen) symmetrisch] falls $f(x) = f(-x)$

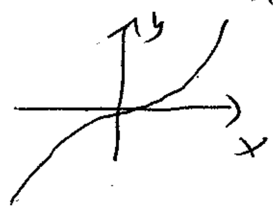
[ungerade oder punktsymmetrisch um den Ursprung oder anti-symmetrisch] falls $f(x) = -f(-x)$

Beispiel:

$f(x) = x^2 = f(-x)$ → gerade




$f(x) = x^3 = -f(-x)$ → ungerade




Eine Funktion heißt <u>periodisch</u> mit Periode $p$ falls

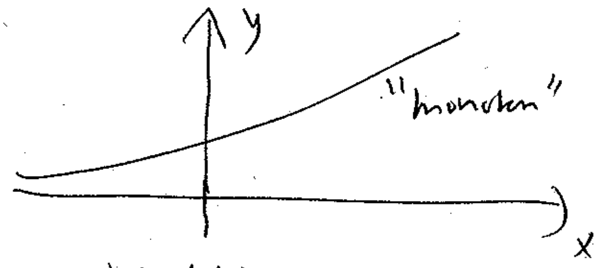
$$f(x+p) = f(x) \qquad \forall x \in D$$

Beispiel: $f(x) = \sin(x+p) = \sin x$ falls $p = 2\pi n, n \in \mathbb{Z}$

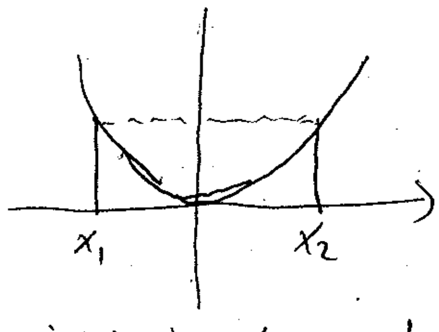
Eine Funktion heißt <u>injektiv</u> falls

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \qquad \forall x_1, x_2 \in D$$

Beispiel:




injektiv

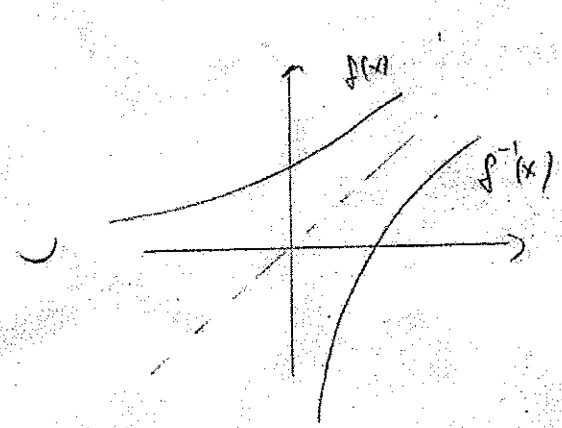


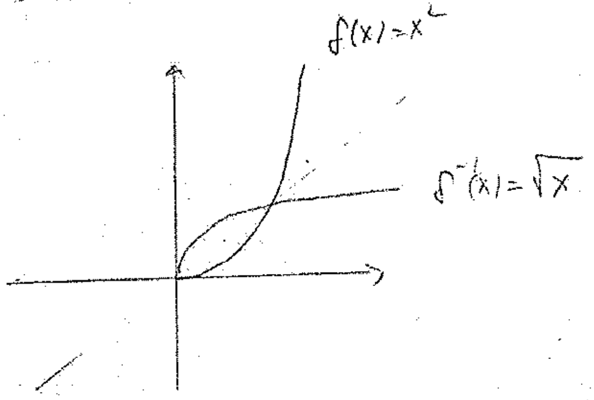

nicht injektiv da $f(x_1) = f(x_2)$
$x_1 \neq x_2$

Für eine injektiven Fkt ex. eine Umkehrfkt

$$f^{-1}: \begin{array}{ccc} W & \longrightarrow & D \\ \Psi & & \Psi \\ y = f(x) & \longmapsto & x = f^{-1}(y) \end{array} \quad \Rightarrow \quad f(f^{-1}(x)) = x = f^{-1}(f(x))$$

⇒ Spiegelung der Graphen an Winkelhalbierenden



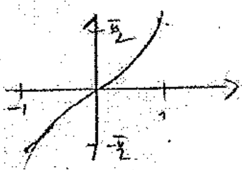

Beispiel: $f(x) = x^2$, $D = \mathbb{R}^+$

$\Rightarrow f^{-1} = \sqrt{x}$

check: $f(f^{-1}(x)) = (\sqrt{x})^2 = x$ ✓

Inverse trigonometrische Fkt:

$f(x) = \arcsin x$



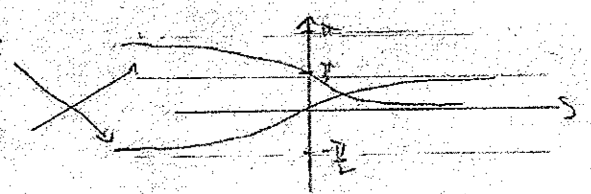
$D = [-1, 1]$, $W = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$f(x) = \arccos x$

$D = [-1, 1]$, $W = [0, \pi]$

$f(x) = \arctan x$

$f(x) = \operatorname{arccot} x$



$D = \mathbb{R}$

$W = \begin{cases} (-\frac{\pi}{2}, \frac{\pi}{2}) \\ (0, \pi) \end{cases}$