$$= \frac{\mu_0}{4\pi} \int \Bigg[\frac{-(y-y')\, j_z(\vec{r}', t - |\vec{r}-\vec{r}'|/c_0) + (z-z')\, j_y(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0)}{|\vec{r}-\vec{r}'|^3}$$

$$\frac{-(y-y')\, \dot{j}_z(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0) + (z-z')\, \dot{j}_y(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0)}{c_0 |\vec{r}-\vec{r}'|^2} \Bigg] d^3 r'$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \int \Bigg[\frac{\vec{j}(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0) \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} + \frac{\dot{\vec{j}}(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0) \times (\vec{r}-\vec{r}')}{c_0 |\vec{r}-\vec{r}'|^2} \Bigg] d^3 r'$$

"Nahfeld" $\propto \frac{1}{r^2}$ "Fernfeld" oder "Strahlungsfeld" $\propto \frac{1}{r}$

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} =$$

$$= \frac{1}{4\pi\varepsilon_0} \int \Bigg[\frac{\rho_e(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0)\,(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} + \frac{\dot{\rho}_e(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0)\,(\vec{r}-\vec{r}')}{c_0 |\vec{r}-\vec{r}'|^2}$$

$$- \frac{\dot{\vec{j}}(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0)}{c_0^2 |\vec{r}-\vec{r}'|} \Bigg] d^3 r'$$

$$= \frac{1}{4\pi\varepsilon_0} \int \Bigg[\frac{\rho_e(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0)(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} + \frac{c_0\, \dot{\rho}_e(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0)\, \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|} - \dot{\vec{j}}(\vec{r}', t-|\vec{r}-\vec{r}'|/c_0)}{c_0^2 |\vec{r}-\vec{r}'|} \Bigg] d^3 r'$$

Nahfeld Fernfeld

abkürzende Schreibweise:

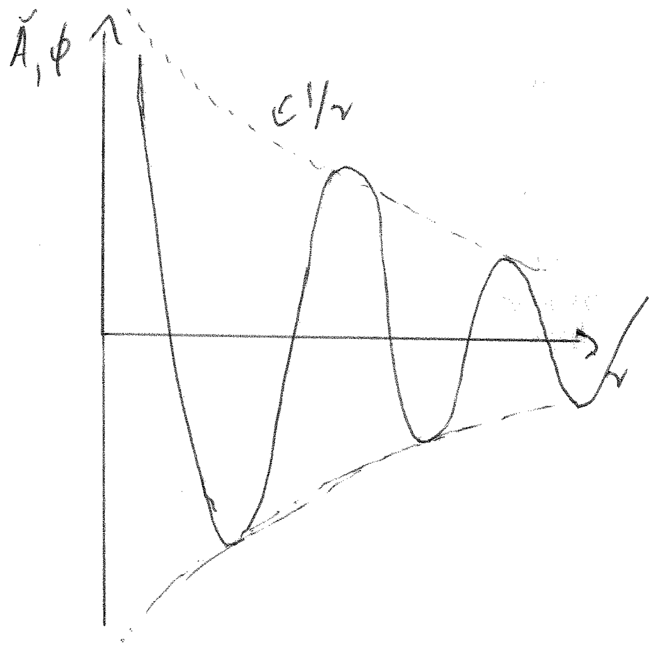
$$\vec{e} := \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|} \qquad t_{ret} := t - |\vec{r}-\vec{r}'|/c_0$$

$$\Rightarrow \quad \breve{B} = \frac{\mu_0}{4\pi} \int \left[\frac{\breve{j}(\breve{r}', t_{ret}) \times \breve{e}}{|\breve{r}-\breve{r}'|^2} + \frac{\dot{\breve{j}}(\breve{r}', t_{ret}) \times \breve{e}}{c_0 |\breve{r}-\breve{r}'|} \right] d^3\breve{r}'$$

$$\breve{E} = \frac{1}{4\pi\varepsilon_0} \int \left[\frac{\rho_e(\breve{r}', t_{ret})\breve{e}}{|\breve{r}-\breve{r}'|^2} + \frac{c_0 \dot{\rho}_e(\breve{r}', t_{ret})\breve{e} - \dot{\breve{j}}(\breve{r}', t_{ret})}{c_0^2 |\breve{r}-\breve{r}'|} \right] d^3\breve{r}'$$

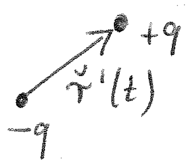
Nahfeld: für $r \to 0$ $t_{ret} \to t$ geht dies in alte ~~stationäre~~ "instantane" Formeln über

Fernfeld, Strahlung $\propto$ Zeitableitungen von Ladungs- und Stromdichten



$\rightarrow$ maximale Ableitungen $\propto \frac{1}{r}$

4.2. Anwendung: Der nicht-relativistische, schwingende Dipol



$r' \ll r$

$$\rho_e(\breve{r}', t) = -q\,\delta(\breve{r}') + q\,\delta(\breve{r}'(t) - \breve{r}')$$

$$\Rightarrow \quad q_{tot} = \int \rho_e(\breve{r}')\, d^3\breve{r}' = 0$$

$$\breve{p}(t) = \int \rho_e(\breve{r}')\, \breve{r}'\, d^3\breve{r}' = q\breve{r}'(t)$$

$$\breve{j}(\breve{r}', t) = \dot{\breve{r}}'(t)\, q\, \delta(\breve{r}'(t) - \breve{r}')$$

$$\Rightarrow \quad \int \breve{j}(\breve{r}', t)\, d^3\breve{r}' = q\dot{\breve{r}}'(t) = \dot{\breve{p}}(t)$$

$$\Rightarrow \boxed{\vec{A}(\vec{r},t) \simeq \frac{\mu_0}{4\pi} \frac{\dot{\vec{p}}(t-r/c_0)}{r}}$$

Für $\phi(\vec{r},t)$ müssen wir das Argument unter dem Integral, $\frac{\rho_e(\vec{r}\,',t-|\vec{r}-\vec{r}\,'|/c_0)}{|\vec{r}-\vec{r}\,'|}$, nach $\vec{r}\,'$ entwickeln:

$$\frac{\rho_e(\vec{r}\,',t-|\vec{r}-\vec{r}\,'|/c_0)}{|\vec{r}-\vec{r}\,'|} = \frac{\rho_e(\vec{r}\,',t-r/c_0)}{r} + \frac{\rho_e(\vec{r}\,',t-r/c_0)}{r^3}\vec{r}\cdot\vec{r}\,'$$

$$+ \frac{\dot{\rho}_e(\vec{r}\,',t-r/c_0)}{c_0 r}\frac{\vec{r}\cdot\vec{r}\,'}{r}$$ + höhere Zeitableitungen die durch Faktoren $\left(\frac{v}{c_0}\right)^n$, $n\geq 2$, unterdrückt sind

Integration über $\vec{r}\,'$ liefert dann $(t_{ret} := t - r/c_0)$:

$$\boxed{\phi(\vec{r},t) \simeq \frac{1}{4\pi\varepsilon_0}\frac{[\vec{p}(t_{ret}) + (r/c_0)\dot{\vec{p}}(t_{ret})]\cdot\vec{r}}{r^3}}$$

da $\int \rho_e(\vec{r}\,',t_{ret})\,\vec{r}\,' d^3\vec{r}\,' = \vec{p}(t_{ret})$

$\int \dot{\rho}_e(\vec{r}\,',t_{ret})\,\vec{r}\,' d^3\vec{r}\,' = \dot{\vec{p}}(t_{ret})$

$$\Rightarrow B_x(\vec{r},t) = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = \frac{\mu_0}{4\pi}\Big[-\frac{y}{r^3}\dot{p}_z(t_{ret}) + \frac{z}{r^3}\dot{p}_y(t_{ret})$$

$$-\frac{y}{c_0 r^2}\ddot{p}_z(t_{ret}) + \frac{z}{c_0 r^2}\ddot{p}_y(t_{ret})\Big]$$

$$\Rightarrow \vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi}\frac{[\dot{\vec{p}}(t_{ret}) + (r/c_0)\ddot{\vec{p}}(t_{ret})]\times\vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi}\Bigg[\underbrace{\frac{\dot{\vec{p}}(t_{ret})\times\vec{e}_r}{r^2}}_{\text{Nahfeld} \propto \frac{1}{r^2}} + \underbrace{\frac{\ddot{\vec{p}}(t_{ret})\times\vec{e}_r}{c_0 r}}_{\text{Strahlung} \propto \frac{1}{r}}\Bigg]$$

$$\vec E(\vec r,t) = -\vec\nabla\phi - \frac{\partial \vec A}{\partial t}$$

Definiere zur Abkürzung

$$\vec p^{\,*}(t) := \vec p(t-r/c_0) + \frac{r}{c_0}\dot{\vec p}(t-r/c_0) = \vec p(t_{ret}) + \frac{r}{c_0}\dot{\vec p}(t_{ret})$$

$$\Rightarrow \vec\nabla\phi = \frac{1}{4\pi\varepsilon_0 r^3}\Big[\vec p^{\,*}(t) - \frac{3(\vec p^{\,*}(t)\cdot\vec r)\vec r}{r^2} + \cancel{\dot{\vec p}(t_{ret})\cdot\vec r\left(-\frac{\vec r}{c_0^2 r}\right)}$$

$$+ \cancel{\dot{\vec p}(t_{ret})\cdot\vec r\left(\frac{\vec r}{c_0 r}\right)} + \ddot{\vec p}(t_{ret})\cdot\vec r\left(-\frac{\vec r}{c_0^2}\right)\Big]$$

$$= \frac{1}{4\pi\varepsilon_0 r^3}\left[\vec p^{\,*}(t) - \frac{3(\vec p^{\,*}(t)\cdot\vec r)\vec r}{r^2} - \frac{(\ddot{\vec p}(t_{ret})\cdot\vec r)\vec r}{c_0^2}\right]$$

$$\frac{\partial\vec A}{\partial t} = \frac{1}{4\pi\varepsilon_0 c_0^2 r}\ddot{\vec p}(t_{ret})$$

Da $\dfrac{(\ddot{\vec p}(t_{ret})\cdot\vec r)\vec r}{r^3} - \dfrac{\ddot{\vec p}(t_{ret})}{r} = \dfrac{\vec r\times(\vec r\times\ddot{\vec p}(t_{ret}))}{r^3}$ hat man

$$\vec E(\vec r,t) = \frac{1}{4\pi\varepsilon_0 r^3}\left(3(\vec p^{\,*}(t)\cdot\vec e_r)\vec e_r - \vec p^{\,*}(t)\right) \rightarrow \text{Nahfeld eines Dipols} \propto \frac{1}{r^3}$$

$$+ \frac{\mu_0}{4\pi r}\vec e_r\times(\vec e_r\times\ddot{\vec p}(t_{ret})) \rightarrow \text{Strahlungsfeld} \propto \frac{1}{r}$$

Zusammenfassung:

Das Strahlungsfeld des nicht-relativistischen schwingenden Dipols ist

$$\vec E(\vec r,t) = \frac{\mu_0}{4\pi r}\vec e_r\times\left[\vec e_r\times\ddot{\vec p}(t-r/c_0)\right]$$

$$\vec B(\vec r,t) = \frac{\mu_0}{4\pi c_0 r}\ddot{\vec p}(t-r/c_0)\times\vec e_r$$

bestimmt durch Beschleunigung der Ladungen