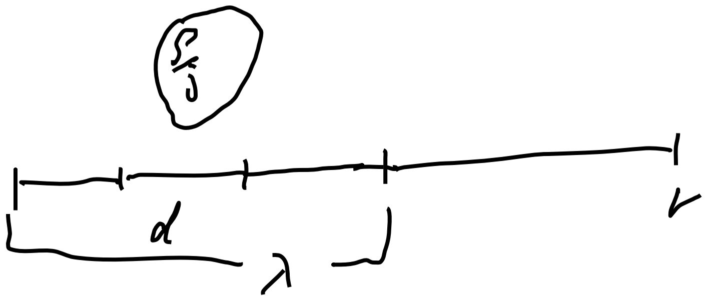
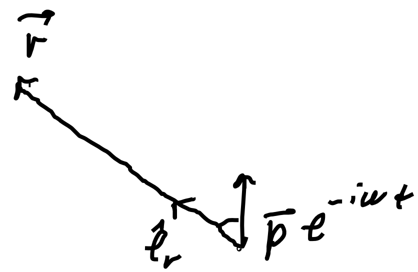
Hertz'scher Dipol.



Strahlfeld: $r \gg \lambda \gg d$.

$$\vec{B}(\vec{r},t) = \frac{k^2}{4\pi\varepsilon_0 c}\left(\hat{e}_r \times \vec{p}\right)\frac{e^{i(kr-\omega t)}}{r}$$

$$\vec{E}(\vec{r},t) = \frac{k^2}{4\pi\varepsilon_0}\left(\hat{e}_r \times \vec{p}\right)\times \hat{e}_r\,\frac{e^{i(kr-\omega t)}}{r} = c\vec{B}\times\hat{e}_r$$



Poynting-Vektor $\quad \vec{S} = \vec{E}\times\vec{H} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

$T = \frac{2\pi}{\omega}$

$$\langle\vec{S}\rangle_T = \frac{1}{T}\int_0^T \vec{S}(\vec{r},t)\,dt = \frac{1}{2\mu_0}\mathrm{Re}\left(\vec{E}\times\vec{B}^*\right).$$

Physikalisch: $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})\cos(\omega t - kr)$, $\quad \vec{B}(\vec{r},t) = \vec{B}(\vec{r})\cos(\omega t - kr)$.

$$\frac{1}{T}\int_0^T \cos^2(\omega t - kr)\,dt = \frac{1}{2}.$$

$$\vec{E}(\vec{r}) = c\cdot\vec{B}(\vec{r})\times\hat{e}_r\,, \qquad [\vec{B}(\vec{r},t) = \vec{B}(\vec{r})\, e^{i(kr-\omega t)}]\,,\ \vec{B}(\vec{r})\in\mathbb{R}$$

$$\vec{E}\times\vec{B}^* = c\,[\vec{B}(\vec{r})\times\hat{e}_r]\times\vec{B}(\vec{r}) = -c\,\vec{B}(\vec{r})\times[\vec{B}(\vec{r})\times\hat{e}_r] =$$

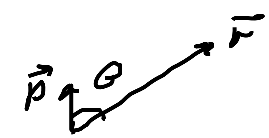
$$= -c\,\{\vec{B}(\vec{B}\cdot\hat{e}_r) - \hat{e}_r\vec{B}^2\} = c\,\{\hat{e}_r\vec{B}^2(\vec{r}) - \vec{B}(\vec{r})(\vec{B}(\vec{r})\cdot\hat{e})\}$$

$$\vec{B}(r) = \frac{k^2}{4\pi\varepsilon_0 c}\,\frac{\hat{e}_r\times\vec{p}}{r}$$

$$\hat{e}_r\cdot\vec{B}^2 = \frac{k^4}{(4\pi\varepsilon_0 c)^2}\,\frac{(\hat{e}_r\times\vec{p})^2}{r^2}\,\hat{e}_r$$

$$\vec{B}(\vec{r})\cdot(\vec{B}(\vec{r})\cdot\hat{e}_r) = \frac{k^4}{(4\pi\varepsilon_0 c)^2}\cdot\frac{\overbrace{(\hat{e}_r\times\vec{p})\cdot\hat{e}_r}^{=0}}{r^2}\cdot(\hat{e}_r\times\vec{p}) = 0\,.$$

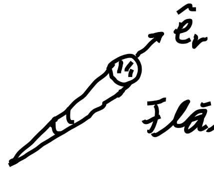
$$c\,[\hat{e}_r\vec{B}^2 - (\vec{B}\cdot\hat{e}_r)\cdot\vec{B}] = \frac{k^4}{16\pi^2\varepsilon_0^2 c\, r^2}\,(\hat{e}_r\times\vec{p})^2\,\hat{e}_r = \Big|$$



$$= \frac{k^4}{16\pi^2\varepsilon_0^2 c}\,\frac{|\vec{p}|^2\sin^2\theta}{r^2}\,\hat{e}_r$$

$$\langle\vec{S}(\vec{r})\rangle = \frac{k^4|\vec{p}|^2\,\hat{e}_r}{32\pi^2\varepsilon_0^2\mu_0 c\, r^2}\sin^2\theta = \left|\begin{array}{l}\varepsilon_0\mu_0 = \frac{1}{c^2}\\ k = \frac{\omega}{c}\end{array}\right| = \frac{\mu_0 p^2\omega^4}{32\pi^2 c}\,\frac{\sin^2\theta}{r^2}\,\hat{e}_r$$

Die Leistung in Raumwinkel.



Fläche $dA = r^2 \cdot d\Omega$

$$\frac{dP(\Omega)}{d\Omega} = r^2 \langle \vec{S} \cdot \hat{e}_r \rangle$$

$$\frac{dP(\Omega)}{d\Omega} = \frac{\mu_0 p^2 \omega^4}{32\pi^2 c} \sin^2\theta .$$

Volle ausgestrahlte Leistung.

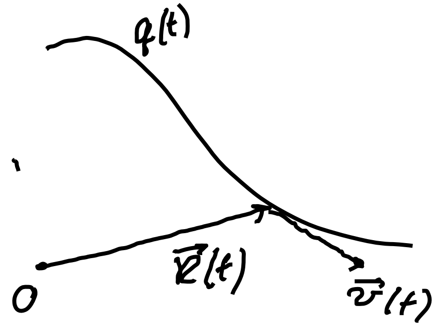
$$P = \int \langle \vec{S} \rangle \cdot \vec{n}\, da = \frac{\mu_0 p^2 \omega^4}{32\pi^2 c} \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta \cdot \sin^2\theta =$$

$$= \frac{\mu_0 p^2 \omega^4}{32\pi^2 c}\, 2\pi \int_0^{\pi} (1-\cos^2\theta) \sin\theta\, d\theta =$$

$$= \frac{\mu_0 p^2 \omega^4}{16\pi c} \left\{ -\cos\theta \Big|_0^{\pi} + \frac{\cos^3\theta}{3} \Big|_0^{\pi} \right\} = \frac{\mu_0 p^2 \omega^4}{12\pi c} .$$

$$\left\langle -\frac{\partial w}{\partial t} \right\rangle_T = \langle P \rangle_T = \frac{\mu_0 p^2 \omega^4}{12\pi c} \sim \omega^4 \leftarrow$$ (blauen Himmel).

Bewegende Punktladung



die Bahn $\vec{R}(t)$: $\vec{E}(\vec{r},t),\ \vec{B}(\vec{r},t)$ - ?

$\vec{A}(\vec{r},t),\ \varphi(\vec{r},t)$ - ?

$$\rho(\vec{r}) = q\,\delta(\vec{r} - \vec{R}(t)).$$

$$\vec{j}(\vec{r}) = q \cdot \vec{v}(t)\,\delta(\vec{r} - \vec{R}(t))$$

EM - Potentiale :

$$\varphi(\vec{r},t) = \frac{1}{\varepsilon_0} \int d^3\vec{r}\,' \int dt'\, G^R(\vec{r}-\vec{r}\,', t-t')\,\rho(\vec{r}\,'; t') =$$

$$= \frac{1}{4\pi\varepsilon_0} \int d^3\vec{r}\,' \int dt'\, \frac{\delta\left(t - t' - \frac{|\vec{r}-\vec{r}\,'|}{c}\right)}{|\vec{r}-\vec{r}\,'|}\, q\,\delta(\vec{r}\,' - \vec{R}(t)) =$$

$$= \frac{q}{4\pi\varepsilon_0} \int dt'\, \frac{\delta\left(t' - \left(t - \frac{1}{c}|\vec{r} - \vec{R}(t')|\right)\right)}{|\vec{r} - \vec{R}(t')|}$$

$$\delta(f(t')) = \delta\left(t' + \frac{1}{c}|\vec{r} - \vec{R}(t')| - t\right)$$

$$f(t') = \frac{1}{c}(\vec{r} - \vec{R}(t'))| + t' - t.$$

$$\delta(f(t')) = \sum_{j=1}^{n} \frac{\delta(t'-t_j)}{\left|\frac{df}{dt'}\right|_{t'=t_j}}$$

$$f(t_j) = 0$$

$$\frac{df}{dt'} = 1 + \frac{1}{c}\frac{d}{dt'}|\vec{r}-\vec{R}(t')| = 1 - \frac{1}{c}\frac{(\vec{r}-\vec{R}(t'))}{|\vec{r}-\vec{R}(t')|}\cdot\vec{v}(t')$$

$$\frac{df}{dt'} = 1 - \frac{1}{c}\hat{e}_{\vec{r}-\vec{R}(t')}\cdot\vec{v}(t') \quad , \quad |\vec{v}| < c$$

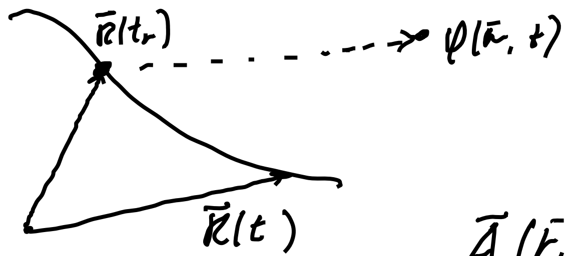
$$\Rightarrow \quad 1 - \frac{1}{c}|\vec{v}| \leq \frac{df}{dt'} << 1 + \frac{1}{c}|\vec{v}| \Rightarrow \frac{df}{dt} > 0$$

$\rightarrow$ $f(t)$ – höchstens eine 0-Stelle:

$$f(t') = 0 \Rightarrow t' = t_r = t - \frac{1}{c}|\vec{r}-\vec{R}(t')|$$

$$\varphi(\vec{r},t) = \frac{q}{4\pi\varepsilon_0} \frac{1}{|\vec{r}-\vec{R}(t_r)|\,\left|1-\frac{1}{c}\frac{(\vec{r}-\vec{R}(t_r))\cdot\vec{v}(t_r)}{|\vec{r}-\vec{R}(t_r)|}\right|} =$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{|\vec{r}-\vec{R}(t_r)-\frac{1}{c}(\vec{r}-\vec{R}(t_r))\cdot\vec{v}(t_r)|}$$



$$\vec{A}(\vec{r},t) = \frac{\mu_0\, q\, \vec{v}(t_r)}{4\pi\,|\vec{r}-\vec{R}(t_r)-\frac{1}{c}(\vec{r}-\vec{R}(t_r))\cdot\vec{v}(t_r)|}.$$

<u>Lienard - Wiechert - Potentiale.</u>

$$t' = t - \frac{1}{c}|\vec{r}-\vec{R}(t')| \longmapsto t'(t) \to t_r = t - \frac{1}{c}|\vec{r}-\vec{R}[t'(t)]|.$$

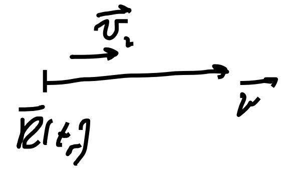
$$\Downarrow$$

$$\underline{t_r(t)}$$

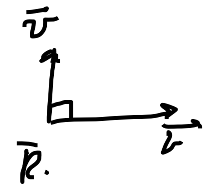
$$\varphi(\vec r, t) = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{|\vec r - \vec R(t_r)|\underbrace{\left[1 - \frac{\vec v(t_r)\cdot\hat e_r}{c}\right]}_{\text{Verkürzung des Abstandes}}}\,.$$

$$\vec D_r(\vec r, t) \equiv \vec r - \vec R(t_r) \text{ – retardierte Abstandsvektor}$$

$$\hat e_r = \frac{\vec D_r}{|\vec D_r|}, \qquad \varkappa = 1 - \frac{1}{c}\,\hat e_r\cdot\vec v$$



$\varkappa = 1 - \frac{v}{c}$



$\varkappa = 1.$

$$\varphi(\vec r, t) = \frac{q}{4\pi\varepsilon_0 |\vec D_r|\cdot\varkappa}$$

$$\vec A = \frac{\mu_0 q \vec v}{4\pi\,|\vec D_r|\cdot\varkappa}$$

Kovariante Maxwell-Gln.

4-Vektor : $X^\mu = (x^0, \underbrace{x^1, x^2, x^3}_{\vec{r}})^T$, $x^0 \equiv ct$

$X_\mu = (ct, -\vec{r})$

$|\vec{X}|^2 = X_\mu X^\mu = c^2 t^2 - r^2 = s^2$

Metr. Tensor : $g_{\mu\nu} = g^{\mu\nu} = \begin{bmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{bmatrix}$

$X_\mu = g_{\mu\nu} x^\nu$

4 - Gradient: $\quad \partial_\alpha \equiv \dfrac{\partial}{\partial x^\alpha}$

$$\partial_0 = \frac{1}{c}\frac{\partial}{\partial t} \qquad \partial^\alpha \equiv \frac{\partial}{\partial x_\alpha}$$

$$\partial_1 = \frac{\partial}{\partial x^1}, \quad \partial^1 = -\frac{\partial}{\partial x}$$

$$\partial_\alpha \cdot \partial^\alpha = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = -\Box$$

$$j^\alpha = (c\rho, j_x, j_y, j_z) \equiv (c\rho, \vec{j})$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

$$\underline{\partial_\alpha \cdot j^\alpha = 0}$$

$$\{\varphi, \vec{A}\} \longmapsto A^{\alpha} = \{\tfrac{1}{c}\varphi, \vec{A}\}.$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\varphi = -\frac{1}{\varepsilon_0}\rho$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\mu_0 \vec{j}$$

$$\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t} = 0$$

$$\Box = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = -\partial_\alpha \partial^\alpha$$

$$\Box A^{\alpha} = \frac{1}{c^2\varepsilon_0} j^{\alpha}$$

$$\begin{cases} \Box A^{\alpha} = \mu_0 j^{\alpha}, & \alpha = 0, 1, 2, 3 \\ \partial_\alpha A^\alpha = 0 \end{cases}$$