Hertz'scher Dipol

$$\vec{A}(\vec{r},t) = -i\,\frac{k}{4\pi\varepsilon_0 c}\,\frac{e^{i(kr-\omega t)}}{r}\,\vec{p}$$

$$\vec{B} = \nabla\times\vec{A}, \qquad \nabla\times\vec{B} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \longmapsto \vec{E}$$

$$\vec{B}(\vec{r},t) = \nabla\times\vec{A}(\vec{r},t) = i\,\frac{k}{4\pi\varepsilon_0 c}\,\vec{p}\times\nabla\left(\frac{e^{i(kr-\omega t)}}{r}\right) =$$

$$\left[\ \nabla f(r) = \frac{\partial f}{\partial r}(\nabla r) = \frac{\partial f}{\partial r}\,\hat{e}_r\ \right]$$

$$= i\,\frac{k}{4\pi\varepsilon_0 c}\,(\vec{p}\times\hat{e}_r)\,\frac{\partial}{\partial r}\left(\frac{e^{i(kr-\omega t)}}{r}\right) =$$

$$= \frac{k^2}{4\pi\varepsilon_0 c}\,\frac{e^{i(kr-\omega t)}}{r}\left(1-\frac{1}{ikr}\right)(\hat{e}_r\times\vec{p}).$$

$$\left[\ \frac{\partial}{\partial r}\left(\frac{e^{i(kr-\omega t)}}{r}\right) = \left(\frac{ik}{r}-\frac{1}{r^2}\right)e^{i(kr-\omega t)}\ \right].$$

$$\vec{B}(r,t) = \frac{k^2}{4\pi\varepsilon_0 c}\,\frac{e^{i(kr-\omega t)}}{r}\left(1-\frac{1}{ikr}\right)\hat{e}_r\times\vec{p}.$$

$\vec{E}$- Feld : $\left(\vec{E}(\bar{r},t) = \vec{E}(\bar{r})\,e^{-i\omega t}\right)$.

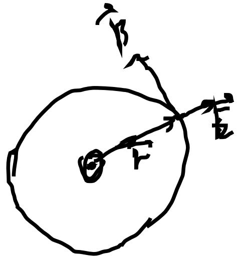
$$\nabla\times\vec{B} = \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E}, \qquad \frac{\partial\vec{E}}{\partial t} = -i\omega\vec{E}$$

$$\nabla\times\vec{B} = -\frac{i\omega}{c^2}\vec{E} = -\frac{ik}{c}\vec{E} = \frac{k}{ic}\vec{E} \qquad \left(\frac{\omega}{c}=k\right).$$

$$\vec{E}(\bar{r},t) = \frac{ic}{k}\nabla\times\vec{B} = \frac{ic}{k}\cdot\frac{ik}{4\pi\varepsilon_0 c}\nabla\times\left[(\bar{p}\times\hat{e}_r)\frac{\partial}{\partial r}\left(\frac{e^{i(kr-\omega t)}}{r}\right)\right] =$$

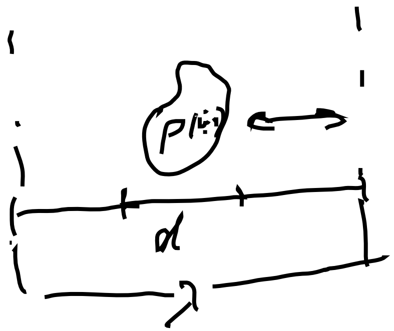
$$= -\frac{1}{4\pi\varepsilon_0}\nabla\times\left[(\bar{p}\times\hat{e}_r)\frac{\partial}{\partial r}\left(\frac{e^{i(kr-\omega t)}}{r}\right)\right].$$

$$\vec{E}(\bar{r},t) = \frac{1}{4\pi\varepsilon_0}\,\frac{e^{i(kr-\omega t)}}{r}\Big\{k^2(\hat{e}_r\times\vec{p})\times\hat{e}_r + \\ + \frac{1}{r}\left(\frac{1}{r}-ik\right)\left[3\,\hat{e}_r(\hat{e}_r\cdot\vec{p})-\vec{p}\right]\Big\}.$$



$$\vec E(\vec r, t) = \vec E(\vec r)\, e^{-i\omega t}$$
$$\vec B(\vec r, t) = \vec B(\vec r)\, e^{-i\omega t}$$

1) Nahfeld $(kr \ll 1 \iff r \ll \lambda)$



$$\vec E(\vec r) \propto \left\{ \frac{k^2 r^2}{r^2} (\hat e_r \times \vec p) \times \hat e_r + \frac{1}{r^2}(1 - ikr)\left[3\hat e_r(\hat e_r \cdot \vec p) - \vec p\right] \right\} =$$
$$\approx \frac{1}{r^2}\left[3\hat e_r(\hat e_r \cdot \vec p) - \vec p\right].$$

$$\frac{e^{ikr}}{r} \approx \frac{1+ikr}{r} \approx \frac{1}{r} \quad (kr \ll 1)$$

$$\vec E(\vec r) \cong \frac{1}{4\pi\varepsilon_0}\,\frac{3\hat e_r(\hat e_r \cdot \vec p) - \vec p}{r^3}.$$

$$\vec B(\vec r) = \frac{1}{4\pi\varepsilon_0 c}\,\frac{ik(\hat e_r \times \vec p)}{r^2} = \frac{i\omega}{4\pi\varepsilon_0 c^2}\,\frac{(\hat e_r \times \vec p)}{r^2} = \frac{i\omega\mu_0}{4\pi}\,\frac{(\hat e_r \times \vec p)}{r^2}$$

$$\vec p(t) = \vec p\, e^{-i\omega t} \Rightarrow \frac{\partial \vec p}{\partial t} = -i\omega \vec p\, e^{-i\omega t} \longleftarrow \vec j : \quad \vec B(\vec r) = \frac{\mu_0}{4\pi}\,\frac{(\hat e_r \times \vec j)}{r^2}.$$

$$\underline{\vec j = -i\omega \vec p}$$

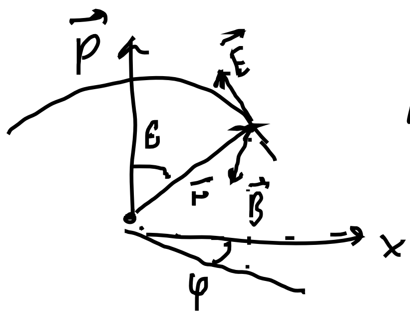
2) Strahlungsfeld $(kr \gg 1) \Leftrightarrow (r \gg \lambda)$.

$$\begin{cases} \vec{E}(\vec{r}) \cong \dfrac{1}{4\pi\varepsilon_0} k^2 (\hat{e}_r \times \vec{p}) \times \hat{e}_r \dfrac{e^{ikr}}{r} \\ \vec{B}(\vec{r}) \cong \dfrac{k^2}{4\pi\varepsilon_0 c} (\hat{e}_r \times \vec{p}) \dfrac{e^{ikr}}{r}. \end{cases}$$

$$\vec{E}(\vec{r}) = c\, \vec{B}(\vec{r}) \times \hat{e}_r$$

$\vec{E} \perp \vec{B}$, $\vec{E} \perp \hat{e}_r$

$\vec{B} \perp \hat{e}_r$, $\vec{B} \perp \vec{p}$



$\vec{E} \parallel \hat{e}_\theta$. $\vec{B} \parallel \hat{e}_\varphi$

$|E| \propto \sin(\widehat{\hat{e}_r, p}) = \sin\theta$

$|B| \propto \sin\theta$

Poynting - Vektor für den

Hertz'schen Dipol.

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad Re(\vec{E}) \propto Re(e^{-i\omega t}) = \cos\omega t$$

$$\langle \vec{S} \rangle_{T=\frac{2\pi}{\omega}} = \frac{1}{T} \int_0^T \vec{S}(\vec{r},t)\,dt = \frac{1}{2\mu_0} Re(\vec{E} \times \vec{B}^*)$$

$$Re\,\vec{E}(r,t) \propto \cos(kr - \omega t) \leftarrow \left[e^{-i(kr-\omega t)} \right]$$

$$Re\,\vec{B}(r,t) \propto \cos(kr - \omega t)$$

$$\frac{1}{T} \int_0^T \left(Re\,\vec{E}\right) \times \left(Re\,\vec{B}\right) dt \propto \frac{1}{T} \int_0^T \cos^2(kr - \omega t)\,dt =$$

$$= \frac{1}{T} \int_0^T \frac{1 + \cos[2(kr-\omega t)]}{2}\,dt = \frac{1}{T}\Big(\frac{1}{2}T + \underbrace{\frac{1}{2}\int_0^T \cos[2(kr-\omega t)]\,dt}_{=0} \Big) =$$

$$= \frac{1}{2}$$