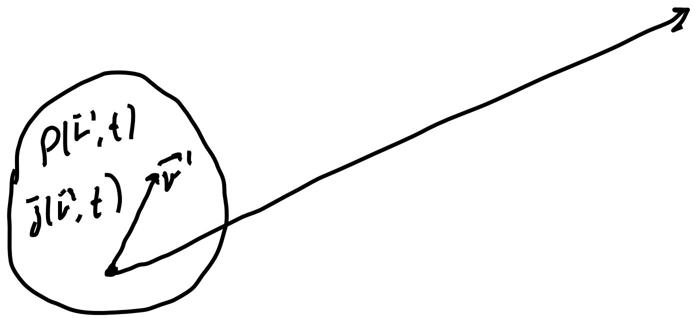
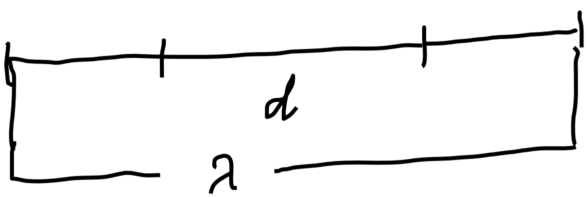
Strahlung

$\vec{E}(\vec{r},t)$
$\vec{B}(\vec{r},t)$



$$\rho(\vec{r},t) = \rho(\vec{r})\, e^{-i\omega t}$$

$$\vec{j}(\vec{r},t) = \vec{j}(\vec{r})\, e^{-i\omega t}$$



$$\frac{\partial \rho}{\partial t} + \nabla\cdot\vec{j} = 0 \quad \Rightarrow \quad \nabla\cdot\vec{j} = i\omega\rho$$

$$\left(\frac{\partial \rho}{\partial t} = -i\omega\rho\right)$$

$\int_V \rho(\vec{r})\, d^3r = 0$ - keine Monopolladung

$$\varphi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\, \frac{\rho(\vec{r}', t_r)}{|\vec{r}-\vec{r}'|}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int d^3r'\, \frac{\vec{j}(\vec{r}', t_r)}{|\vec{r}-\vec{r}'|}, \qquad t_r = t - \frac{|\vec{r}-\vec{r}'|}{c}.$$

$$\vec{j}(\vec{r}, t_r) = \vec{j}(\vec{r})\, e^{-i\omega t}\, e^{ik|\vec{r}-\vec{r}'|}$$

$$\rho(\vec{r}, t_r) = \rho(\vec{r})\, e^{-i\omega t}\, e^{ik|\vec{r}-\vec{r}'|}, \qquad k = \frac{\omega}{c}$$

$k = \frac{2\pi}{\lambda}$, $\quad \lambda = \frac{2\pi}{k}$ – die Wellenlänge.

$$\underline{\lambda \gg d} \iff \frac{2\pi}{k} = \frac{2\pi c}{\omega} \iff cT \gg d$$

$$\frac{d}{T} \ll c$$

Bedingungen: $\quad r \gg d \Rightarrow r \gg r'$

$\lambda \gg d \Rightarrow \lambda \gg r'$

$\Rightarrow$ Multipolentwicklung

Hier – nur Dipolnäherung

Dipolnäherung.

$$|\vec{r}-\vec{r}'| = \sqrt{r^2+r'^2-2\vec{r}\cdot\vec{r}'} = r\sqrt{1-2\frac{\vec{r}\cdot\vec{r}'}{r^2}+\underbrace{\frac{r'^2}{r^2}}_{\approx 0}} \approx$$

$$\approx r\left(1-\frac{\vec{r}\cdot\vec{r}'}{r^2}\right) = r - \frac{\vec{r}}{|r|}\cdot\vec{r}' = r - \hat{e}_r\cdot\vec{r}'.$$

Skalarpotential:

$$\varphi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int d^3\vec{r}'\,\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\,e^{-i\omega\left(t-\frac{|\vec{r}-\vec{r}'|}{c}\right)} \approx$$

$$\approx \frac{1}{4\pi\varepsilon_0}\,e^{-i\omega t}\int d^3\vec{r}'\,\frac{\rho(\vec{r}')\,e^{ik(r-\hat{e}_r\cdot\vec{r}')}}{r-\hat{e}_r\cdot\vec{r}'} =$$

$$= \frac{1}{4\pi\varepsilon}\,e^{i(kr-\omega t)}\int d^3\vec{r}'\,\frac{\rho(\vec{r}')\,e^{-ik\,\hat{e}_r\cdot\vec{r}'}}{r\left(1-\frac{\hat{e}_r\cdot\vec{r}'}{r}\right)} \approx$$

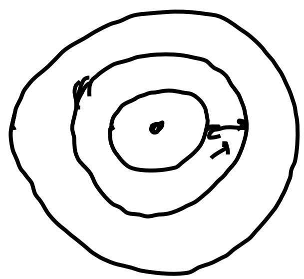
$$= \left|\lambda \gg r' \Rightarrow k\,\hat{e}_r\cdot\vec{r}' \leq \frac{2\pi}{\lambda}r' \ll 1\right| \approx \longmapsto$$

$$\mapsto \approx \frac{1}{4\pi\varepsilon_0}\,\frac{e^{i(kr-\omega t)}}{r}\int d^3\bar{r}'\,\rho(\bar{r}')\left(1+\frac{\hat{e}_r\cdot\bar{r}'}{r}\right)\left(1-ik\,\hat{e}_r\cdot\bar{r}'\right) =$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{e^{i(kr-\omega t)}}{r}\left\{\underbrace{\int d^3\bar{r}'\,\rho(\bar{r}')}_{\overset{\shortparallel}{0}\ (\text{monopol})} + \int d^3\bar{r}'\,\rho(\bar{r}')(\hat{e}_r\cdot\bar{r}')\left(\frac{1}{r}-ik\right)+\dots\right\}$$

$\int d^3\bar{r}'\,\rho(\bar{r}')\,(\bar{r}'\cdot\hat{e}_r) = p_r$ – Projektion von $\bar{p}$ auf die Richtung $\bar{r}$.

$$\frac{e^{i(kr-\omega t)}}{r} \longleftrightarrow$$



$\rightarrow$ Kugelwelle

Hertz'scher Dipol

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\,\frac{e^{i(kr-\omega t)}}{r}\int d^3\vec{r}\,'\,\frac{e^{-ik\,\hat{e}_r\cdot\vec{r}\,'}}{\left|1-\frac{\hat{e}_r\cdot\vec{r}\,'}{r}\right|}\,\vec{j}(\vec{r}\,') \approx$$

$$\approx \frac{\mu_0}{4\pi}\,\frac{e^{i(kr-\omega t)}}{r}\int d^3\vec{r}\,'\,\vec{j}(\vec{r}\,')$$

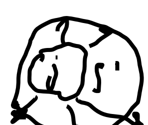
$\underline{\int d^3\vec{r}\,'\,\vec{j}(\vec{r}\,') \;-\; ?}$

$$\nabla\cdot(x_i\vec{j}(\vec{r})) = x_i(\nabla\cdot\vec{j}) + \vec{j}\cdot(\nabla x_i) = x_i(\nabla\cdot\vec{j}) + \underbrace{\hat{e}_i\cdot\vec{j}}_{j_i}$$

$$\underline{j_i = \nabla\cdot(x_i\vec{j}) - x_i(\nabla\cdot\vec{j})}$$

$$\int d^3\vec{r}\,'\,j_i(\vec{r}\,') = \int d^3\vec{r}\,'\,\nabla\cdot(x_i'\vec{j}) - \int d^3\vec{r}\,'\,x_i'(\nabla\cdot\vec{j}) =$$

$$= \int_{\partial V'}(x_i'\vec{j})\cdot\vec{n}\,dS' \xrightarrow[S'\to\infty]{} 0 \quad - \int d^3\vec{r}\,'\,x_i'(\nabla\cdot\vec{j}).$$



Somit

$$\int_{V'} d^3\vec{r}\,'\,\vec{j}(\vec{r}\,') = -\int d^3\vec{r}\,'\,\vec{r}\,'(\nabla\cdot\vec{j}) = \left|\nabla\cdot\vec{j} = -\frac{\partial\rho}{\partial t} = i\omega\rho\right| =$$

$$= \int d^3\vec{r}\,'\,\vec{r}\,'\cdot\frac{\partial\rho(\vec{r}\,',t)}{\partial t} = -i\omega\underbrace{\int d^3\vec{r}\,'\,\rho(\vec{r}\,')\cdot\vec{r}\,'}_{\vec{p}\text{ - Dipolmoment.}}$$

Hertz'schen Dipol

$$\vec{A}(\vec{r},t) = -i\,\frac{\omega\mu_0}{4\pi}\,\frac{e^{i(kr-\omega t)}}{r}\,\vec{p}$$

$$\left(\varepsilon_0\mu_0 = \frac{1}{c^2},\ \frac{\omega}{c} = k\right)$$

$$\vec{A}(\vec{r},t) = -i\,\frac{k}{4\pi\varepsilon_0 c}\,\frac{e^{i(kr-\omega t)}}{r}\,\vec{p}.$$

$$e^{i(kr-\omega t)} = \cos(kr-\omega t) + i\sin(kr-\omega t)$$