EM-Wellen im Vakuum.

$$\begin{cases} \Box \vec{E} = 0 \\ \Box \vec{B} = 0 \end{cases}$$

$\rho = 0\,,\quad \vec{j} = 0$

$$\Box = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

$$\frac{1}{c^2} = \varepsilon_0 \mu_0$$

$$\vec{E}(\vec{r},t) = \vec{E}_+(|\vec{r}|, ct) + \vec{E}_-(|\vec{r}|, ct)$$

$$\vec{B}(\vec{r},t) = \vec{B}_+(|\vec{r}|, ct) + \vec{B}_-(|\vec{r}|, ct).$$

1D: $\vec{E}(x,t) = \vec{E}_+(x-ct) + \vec{E}_-(x+ct)$

$$\vec{E}(\vec{r},t) = \int (d^3\vec{k}) \left\{ \vec{E}_+(\vec{k})\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} + \vec{E}_-(\vec{k})\, e^{i(\vec{k}\vec{r} + \omega t)} \right\}.$$

$|\vec{k}| = \frac{\omega}{c}$

$$\vec{B}(\vec{r},t) = \int (d^3\vec{k}) \left\{ \vec{B}_+(\vec{k})\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} + \vec{B}_-(\vec{k})\, e^{i(\vec{k}\vec{r} + \omega t)} \right\}$$

z. B.: $\vec{E}_+(\vec{r},ct) = \int (d^3\vec{k})\, \vec{E}_+(\vec{k})\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$.

$$\nabla\cdot\bar{E}=0\ ,\qquad \nabla\cdot\bar{B}=0$$

$$\nabla\times\bar{E}=-\frac{\partial\bar{B}}{\partial t}\ ,\qquad \nabla\times\bar{B}=\varepsilon_0\mu_0\frac{\partial\bar{E}}{\partial t}.$$

$$\vec{E}_t=\vec{E}_t(\bar{k})\,e^{i(\bar{k}\cdot\bar{r}-\omega t)}$$

$$\bar{B}_t=\bar{B}_t(\bar{k})\,e^{i(\bar{k}\cdot\bar{r}-\omega t)}$$

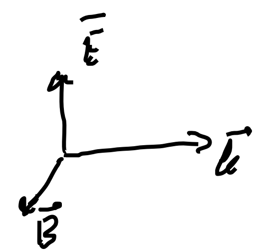
$$\nabla\cdot\left(\vec{E}_k\,e^{i(\bar{k}\cdot\bar{r}-\omega t)}\right)=i\vec{k}\cdot\vec{E}_k\,e^{i(\bar{k}\cdot\bar{r}-\omega t)}$$

$$\nabla\times\left(\bar{E}_k\,e^{i(\bar{k}\cdot\bar{r}-\omega t)}\right)=i\vec{k}\times\bar{E}_k\,e^{i(\bar{k}\cdot\bar{r}-\omega t)}$$

$$\frac{\partial}{\partial t}\left(\bar{E}_k\,e^{i(\bar{k}\cdot\bar{r}-\omega t)}\right)=-i\omega\bar{E}_k\,e^{i(\bar{k}\cdot\bar{r}-\omega t)}$$

$$\nabla\cdot\bar{E}=0 \Rightarrow \vec{k}\cdot\vec{E}_k=0 \qquad\qquad \bar{E}_k\perp\vec{k}$$

$$\nabla\cdot\bar{B}=0 \Rightarrow \vec{k}\cdot\vec{B}_k=0 \qquad \Rightarrow \qquad \vec{B}_k\perp\vec{k}$$



$$\left.\begin{aligned}\nabla\times\bar{E}=-\frac{\partial\bar{B}}{\partial t} &\Rightarrow i\vec{k}\times\bar{E}_k=i\omega\bar{B}_k\\ \nabla\times\bar{B}=\frac{1}{c^2}\frac{\partial\bar{E}}{\partial t} &\Rightarrow i\bar{k}\times\bar{B}_k=-\frac{i\omega}{c^2}\vec{E}_k\end{aligned}\right\}\Rightarrow \vec{E}_k\perp\vec{B}_k$$

$$|\bar{k}|\cdot|\bar{E}|=\omega|\bar{B}|$$

$$|\bar{E}|=\frac{\omega}{|\bar{k}|}|\bar{B}|=c\,|\bar{B}|.$$

Allgemeine Lösung der inhomogenen Wellengleichung.

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G(\bar r - \bar r'; t - t') = -\delta(\bar r - \bar r')\,\delta(t - t')$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) \psi(\bar r, t) = f(r, t), \qquad \psi(\bar r, t) - ?$$

$$f(r,t) = -\int (d\bar r')\int dt'\, f(\bar r'; t')\,\delta(\bar r - \bar r')\,\delta(t - t').$$

$$\psi(\bar r, t) = -\int (d\bar r')\int dt'\, G(\bar r - \bar r'; t - t')\, f(\bar r'; t').$$

F. T. $\quad G(\bar r - \bar r', t - t') = \frac{1}{(2\pi)^4}\int d^3\bar k \int d\omega\, \tilde G(\bar k, \omega)\, e^{i\bar k(\bar r - \bar r')}\, e^{-i\omega(t - t')}.$

$$\delta(\bar r - \bar r') = \frac{1}{(2\pi)^3}\int (dk)\, e^{i\bar k(\bar r - \bar r')}, \qquad \delta(t - t') = \frac{1}{2\pi}\int d\omega\, e^{-i\omega(t - t')}.$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G(\bar r - \bar r'; t - t') = \frac{1}{(2\pi)^4}\int d^3\bar k \int d\omega\, e^{i[\bar k(\bar r - \bar r') - \omega(t - t')]} \times \left(-k^2 + \frac{\omega^2}{c^2}\right)\tilde G(\bar k, \omega) = -\frac{1}{(2\pi)^4}\int d^3\bar k\int d\omega\, e^{i[\bar k(\bar r - \bar r') - \omega(t - t')]}.$$

$\nabla \Leftrightarrow i\vec k \Rightarrow \nabla^2 \Leftrightarrow -k^2$

$\partial_t \Leftrightarrow -i\omega$

$$\left(-k^2 + \frac{\omega^2}{c^2}\right) \hat{G}(\vec{k},\omega) = -1.$$

$$\boxed{\hat{G}(\vec{k},\omega) = \frac{1}{k^2 - \omega^2/c^2}}.$$

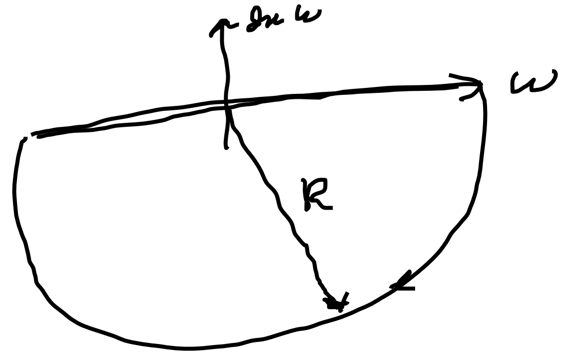
$|\vec{k}| = \pm \frac{\omega}{c} \to$ Singularität $\hat{G}(\vec{k},\omega)$.

Die Reaktion erfolgt nach der Störung.

$$G(t-t') = 0, \quad t < t'$$

$$G(\vec{r}-\vec{r}', t-t') = \int \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}(\vec{r}-\vec{r}')}\, e^{-i\omega(t-t')}}{k^2 - \frac{\omega^2}{c^2}}.$$

$$\int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{k^2 - \omega^2/c^2}$$



1) $t - t' > 0$

$$e^{-i(\omega - iR)(t-t')} = e^{-i\omega(t-t')} \cdot e^{-R(t-t')} \xrightarrow{R\to\infty} 0$$