Elektrodynamik.

$$\begin{cases} \nabla \times \bar{H} = \bar{J} + \dfrac{\partial \bar{D}}{\partial t} \\ \nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t} \\ \nabla \cdot \bar{B} = 0 \\ \nabla \cdot \bar{D} = \rho \end{cases}$$

$$\bar{B} = \mu_0 (\bar{H} + \vec{M}) = \mu_r \mu_0 \vec{H}.$$

$$\bar{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_r \varepsilon_0 \vec{E}.$$

$\rho(\bar{r}, t)$, $\quad \bar{j}(\bar{r}, t)$.

$$\underline{\vec{B} = \nabla \times \vec{A}} \quad \Rightarrow \quad \nabla \cdot \bar{B} = \nabla \cdot (\nabla \times \bar{A}) = 0$$

$$\underline{\nabla \times \bar{E}} = -\frac{\partial \bar{B}}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \bar{A}) = \underline{-\nabla \times \left(\frac{\partial \bar{A}}{\partial t}\right)}$$

$$\underline{\bar{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \varphi}, \qquad \nabla \times (\nabla \varphi) \equiv 0$$

$\bar{A}(\bar{r}, t)$. $\quad \varphi(\bar{r}, t)$.

Eichtransformation.

$$\vec{A}'(\vec{r},t) = \vec{A}(\vec{r},t) + \nabla\chi(\vec{r},t) \qquad [\,\vec{B}' = \vec{B}\,]$$

$$\varphi'(\vec{r},t) = \varphi(\vec{r},t) - \frac{\partial}{\partial t}\chi(\vec{r},t)$$

$$\vec{E}' = -\nabla\varphi' - \frac{\partial\vec{A}'}{\partial t} = -\nabla\varphi + \nabla\left(\frac{\partial}{\partial t}\chi\right) - \frac{\partial\vec{A}}{\partial t} - \frac{\partial}{\partial t}(\nabla\chi) =$$

$$= -\nabla\varphi - \frac{\partial\vec{A}}{\partial t} = \vec{E}.$$

$$\nabla\cdot\vec{B} = 0 \quad \Rightarrow \quad \nabla\cdot(\nabla\times\vec{A}) \equiv 0$$

$$\nabla\times\vec{H} = \vec{j} + \varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \varepsilon_r = 1, \quad \mu_r = 1.$$

$$\vec{B} = \nabla\times\vec{A}, \qquad \vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0\vec{j} + \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\vec{B} = \nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$$

$$\frac{\partial\vec{E}}{\partial t} = -\frac{\partial}{\partial t}\nabla\varphi - \frac{\partial^2\vec{A}}{\partial t^2}$$

$$\nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0\vec{j} - \varepsilon_0\mu_0\left[\frac{\partial^2\vec{A}}{\partial t^2} + \nabla\frac{\partial\varphi}{\partial t}\right].$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \left(-\nabla \varphi - \frac{\partial \bar{A}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \times \bar{A})$$

$$-\underbrace{\nabla \times (\nabla \varphi)}_{= 0} - \nabla \times \left(\frac{\partial \bar{A}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \times \bar{A})$$ – ~~Identität~~.

$$\varepsilon_0 \nabla \cdot \vec{E} = \rho$$

$$\varepsilon_0 \nabla \cdot \left(-\nabla \varphi - \frac{\partial \bar{A}}{\partial t} \right) = \rho$$

$$\nabla^2 \varphi + \frac{\partial}{\partial t} (\nabla \cdot \bar{A}) = -\frac{\rho}{\varepsilon_0}$$

$$\begin{cases} \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} + \varepsilon_0\mu_0 \dfrac{\partial^2\vec{A}}{\partial t^2} + \varepsilon_0\mu_0 \nabla\left(\dfrac{\partial\varphi}{\partial t}\right) = \mu_0\vec{j} \\ \nabla^2\varphi + \dfrac{\partial}{\partial t}(\nabla\cdot\vec{A}) = -\dfrac{\rho}{\varepsilon_0} \end{cases}$$

Mögliche Eichtransformationen

$$\begin{cases} \vec{A}' = \vec{A} + \nabla\chi \\ \varphi' = \varphi - \dfrac{\partial\chi}{\partial t} \end{cases} \qquad (\chi = \chi(\vec{r}, t)).$$

Coulomb-Eichung: $\nabla\cdot\vec{A} = 0$, – nicht Eichinvariant.

$$\nabla\cdot\vec{A}' = \nabla\cdot(\vec{A} + \nabla\chi) = \nabla\cdot\vec{A} + \nabla^2\chi$$

$$\begin{cases} \nabla^2\vec{A} - \varepsilon_0\mu_0 \dfrac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{j} + \varepsilon_0\mu_0 \dfrac{\partial}{\partial t}(\nabla\varphi) \\ \nabla^2\varphi = -\dfrac{\rho}{\varepsilon_0} \end{cases}$$

Lorentz - Eichung

$$\nabla \cdot \vec{A} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\varphi = 0 \quad \Longleftrightarrow \quad \nabla \cdot \vec{A} = -\mu_0 \varepsilon_0 \frac{\partial \varphi}{\partial t}.$$

$$\begin{cases} \nabla \underbrace{\left(\nabla \cdot \vec{A} + \varepsilon_0 \mu_0 \frac{\partial \varphi}{\partial t}\right)}_{0} - \nabla^2 \vec{A} + \varepsilon_0 \mu_0 \frac{\partial^2 \vec{A}}{\partial t^2} = \mu_0 \vec{j} \\ \nabla^2 \varphi + \frac{\partial}{\partial t}(\nabla \cdot \vec{A}) = \nabla^2 \varphi - \mu_0 \varepsilon_0 \frac{\partial^2 \varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0} \end{cases}$$

$$\begin{cases} \nabla^2 \vec{A} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{j} \\ \nabla^2 \varphi - \mu_0 \varepsilon_0 \frac{\partial^2 \varphi}{\partial t^2} = -\frac{1}{\varepsilon_0}\rho \end{cases} \qquad \begin{cases} \Box \varphi = -\frac{\rho}{\varepsilon_0} \\ \Box \vec{A} = -\mu_0 \vec{j} \end{cases}$$

d'Alambert - Operator

$$\Box \equiv \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

$$\frac{1}{c^2} = \mu_0 \varepsilon_0 .$$

Energetische Beziehungen fürs EM-Feld.

$$
-\quad
\begin{array}{l}
\bar{E}\cdot \\[2ex]
\bar{H}\cdot
\end{array}
\left\{
\begin{array}{l}
\nabla\times\bar{H} = \vec{J} + \dfrac{\partial\bar{D}}{\partial t} \\[2ex]
\nabla\times\bar{E} = -\dfrac{\partial\bar{B}}{\partial t}
\end{array}
\right.
$$

$$\bar{E}(\nabla\times\bar{H}) - \bar{H}\cdot(\nabla\times\bar{E}) = \bar{E}\cdot\vec{J} + \bar{E}\,\frac{\partial\bar{D}}{\partial t} + \vec{H}\cdot\frac{\partial\bar{B}}{\partial t}$$

$$(\,\bar{D} = \varepsilon_0\varepsilon_r\bar{E}, \quad \bar{B} = \mu_0\mu_r\bar{H}\,)$$

$$\vec{E}\cdot(\nabla\times\bar{H}) - \bar{H}\,(\nabla\times\bar{E}) = \nabla\cdot(\bar{H}\times\bar{E}) - \nabla\cdot(\bar{E}\times\bar{H}) =$$

$$= -\nabla\cdot(\bar{E}\times\vec{H}) - \nabla\cdot(\vec{E}\times\vec{H}) = -\nabla\cdot(\bar{E}\times\bar{H}) \equiv -\nabla\cdot\vec{S}$$

$$\vec{S} = (\vec{E}\times\vec{H})$$ – der Pointing-Vektor.