Magnetostatik.

$$\nabla^2 \vec{A} = \mu_0 \vec{j} \qquad (\vec{B} = \nabla \times \vec{A}).$$

$$\nabla^2 A_i = \mu_0 j_i \qquad (\text{in kartesischen Koordinaten } i = x, y, z).$$

$$A_i = \frac{\mu_0}{4\pi} \int \frac{j_i(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3r'$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3r'.$$

$$\vec{B} = \nabla \times \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \nabla_r \times \frac{\vec{j}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \, d^3r'$$

$$\nabla_r \times \frac{\vec{j}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} = -\vec{j}(\vec{r}\,') \times \nabla_r \left(\frac{1}{|\vec{r} - \vec{r}\,'|} \right) = \frac{\vec{j}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

$$\left[\vec{\nabla}_R \left(\frac{1}{R} \right) = - \frac{\vec{R}}{R^3} = - \frac{1}{R^2} \cdot \hat{e}_R \right]$$

$$\vec B(\vec r) = \frac{\mu_0}{4\pi}\int_{V'} \frac{\vec j(\vec r')\times(\vec r-\vec r')}{|\vec r-\vec r'|^3}\, d^3r'$$

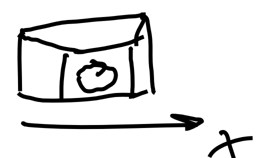
Biot-Savart-Gesetz.

$$\left|\frac{r'}{r}\right| \ll 1.$$

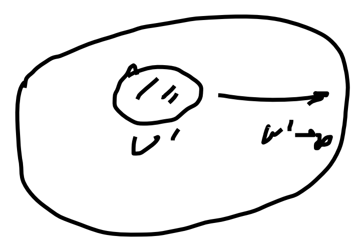
$$\frac{1}{|\vec r-\vec r'|} = \frac{1}{r}\,\frac{1}{\left|1-\frac{\vec r\cdot\vec r'}{r^2}\right|} \simeq \frac{1}{r} + \frac{\vec r\cdot\vec r'}{r^2}$$

$$\vec A(\vec r) \cong \frac{\mu_0}{4\pi}\int_{V'}\left\{\frac{\vec j(\vec r')}{r} + \vec j(\vec r')\cdot\frac{(\vec r\cdot\vec r')}{r^3}\right\} d^3r'$$

1) $\int \frac{\vec j(\vec r')}{r}\, d^3r' = \frac{1}{r}\int_{V'} \vec j(\vec r')\, d^3r' = \frac{1}{r}\int_{\partial V'} \vec j\cdot\vec n\, dS = 0$

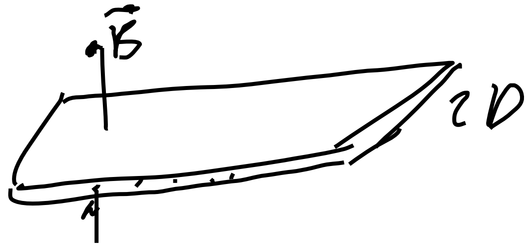


$\int j_x\, dV'$



$j(\vec r'\to\infty) = 0$

$$\boxed{\int \nabla\cdot\vec j\, d^3r = \int \vec j\cdot\vec n\, dS}$$



$$\vec A(\vec r) \approx \frac{\mu_0}{4\pi r^3}\int_{V'} \vec J(\vec r')\cdot(\vec r\cdot\vec r')\, d^3\vec r'$$

$$A_i(\vec r) = \frac{\mu_0}{4\pi r^3}\int_{V} J_i(\vec r')\sum_j r_j r_j'\, d^3\vec r'$$

$\forall\ g(\vec r'),\ f(\vec r')$.

$$\int_{V'} \mathrm{div}\,(g\cdot f\vec J)\, d^3\vec r' = \int_{\partial V'} g f \underbrace{\vec J}_{\to\, 0\ (\vec r'\to\infty)}\cdot\vec n\, dS = 0.$$

$$\nabla\cdot(g f\vec J) = g f(\nabla\cdot\vec J) + \vec J\cdot(g\nabla f + f\nabla g).$$

magnetostatik, $\underline{\nabla\cdot\vec J = 0}$

$\nabla \cdot \vec{J} = 0$ (Magnetostatik)

$$\int_{V'} \operatorname{div}(g f \vec{J}) = \int_{V'} [\, g(\vec{J} \cdot \nabla f) + f(\vec{J} \cdot \nabla g)\,] d^3\bar{r}' = 0.$$

Wähle: $g(\bar{r}') = r_i'$, $f(\bar{r}') = r_j' \Rightarrow$ $\underline{\nabla f = \hat{e}_j, \quad \nabla g = \hat{e}_i}$

$$\int_{V'} [\, r_i'(\vec{J} \cdot \hat{e}_j) + r_j'(\vec{J} \cdot \hat{e}_i)\,] d^3\bar{r}' = \int_{V'} (r_i' J_j + r_j' J_i) d^3\bar{r}' = 0$$

$$\int_{V'} r_j' J_i(\bar{r}') d^3\bar{r}' = - \int_{V'} r_i' J_j(\bar{r}') d^3\bar{r}'$$

$$\int_{V'} J_i(\bar{r}') \sum_j r_j r_j' d^3\bar{r}' = \frac{1}{2} \sum_j r_j \int_{V'} [\, J_i(\bar{r}') r_j' + J_i(\bar{r}') r_j'\,] d^3\bar{r}' =$$

$$= \frac{1}{2} \sum_j r_j \int_{V'} (J_i(\bar{r}') r_j' - J_j(\bar{r}') r_i') d^3\bar{r}' =$$

$$= \frac{1}{2} \sum_j r_j \int_{V'} \sum_k \varepsilon_{kji} r_j' J_i = \frac{1}{2} \int_{V'} d^3\bar{r}' \sum_{k,i} r_j \varepsilon_{kji} r_j' J_i =$$

$$= \frac{1}{2}\int \sum_{k,j} \varepsilon_{ikj} \, [\vec{r}\,' \times \vec{j}\,]_k \cdot r_j \, d^3\vec{r}\,' = \frac{1}{2}\int d^3\vec{r}\,' \, [[\vec{r}\,' \times \vec{j}\,] \times \vec{r}\,]_i \, .$$

$$\int_{V'} j_i(\vec{r}\,') \sum_j r_j r_j' \, d^3\vec{r}\,' = \frac{1}{2}\int [[\vec{r}\,' \times \vec{j}(\vec{r}\,')] \times \vec{r}\,]_i \, d^3\vec{r}\,'$$

$$A_i(\vec{r}) = \frac{\mu_0}{4\pi r^3} \int_{V'} j_i(\vec{r}\,') \sum_j r_j r_j' \, d^3 r' =$$

$$= \frac{\mu_0}{4\pi} \int_{V'} \frac{\frac{1}{2}[[\vec{r}\,' \times \vec{j}\,] \times \vec{r}\,]_i}{r^3} \, d^3\vec{r}\,'$$

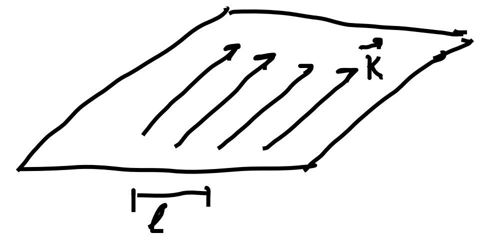
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \, \frac{\vec{m} \times \vec{r}}{r^3} \, .$$

$$\vec{m} = \frac{1}{2}\int_{V'} [\vec{r}\,' \times \vec{j}(\vec{r}\,')] \, d^3\vec{r}\,'$$

$$\vec{B} = \nabla \times \vec{A} = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{r}\cdot\vec{m})\,\vec{r}}{r^5} - \frac{\vec{m}}{r^3}\right].$$

Randbedingungen für das Magnetfeld an der Grenze zweier Medien.

$$\begin{cases} \nabla \times \vec{B} = \mu_0 \vec{J} \\ \nabla \cdot \vec{B} = 0 \end{cases}$$



Oberflächenstromdichte $\vec{I} = \vec{K} \cdot l$