# Fourier-Transformation in d Dimensionen.

$$F(k) = \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx \qquad \partial_x \longmapsto k.$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\; F(k)\, e^{ikx}$$

$d>1.$

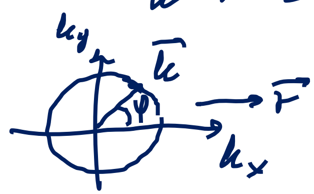
$d=2$

$$f(x,y) = \int_{-\infty}^{\infty} \frac{dk_x\, dk_y}{(2\pi)^2}\, F(k_x,k_y)\, e^{i(k_x\cdot x + k_y \cdot y)} =$$

$$= \int_{-\infty}^{\infty} \frac{d^2\vec{k}}{(2\pi)^2}\, F(\vec{k})\, e^{i\vec{k}\cdot\vec{r}}$$

Polarkoordinaten: $(r,\varphi) \longmapsto (|k|,\varphi)$

$$\vec{k}\cdot\vec{r} = |\vec{k}|\cdot|\vec{r}|\cos\varphi$$



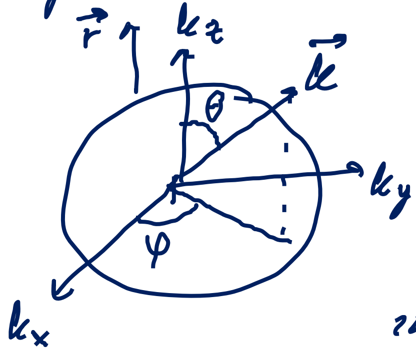

$$d^2\vec{k} = k\, dk\, d\varphi$$

$$\int_{-\infty}^{\infty} \frac{d^2\bar{k}}{(2\pi)^2}\, F(\bar{k})\, e^{i\bar{k}\cdot\bar{r}} = \int_0^{\infty}\frac{k\,dk}{2\pi}\int_0^{2\pi}\frac{d\varphi}{2\pi}\, F(\bar{k})\, e^{ik\cdot r\cos\varphi}.$$

$d = 3$

$$f(\vec r) = \int \frac{d^3\vec k}{(2\pi)^3}\, F(\vec k)\, e^{i\vec k\cdot\vec r}$$

Kugelkoordinaten: $(r, \theta, \varphi) \longmapsto (k, \theta, \varphi)$




$\vec k \cdot \vec r = k r \cos\theta$

$d^3\vec k = k^2 \sin\theta \, dk \, d\theta \, d\varphi$

$$\int \frac{d^3\vec k}{(2\pi)^3}\, F(\vec k)\, e^{i\vec k\cdot\vec r} = \int_0^{2\pi} \frac{d\varphi}{2\pi} \int_0^{\pi} \frac{\sin\theta\, d\theta}{2\pi} \int_0^{\infty} \frac{k^2\, dk}{2\pi}\, F(\vec k)\, e^{ikr\cos\theta}$$

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$d$: $\quad f(\vec r) = \int \frac{d^d\vec k}{(2\pi)^d}\, F(\vec k)\, e^{i\vec k\cdot\vec r}.$

# „Coulomb"-Potential in $d < 3$.

$D = 3$.

$$\nabla^2 \varphi(\vec r) = -\frac{\rho(\vec r)}{\varepsilon_0} \quad \Longleftrightarrow \quad \varphi(\vec r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

$$\rho(\vec r) = q\,\delta(\vec r)$$

$D = 2$:

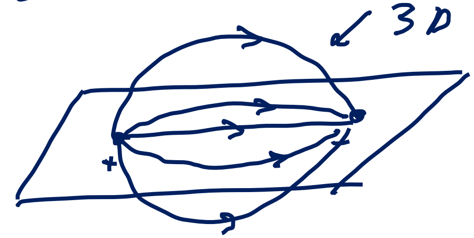
$$\nabla^2 \varphi(\vec r) = (\partial_x^2 + \partial_y^2)\,\varphi(\vec r) = -\frac{\rho(\vec r)}{\varepsilon_0}$$

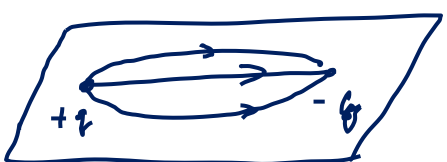
$$\Big\downarrow \text{FT}$$

$$(k_x^2 + k_y^2)\,\varphi(\vec k) = +\frac{\rho(\vec k)}{\varepsilon_0} \rightarrow \varphi(\vec k)$$

$$\varphi(x,y) = \int \frac{dk_x\,dk_y}{(2\pi)^2}\,\varphi(k_x, k_y)\,e^{i(k_x x + k_y y)}$$

$3D$: $\varphi(\vec r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

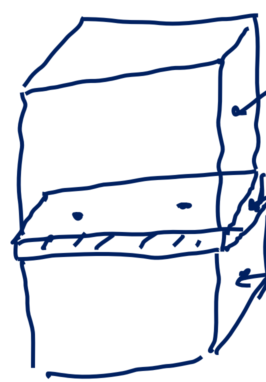



2D-„Coulomb"

$\varepsilon_2 \gg \varepsilon_1$

$E_{2n} \ll E_{1n}$



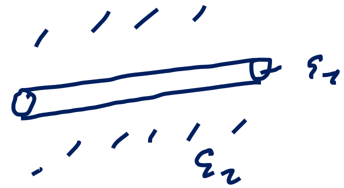

$$\vec D = \varepsilon \vec E$$

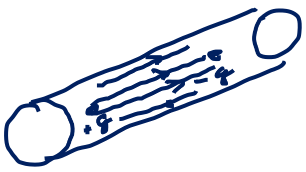
$$D_{1n} = D_{2n}$$

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} E_{1n}, \quad \varepsilon_2 \gg \varepsilon_1$$

$D = 1$: $\quad \partial_x^2 \varphi(x) = -\frac{\rho(x)}{\varepsilon_0}$ „1D-Coulomb".



# Faltungssatz

Faltung: $F(x) = (f * g)(x) = \int_{-\infty}^{\infty} f(x')\, g(x-x')\, dx' =$

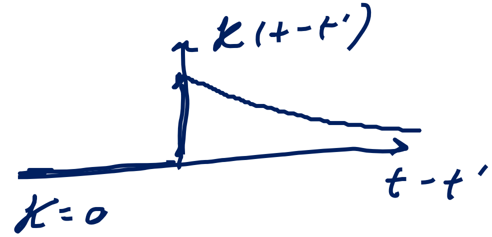
$$= \int_{-\infty}^{\infty} g(x')\, f(x-x')\, dx' .$$

$x \to t$.

$$Q(t) = \int_{-\infty}^{t} K(t,t')\, F(t')\, dt' = \int_{-\infty}^{t} K(t-t')\, F(t')\, dt'$$

$K(t-t')$


$\Leftrightarrow$




$$Q(t) = \int_{-\infty}^{\infty} K(t-t')\, F(t')\, dt' .$$

$$F(x) = \int_{-\infty}^{\infty} f(x')\, g(x-x')\, dx'.$$

$$F.T.(F(x)) = F(k) = \int_{-\infty}^{\infty} F(x)\, e^{-ikx}\, dx =$$

$$= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' f(x')\, g(x-x')\, e^{-ikx} =$$

$$= \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx'\, \underline{e^{-ikx}} \int_{-\infty}^{\infty} \frac{dk'}{2\pi} f(k')\, \underline{\underline{e^{ik'x'}}} \int_{-\infty}^{\infty} \frac{dp'}{2\pi} g(p')\, e^{i\underline{p'(x} - \underline{\underline{x'}})} =$$

$$= \int_{-\infty}^{\infty} \frac{dk'}{2\pi} \int_{-\infty}^{\infty} \frac{dp'}{2\pi} f(k')\, g(p') \underbrace{\int_{-\infty}^{\infty} dx\, e^{ix(p'-k)}}_{2\pi\,\delta(p'-k)} \underbrace{\int_{-\infty}^{\infty} dx'\, e^{ix'(k'-p')}}_{2\pi\,\delta(k'-p')} =$$

$$= \int dk' \int_{-\infty} dp' f(k')\, g(p') \cdot \delta(p'-k)\, \delta(k'-p') =$$

$$= \int_{-\infty}^{\infty} dp' f(p')\, g(p')\, \delta(p'-k) = f(k)\, g(k).$$

$$\boxed{F(k) = f(k) \cdot g(k).}$$

$$\int_{-\infty}^{\infty} dk' f(k')\, \delta(k'-p') = f(p').$$

Faltung

$$\int_{-\infty}^{\infty} f(x')\, g(x-x')\, dx' = \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, f(k)\, g(k)\, e^{ikx} .$$