Fourier - Transformation.

$\forall x: \quad f(x) = f(x+a)$

$$f(x) = \frac{1}{2} A_0 + \sum_{m=1}^{\infty} \left[A_m \cos\left(\frac{2\pi m x}{a}\right) + B_m \sin\left(\frac{2\pi m x}{a}\right) \right]. -$$

\- Fourier - Reihe.

A_0, A_m, B_m - Koeffizienten.

$$\cos\left(\frac{2\pi m(x+a)}{a}\right) = \cos\left(\frac{2\pi m x}{a} + \frac{2\pi m a}{a}\right) = \cos\left(\frac{2\pi m x}{a} + 2\pi m\right) =$$

$$= \cos\left(\frac{2\pi m x}{a}\right).$$

$$A_m = \frac{2}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} f(x) \cos\left(\frac{2\pi m x}{a}\right) dx.$$

$$B_m = \frac{2}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} f(x) \sin\left(\frac{2\pi m x}{a}\right) dx.$$

$$a \longmapsto \infty : \quad \frac{2\pi m}{a} - \frac{2\pi(m-1)}{a} = \frac{2\pi}{a} \xrightarrow[a\to\infty]{} 0$$

$$\frac{2\pi m}{a} \longmapsto k \in R$$

$$\sum_{m=1}' \xrightarrow[a\to\infty]{} \int dm = \int \frac{a}{2\pi} dk = \frac{a}{2\pi} \int dk$$

$$\cos\left(\frac{2\pi m x}{a}\right) \longmapsto \cos(kx) = \frac{e^{ikx} + e^{-ikx}}{2} .$$

$$\sin\left(\frac{2\pi m}{a} x\right) \longmapsto \sin(kx) = \frac{e^{ikx} - e^{-ikx}}{2i} .$$

$$A_m \longmapsto \sqrt{\frac{2\pi}{a}} A(k) \quad : \quad B_m \longmapsto \sqrt{\frac{2\pi}{a}} B(k)$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) e^{ikx} dk$$

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

Alternativ (equivalent)

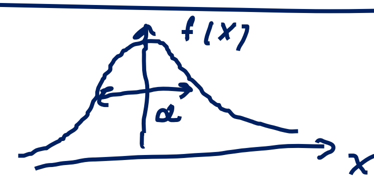
$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k)\, e^{ikx}\, dk$$

$$F(k) = \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx$$

$\delta(x)$:
$$F.T.(\delta(x)) = \int_{-\infty}^{\infty} \delta(x)\, e^{-ikx}\, dx = 1.$$

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx}\, dk .$$

Bsp : $f(x) = e^{-\frac{x^2}{2\alpha^2}}$

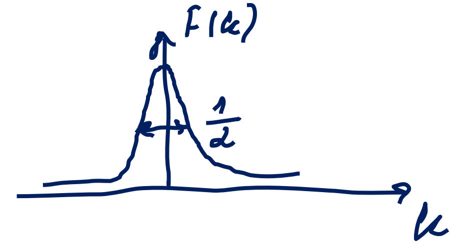


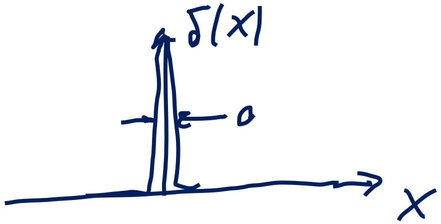
$$F(k) = \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\alpha^2}}\, e^{-ikx}\, dx = \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\alpha^2}\left(x^2 + 2\alpha^2 ikx + (i\alpha^2 k)^2 - (i\alpha^2 k)^2\right)\right] dx =$$

$$= \int_{-\infty}^{\infty} dx\, \exp\left[-\frac{1}{2\alpha^2}(x + i\alpha^2 k)^2\right] \cdot e^{-\frac{\alpha^2 k^2}{2}} = \left| \int_{-\infty}^{\infty} e^{-ax^2}\, dx = \sqrt{\frac{\pi}{a}} \right| =$$

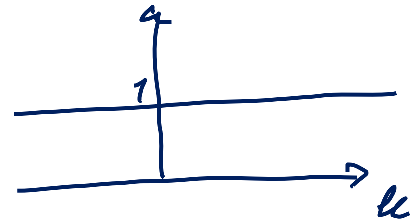
$$= \sqrt{2\pi\alpha}\; e^{-\frac{\alpha^2 k^2}{2}}$$

$$\text{F.T.}\left(e^{-\frac{x^2}{2\alpha^2}}\right) = \sqrt{2\pi}\,\alpha\; e^{-\frac{\alpha^2}{2}k^2}.$$





$$\xmapsto{\text{F.T.}}$$



F.T. der Ableitung.

$$\frac{df(x)}{dx} = \frac{d}{dx}\left[\int_{-\infty}^{\infty}\frac{dk}{2\pi}F(k)\,e^{ikx}\right] = \int_{-\infty}^{\infty}\frac{dk}{2\pi}\underbrace{ik\,F(k)}_{\text{F.T.}\left(\frac{df}{dx}\right)}e^{ikx}$$

$$\text{F.T.}\left(\frac{df}{dx}\right) = ik\,F(k)$$

$$\partial_x \xmapsto{\text{F.T.}} ik$$

$$\nabla = \begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix} \xmapsto{\text{F.T.}} i\begin{pmatrix}k_x\\ k_y\\ k_z\end{pmatrix} = i\vec{k}.$$

$$\nabla^2 \varphi(\vec{r}) = -\frac{\rho(\vec{r})}{\varepsilon_0}$$ - Poisson - Glch.

$$F.T.\left(\varphi(\vec{r})\right) = \varphi(\vec{k}) \qquad F.T.\left(\rho(\vec{r})\right) = \rho(\vec{k})$$

$$\nabla^2 \xrightarrow{F.T.} (i\vec{k})^2 = -k^2$$

$$-k^2 \varphi(\vec{k}) = -\frac{1}{\varepsilon_0}\rho(\vec{k})$$

$$\varphi(\vec{k}) = \frac{1}{\varepsilon_0 k^2}\rho(\vec{k}).$$

$$\varphi(\vec{r}) = \int \frac{d^3\vec{k}}{(2\pi)^3}\,\frac{\rho(\vec{k})}{\varepsilon_0 k^2}\,e^{i\vec{k}\cdot\vec{r}} = \int_{-\infty}^{\infty} \frac{dk_x\,dk_y\,dk_z}{(2\pi)^3}\,\frac{\rho(\vec{k})}{\varepsilon_0 k^2}\,e^{i(k_x\cdot x + k_y\cdot y + k_z\cdot z)}.$$