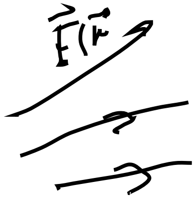
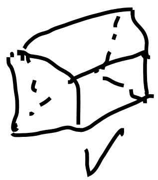
Gauß'sche Satz

Zusammenhang $\nabla\cdot\vec{E} \longleftrightarrow \Phi$

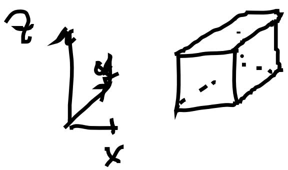




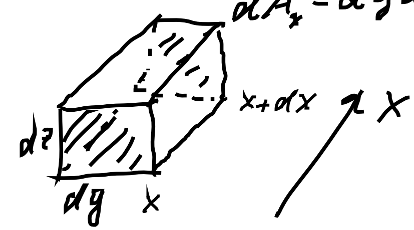
$A = \partial V \qquad \iiint\limits_V (\nabla\cdot\vec{E})\,dV = \oiint\limits_{\partial V} (\vec{E}\cdot\vec{n})\,dA = \Phi$

$V \longrightarrow \sum dV$

$dV = dx\,dy\,dz$



$dA_x = dy\,dz$



$$d\Phi = d\Phi_x + d\Phi_y + d\Phi_z$$

$$d\Phi_x = \Phi_{x+dx} - \Phi_x = E_x(x+dx, y, z)\,dA_x - E_x(x,y,z)\,dA_x =$$

$$= \left[E_x(x+dx,y,z) - E_x(x,y,z)\right] dy\,dz = \frac{\partial E_x}{\partial x}\,dx\,dy\,dz$$

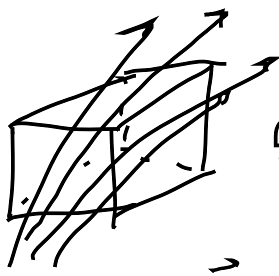
Analog $d\Phi_y$, $d\Phi_z$

$$d\Phi = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)\underbrace{dx\,dy\,dz}_{dV} = (\nabla\cdot\vec{E})\,dV$$

$$\Phi = \iiint_V (\nabla \cdot \vec{E}) \, dV$$

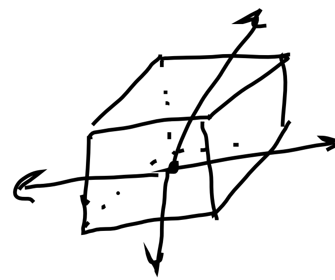
Def.: $\Phi = \oiint_{\partial V} (\vec{E} \cdot \vec{n}) \, dA$

Elektrostatik: $\nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0}$



$\vec{E}(\vec{r})$

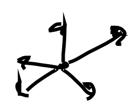
$\Phi = 0$



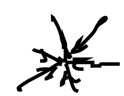
$\Phi > 0$

$\nabla \cdot \vec{E}(\vec{r})$ – räumlich lokal.

$\nabla \cdot E = 0$

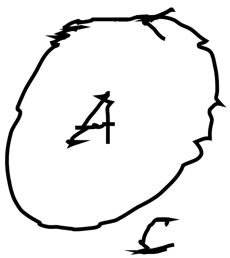


$\nabla \cdot \vec{E} > 0$



$\nabla \cdot \vec{E} < 0$

Der Satz von Stokes

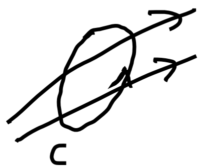


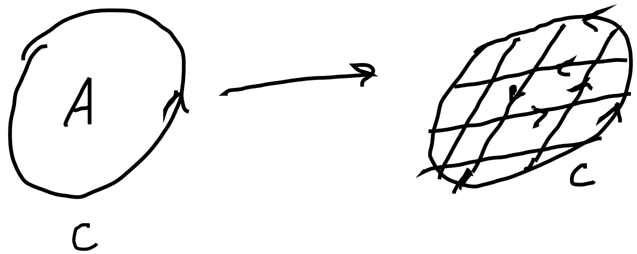
$$\oint_{C} \vec{E}\, d\vec{r} = \iint_{A} (\nabla \times \vec{E}) \cdot \vec{n}\, d\vec{A}$$

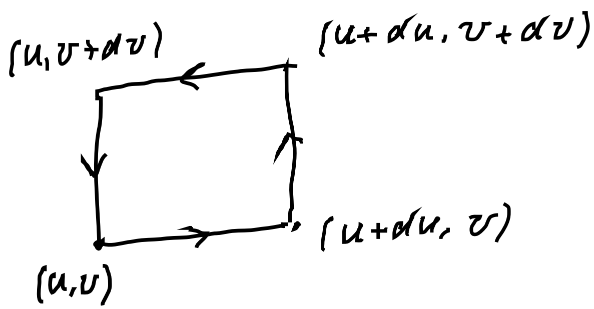
$C = \partial A$

$$\oint_{C} \vec{E}\cdot d\vec{r} = \iint_{A} (\nabla\times\vec{E})\cdot\vec{n}\, dA$$

$C = \partial A$







$(du \perp dv)$

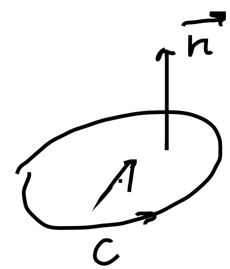
$$\oint_{dC} \vec{E}\cdot d\vec{r} = \frac{1}{2}\left[E_u(u,v) + E_u(u+du,v)\right]du + \frac{1}{2}\left[E_v(u+du, v) + E_v(u+du, v+dv)\right]dv - \frac{1}{2}\left[E_u(u+du, v+dv) + E_u(u, v+dv)\right]du - \frac{1}{2}\left[E_v(u, v+dv) + E_v(u,v)\right]dv =$$

$$= \frac{1}{2}\left[E_u(u,v) - E_u(u, v+dv) + E_u(u+du, v) - E_u(u+du, v+dv)\right] du +$$

$$+ \frac{1}{2}\left[\underbrace{E_v(u+du, v) - E_v(u,v)}_{\frac{\partial E_v}{\partial u} du} + \underbrace{E_v(u+du, v+dv) - E_v(u, v+dv)}_{\frac{\partial E_v}{\partial u} du}\right] dv =$$

$$= \left(\frac{\partial E_v}{\partial u} - \frac{\partial E_u}{\partial v}\right) du\, dv = \left(\nabla \times \vec{E}\right)_{(uv)} du\, dv$$

$$\oint_{C=\partial A} \vec{E} \cdot d\vec{F} = \iint_A (\nabla \times \vec{E}) \cdot \vec{n}\, dA$$



$|\vec{n}| = 1.$