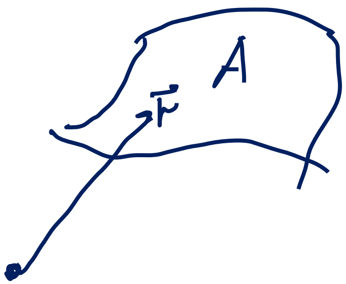
Oberflächenintegral

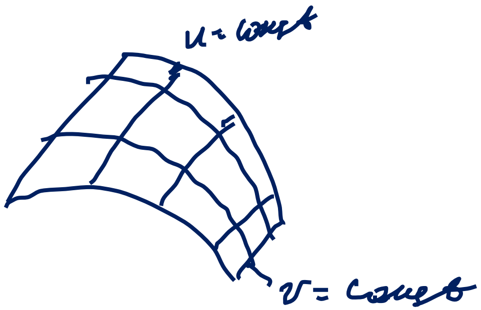


$$f(x,y,z)$$

$$\int f(\vec{r}) \cdot dA$$

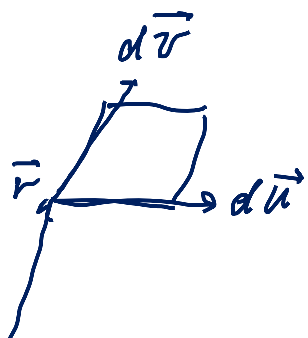
$$A = \{ \vec{r}(u,v) \}$$

$$\vec{r}(u,v) = \begin{pmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{pmatrix}$$



$$f(\vec{r}) \mapsto f(\vec{r}(u,v))$$

$$dA - ?$$



$$dA = |d\vec{u} \times d\vec{v}|$$

$$dx = \frac{\partial x}{\partial u} du + \frac{\partial x}{\partial v} dv$$

analog dy, dz

$$d\vec{u} = \begin{pmatrix} \frac{\partial x}{\partial u} \\ \frac{\partial y}{\partial u} \\ \frac{\partial z}{\partial u} \end{pmatrix} du = \frac{\partial \vec{r}}{\partial u} du \,, \qquad d\vec{v} = \frac{\partial \vec{r}}{\partial v} dv$$

$$dA = |d\vec{u} \times d\vec{v}| = \left| \frac{\partial \vec{r}}{\partial u} du \times \frac{\partial \vec{r}}{\partial v} dv \right| = \left| \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right| du\, dv$$

$$A: \quad \vec{r} = \begin{pmatrix} \sin\theta \cos\varphi \\ \sin\theta \sin\varphi \\ \cos\theta \end{pmatrix} \qquad [\, u \to \theta; \quad v \to \varphi \,]$$

$$\frac{\partial \vec{r}}{\partial \theta} = \begin{pmatrix} \cos\theta \cos\varphi \\ \cos\theta \sin\varphi \\ -\sin\theta \end{pmatrix}, \qquad \frac{\partial \vec{r}}{\partial \varphi} = \begin{pmatrix} -\sin\theta \sin\varphi \\ \sin\theta \cos\varphi \\ 0 \end{pmatrix}$$

$$\frac{\partial \vec{r}}{\partial \theta} \times \frac{\partial \vec{r}}{\partial \varphi} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\theta\sin\varphi & \sin\theta\cos\varphi & 0 \end{vmatrix} = \hat{e}_x \sin^2\theta \cos\varphi + \hat{e}_y \sin^2\theta \sin\varphi + \hat{e}_z (\cos\theta \sin\theta \cos^2\varphi + \cos\theta \sin\theta \sin^2\varphi)$$

$$\frac{\partial \vec{F}}{\partial \theta} \times \frac{\partial \vec{F}}{\partial \varphi} = \begin{pmatrix} \sin^2\theta \, \cos\varphi \\ \sin^2\theta \, \sin\varphi \\ \cos\theta \, \sin\theta \end{pmatrix}.$$

$$\left| \frac{\partial \vec{F}}{\partial \theta} \times \frac{\partial \vec{F}}{\partial \varphi} \right| = \left[\underline{\sin^4\theta \cos^2}\varphi + \underline{\sin^4\theta \sin^2}\varphi + \cos^2\theta \sin^2\theta \right]^{1/2} =$$

$$= \left[\sin^4\theta + \cos^2\theta \sin^2\theta \right]^{1/2} = \sqrt{\sin^2\theta} = |\sin\theta|$$

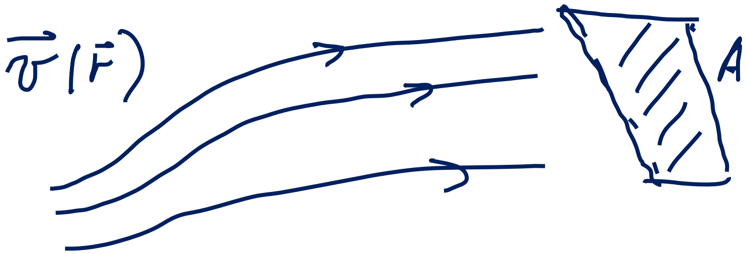
$dA = |\sin\theta| \, d\theta \, d\varphi$, $\quad 0 \le \theta \le \pi \Rightarrow \sin\theta \geq 0$

$0 \le \varphi < 2\pi$

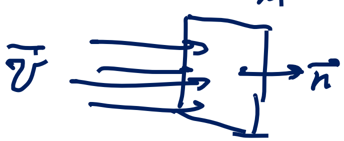
$\underline{dA = \sin\theta \, d\theta \, d\varphi}$

$$\iint_A f(\vec{F}) \, dA = \iint_{A[u,v]} f(\vec{F}(u,v)) \left| \frac{\partial \vec{F}}{\partial u} \times \frac{\partial \vec{F}}{\partial v} \right| du \, dv.$$

Fluss des Vektorfeldes.

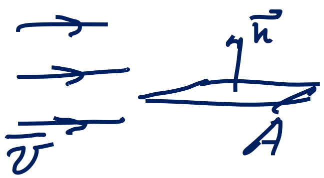


Fluß = Flüssigkeitsmenge pro Zeiteinheit

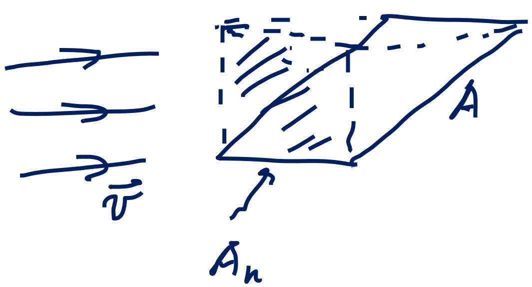


$\vec{v} \parallel \vec{n}$ $\qquad \varphi = \rho \cdot v A$

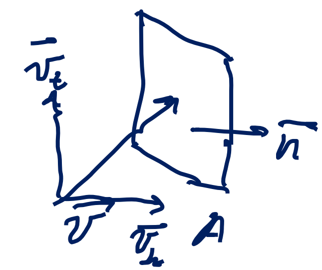
Dichte



$\vec{n} \perp \vec{v} \longrightarrow \varphi = 0$



$\varphi = v \cdot A_n \cdot \rho \iff$



$|\vec{n}| = 1.$

$\varphi = \rho\, v_n \cdot A = \rho(\vec{v} \cdot \vec{n}) A$

$$\Phi = \iint_A \left(\vec{v}(\vec{r}) \cdot \vec{n}(\vec{r})\right) \cdot dA$$

$[\rho = 1]$

Fluß des Feldes $\vec{E}(\vec{r})$ durch die Oberfläche A.

$$\Phi = \iint_A (\vec{E} \cdot \vec{n})\, dA$$

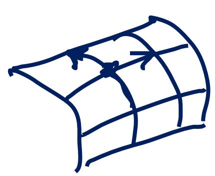
$[\vec{E}(\vec{r}),\ \vec{n}(\vec{r})]$

$A: \vec{r}(u, v)$

$\vec{E}(\vec{r}) = \vec{E}(\vec{r}(u,v))$ auf der Fläche A.

$\vec{n}(\vec{r})$ – Einheitsvektor normal zur Oberfläche A.

$\vec{n}(\vec{r})$ – ? $d\vec{u}, d\vec{v} \in A$



$$(d\vec{u} \times d\vec{v}) \parallel \vec{n}$$

$$\vec{n} = \frac{(d\vec{u} \times d\vec{v})}{|d\vec{u} \times d\vec{v}|}.$$

$$\Phi = \int_A \vec{E}(F(u,v)) \cdot \frac{d\vec{u} \times d\vec{v}}{|d\vec{u} \times d\vec{v}|} \underbrace{|d\vec{u} \times d\vec{v}|}_{dA} =$$

$$= \int_A \vec{E}(F(u,v)) \cdot (d\vec{u} \times d\vec{v}) = \int_A \underbrace{(\vec{E}\, d\vec{u}\, d\vec{v})}_{\text{Spatprodukt}} .$$

$$(\vec{E}\, d\vec{u}\, d\vec{v}) = \vec{E} \cdot (d\vec{u} \times d\vec{v}) .$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \overbrace{\underbrace{\vec{c} \cdot (\vec{a} \times \vec{b})}_{\substack{\| \\ (\vec{a} \times \vec{b}) \cdot \vec{c}}}}^{\text{Spatprodukt}} = \vec{b} \cdot (\vec{c} \times \vec{a}) = (\vec{a}\,\vec{b}\,\vec{c})$$

$$(a\,b\,c) = -(b\,a\,c) ;$$

$$(\vec{a}\,\vec{b}\,\vec{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} .$$