Floquet formulation for the investigation of multiphoton quantum interference in a superconducting qubit driven by a strong field

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Abstract

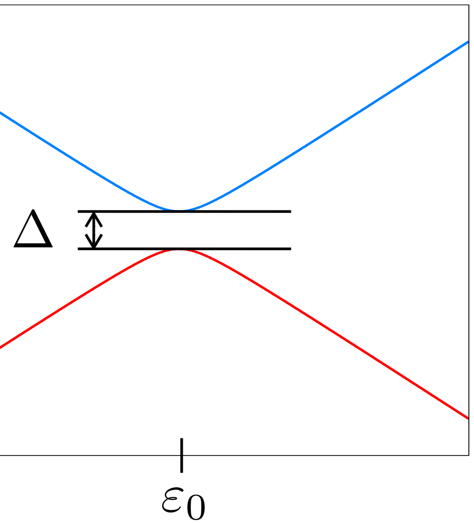
We present a Floquet investigation of multiphoton quantum interference in a strongly driven superconducting qubit. The procedure involves a transformation of a time-dependent problem into an equivalent time-independent infinite-dimensional Floquet matrix eigenvalue problem. The results of a two-level qubit system show quantum interference fringes around multiphoton resonance positions in agreement with the experimental results[1)]. We further explore the interference patterns in terms of quasienergies and the resonance position shifts as the tunneling strength increased. The Floquet formulation promises a new and accurate approach for the investigation of quantum interference phenomenon in the qubits.

Introduction

- SQUID: Superconducting Quantum Interference Devices
- Superconducting qubit which has two magnetic flux states is a promising candidate for quantum computing.
- Recent experiments demonstrate quantum interference fringes around multiphoton resonance positions in a strongly driven superconducting qubit[1,2].

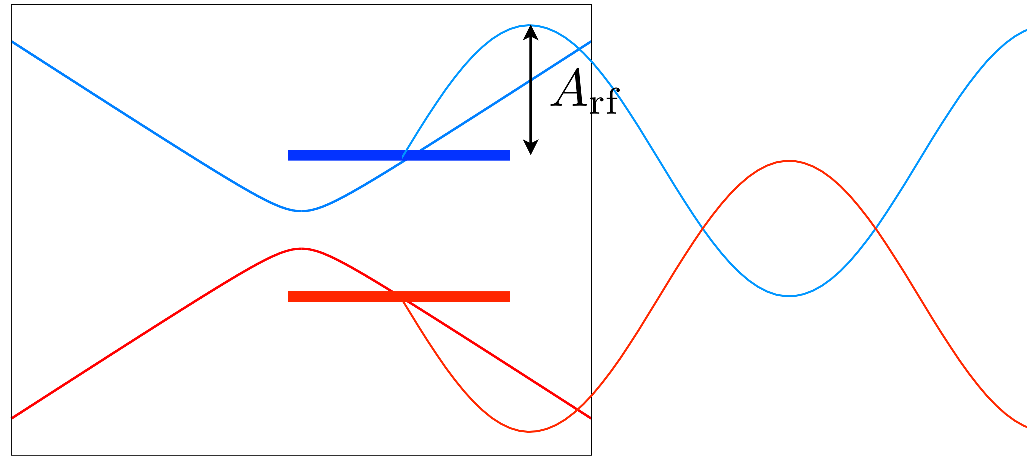
Computational Details

Two-level qubit without RF field



$$H_0 = -\frac{1}{2}\begin{pmatrix} \varepsilon_0 & \Delta \\ \Delta & -\varepsilon_0 \end{pmatrix}$$

Two-level qubit driven by RF field



$$H(t) = -\frac{1}{2}\begin{pmatrix} \varepsilon(t) & \Delta \\ \Delta & -\varepsilon(t) \end{pmatrix}$$

where $\varepsilon(t) = \varepsilon_0 + A_{\mathrm{rf}} \cos \omega t$

Δ: tunneling strength, ε_0: flux detuning, A_{rf}: RF field amplitude

Floquet theorem[3)]

$$i\hbar\frac{\partial}{\partial t}\Psi(t) = \hat{H}(t)\Psi(t) \quad \text{where } \hat{H}(t) = \hat{H}(t+\tau)$$

$$\Rightarrow \Psi(t) = e^{-i\varepsilon t/\hbar}\Phi(t) \quad \text{where } \varepsilon \text{ is the quasienergy}$$

$$\text{and } \Phi(t) = \Phi(t+\tau)$$

Equivalent time-independent eigenvalue problem

$$\hat{\mathcal{H}}(t) \equiv \hat{H}(t) - i\hbar\frac{\partial}{\partial t} \quad \Rightarrow \quad \hat{\mathcal{H}}(t)\Phi(t) = \varepsilon\Phi(t)$$

$$\hat{H}(t) = \sum_n \hat{H}^{[n]} e^{-in\omega t} \text{ and } \Phi(t) = \sum_n \Phi^{[n]} e^{-in\omega t}$$

$$\text{Floquet state: } |\alpha n\rangle = |\alpha\rangle \otimes |n\rangle$$

$$\Rightarrow \langle \alpha n|\hat{H}_F|\beta m\rangle = H_{\alpha\beta}^{[n-m]} + n\omega\delta_{\alpha\beta}\delta_{nm}$$

Floquet matrix structure

$$
\begin{array}{c|cc|cc|cc|}
 & [-1] & & [0] & & [+1] & \\
\hline
[-1] & -\frac{\varepsilon_0}{2}-\omega & -\frac{\Delta}{2} & -\frac{A_{\mathrm{rf}}}{4} & & & \\
 & -\frac{\Delta}{2} & +\frac{\varepsilon_0}{2}-\omega & & +\frac{A_{\mathrm{rf}}}{4} & & \\
\hline
[0] & -\frac{A_{\mathrm{rf}}}{4} & & -\frac{\varepsilon_0}{2} & -\frac{\Delta}{2} & -\frac{A_{\mathrm{rf}}}{4} & \\
 & & +\frac{A_{\mathrm{rf}}}{4} & -\frac{\Delta}{2} & +\frac{\varepsilon_0}{2} & & +\frac{A_{\mathrm{rf}}}{4} \\
\hline
[+1] & & & -\frac{A_{\mathrm{rf}}}{4} & & -\frac{\varepsilon_0}{2}+\omega & -\frac{\Delta}{2} \\
 & & & & +\frac{A_{\mathrm{rf}}}{4} & -\frac{\Delta}{2} & +\frac{\varepsilon_0}{2}+\omega \\
\hline
\end{array}
\quad \ddots
$$

Time-dependent Hamiltonian

$$H(t) = -\frac{1}{2}\begin{pmatrix} \varepsilon(t) & \Delta \\ \Delta & -\varepsilon(t) \end{pmatrix}$$

where $\varepsilon(t) = \varepsilon_0 + A_{\mathrm{rf}} \cos \omega t$

Fourier components

$$H^{[0]} = -\frac{1}{2}\begin{pmatrix} \varepsilon_0 & \Delta \\ \Delta & -\varepsilon_0 \end{pmatrix}$$

$$H^{[+1]} = -\frac{1}{4}\begin{pmatrix} A_{\mathrm{rf}} & 0 \\ 0 & -A_{\mathrm{rf}} \end{pmatrix}$$

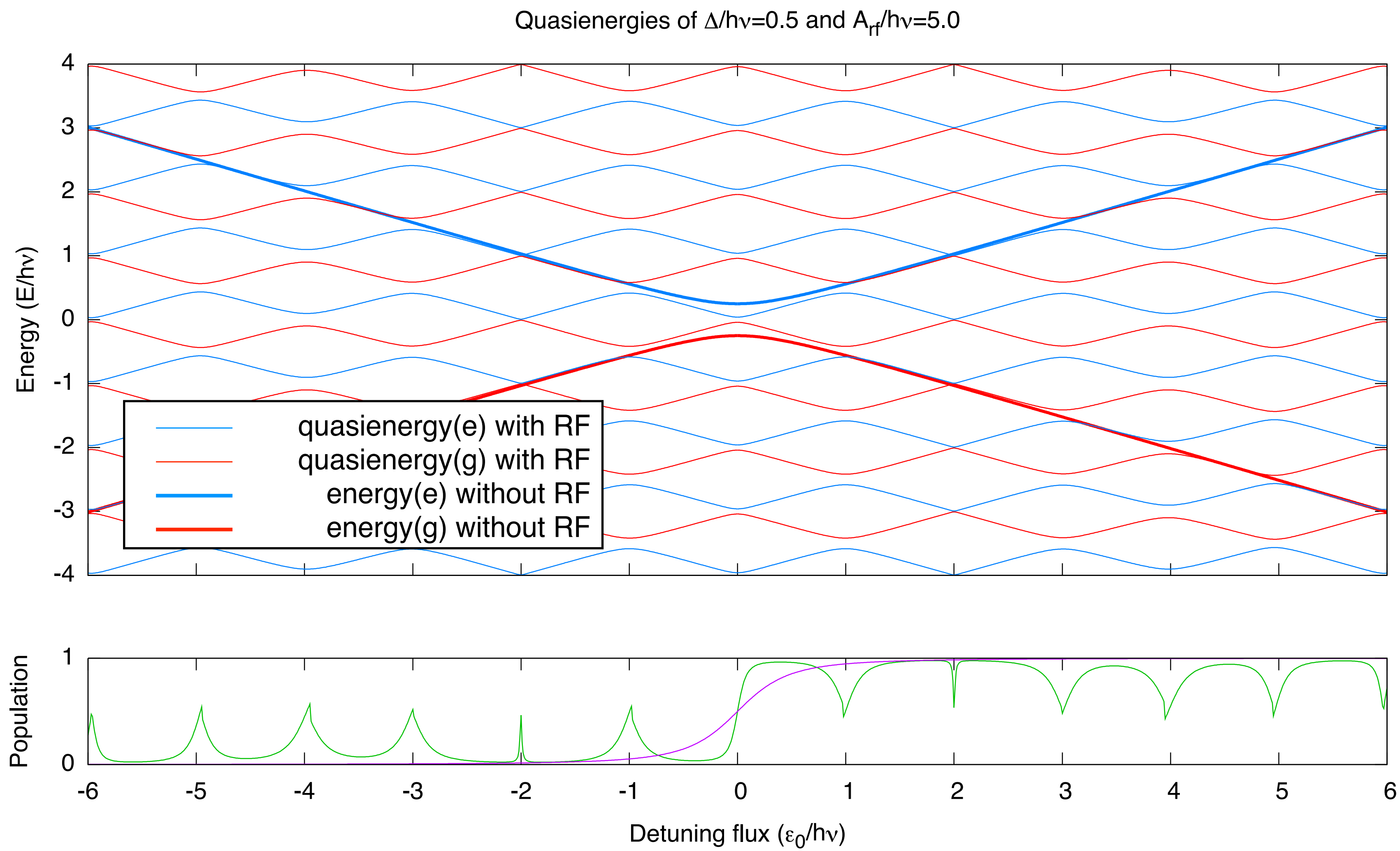
$$H^{[-1]} = -\frac{1}{4}\begin{pmatrix} A_{\mathrm{rf}} & 0 \\ 0 & -A_{\mathrm{rf}} \end{pmatrix}$$

Time-dependent wavefunction: $\Psi(t) = |\alpha\rangle \sum_n a_n e^{i(n\omega-\lambda)t} + |\beta\rangle \sum_n b_n e^{i(n\omega-\lambda)t}$

Time-averaged switching probability: $\bar{P}_{\alpha\to\beta} = 2\left(\sum_n a_n^2\right)\left(\sum_n b_n^2\right)$

Results

Plots of quasienergies as a function of ε_0

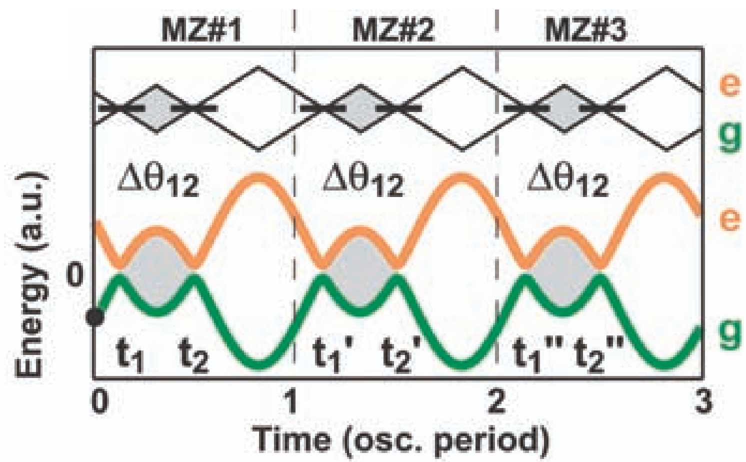


Mean energy[4)] is computed from eigenvectors:

$$\bar{E} \equiv \frac{1}{\tau}\int_0^{\tau} \varepsilon(t)dt = \lambda + \left\langle\!\left\langle \Phi(t) \left| i\frac{\partial}{\partial t} \right| \Phi(t) \right\rangle\!\right\rangle = \lambda - \sum_n \left(a_n^2 + b_n^2\right) n\omega$$

(λ: quasienergy)

Analogue of Mach-Zehnder interference*



Constructive interference condition with the phase:

$$\Delta\theta = 2\pi n$$

$$\Delta\theta = \frac{1}{\hbar}\int_0^{\tau} \left[\varepsilon_e(t) - \varepsilon_g(t)\right] dt = \frac{2\bar{E}\tau}{\hbar} = \frac{4\pi\bar{E}}{h\nu}$$

Constructive interference condition with the mean energy:

$$\frac{\bar{E}}{h\nu} = \frac{n}{2} \quad (n:\ \text{integer})$$

* Picture taken from Oliver *et al.*, *Science* **310**, 1653 (2005)[1)]

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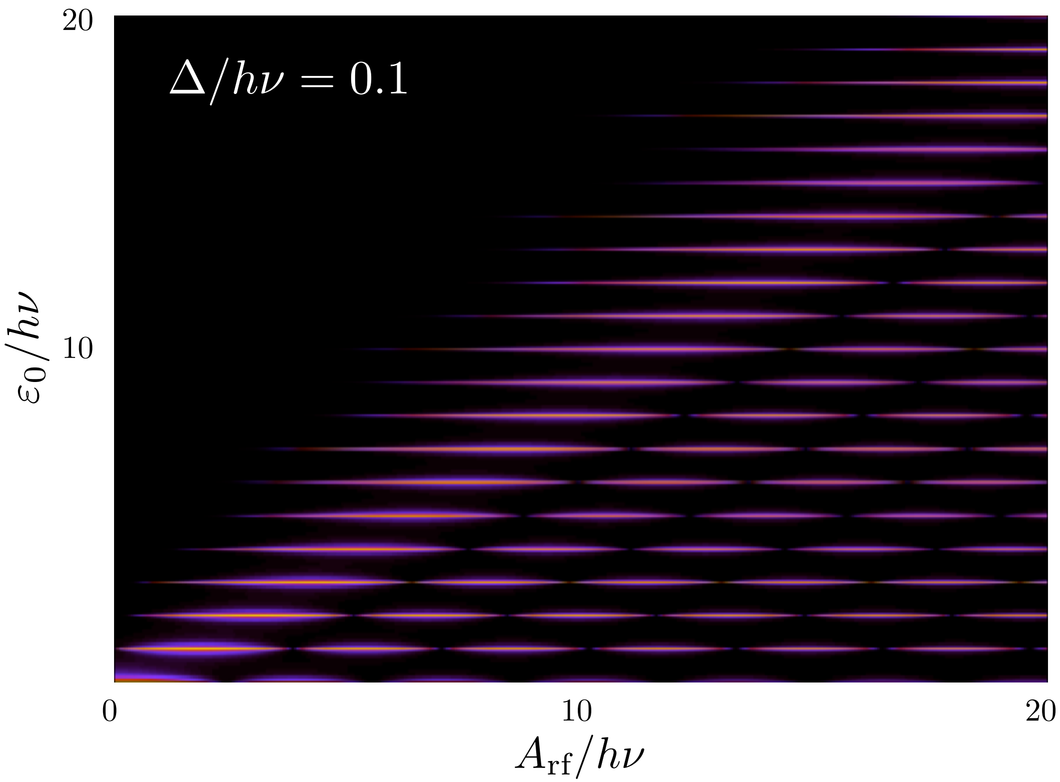
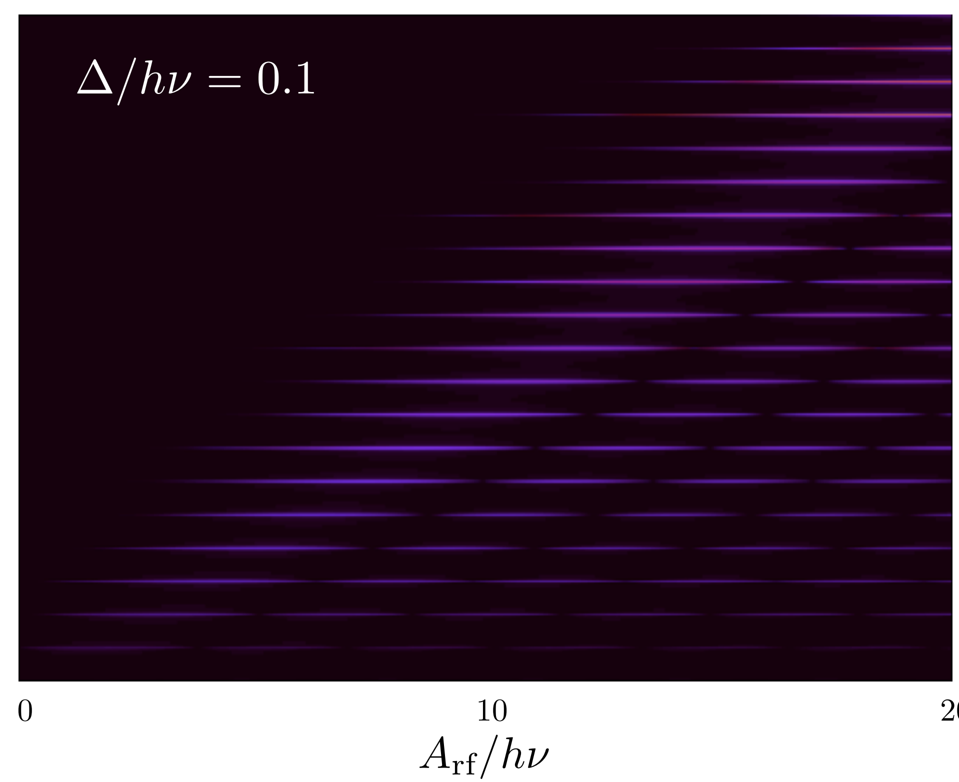
Comparison with switching probability and mean energy

Switching probability $\bar{P}_{\alpha\to\beta}$



Mean energy $\left(\bar{E} + \frac{\varepsilon_0}{2}\right)/h\nu$



Plots of switching probabilities are agreed with experimental results[1]).

Plots of mean energies show the same interference fringes.

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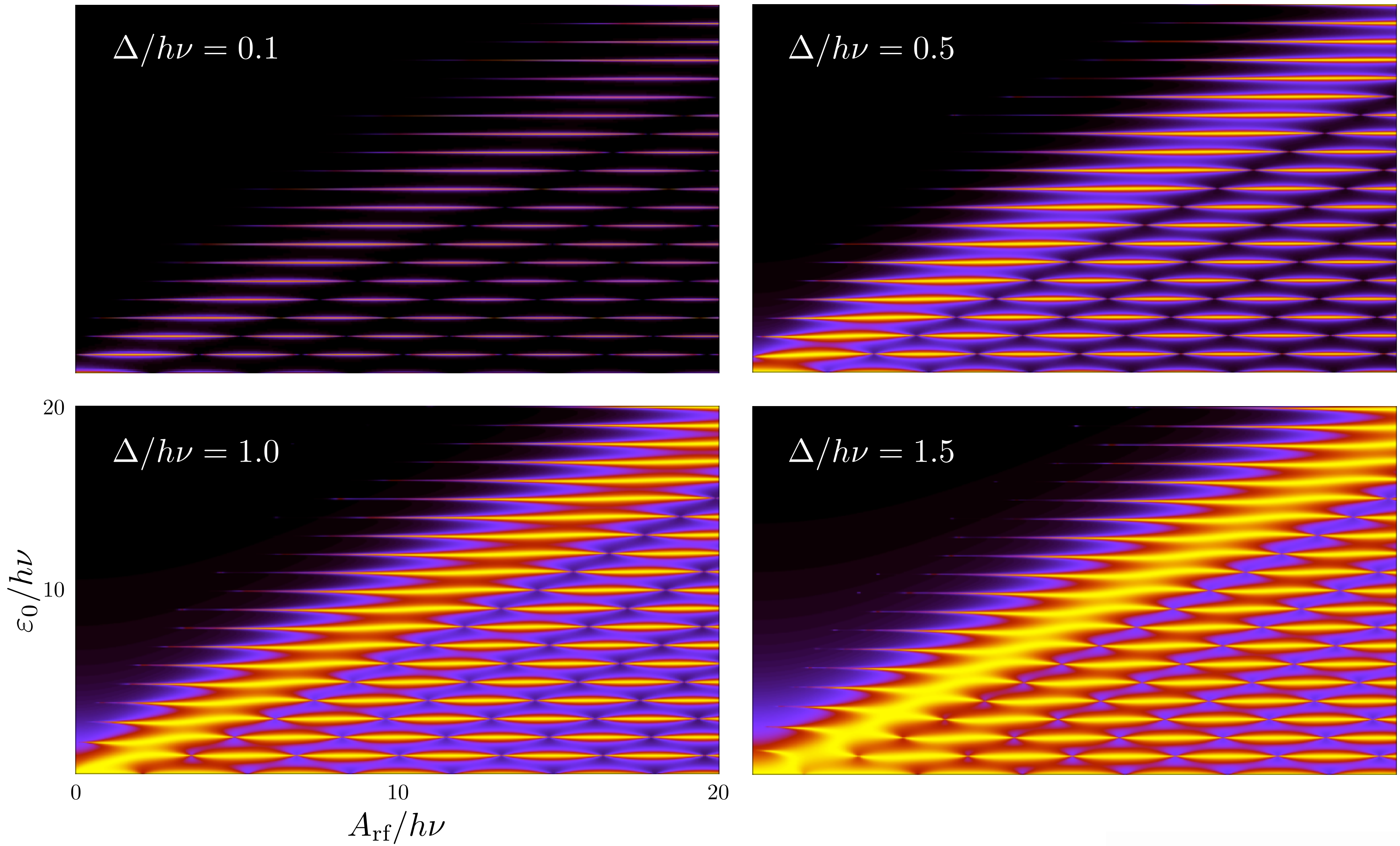
Plots of switching probability $\bar{P}_{\alpha\to\beta}$ changing $\Delta/h\nu$



Conclusion

- The plots of switching probabilities show quantum interferences due to accumulated phase difference in agreement with the experimental results.
- The phase difference can be derived from the mean energy (quasi-energy), and the mean energy plot shows the same interference patterns.
- Resonance position shifts are observed as the tunneling strength increases.
- The Floquet formalism is extended to investigate quantum interference phenomenon in the qubits.

References

1. W. D. Oliver, *et al.*, *Science* **310**, 1653 (2005).
2. D. M. Berns, *et al.*, *Phys. Rev. Lett.* **97**, 150502 (2006).
3. S. I. Chu and D. A. Telnov, *Phys. Rep.* **390**, 1 (2004).
4. S. I. Chu, *Adv. Chem. Phys.* **73**, 739 (1989).