Instanton Physics

A. Ringwald

http://www.desy.de/~ringwald

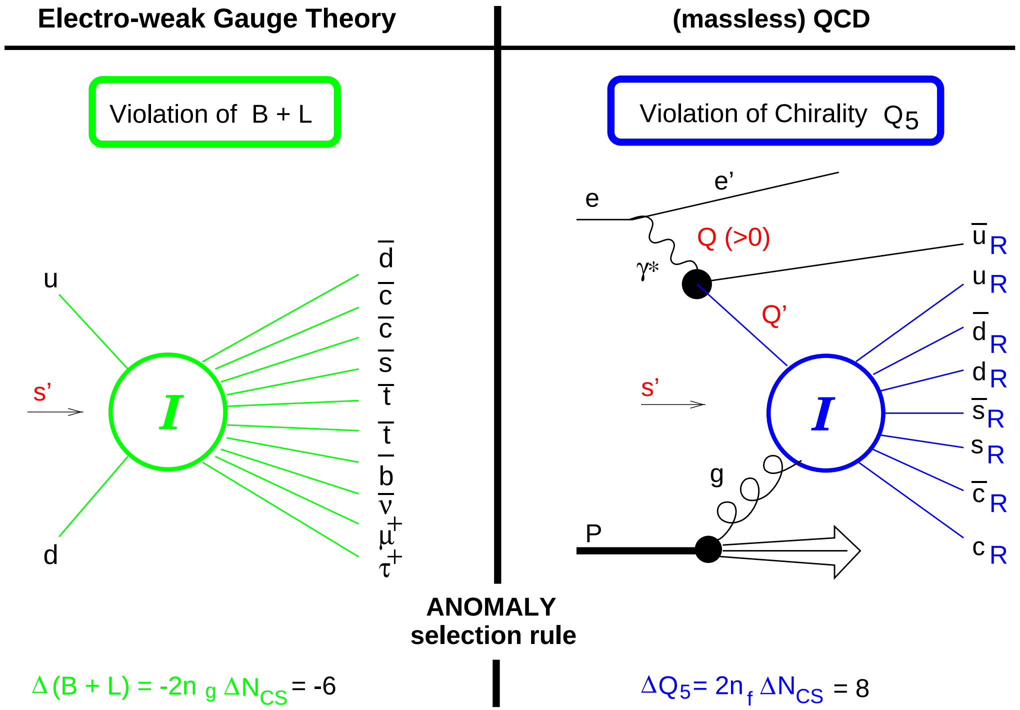
DESY

QCD at High energies/Frontiers in QCD
Sino-German Workshop
Beijing/China, September 2004

1. Introduction

- Standard Model of **electroweak** (**QFD**) and **strong** (**QCD**) interactions remarkably successfull
- There are processes that cannot be described by ordinary perturbation theory: [Adler '69; Bell,Jackiw '69; Bardeen '69]]

  B+L/**Chirality**-violating processes in **QFD**/**QCD**
- **Anomalous** processes induced by **topological fluctuations** of the non-abelian gauge fields, notably by **instantons** [Belavin *et al.* '75; 't Hooft '76]



- Topological gauge field fluctuations and associated anomalous processes play important role in
  - **QCD** in various **long-distance** aspects:
    - $U(1)_A$ problem ($m_{\eta'} \gg m_\eta$) ['t Hooft '76]
    - $SU(n_f)$ chiral symmetry breaking [Shuryak '82; Diakonov,Petrov '86]
  - **QFD** at **high temperatures**: [Kuzmin,Rubakov,Shaposhnikov '85]
    - Impact on baryon and lepton asymmetries of the universe
- Are they directly observable in **high energy reactions**?
  - **QFD**: Intense studies in early 1990s; inconclusive [AR '90; Espinosa '90; ...]
  - **QCD**:
    - **Hard QCD**-instanton induced events in **deep inelastic scattering**
      - · reliably calculable and sizeable rate [Moch,AR,F.Schrempp '97; AR,F.Schrempp '98]
      - · characteristic final state signature [AR,F.Schrempp '94–'01]
    - **Soft QCD**-instanton induced events might be responsible for the bulk of inelastic processes [E.Levin *et al.*; Shuryak *et al.*; F.Schrempp,Utermann '02]

- **Further content:**

  2. **Axial anomaly and topology**
  3. **Instanton perturbation theory**
  4. **QCD-instantons at HERA**
  5. **QFD-instantons at VLHC**
  6. **Conclusions**

2. Axial anomaly and topology

- In absence of quark masses, QCD Lagrangian $[q = \text{column}(\underbrace{u, d, s, \ldots}_{n_f})]$

$$\mathcal{L}^0_{\text{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + i\bar{q}_L \gamma^\mu D_\mu q_L + i\bar{q}_R \gamma^\mu D_\mu q_R$$

invariant under independent **global transformations**

$$\boldsymbol{G \equiv \underbrace{SU(n_f)_L \otimes SU(n_f)_R}_{\text{chiral symmetry}} \otimes \underbrace{U(1)_{L+R}}_{\text{vector}} \otimes \underbrace{U(1)_{L-R}}_{\text{axial vector}}}$$

$$SU(n_f)_{L,R} \quad : \quad q_{L,R} \xrightarrow{G} g_{L,R}\, q_{L,R}$$

$$U(1)_{L+R} \equiv U(1)_V \quad : \quad q_{L,R} \xrightarrow{G} \mathrm{e}^{\mathrm{i}\theta}\, q_{L,R}$$

$$U(1)_{L-R} \equiv U(1)_A \quad : \quad q_L \xrightarrow{G} \mathrm{e}^{-\mathrm{i}\theta}\, q_L, \qquad q_R \xrightarrow{G} \mathrm{e}^{\mathrm{i}\theta}\, q_R$$

- How are these symmetries realized in Nature?

- **Chiral symmetry** $\boldsymbol{SU(n_f)_L \otimes SU(n_f)_R}$ should be approximately good in light quark sector (u,d,s), but **not seen in hadronic spectrum:**
  - Hadrons can be classified in $\boldsymbol{SU(3)_V \equiv SU(3)_{L+R}}$ representations, but degenerate multiplets with opposite parity do not exist
  - Octet of pseudoscalar mesons (π^+, π^-, π^0, η, K^+, K^-, K^0, and $\bar{K}^0$) much lighter than all other hadronic states

⇒ **Ground state of the theory not symmetric under the chiral group:**

$$\boldsymbol{SU(3)_L \otimes SU(3)_R \stackrel{\rm SCSB}{\longrightarrow} SU(3)_{L+R}}$$

and an **octet $(3^2 - 1)$ of pseudoscalar massless bosons appears** in the theory; their small masses generated by the quark-mass matrix

- What about the **axial symmetry**?

$U(1)_A$ problem

- If $\boldsymbol{U(1)_A}$ unbroken in vacuum $\Rightarrow$ in chiral limit all massless hadrons would have massless partners of opposite parity $\Rightarrow$ not realized in Nature.

- If $\boldsymbol{U(1)_A}$ spontaneously broken, then there should be an $I = 0$ pseudoscalar Goldstone boson, whose perturbed state should have about the same mass as the pion. Only candidate, $\eta'(958)$, too heavy: $M_{\eta'} \gg M_\eta$.

- Solution involves **$\boldsymbol{U(1)_A}$ anomaly and topological (instanton) effects**

- **$U(1)_A$ is not a symmetry of the theory at the quantum level:**

  The $U(1)_A$ axial current is not conserved due to an **axial anomaly:**

  [Adler '69; Bell,Jackiw '69]

$$\partial_\mu \left(\bar{q}\gamma^\mu\gamma_5 q\right) = 2\, n_f\, \nu \qquad ; \qquad \nu \equiv \frac{\alpha_s}{16\pi}\,\epsilon^{\mu\nu\rho\sigma}\, G^a_{\mu\nu} G^a_{\rho\sigma} \equiv \frac{\alpha_s}{8\pi}\, G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu}$$

- ν so-called **topological charge density**. Can be written as total divergence

$$\nu = \partial^\mu K_\mu(x) \;\; ; \;\; K_\mu = \frac{\alpha_s}{4\pi}\epsilon_{\mu\alpha\beta\gamma} A^\alpha_a \left(\partial^\beta A^\gamma_a + \frac{1}{3} g f_{abc} A^\beta_b A^\gamma_c \right)$$

K_μ so-called **Chern-Simons current**

$\Rightarrow$ For sufficiently rapidly vanishing fields can write for integral of anomaly over space-time, $\int \mathrm{d}^4x\, \partial_\mu\, (\bar{q}\gamma^\mu\gamma_5 q) = 2\, n_f\, \int \mathrm{d}^4x\, \nu \equiv 2\, n_f\, \int \mathrm{d}^4x\, \partial_\mu K^\mu$,

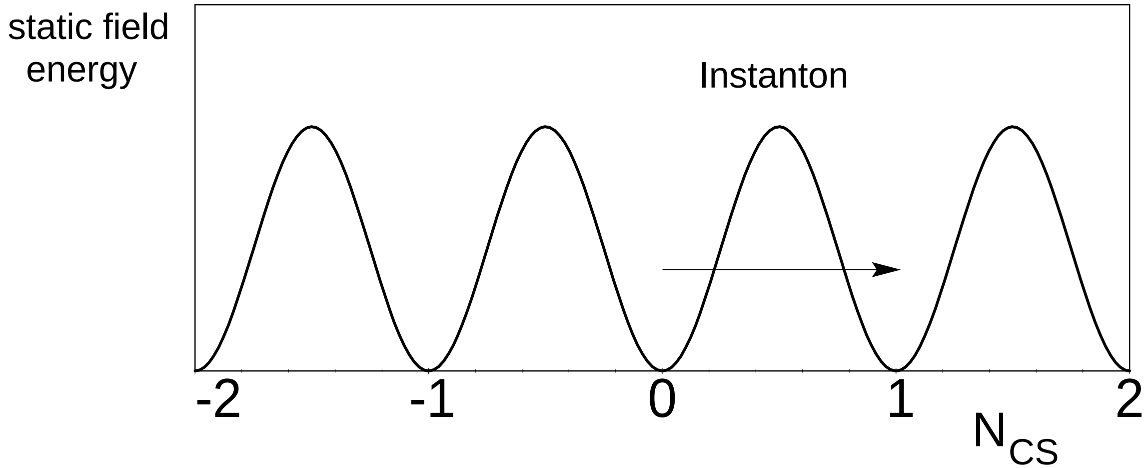
$$\underbrace{\int_{-\infty}^{+\infty} \mathrm{d}t\, \partial_t \int \mathrm{d}^3x\, \bar{q}\gamma^0\gamma_5 q}_{Q_5(t=\infty)-Q_5(t=-\infty)} = 2\, n_f\, \underbrace{\int_{-\infty}^{+\infty} \mathrm{d}t\, \partial_t \int \mathrm{d}^3x\, K^0}_{N_{\mathrm{CS}}(t=\infty)-N_{\mathrm{CS}}(t=-\infty)}$$

in short

$$\boldsymbol{\triangle Q_5 = 2\, n_f\, \triangle N_{\mathrm{CS}}}$$

i.e. in the background of gauge fields which evolve in time such that their Chern-Simons number $N_{\mathrm{CS}} = \int \mathrm{d}^3x\, K^0$ changes by $\triangle N_{\mathrm{CS}} \Rightarrow$ the fermionic $U(1)_A$ charge Q_5 changes by $2\, n_f\, \triangle N_{\mathrm{CS}}$

- Classical gauge fields with zero energy have **integer** N_{CS}.
- There are topological obstructions to deform such zero energy fields differing in their Chern-Simons numbers smoothly into each other $\Rightarrow$ separated by **energy barrier** [Jackiw,Rebbi 176; Callan,Dashen,Gross '76]



- **Instanton** describes $\triangle N_{\text{CS}} = 1$ **tunneling transition**. Associated with anomalous violation of axial charge conservation, $\triangle Q_5 = 2\,n_f$. ['t Hooft '76]

$\Rightarrow$ No reason to expect $U(1)_A$ to be a symmetry $\Rightarrow$ no $U(1)_A$ problem.

3. Instanton perturbation theory

- Generalized saddle-points in the Euclidean functional integral formulation of **QCD**,

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int [dA][d\psi][d\overline{\psi}]\, \mathcal{O}[A,\psi,\overline{\psi}]\, \mathrm{e}^{-S[A,\psi,\overline{\psi}]}\,,$$
$$Z = \int [dA][d\psi][d\overline{\psi}]\, \mathrm{e}^{-S[A,\psi,\overline{\psi}]}\,.$$

- Perturbation theory:
  - **perturbative QCD:** expansion about trivial vacuum solution, i.e. vanishing gluon field and vanishing quark fields and thus vanishing Euclidean action, $S = 0$.
  - **instanton perturbation theory:** generalized saddle-point expansion of the Euclidean functional integral about non-trivial minima of the Euclidean action with $S \neq 0$.

- Non-trivial minima ($\Leftrightarrow$ solutions) have **integer** **Pontryagin index (topological charge)**

$$Q \equiv \int d^4x \, \nu(x) \equiv \triangle N_{\rm CS} = \pm 1, \pm 2, \ldots ,$$

and their **action** is a multiple of $2\pi/\alpha_s$,

$$S \equiv \int d^4x \, \frac{1}{2} {\rm tr}(G_{\mu\nu} G_{\mu\nu}) = \frac{2\,\pi}{\alpha_s} \, |Q| = \frac{2\,\pi}{\alpha_s} \cdot (1, 2, \ldots).$$

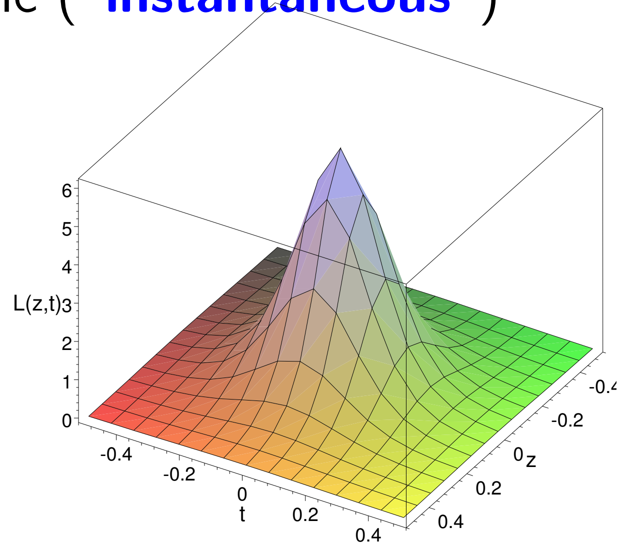
$\Rightarrow$ **Dominant** saddle-point for $\alpha_s \ll 1$: **Instanton ($Q = 1$):** [Belavin *et al.* '75]

$$A_\mu^{(I)}(x;\rho,U,x_0) = -\frac{\mathrm{i}}{g}\,\frac{\rho^2}{(x-x_0)^2}\,U\,\frac{\sigma_\mu\,(\overline{x}-\overline{x_0}) - (x_\mu - x_{0\mu})}{(x-x_0)^2+\rho^2}\,U^\dagger$$

- **size** ρ, **color orientation** U, **position** x_0
- localized in Euclidean space and time ("**instantaneous**")

$$\mathcal{L}\left(A_\mu^{(I)}(x;\rho,U,0)\right) = \frac{12}{\pi\alpha_s}\cdot\frac{\rho^4}{(x^2+\rho^2)^4}$$

$$\Rightarrow\quad S\left[A_\mu^{(I)}\right] = \frac{2\,\pi}{\alpha_s}$$



- **Instanton**-contribution to vacuum-to-vacuum amplitude Z:

$$\frac{1}{Z^{(0)}}\frac{dZ^{(I)}}{d^4x} = \int_0^\infty d\rho\, D_m(\rho) \int dU$$

- **Size distribution** $D_m(\rho)$ known in instanton perturbation theory

['t Hooft '76; Bernard '79]

$$\alpha_s(\mu_r)\ln(\rho\,\mu_r) \ll 1, \qquad \rho\, m_i(\mu_r) \ll 1,$$

in 2-loop renormalization-group invariant form, [Morris *et al.* '85]

$$\frac{dn_I}{d^4x\, d\rho} = D_m(\rho) = D(\rho)\prod_{i=1}^{n_f}(\rho\, m_i(\mu_r))\,(\rho\,\mu_r)^{n_f\,\gamma_0 \frac{\alpha_s(\mu_r)}{4\pi}}$$

- **Reduced size distribution $D(\rho)$:**

$$D(\rho) = \frac{d}{\rho^5}\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{2\,N_c} \mathrm{e}^{-\frac{2\pi}{\alpha_s(\mu_r)}}\,(\rho\,\mu_r)^{\beta_0+(\beta_1-4\,N_c\beta_0)\frac{\alpha_s(\mu_r)}{4\pi}}$$

- Clearly non-perturbative, $\propto \mathrm{e}^{-\frac{2\pi}{\alpha_s(\mu_r)}}$
- Power-law behaviour of (reduced) size distribution,

$$D(\rho) \sim \rho^{\beta_0-5+\mathcal{O}(\alpha_s)}; \qquad \beta_0 = \frac{11}{3}\,N_c - \frac{2}{3}\,n_f$$

  - dominant contribution to ρ integral generically originates from large ρ
  - spoils the applicability of instanton perturbation theory, $\alpha_s(1/\rho) \ll 1$

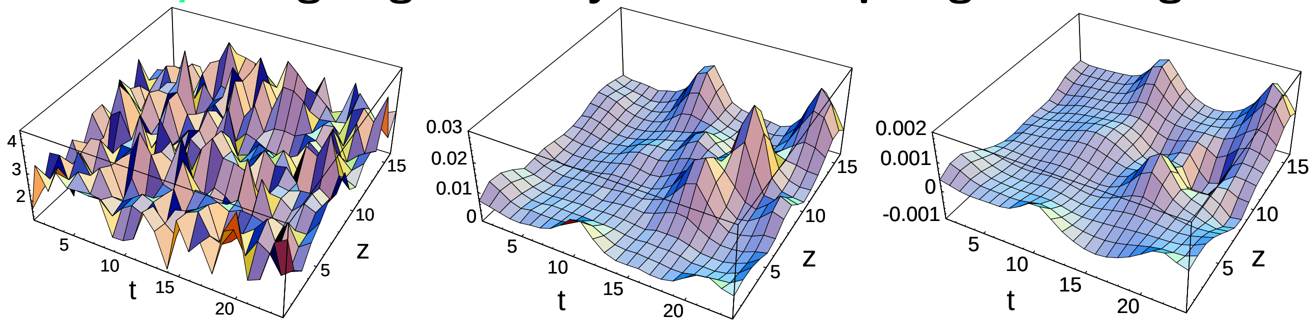
- Size distribution basic building block of **instanton** perturbation theory:

  ◦ appears in generic instanton contributions to Green's functions

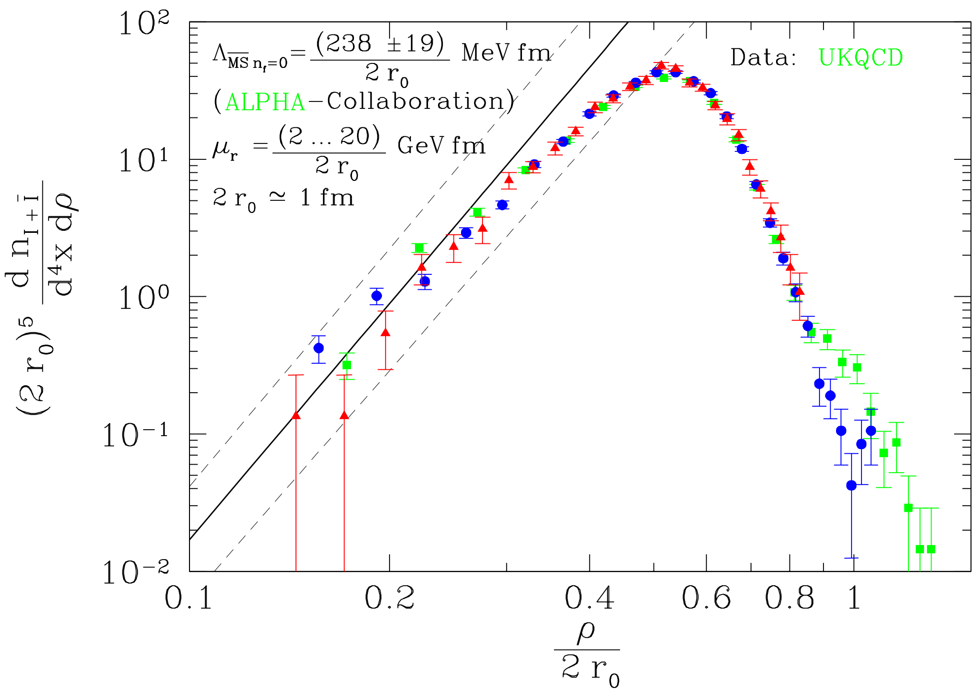
  ⇒ important to know the region of validity of the perturbative result

- Crucial information from lattice investigations on the topological structure of the QCD vacuum

[Chu *et. al* '94] **Lagrange Density** **Topological Charge Density**



- Confronting lattice data ($n_f = 0$) [Smith, Teper (UKQCD) '98] on size distribution with perturbative result [AR, F. Schrempp '99]:



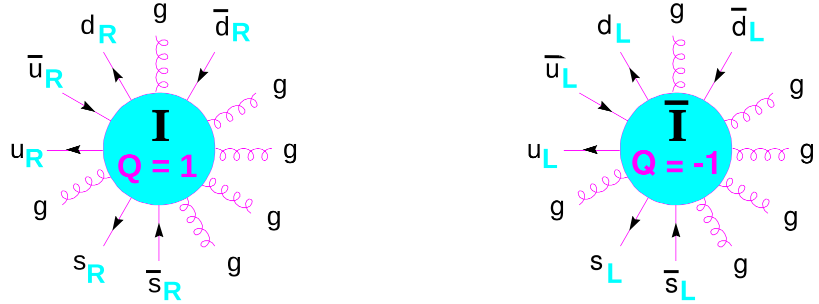
$\Rightarrow$ **Instanton perturbation theory reliable for $\rho\,\Lambda \lesssim 0.4$**

- Suppression of instanton contribution to the vacuum-to-vacuum amplitude for small quark masses $\rho m_i \ll 1$:

$\Leftarrow$ **Axial anomaly**: [Adler '69; Bell, Jackiw '69]
Any gauge field fluctuation with topological charge Q must be accompanied by a corresponding change in axial charge, $\triangle Q_5 = 2\, n_f\, Q$

$\Rightarrow$ pure vacuum-to-vacuum transitions vanish in chiral limit

$\Rightarrow$ Green's functions corresponding to anomalous Q_5 violation: ['t Hooft '76]



- $*$ main contribution due to instantons
- $*$ do not suffer from any mass suppression

- Simplest **example** of one light flavour ($n_f = 1$): Fermionic two-point function

$$\langle \psi(x_1)\overline{\psi}(x_2)\rangle^{(I)} \simeq \int d^4x \int\limits_0^\infty d\rho\, D(\rho) \int dU\ (\rho\, m)\ S^{(I)}(x_1, x_2; x, \rho, U)$$

Quark propagator in the I-background $S^{(I)}$,

$$S^{(I)}(x_1, x_2; \ldots) = \frac{\kappa_0(x_1;\ldots)\,\kappa_0^\dagger(x_2;\ldots)}{m} + \sum_{n\neq 0} \frac{\kappa_n(x_1;\ldots)\,\kappa_n^\dagger(x_2;\ldots)}{m + i\lambda_n},$$

in terms of the spectrum of the Dirac operator in the I-background, which has exactly one right-handed zero mode κ_0, ['t Hooft '76]

$$-\mathrm{i}\gamma_\mu D_\mu^{(I)}\kappa_n = \lambda_n \kappa_n; \quad \text{with } \lambda_0 = 0 \text{ and } \lambda_n \neq 0 \text{ for } n \neq 0,$$

for $m \longrightarrow 0$ only the zero mode contribution survives,

$$\langle \psi(x_1)\overline{\psi}(x_2)\rangle^{(I)} \simeq \int d^4x \int\limits_0^\infty d\rho\, D(\rho) \int dU\, \rho\, \kappa_0(x_1; x, \rho, U)\, \kappa_0^\dagger(x_2; x, \rho, U)$$

- First principle calculations of instanton contributions only possible for quantities to which large size instantons do not contribute:
  - **Short-distance coefficient functions** in the operator product expansion of two-point functions. [Andrei,Gross '78;...;Novikov *et al.* '80;...; Balitsky,Braun '93]
    Problem:
    * No physically relevant two-point function known which receives contribution solely from instantons.

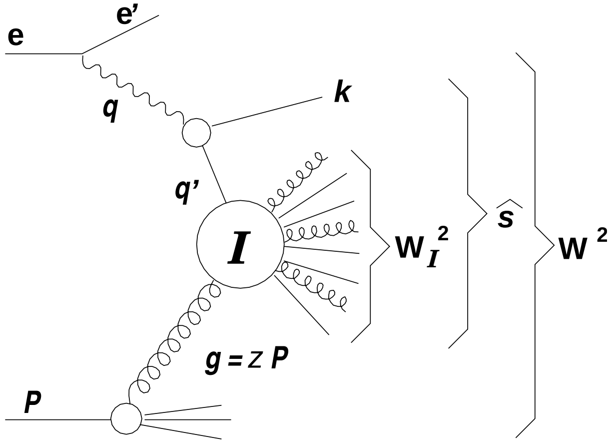
    → Instanton contribution typically hidden beyond large perturbative background.
  - **Unique possibility:**
    Anomalous $\triangle Q_5 = \pm 2\, n_f$ processes in deep-inelastic scattering at large momentum transfer [Moch,AR,F.Schrempp '97]

    → In $m_q = 0$ limit, receive contributions only from (anti-)instantons.

4. QCD-Instantons at HERA

[AR,F.Schrempp '94–'01]

- **Kinematics:**



Deep-inelastic scattering variables:

$S = (e+P)^2$
$Q^2 = -q^2 = -(e-e')^2$
$x_{\rm Bj} = Q^2/\ (2P\cdot q)$
$y_{\rm Bj} = Q^2/\ (S\, x_{\rm Bj})$
$W^2 = (q+P)^2 = Q^2(1/x_{\rm Bj}-1)$
$\hat{s} = (q+g)^2$
$z = x_{\rm Bj}\ (1+\hat{s}/Q^2)$

Variables of instanton-subprocess:

$Q'^2 = -q'^2 = -(q-k)^2$
$x' = Q'^2/\ (2\ g\cdot q')$
$W_I^2 = (q'+g)^2 = Q'^2(1/x'-1)$

- "Fiducial" kinematical region from lattice constraints: [AR,F.Schrempp '99;'01]

$$\left(\rho^*\Lambda^{(0)}_{\overline{\rm MS}}\lesssim 0.4, \frac{R^*}{\rho^*}\gtrsim 1.0\right) \Rightarrow \left(Q'/\Lambda^{(n_f)}_{\overline{\rm MS}}\gtrsim 30.8, x'\gtrsim 0.35\right)$$

Instanton-Antiinstanton Estimate

[AR,F.Schrempp '98]; also [Zakharov '90; Khoze,AR '91]

$$\hat{\sigma}^{(I)}_{\rm p_1p_2} \sim \int d^4R \int_0^\infty d\rho \int_0^\infty d\overline{\rho}\, D(\rho) D(\overline{\rho}) \int dU\, e^{-\frac{4\pi}{\alpha_g}\Omega\left(U, \frac{R^2}{\rho\overline{\rho}}, \dots\right)}\; e^{i(p_1+p_2)\cdot R - \sum_{i=1}^2 \sqrt{-p_i^2}\,(\rho+\bar\rho)}$$

- **Ingredients:**
  - Instanton-size distribution $D(\rho) \propto e^{-2\pi/\alpha_g}$
  - $\Omega\left(U, R^2/(\rho\bar\rho), \dots\right)$:
    - ∗ Exponentiation of $\mathcal{O}(1/\alpha_g)$ final state gauge bosons [AR '90;Espinosa '90]
    - ∗ Anti-instanton-instanton interaction

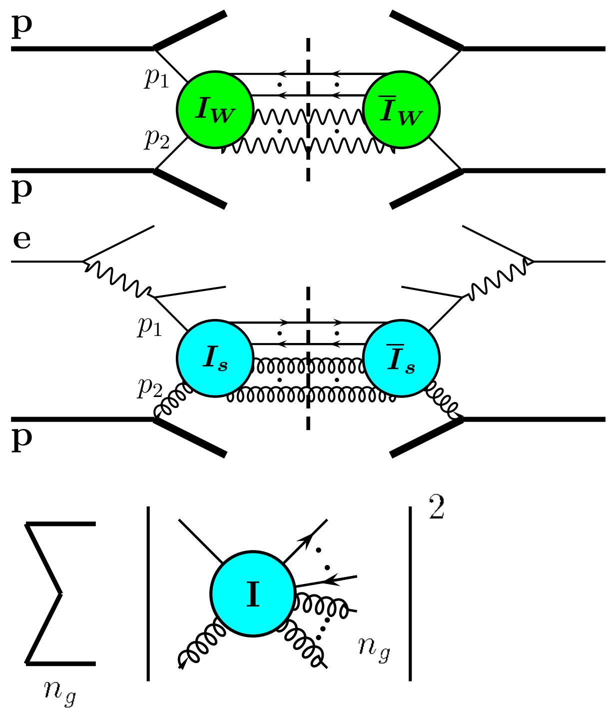
[Khoze,AR '91; AR,F.Schrempp '98]

- **General form:**

$$\hat{\sigma}^{(I)} \sim e^{-\frac{4\pi}{\alpha_g} F_g(\epsilon)}; \quad \text{with } \epsilon = \sqrt{\hat{s}}/M_{\rm sp}$$

"Holy-Grail" function $F_g(\epsilon) \searrow$ for $\epsilon \nearrow$, with

$$0 < F_g(1) < F_g(0) = 1\,.$$



- **Saddle point evaluation:**

$$\hat{\sigma}^{(I)} \propto \mathrm{e}^{-\Gamma_*} \equiv \mathrm{e}^{-\frac{4\pi}{\alpha g} F_g(\epsilon)},$$

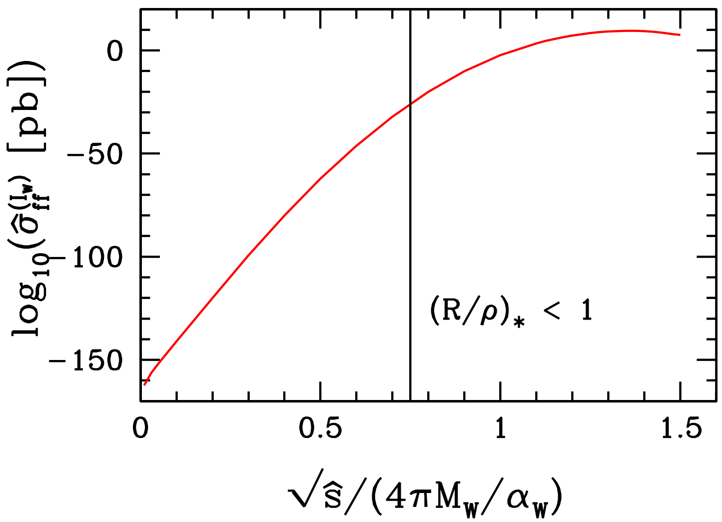
where

$$\epsilon \equiv \begin{cases} \sqrt{\hat{s}}/(4\pi M_W/\alpha_W) & \text{(QFD)} \\ \sqrt{\hat{s}}/Q' \equiv \sqrt{1/x' - 1} & \text{(QCD)} \end{cases}$$

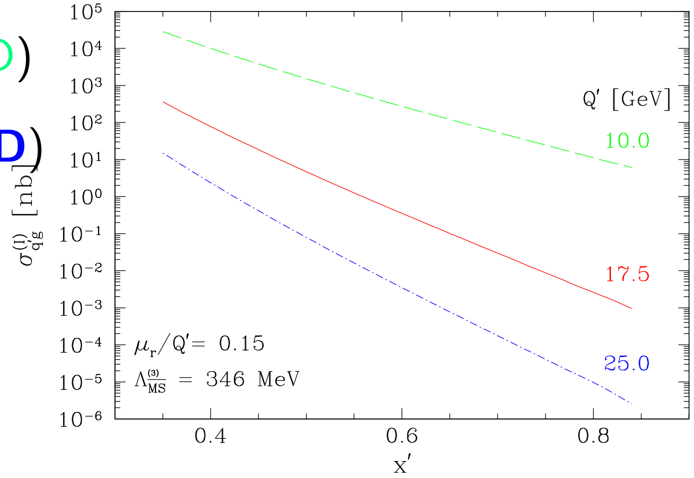
is a scaled cm energy and

$$F_g = 1 + \Omega_g(1,\xi_*) + \begin{cases} -(\xi_* - 2)\,\frac{\partial}{\partial \xi_*}\Omega_g(1,\xi_*)_{|\xi_*=2+\left(\frac{R}{\rho}\right)^2_*} & \text{(QFD)} \\ 0 & \text{(QCD)} \end{cases}$$

- Increasing $\epsilon \Rightarrow$ smaller $(R/\rho)_*$ probed $\Rightarrow$ cross-section grow due to attractive nature of Ω_g in perturbative semi-classical regime



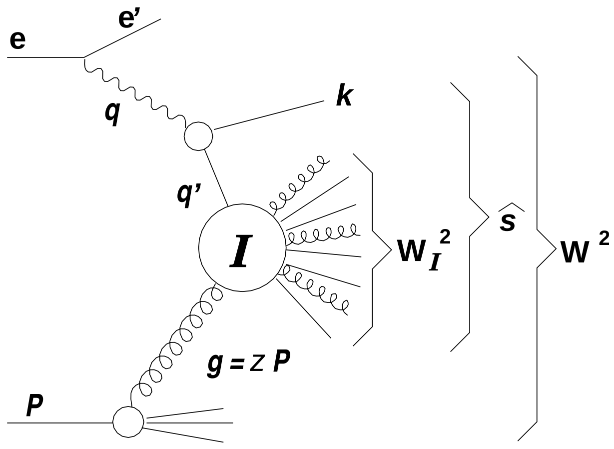
[AR '02]



[AR,F.Schrempp '98,'00]

Event generator QCDINS 2.0:

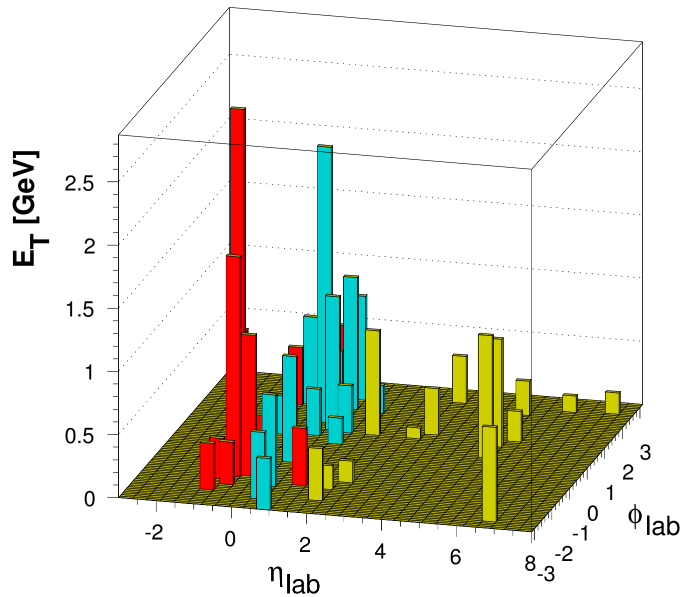
[Gibbs,AR,F.Schrempp '95; AR,F.Schrempp '00]



- **Hard subprocess:**
  - isotropic in $q'g$ CM
  - flavour democratic
  - large parton multiplicity

$$\langle n_q + n_g \rangle = 2\,n_f - 1 + \mathcal{O}(1)/\alpha_s \gtrsim 8,$$

- **Parton shower** (**HERWIG**)
- **Hadronization** (**HERWIG** or **JETSET**)



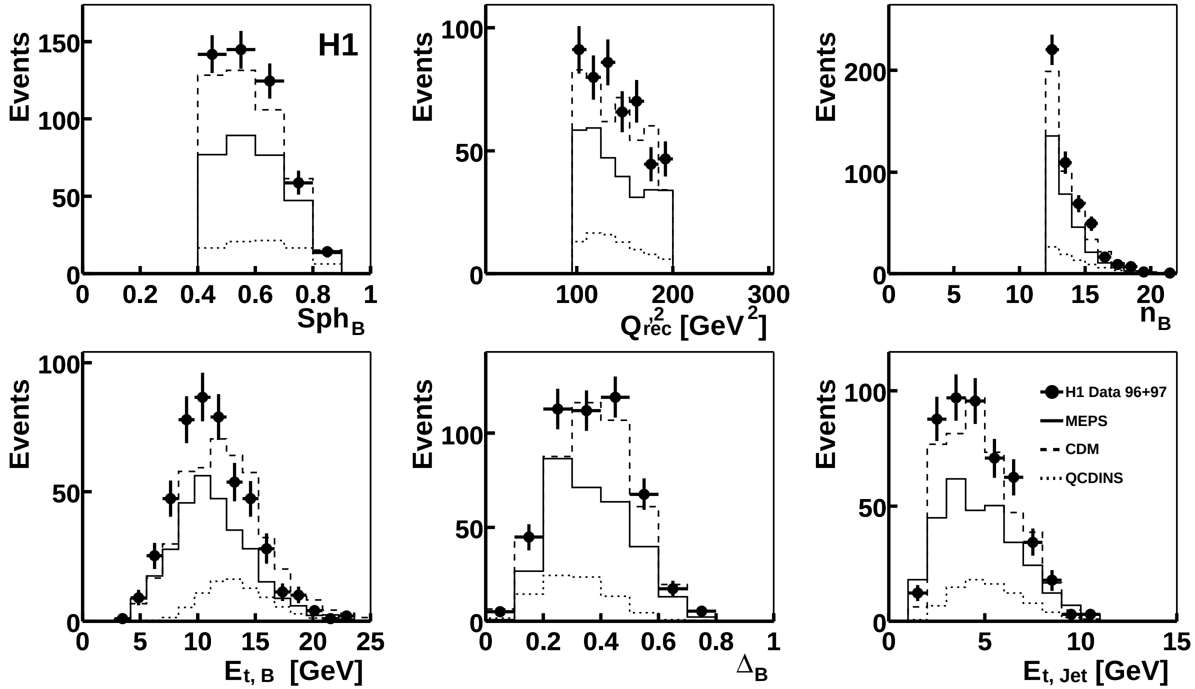
Pioneering search by H1 collaboration

[H1 collab. '02]

- Based on **QCDINS 2.0**, compared to **MEPS** and **CDM**
  - **Excess** with instanton-like topology, compatible with instanton signal
  - Statistical **significant** in comparison to **MEPS**
  - **Uncertainties** in background simulations?

  $\Rightarrow$ **Upper limit on σ**

- Data do not exclude cross-section predicted for small $(R_*/\rho_*) \gtrsim 0.5$, as long as one probes small $\rho_* \ll 0.3$ fm



Summary of H1/ZEUS searches at HERA I

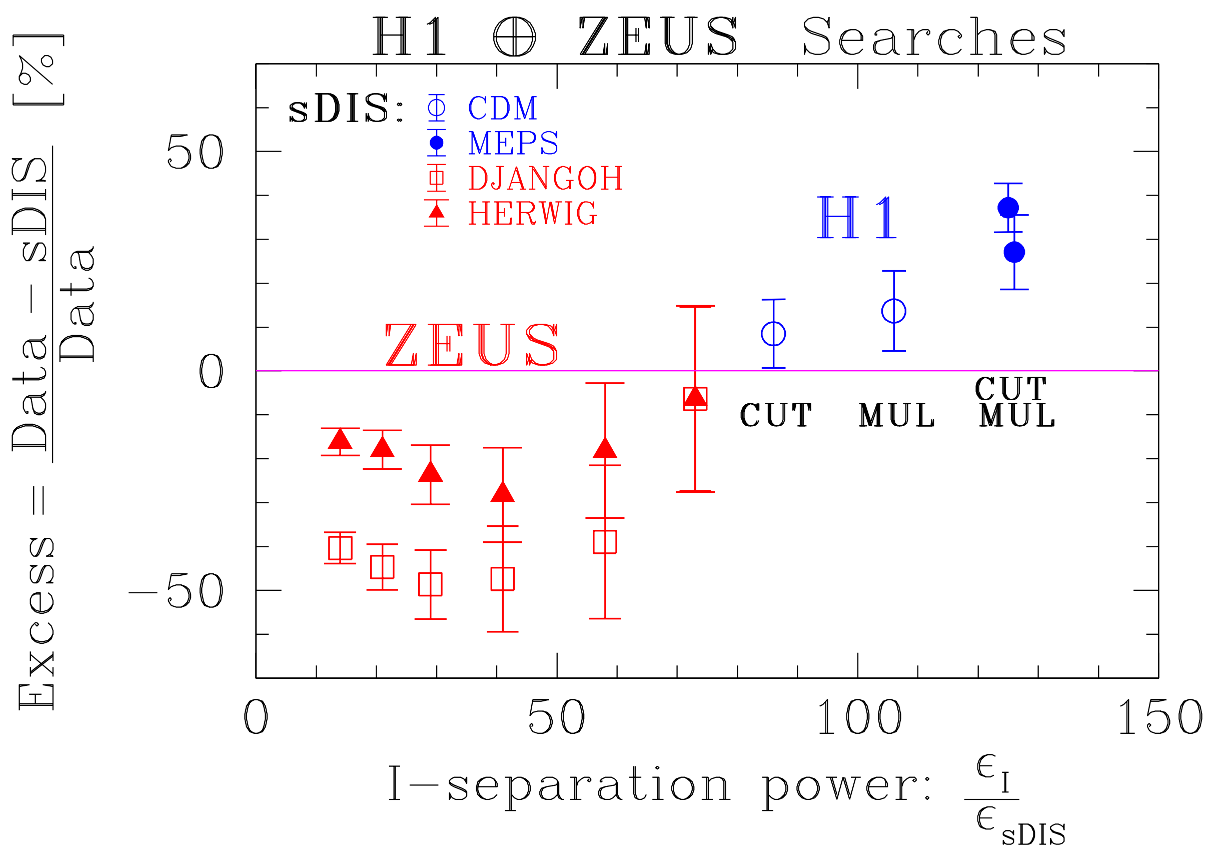
[H1 collab. '02; ZEUS collab. '04]

- Instanton-enriched samples by cutting on discriminating observables

- Large uncertainties due to different event generators

- Upper limits on instanton-induced cross section factor ~ 5 above predictions

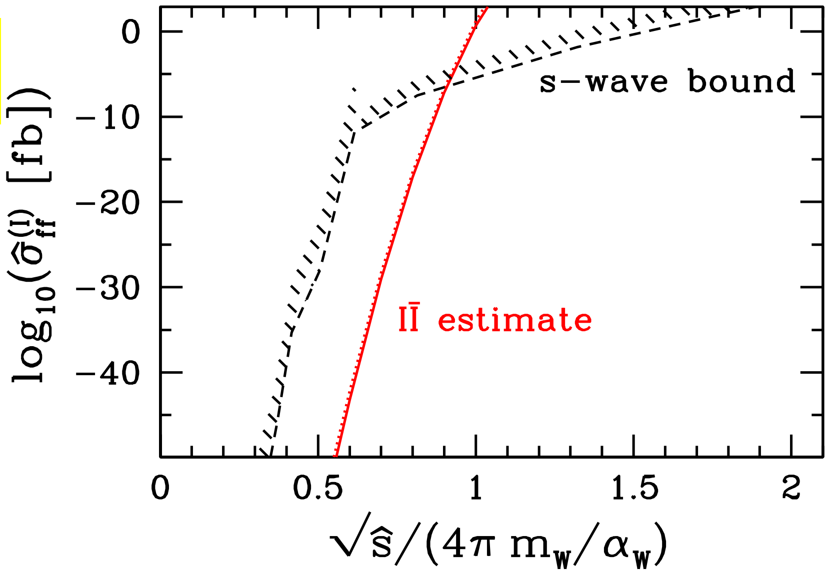
- **HERA II** may allow to reach higher instanton-separation power $\epsilon_{\rm I}/\epsilon_{\rm sDIS}$

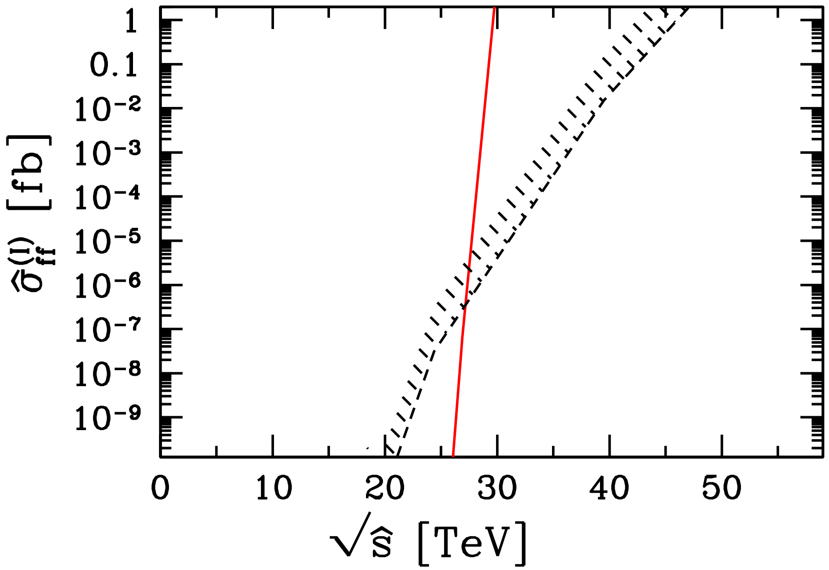


[F. Schrempp '04]

5. QFD-Instantons at VLHC?

- Lattice data as well as **H1/ZEUS** limit on small-size **QCD**-instantons suggest:
  - $I\bar{I}$ estimate reliable, as long as $(R/\rho)_* \geq 1$
  - For $(R/\rho)_* < 0.5 \div 1$, rapid growth, as implied by Ω, stops.
- Implications for **QFD**-instantons:
  - $(R/\rho)_* < 0.5 \div 1$ corresponds to $\epsilon < 0.75 \div 1.15$, $\sqrt{\hat{s}} < 22 \div 35$ TeV
  - At these energies, parton-parton cross-section estimates and bounds reach observable values





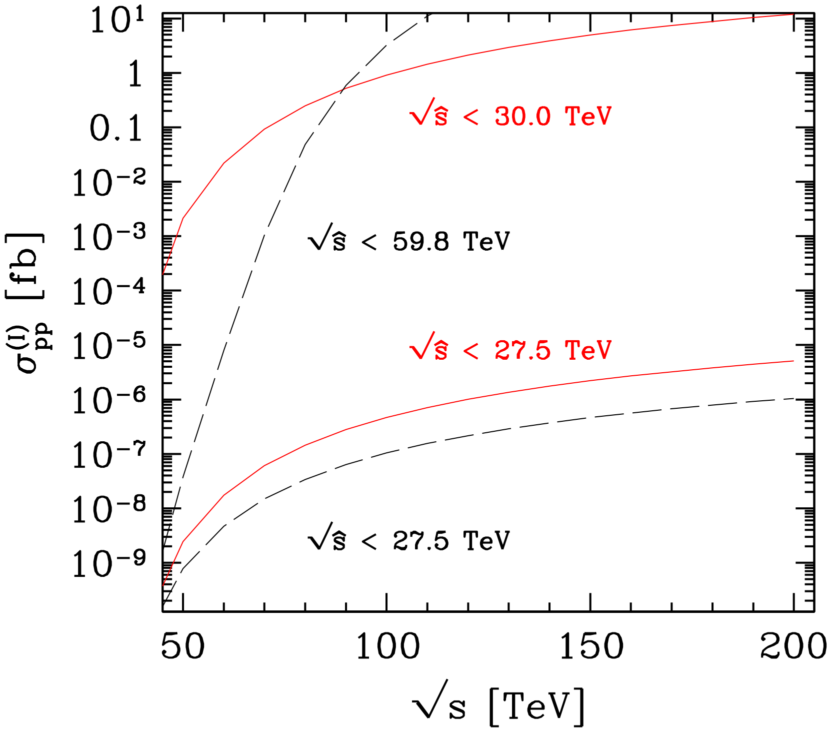
- **Estimate of $\sigma_{\rm pp}^{(I_W)}$**

[AR,Tu (unpublished)]

– **VLHC:**

$$\sqrt{s_{PP}} \approx 200 \text{ TeV}$$
$$\mathcal{L} \approx 6 \cdot 10^2 \text{ fb}^{-1} \text{ yr}^{-1}$$

– **Observable, $\gtrsim 10^{-3}$ fb**, if estimate valid/bound saturated up to $\sqrt{\hat{s}} \approx 28/35$ TeV.



⇒ Further study worthwhile

Phenomenology of QFD-instantons

[AR,F.Schrempp,Wetterich '91; Gibbs,AR,Webber,Zadrozny '94]

- No background from perturbative Standard Model processes by requiring
  - ≥ 4 identified charged e's or μ's
  - $E_T \geq$ several TeV

- **Event generator HERBVI:**

[Gibbs,Webber '95]

  - B-violation cannot be established
  - **L-violation verifiable:** measure

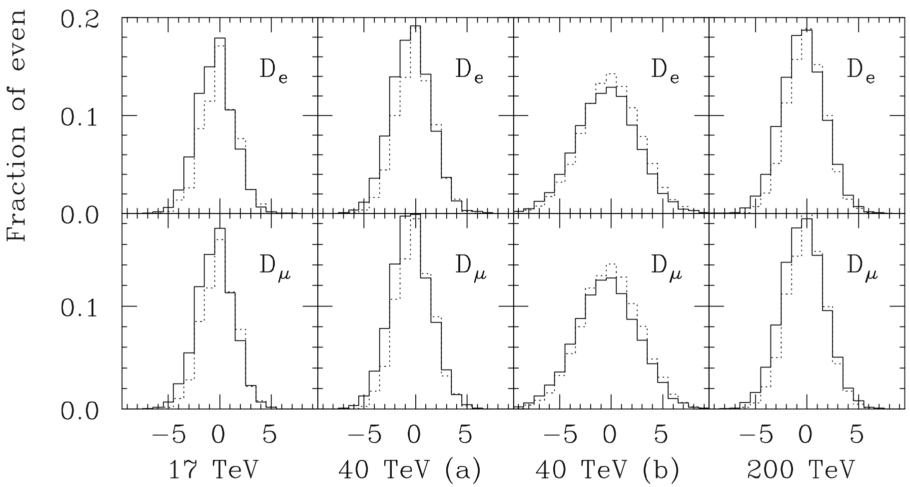
$$D_\ell = N_{\ell^-} - N_{\ell^+} \, ;$$

need $\sim 10^3$ events

[Gibbs,AR,Webber,Zadrozny '94]

Simulations performed			
Energy (TeV)		n_B estimate	$\sqrt{\hat{s}_0}$ (TeV)
17		$1/\alpha_W$	5
40	(a)	$1/\alpha_W$	18
40	(b)	LOME	18
200		$1/\alpha_W$	18

[Gibbs,AR,Webber,Zadrozny '94]



6. Conclusions

- Opportunities to study **instanton-induced hard scattering processes**

- Need **future hadron collider** to explore **QFD** instantons and electroweak $B + L$ violation

  – If cross-section exceeds even $\mathcal{O}$(10 nb/mb), first signs of electroweak sphaleron production may be/may already have been seen in neutrino nucleon scattering at **cosmic ray facilities and neutrino telescopes**

  [Morris,AR '94; Fodor,Katz,AR,Tu '03; Han,Hooper '03]

- Hard **QCD** instanton-induced scattering processes

  – can and are being probed presently in DIS at **HERA**
  – study of prospects at **LHC** underway [Carli,Petermann,**F.Schrempp**]
  – yield insight into fait of **QFD** instanton-induced processes at multi-TeV