DESY Theory Group, Hamburg

Tutorials for the Lecture

Quantum Mechanics II

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Tutorial 13

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Home Exercise 34 Helicity amplitudes

For massless fermions, efficient and modern ways to calculate quantum mechanical amplitudes, are via so-called two-spinor products, named after Weyl and van der Waerden. For this we introduce two auxiliary vectors, k_0^μ, k_1^μ with $k_0^2 = 0$, $k_1^2 = -1$, $k_0 \cdot k_1 = 0$. Next, we define basis spinors, u_L^0 as the left-handed spinor for a fermion with momentum k_0, and $u_R^0 = \not{k}_1 u_L^0$. We then define for any light-like momentum $p, p^2 = 0$, the spinors

$$u_L(p) = \frac{1}{\sqrt{2\,p\cdot k_0}} \not{p}\, u_R^0 \qquad \text{and} \qquad u_R(p) = \frac{1}{\sqrt{2\,p\cdot k_0}} \not{p}\, u_L^0 \qquad . \tag{1}$$

Except for the case that p is proportional to k_0 this fixes the phase convention for the spinors.

(a) Show that $u_L(p)$ and $u_R(p)$ with that definition are indeed left- and right-handed spinors, respectively.

(b) Show that $\not{k}_0 u_R^0 = 0$. Proof that for any light-like momentum p, $\not{p} u_L(p) = \not{p} u_R(p) = 0$.

(c) Now take the explicit choices $k_0 = (E, 0, 0, -E)$ and $k_1 = (0, 1, 0, 0)$. Construct $u_L^0, u_R^0, u_L(p), u_R(p)$ explicitly.

(d) We define the spinor product to $[pq]$ and $\langle pq \rangle$ for light-like momenta p, q by

$$[pq] := \overline{u}_R(p) u_L(q) \qquad , \langle pq \rangle := \overline{u}_L(p) u_R(q) \qquad . \tag{2}$$

Use the explicit forms for the spinors from the former part and compute the spinor products explicitly. Show that

$$\langle pq \rangle = [qp]^* \qquad , [pq] = -[qp] \qquad . \tag{3}$$

The two relations then also imply $\langle pq \rangle = -\langle qp \rangle$. Furthermore, proof

$$|[pq]|^2 = 2\,p\cdot q \qquad . \tag{4}$$

Thus the spinor products are the square roots of the 4-vector products.

(e) Show that

$$u_L^0 \overline{u}_L^0 = \frac{1-\gamma^5}{2} \not{k}_0 \ , \qquad u_R^0 \overline{u}_R^0 = \frac{1+\gamma^5}{2} \not{k}_0 \qquad . \tag{5}$$

(f) Use the identities of part (e), and the Dirac matrix trace formulae to show again (and independent of the special choice of k_0 and k_1) the relation

$$|[pq]|^2 = 2\,p\cdot q \qquad . \tag{6}$$

Home Exercise 35 Recap

Recap all of the exercises and lectures, collect and ask questions, on technical points like calculations etc. or on conceptual points and problems of understanding.