Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Supersymmetrie

WS 07/08

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Blatt 9

Hausaufgabe 8 Modified O'Raifeartaigh Model

Consider a model with three left-chiral superfields $\hat{\Phi}_1$, $\hat{\Phi}_2$ and $\hat{\Phi}_3$, with the superpotential

$$\mathcal{W}(\hat{\Phi}_1, \hat{\Phi}_2, \hat{\Phi}_3) = \hat{\Phi}_1 \hat{\Phi}_3^2 + \hat{\Phi}_2 \left(\hat{\Phi}_3^2 + c \right).$$

Without loss of generality, take c to be a positive real constant.

a) Show that this is a theory where SUSY is spontaneously broken by finding the absolute minimum of the scalar potential.

b) Determine the masses of all scalar fields.

c) Diagonalize the fermion mass terms and express the goldstino field in terms of the fermionic component fields ψ_1, ψ_2 and ψ_3.

Präsenz-Aufgabe 10 Supersymmetric Noether Current

The Noether procedure allows to calculate the conserved current of SUSY as

$$\mathcal{J}_\alpha^\mu = \mathcal{K}_\alpha^\mu + \mathcal{N}_\alpha^\mu, \quad \text{with}$$

$$\sum_{\text{all fields}} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta_\xi \phi = -\left(\xi^\alpha \mathcal{N}_\alpha^\mu + h.c. \right)$$

and $\mathcal{K}_\alpha^\mu$ coming from the SUSY transformation of the Lagrangian:

$$\delta_\xi \mathcal{L} = \xi^\alpha \partial_\mu \mathcal{K}_\alpha^\mu + h.c.$$

a) Verify that $\partial_\mu \mathcal{J}_\alpha^\mu = 0$.

b) Calculate $\mathcal{K}_\alpha^\mu$.

Hint:

Use $\mathcal{L} = \phi^\dagger \phi|_{\text{D-Term}} + (\mathcal{W}|_{\text{F-Term}} + h.c.)$, as well as $\delta_\xi D = \bar{\xi} \bar{\sigma}^\mu \partial_\mu \lambda - \xi \sigma^\mu \partial_\mu \bar{\lambda}$ and $\delta_\xi F = i\sqrt{2} \bar{\xi} \bar{\sigma}^\mu \partial_\mu \psi$. Determine the λ-component of $\phi^\dagger \phi$ and the ψ-component of $\mathcal{W}$.
You should find

$$\mathcal{J}_\alpha^\mu = \sqrt{2} \partial_\nu \phi^\star \left(\sigma^\nu \bar{\sigma}^\mu \psi \right)_\alpha + i\sqrt{2} \left(\sigma^\mu \bar{\psi} \right)_\alpha \left(\frac{\partial \mathcal{W}^\dagger}{\partial \phi^\dagger} \right).$$

c) Calculate the current for the Wess-Zumino model and verify that it is conserved.

d) Construct the charge operator

$$Q_\alpha = \int d^3x \mathcal{J}_\alpha^0(x)$$

and show that

$$[iQ_\alpha, \phi] = \delta_\xi \phi.$$