Übungen zur Vorlesung

Supersymmetrie

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Präsenz-Aufgabe 8 Supersymmetric QED

In the two-spinor formalism with

$$D_\mu C_\pm = (\partial_\mu \pm ieA_\mu)\, C_\pm, \quad \text{where } \; C = \phi, \psi,$$

the Lagrangian of SQED is given by

$$\begin{aligned}
\mathcal{L}_{SQED} = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\left(\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda\right) + \frac{1}{2}D^2 + |F_+|^2 + |F_-|^2 \\
& + |D_\mu\phi_+|^2 + |D_\mu\phi_-|^2 + i\left(\bar{\psi}_+\bar{\sigma}^\mu D_\mu\psi_+\right) + i\left(\bar{\psi}_-\bar{\sigma}^\mu D_\mu\psi_-\right) \\
& - \sqrt{2}ie\left[\left(\bar{\psi}_+\bar{\lambda}\right)\phi_+ - (\psi_+\lambda)\,\phi_+^\dagger - \left(\bar{\psi}_-\bar{\lambda}\right)\phi_- + (\psi_-\lambda)\,\phi_-^\dagger\right] \\
& + eD\left(|\phi_+|^2 - |\phi_-|^2\right) - m\left(\psi_+\psi_- + \bar{\psi}_+\bar{\psi}_-\right) \\
& + m\left(\phi_+F_- + \phi_-F_+ + \phi_+^\dagger F_-^\dagger + \phi_-^\dagger F_+^\dagger\right). \qquad (1)
\end{aligned}$$

a) Find the equations of motion for $D, F_+, F_-, F_+^\dagger, F_-^\dagger$ and eliminate them from the Lagrangian.

b) Use the notation (in conventional normalization)

$$\Psi = \begin{pmatrix} \psi_- \\ \bar{\psi}_+ \end{pmatrix}, \quad \tilde{\chi} = \begin{pmatrix} i\lambda \\ -i\bar{\lambda} \end{pmatrix}$$

for the electron and gaugino bispinor respectively in order to rewrite the Lagrangian (1) in terms of four-spinors.

c) Derive and discuss the Feynman rules.

d) What would be the $\tilde{e}_L$ and $\tilde{e}_R$ in the context of the MSSM?

Präsenz-Aufgabe 9 Aspects of the Generalization to the non-Ablian case

a) Use the Leibniz rule to show that

$$D_\alpha e^{\hat{V}} = \left(D_\alpha \hat{V}\right) e^{\hat{V}}.$$

Does this hold for non-Abelian fields as well?

b) Show that the definition of

$$\hat{W}_\alpha = -\frac{1}{4}\left(\bar{D}\bar{D}\right) D_\alpha \hat{V}$$

has to be generalized to

$$\hat{W}_\alpha = -\frac{1}{4}\left(\bar{D}\bar{D}\right)\left(e^{-\hat{V}} D_\alpha e^{\hat{V}}\right).$$

c) Derive the transformation law

$$\hat{W}_\alpha \rightarrow \hat{W}'_\alpha = e^{-i\hat{\Lambda}} \hat{W}_\alpha e^{i\hat{\Lambda}}$$

for

$$e^{\hat{V}} \rightarrow \mathrm{e}^{\hat{V}'} = e^{-i\hat{\Lambda}^\dagger} e^{\hat{V}} e^{i\hat{\Lambda}}.$$