Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Supersymmetrie

WS 07/08

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Blatt 4

Hausaufgabe 3 Integration Over Grassmann Variables

a) Demand that translational invariance also holds for Grassmann variables:

$$\int_{-\infty}^{\infty} d\theta\ \phi(\theta) = \int_{-\infty}^{\infty} d\theta\ \phi(\theta + \eta)\,,$$

where η is a constant Grassmann number.

What follows from the Taylor expansion for the integrals

$$\int d\theta, \quad \int d\theta\ \theta,$$

if one assumes the most simple normalizations?

b) Compare differentiation and integration.

c) Define

$$d^2\theta \equiv -\frac{1}{4} d\theta^\alpha d\theta_\alpha$$

$$\text{and} \quad d^2\bar{\theta} \equiv -\frac{1}{4} d\bar{\theta}_{\dot{\alpha}} d\bar{\theta}^{\dot{\alpha}}.$$

What follows for

$$\int d^2\theta\,(\theta\theta)\,, \quad \int d^2\bar{\theta}\,\left(\bar{\theta}\bar{\theta}\right)\ ?$$

What for other integrals?

$$\int d^4\theta\,(\theta\theta)\left(\bar{\theta}\bar{\theta}\right) = \int d^2\theta d^2\bar{\theta}\,(\theta\theta)\left(\bar{\theta}\bar{\theta}\right) = ?$$

d) Find expressions for $\delta^2(\theta)$ and $\delta^2\left(\bar{\theta}\right)$.

Präsenz-Aufgabe 4 Superspace Representation, Chiral Superfields

Use the following definitions for supercharges and covariant derivatives:

$$\boxed{\begin{aligned} Q_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu & \qquad D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu \\ \bar Q_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu & \qquad \bar D_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \end{aligned}}$$

a) Calculate all anticommutators.

b) The left-handed superfield Φ is defined by the irreducibility condition $\bar D_{\dot\alpha}\Phi = 0$. (Why is it invariant under SUSY trafos?)
Transform supercharges and covariant derivatives to the variables: θ, $\bar\theta$, $y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta$. What is that good for?

How do the variables $\theta, \bar\theta, y^{\dagger\mu}$ have to look like for the same purpose of right-handed superfields with the condition $D_\alpha\Phi = 0$?

c) Show that the chiral multiplet can be written as (historical normalization):

$$\Phi_L(y,\theta) = \varphi(y) \ + \ \sqrt{2}\,\theta^\alpha\,\psi_\alpha(y) \ + \ (\theta\theta)\,F(y).$$

Expand $\Phi_L(y,\theta)$ back to the old variables.

d) Prove that the SUSY trafos in that representation are (ε infinitesimal):

$$\theta^\alpha \to \theta^\alpha + \varepsilon^\alpha$$

$$x^\mu \to x^\mu + 2\,i\,\theta\sigma^\mu\bar\varepsilon\ .$$

Derive the SUSY transformations of φ, ψ and F.

e) Calculate the products of two and three chiral superfields $\Phi_1\Phi_2$ and $\Phi_1\Phi_2\Phi_3$ in the above representation as well as the product $\Phi^\dagger\Phi$. What kind of superfield is the latter?